Holographic spin fluctuation and Competing order

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General comment: ads/cmt

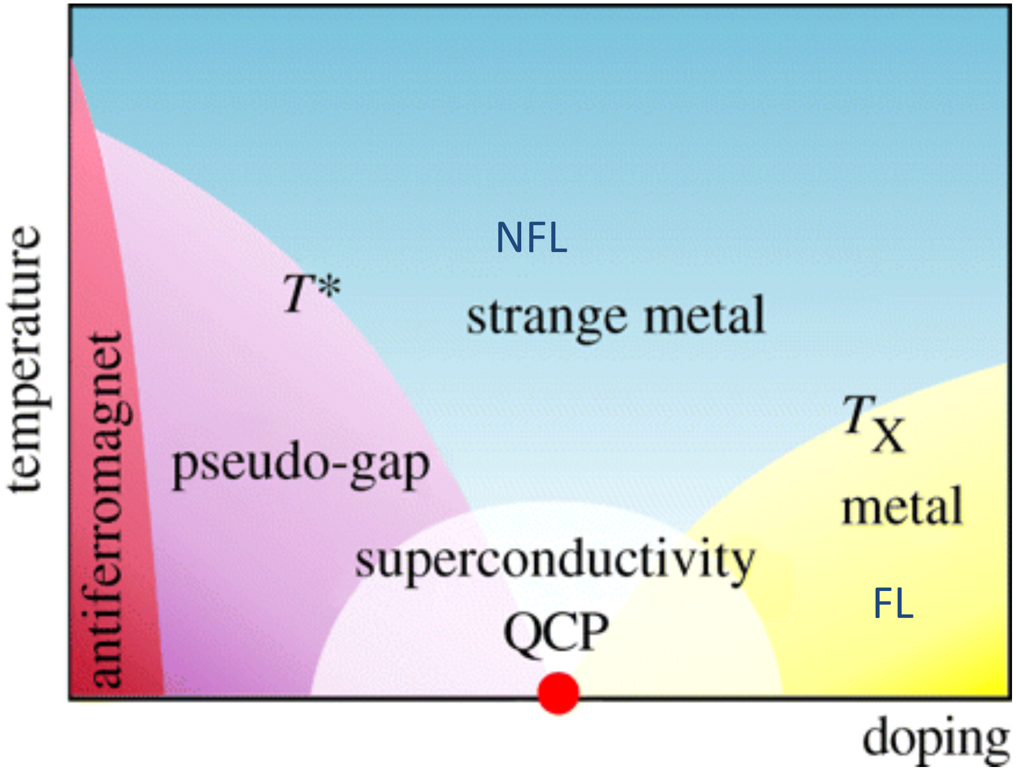
- Try to get scaling exponents →mean field theory value (due to analyticity).

- continuum theory: Details inside a cell are averaged out. → Not a tool for seeking micropic pairing mechanism.

- In gravity limit, AdS/CFT is a mean field theory

A Goal OF AdS/CMT

quantitative understanding the phase diagram



Based on Royal Society publishing, D. Galanakis et.al

To classify phases / To set up MFT

- we need order parameter although here Order is by interaction not by symmetry.
- Finite number of Phases→

