

Behind the geon horizon

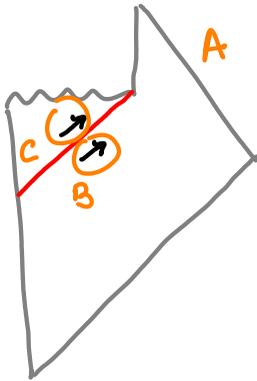
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· based on 1412.1084, w/ Simon Ross

Motivation

- Black hole information paradox

Mathur '09, AMPS '12



- recovery of information $\Rightarrow S_{AB} < S_A$

- smoothness of horizon $\Rightarrow S_{BC} \approx 0$

- strong subadditivity $S_A + S_C \leq S_{AB} + S_{BC}$

↓

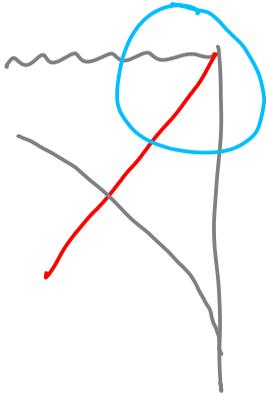
contradiction!

- possible way out: $C \subset A \rightarrow$ black hole complementarity

\rightarrow How is the black hole interior encoded outside?

The black hole interior in AdS/CFT

- large black hole in AdS w/ a single exterior



- dual to pure state $|\psi\rangle \subset$ CFT thermalises
- i.e. when probed by a small algebra of observables \mathcal{O}_i

$$\langle \psi | \mathcal{O}_i \dots | \psi \rangle = \text{Tr}(\rho_{\text{th}} \mathcal{O}_i \dots) + \mathcal{O}(e^{-S})$$

- Papadodimas - Raju (PR) \rightarrow quantitative proposal for reconstructing the b.h. interior
- this talk \rightarrow concrete example and check of the PR construction ($\mathbb{R}P^2$ geon black hole)

Plan

- review : reconstruction of bulk from the boundary
 - the PR proposal
 - the \mathbb{RP}^2 geon & properties
- construction of mirror operators
- modifications of the geon state
- future directions

Reconstructing the black hole

interior

Reconstructing the bulk from the boundary in AdS/CFT

- CFT \rightarrow large N , few operators of low dimension
- correlation functions of single-trace operators **factorize**

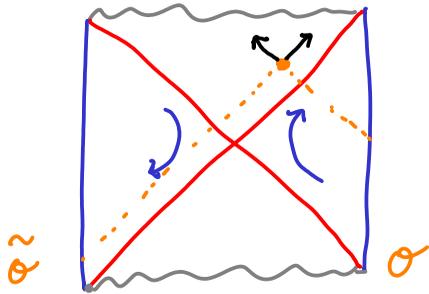
$$\langle \mathcal{O} \mathcal{O} \mathcal{O} \mathcal{O} \rangle = \langle \mathcal{O} \mathcal{O} \rangle \langle \mathcal{O} \mathcal{O} \rangle + \text{perm.} + \mathcal{O}(1/N)$$

- **generalized free field** operators \rightarrow free scalar in AdS
 $(\square_{\text{AdS}} - m^2)\Phi = 0$

$$\Phi(z, x^{\mu}) = \int d^d x' \mathcal{K}(z, x; x') \mathcal{O}(x')$$

- **reproduces local EFT** in the bulk, pert. in $1/N$, around vacuum + few excitations
- **breaks down** if we compute very "long" correlators

Reconstructing the black hole interior : eternal b.h



- entangled state in 2 copies of the CFT

$$|\Psi_{\text{tfd}}\rangle = \sum_i e^{-\frac{\beta E_i}{2}} |\epsilon_i\rangle |\tilde{\epsilon}_i\rangle$$