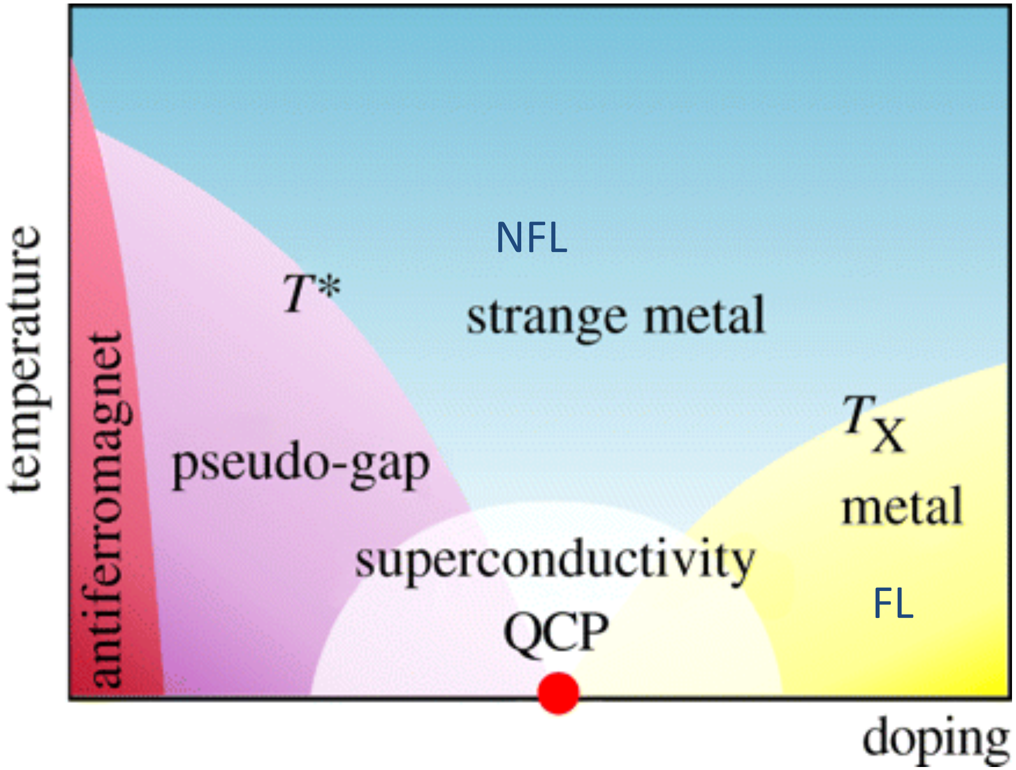
  finite number of order parameters (fields).
- For Pseudo Gap region : Not clear what kind of order. Scalar? Vector? or something else?

Order parameter.

- We need order parameter although it is NOT completely controlled by symmetry.
- What is the hint to characterize the order parameter of theoretically unknown region?
- If we have general results on the competition/collaboration of order parameters in general, it can give a guide.

Only two regions clear

AFM: real scalar; SC: complex scalar



→ Spin fluc. / ← Charge fluc.

Competing order in holography

- Conjecture (Basu et.al 1007.3480)
  Repulsive/attractive int.→ orders coexist/repel
- Ex1. $$V(A,B) = \eta A^2 B^2$$
  $$i)\eta > 0 \rightarrow attraction \rightarrow repel$$
  $$ii)\eta < 0 \rightarrow repulsion \rightarrow coexist$$
- Ex2. Holography without potential.
  SC : complex scalar, pick some order : Real scalar
  If they attract simply by gravity
  → Two orders repel each other. (corollary of conj.)
  Holography confirms!

Q: How about SC v.s vector order?

- 1211.1798 by Takaaki Ishii and SJS

Complex scalar+ extra vector

1211.1798

$$S=\int d^4x\sqrt{-g}\Big(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-|\partial_\mu\Phi-iA_\mu\Phi|^2-M^2\Phi^2$$
$$-\frac{1}{4}G_{\mu\nu}G^{\mu\nu}-\frac{m^2}{2}B_\mu B^\mu-\frac{c}{2}F_{\mu\nu}G^{\mu\nu}\Big),$$

where $F=dA$ and $G=dB$. The scalar field Φ is charged only under A_μ,

no direct coupling between Φ and B_μ

B_μ, represents impurities (in v2) or spin current (in v1)

Title

v1: A ferromagnetic superconductor in holography

v2: Impurity effect in a holographic superconductor

Results

- In the presence of SC condensation of CS, vector condensation (Bt) is actually induced by it→ coexists.
- The sign of Bt condensation is that of coupling c G.F
- As c goes up, SC gap goes like power rather than exponential.
- Without SC, no vector condensation in Bt.

(sign of c is immaterial for the existence of <Bt>)

Result: ac conductivity

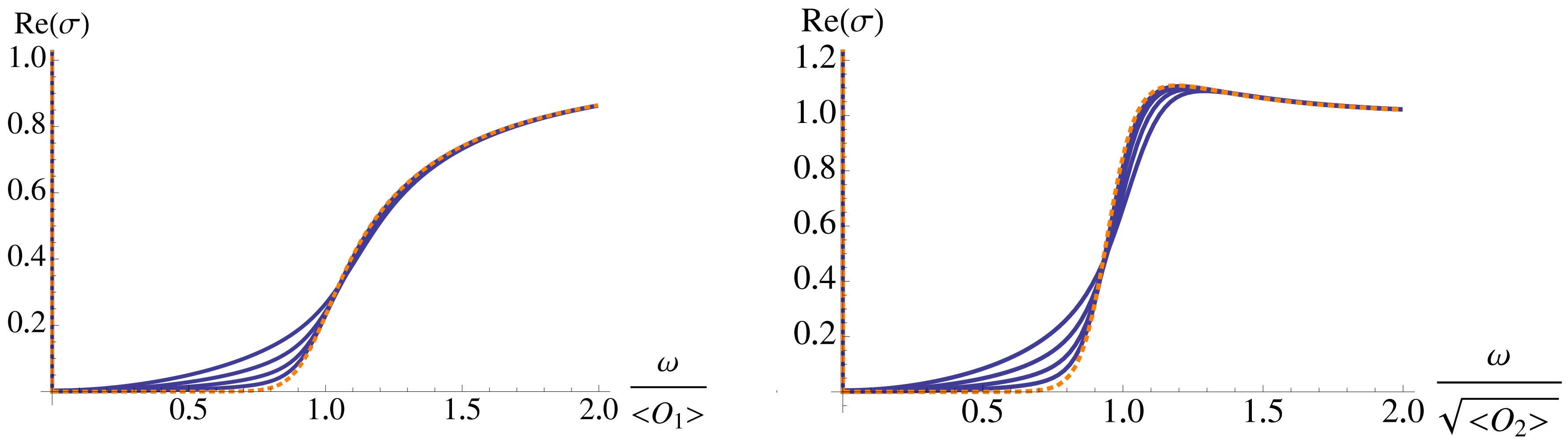


Figure 5. Comparisons of the real part of the conductivity for $c = 0$ (orange dotted lines) and $c = 0.2$, 0.3, 0.4, 0.5 (blue real lines) computed when $T/T_C = 0.20$. The left panel is when $\Delta = 1$, and the right panel is when $\Delta = 2$. Exponential and power-law behaviors are interpolated by changing c.