- $[O, \tilde{O}] = 0$ $\langle 00 \dots \rangle$ - thermal

$$\langle \Psi_{\text{tfd}} | O(t_1, x_1) \dots \tilde{O}(t_n, x_n) \dots | \Psi_{\text{tfd}} \rangle = \frac{1}{Z_p} \text{Tr} [e^{-\beta H} O(t_1, x_1) \dots O(t_n + \frac{i\beta}{2}, x_n) \dots]$$

$$\tilde{O} | \Psi_{\text{tfd}} \rangle = e^{-\frac{\beta H}{2}} O^\dagger e^{\frac{\beta H}{2}} | \Psi_{\text{tfd}} \rangle$$

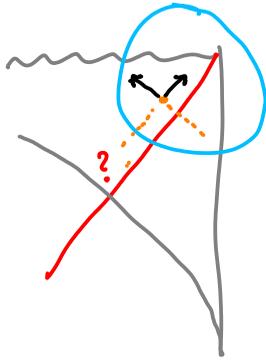
- bulk field in interior

$$\Phi(t, \vec{x}, r) = \int_{\omega > 0} d\omega d^{d-1} k [\tilde{O}_{\omega k} \mathcal{K}_{\omega k}^{(1)}(t, \vec{x}, r) + \tilde{\tilde{O}}_{\omega k} \mathcal{K}_{\omega k}^{(2)}(t, \vec{x}, r) + \text{h.c.}]$$

$\xleftarrow{\text{BTZ}} \xrightarrow{\text{BTZ}}$

- reproduces local EFT in the eternal black hole

Reconstruction of the black hole interior - single-sided b.h



- $|\psi\rangle$ - pure state that thermalizes
- need right-moving modes!
- PR proposal:

same CFT!

$$\Phi(t, \vec{x}, \sigma) = \int_{\omega > 0} d\omega d^d k \left[\sigma_{\omega k} \mathcal{K}_{\text{eternal}}^{(1)}(t, \vec{x}, r) + \tilde{\sigma}_{\omega k} \mathcal{K}_{\text{eternal}}^{(2)}(t, \vec{x}, r) + \text{h.c.} \right]$$

Conditions on $\tilde{\sigma}$:

- "correctly entangled" $\tilde{\sigma}|\psi\rangle = e^{-\frac{\beta_H}{2}} \sigma^\dagger e^{\frac{\beta_H}{2}} |\psi\rangle$ ↖ state dependent!
- commute when acting on $|\psi\rangle$ + excitations $[\tilde{\sigma}, \sigma] \sigma \dots |\psi\rangle = 0$
- solution always exists if $P(\sigma_i) |\psi\rangle \neq 0$

The \mathbb{RP}^2 geom example

BTZ warm-up

AdS_3 : embedding in $\mathbb{R}^{2,2}$

$$ds^2 = -dT_1^2 - dT_2^2 + dX_1^2 + dX_2^2$$

- hyperboloid $-T_1^2 - T_2^2 + X_1^2 + X_2^2 = -\ell^2$

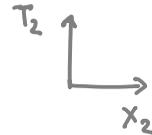
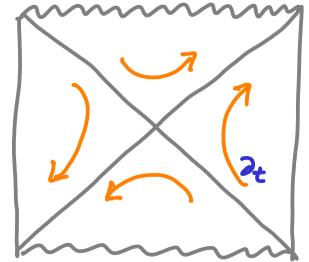
- $\xi \equiv X_1 \partial_{T_1} + T_1 \partial_{X_1}$; $\eta \equiv X_2 \partial_{T_2} + T_2 \partial_{X_2}$

BTZ: quotient by $e^{2\pi r_+ \xi / \ell}$

$$ds^2 = -\frac{(r^2 - r_+^2)}{\ell^2} dt^2 + \frac{\ell^2}{r^2 - r_+^2} dr^2 + r^2 d\varphi^2$$