Result: coexistence of sc & vec

Overlapping phase diagram

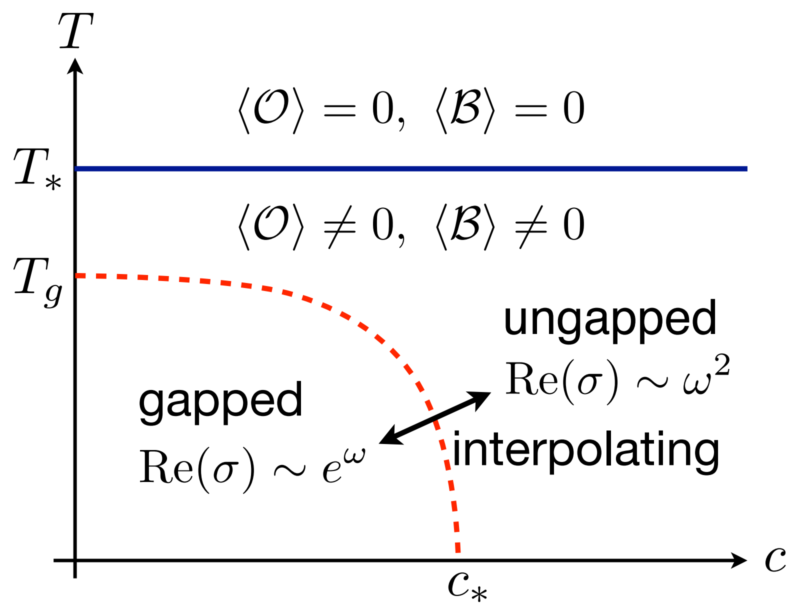
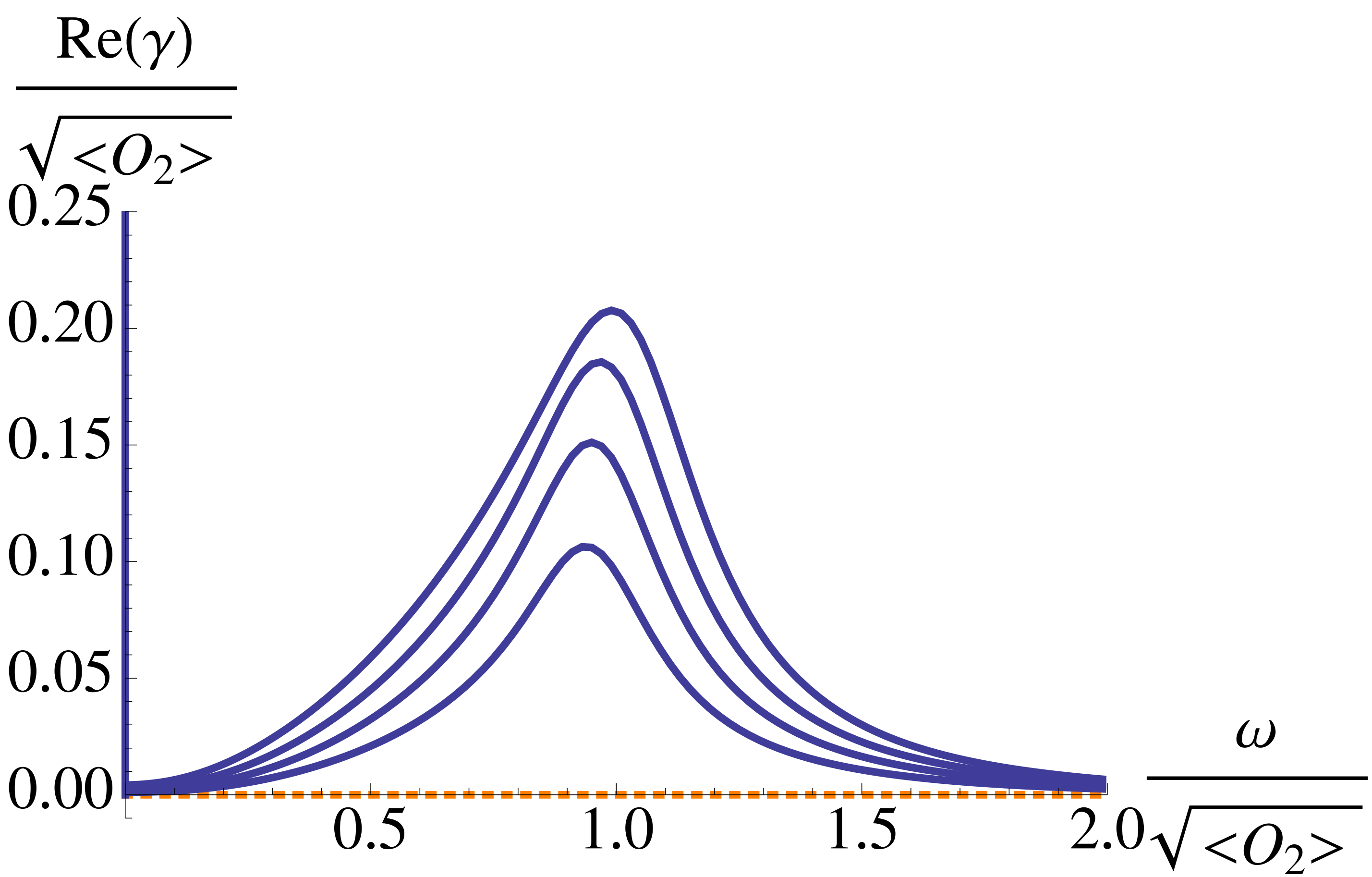


Figure 6. A schematic description of the phase diagram expected. Here $\langle \mathcal{B} \rangle = 0$ if $c = 0$, while $\langle \mathcal{B} \rangle \neq 0$ once $c \neq 0$ and $T < T_C$. Gapped and ungapped phases would be interpolated smoothly. There may be some critical $c = c_*$ above which there is no mass gap. Figure 5 corresponds to looking at a horizontal slice of this diagram at $T/T_C = 0.20$.

Result (spin susceptibility / impurity conductivity)



Implication to PG order parameter.

- If the PG phase is a phase of incoherent paired state(precursor of SC), we should use the vector order parameter for PS. Not a scalar! (Scalar with negative potential is also OK). We need to introduce two scales: One for SC and the other for PS which is of order T*.
- If PG itself is an order competing with SC, its order parameter can be a real scalar. Possibly with attractive interaction.
- If PG is a spin Liquid:➔Not in this talk. (Top.O)

magnetism with axion

- Motivation.
  We want to control the magnetism by changing the dopping parameter.
  Spin is not totally independent of charge.
  So we use the Chern-Simon / Axion term.

with

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References: 1003.0010(Iqbal et.al); 1007.3480 Basu et.al

Action

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{-g}\left\{R + \frac{6}{L^2} - \frac{1}{4}F^2 - \frac{1}{2}(\partial\chi_I)^2\right\} + \frac{q_\chi}{16\pi G}\int_{\mathcal{M}} (\partial\chi_I)^2\, F\wedge F$$
$$- \frac{1}{16\pi G}\int_{\partial\mathcal{M}} d^3x\sqrt{-\gamma}\left(2K + \frac{4}{L} + R[\gamma] - \frac{L}{2}\nabla\chi_I\cdot\nabla\chi_I\right)$$

dyonic ansatz with linear axion

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + r^2(dx^2 + dy^2)$$

$$A = a(r)dt + \frac{1}{2}B\,(xdy - ydx)$$

$$\chi_I = \beta\delta_{Ii}x^i.$$

Solution

$$U(r) = r^2 - \frac{\beta^2}{2} - \frac{m_0}{r} + \frac{q^2 + B^2}{4r^2} + \boxed{\frac{64\beta^4 B^2 q_\chi^2}{5r^6} + \frac{8\beta^2 B q q_\chi}{3r^4}},$$

$$a(r) = \mu - \frac{q}{r} \boxed{- \frac{16\beta^2 B q_\chi}{3r^3}}.$$

In terms of the horizon radius r0,

$$q = r_0\mu - \frac{16\beta^2 B q_\chi}{3r_0^2}$$

$$m_0 = r_0^3\left(1 + \frac{\mu^2 + B^2}{4r_0^2} - \frac{\beta^2}{2r_0^2}\right) + \frac{256\beta^4 B^2 q_\chi^2}{45r_0^5}$$

Thermodynamics

$$4\pi T = U'(r_0) = 3r_0 - \frac{r_0^2\mu^2 + B^2 + 2r_0^2\beta^2}{4r_0^3} - \frac{\beta^2 B q_\chi}{3r_0^4}\left(16\mu + \frac{256\beta^2 B q_\chi}{3r_0^3}\right)$$

$$s = 4\pi r_0^2 \qquad S^E \equiv \frac{\mathcal{V}_2}{T}\mathcal{W} \equiv \frac{\Omega}{T} \qquad \mathcal{V}_2 = \int \mathrm{d}x\mathrm{d}y.$$

$$\mathcal{W} = \frac{\Omega}{\mathcal{V}_2} = -r_0^3 - \frac{r_0}{4}\left(\mu^2 + 2\beta^2 - 3\frac{B^2}{r_0^2}\right) + \frac{32B\beta^2 q_\chi}{45r_0^5}\left(56B\beta^2 q_\chi + 15r_0^3\mu\right)$$

$$= \varepsilon - sT - \mu q\,,$$

$$\varepsilon = 2m_0$$

Magnetization

$$M = -\frac{1}{\mathcal{V}_2}\frac{\delta S}{\delta B}\bigg|_{\partial\mathcal{M}}$$