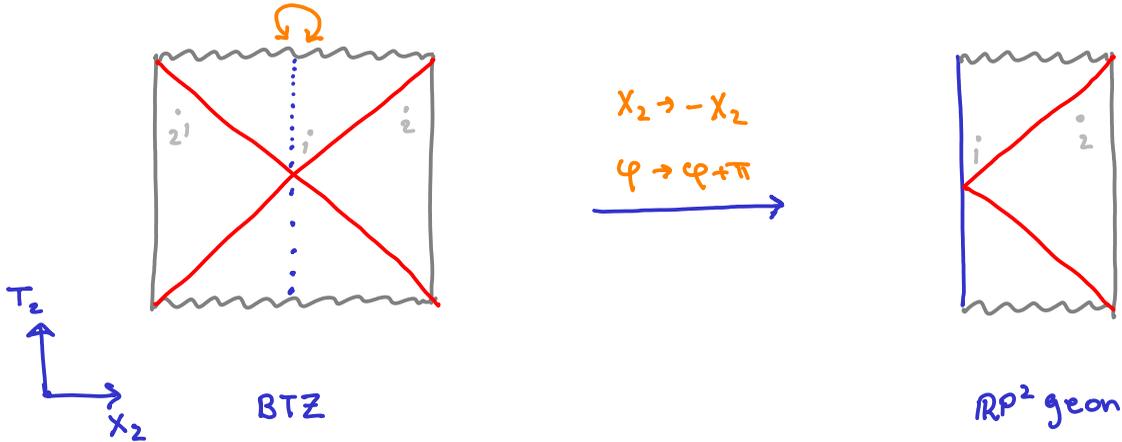
$$\xi = \ell/r_+ \partial_\varphi \quad ; \quad \eta = \ell^2/r_+ \partial_t$$

- r - t plane \leftrightarrow X_2, T_2 plane



Definition of the \mathbb{RP}^2 geon

Louko & Marolf '98



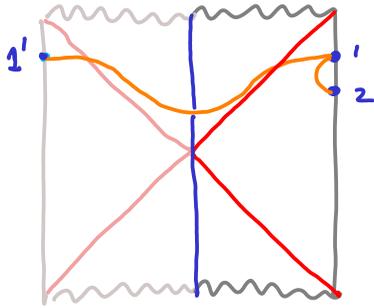
- correlators obtained via method of images

$$\langle \Phi(P_1) \Phi(P_2) \rangle_{\text{geon}} = \langle \Phi(P_1) \Phi(P_2) \rangle_{\text{BTZ}} + \langle \Phi(P_1) \Phi(P_2') \rangle_{\text{BTZ}}$$

- analyticity \rightarrow geodesic approximation

Thermality

- $|\psi_g\rangle \rightarrow$ pure state that thermalizes at late times



- late-time correlators

$$\langle O(t_1) O(t_2) \rangle_{\text{geon}} = \underbrace{\langle O(t_1) O(t_2) \rangle_{\text{BTZ}}}_{\text{thermal}}$$

$$+ \langle O(t'_1) O(t_2) \rangle_{\text{BTZ}}$$

$$\propto e^{-(t_1+t_2)\Delta/\beta}$$

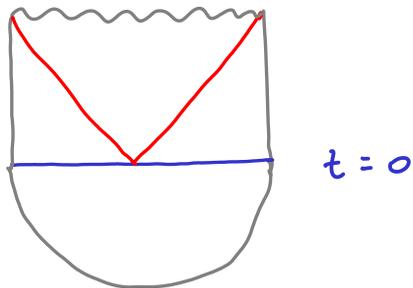
- for $t > t_* = \frac{\beta}{2\pi} \ln S_{\text{BH}}$ scrambling time suppressed

- geon correlators are not thermal for $t \approx 0$

Path integral construction

Maldacena '01

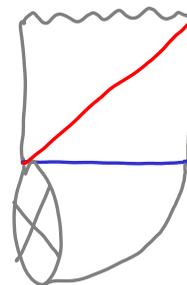
• eternal BTZ



Lorentzian

Euclidean

• \mathbb{RP}^2 geon



$$Z_{\text{CFT}} \left[\text{Cylinder} \right]_{\beta/2} \rightarrow \begin{cases} z \sim \beta/2 - z \\ \varphi \sim \varphi + \pi \end{cases} \rightarrow Z_{\text{CFT}} \left[\text{Cylinder} \right]_{\beta/4}$$

$$\mathcal{Z}_{\text{fd}} = \sum_E e^{-\frac{\beta E}{2}} |E\rangle_1 |E\rangle_2$$

$$|4_g\rangle = e^{-\frac{\beta H}{4}} |c\rangle$$

crosscap

Properties of the geon state

- entanglement structure

$$|\psi_g\rangle = e^{-\beta H/4} |C\rangle ; (L_n - (-1)^n \bar{L}_{-n}) |C\rangle = 0$$

→ entangled state between left & right-movers (in single CFT)

- e.g. free boson CFT $|C\rangle = \exp\left[-\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \alpha_{-n} \bar{\alpha}_{-n}\right] |0\rangle$

- generally, crosscap expected to satisfy

$$\mathcal{A}^+(t, \varphi) |C\rangle = \mathcal{A}(-t, \varphi + \pi) |C\rangle$$

$$\mathcal{A} = e^{\frac{\beta H}{4}} \mathcal{O} e^{-\frac{\beta H}{4}}$$

$$\Rightarrow e^{-\frac{\beta H}{2}} \mathcal{O}^+(t, \varphi) e^{\frac{\beta H}{2}} |\psi_g\rangle = \mathcal{O}(-t, \varphi + \pi) |\psi_g\rangle$$

• exact

• matches gravity

Properties of the geon state

- for GFF operators, can solve

$$|\psi_g\rangle \sim \prod_{\omega, k} \exp[\alpha_{\omega, k} (-1)^k O_{\omega, k}^+ O_{\omega, -k}^+] \sim \prod_{\substack{\omega_0, t_0 \\ k_0, \varphi_0}} \exp[\alpha_{\omega_0, k_0} O_{\omega_0, t_0}^+ O_{\omega_0, -t_0}^+]$$

→ thermal density matrix at late times

- support at high energies

$$|C\rangle = \sum_i c_{i, m_i} |i, m_i\rangle_L |i, m_i\rangle_R \Rightarrow |\psi_g\rangle = \sum_i e^{-\frac{\beta E_i}{4}} c_{i, m_i} |i\rangle_L |i\rangle_R$$

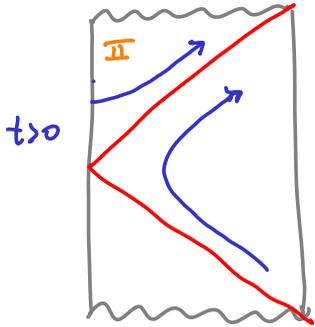
- partition function → CFT on Klein bottle



$$\tilde{d}_K = \langle \text{tr} c | e^{-\frac{\beta K}{2}} |C\rangle = \sum_i e^{-\frac{\beta E_i}{2}} d_c(E_i) \quad d_c = \sum_{m_i} |c_{i, m_i}|^2$$

- "modular invariance" $\Rightarrow d_c(E) \sim e^{\pi \sqrt{cE/3}}$ Cardy growth
- matches geon black hole entropy @ $t=0$, not $t>0$

Mirror operators - method I (direct construction)



• region II wavefunctions \rightarrow symmetric $\begin{cases} t \rightarrow -t \\ \varphi \rightarrow \varphi + \pi \end{cases}$

•
$$\Phi_{\text{geon}}^{\text{II}}(t, r, \varphi) = \sum_m \int d\omega \left[\mathcal{O}_{\omega, m} \left(e^{-i\omega t + im\varphi} + (-1)^m e^{i\omega t + im\varphi} \right) K_{\text{BTZ}}^{(1)} + \text{h.c.} \right]$$