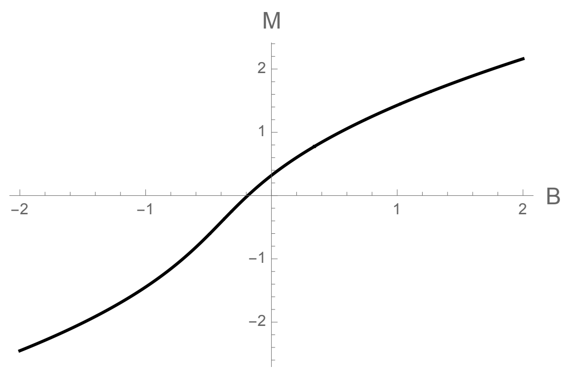
$$= -\frac{15Br_0^4 + 80r_0^2 q\, q_\chi \beta^2 + 768Bq_\chi^2\beta^4}{5r_0^2(3qr_0^2 + 16Bq_\chi\beta^2)} \cdot \frac{q}{r_0}$$

$B = 0$**: Spontaneous Magnetization**

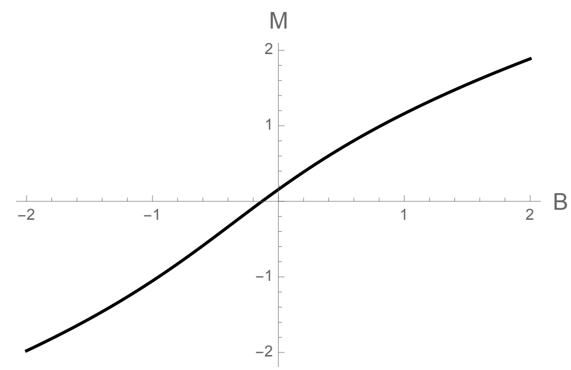
$$M\bigg|_{B\to 0} = -\frac{16\mu q_\chi \beta^2}{3r_0^2} \quad = -\frac{108\mu q_\chi\beta^2}{\left(2\pi T + \sqrt{4\pi^2T^2 + \frac{3}{4}(\mu^2 + 2\beta^2)}\right)^2}$$

$$\sim -\frac{3}{4\pi}\cdot\frac{B}{T} \qquad \text{For large temperature,}$$

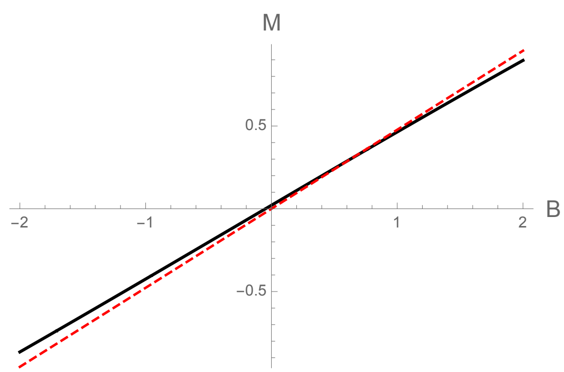
B dependence of M



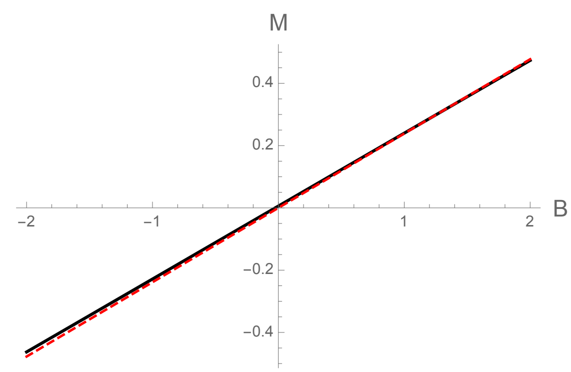
(a) $T = 0$



(b) $T = 0.1$



(c) $T = 0.5$



(d) $T = 1$

Figure 1: B dependence of the magnetization for different temperature with $\mu = 2$, $\beta = 0.1$ and $q_\chi = 1$. Red dashed line is $M = \frac{3B}{4\pi T}$.