• mirror operators

$$\tilde{\mathcal{O}}_{\omega, m} = (-1)^m \mathcal{O}_{\omega, -m}$$

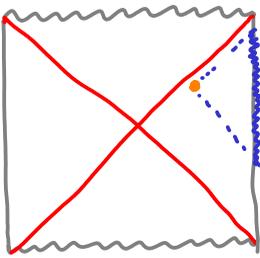
• Fourier transform

$$\tilde{\mathcal{O}}_g(t, \varphi) = \mathcal{O}(-t, \varphi + \pi)$$

\hookrightarrow agrees w/ field theory expectation

• $[\mathcal{O}(-t), \mathcal{O}(t')] \approx 0$ only for t, t' large

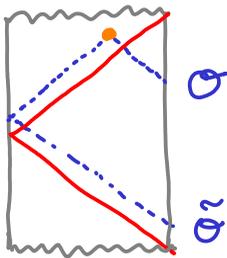
Distinguishing Θ & $\tilde{\Theta}$



- for $r \rightarrow r_+$ $\Phi_{\text{mult}}(t, r, \varphi) = \int dt' d\varphi' \underbrace{k(x, x')}_{\text{boundary support diverges}} \Theta(t', \varphi)$

- $K_{\omega m}(t, r, \varphi) \underset{r \rightarrow r_+}{\sim} \frac{1}{\omega} e^{\frac{\beta |m|}{4}} \cos(\omega r_* + \delta_{\omega m})$

- smear $\Phi_{\omega_0, m_0}(t_0, r) = \int dt d\varphi \left[\xi_{\omega_0 t_0}^*(t) \Phi^\dagger(t, r, \varphi) + \text{h.c.} \right]$



- $\xi_{\omega_0 t_0}$: $e^{-i\omega_0 t}$ $\frac{1}{\epsilon}$ $\epsilon \cdot e^{i\omega_0 t_0}$

- for $\omega_0 \gg e^{-\beta/4}, M \rightarrow$ ray tracing

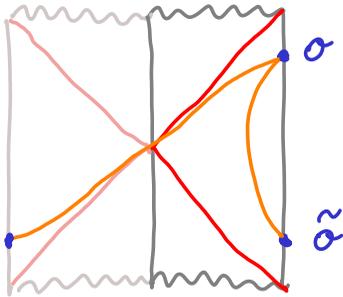
Mirror operators - method II (PR)

- PR conditions: $\left\{ \begin{array}{l} \tilde{\mathcal{O}} |\psi_g\rangle = e^{-\frac{\beta H}{2}} \mathcal{O} e^{\frac{\beta H}{2}} |\psi_g\rangle \\ [\tilde{\mathcal{O}}, \mathcal{O}] |\psi_g\rangle = 0 \end{array} \right.$

stays

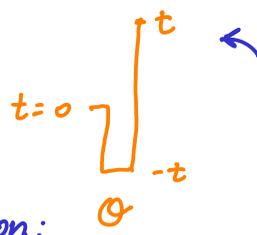
looks dm

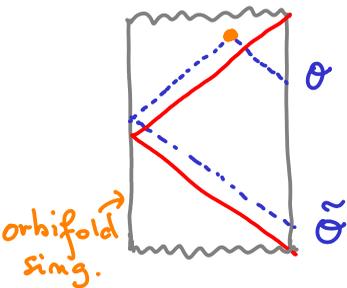
- bulk geon state $d_{\omega, m}^g |\psi_g\rangle \propto (a_{\omega, m} - e^{-\frac{\beta \omega}{2}} (-1)^m a_{\omega, -m}^\dagger) |\psi_g\rangle = 0$
 \hookrightarrow consistent w/ $\tilde{\mathcal{O}} = \mathcal{O}(-t, \varphi + \pi)$



- $\langle \mathcal{O} \tilde{\mathcal{O}} \rangle_{\text{geon}} = \langle \mathcal{O}(t) \mathcal{O}(-t) \rangle_{\text{BTZ}} + \langle \mathcal{O}(t) \tilde{\mathcal{O}}(t) \rangle_{\text{BTZ}}$
 \uparrow time-indep, $[\mathcal{O}, \tilde{\mathcal{O}}]_{\text{BTZ}} = 0$
 $\propto e^{-t/\beta}$
- $\langle [\mathcal{O}, \tilde{\mathcal{O}}] \rangle_{\text{geon}} \sim e^{-t/\beta} \leftarrow$ correct

Comments

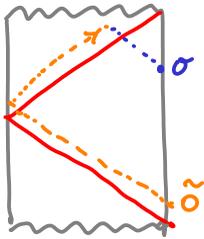
- $\tilde{\mathcal{O}}$ simple \rightarrow precursor sense $t=0$  very complicated
- ray tracing v. special to geom: \mathcal{O}
 - no transplanckian problem
 - reflects back to boundary
- another simple example: \mathcal{J} -quotient of BTZ ($x_2 \rightarrow -x_2$)



$$\mathcal{O}(-t, \varphi) |\psi_0\rangle = e^{-\frac{\beta H}{2}} \mathcal{O}^+(t, \varphi) e^{\frac{\beta H}{2}} |\psi_0\rangle$$

Modifications of the geon state

- PR prescription \rightarrow assumes smooth horizon \rightarrow can one predict when horizon not smooth?
- prescription only applies to equilibrium states



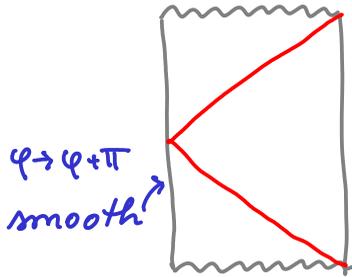
- $\mathcal{O} |\psi_g\rangle \rightarrow$ out of equilibrium
- $\tilde{\mathcal{O}} |\psi_g\rangle \rightarrow \mathcal{H}$. should horizon still be smooth?
- $U(\tilde{\mathcal{O}}) |\psi_g\rangle$, e.g. $e^{i\omega \tilde{\mathcal{O}}_w^\dagger \tilde{\mathcal{O}}_w} |\psi_g\rangle$

$d_{wm}^\circ | \tilde{u} \psi_g \rangle \neq 0 \Rightarrow$ horizon not smooth?

- $e^{i\alpha \mathcal{O}_w^\dagger \mathcal{O}_w} |\psi_g\rangle \rightarrow$ ambiguity of the PR proposal?

A simple example

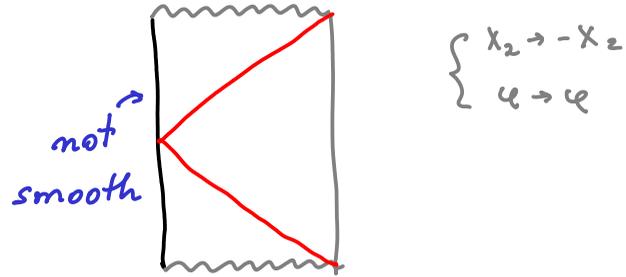
$\mathbb{R}P^2$ geon



$$|\psi_g\rangle = e^{-\frac{\beta H}{4}} |C\rangle$$

$$|\psi_g\rangle = U |\psi_B\rangle$$

J-quotient of BTZ



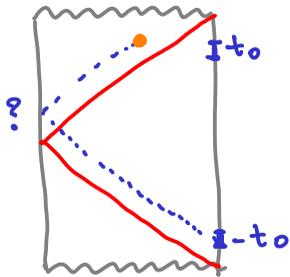
$$|\psi_B\rangle = e^{-\frac{\beta H}{4}} |B\rangle$$

, however: horizon is smooth!

• only interior changes

Future directions

- $| \psi_g \rangle \sim \prod_{\omega_0, t_0} \exp [\alpha_{\omega_0} \mathcal{O}_{\omega_0, t_0}^\dagger \mathcal{O}_{\omega_0, -t_0}^\dagger] | 0 \rangle$ + unitary rot $\alpha_{\omega_0} \rightarrow e^{i\epsilon} \alpha_{\omega_0}$



• what happens to the geometry?

- backreaction $\tilde{\mathcal{O}} | \psi_g \rangle$
- more general geons, w/ non-trivial topology behind the horizon?

Thank you !