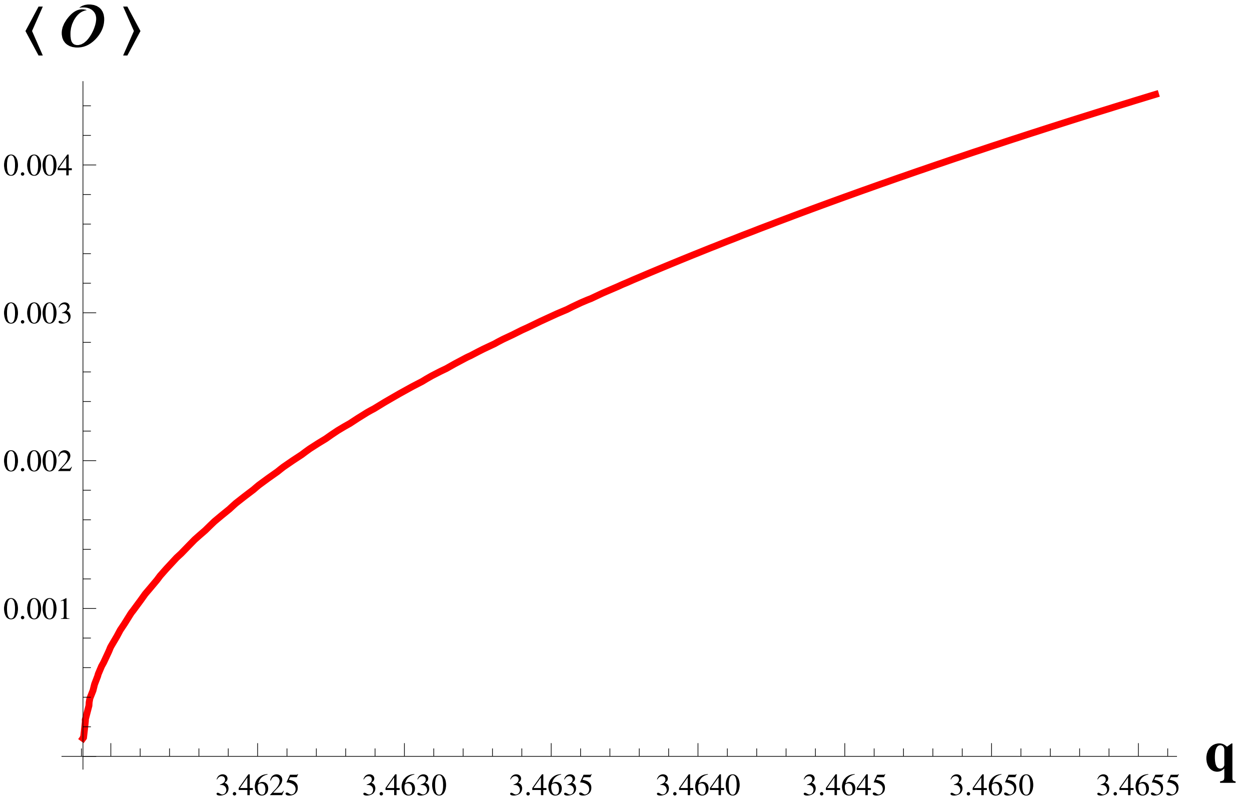
Q: What about the antiferromagnetic spin fluctuation?

- Introduce a scalar field as a spin wave order parameter.
- With the same philosophy as before, introduce CS coupling :

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{-g}\left\{R+\frac{6}{L^2}-\frac{1}{4}F^2-\frac{1}{2}\left(\partial\phi\right)^2-\frac{1}{2}m^2\phi^2\right\} + \frac{q_h}{16\pi G}\int_{\mathcal{M}}\phi^n F\wedge F$$

* scalar represent angular fluctuation in non-linear sigma model.

result



⟨ $\mathcal{O}$ ⟩
0.004
0.003
0.002
0.001
3.4625
3.4630
3.4635
3.4640
3.4645
3.4650
3.4655
q

Interpretation

- Condensation of real scalar: new order is set by coupling it to F^F term.
- This is not by a symmetry breaking.
  The gap was induced by dynamics.
- This may be a new order.
  May be Relevant to

Story2 +Story3

- Chi is the master field of Magnetization
- x,y dependent part is the non-normalizable source term.
- x,y independent part is the

  magnetic charge/B-field term and is described
by phi term is its fluctuation.

Conclusion

- Interaction between the order parameters are very useful guidelines to set up a model.
- In the presence of coupling to F^F
Momentum dissipating impurity induces a permanent magnetization.
- In the presence of coupling to F^F,
real scalar
- Neither are not related to symmetry breaking.