Position space analysis of AdS (in)stability

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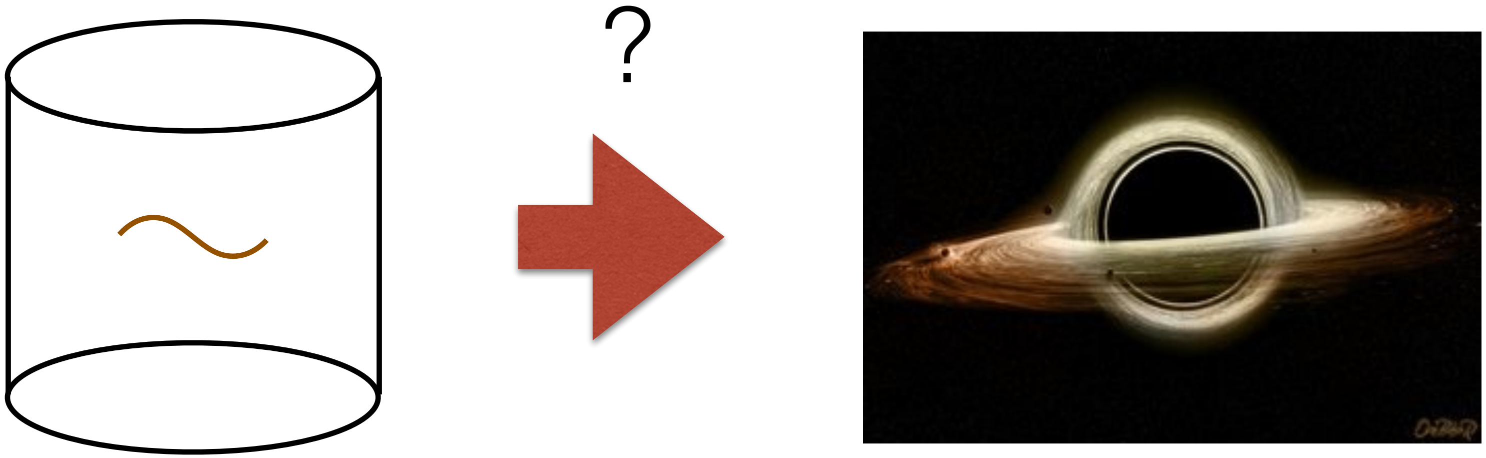
based on 1410.1880

GGI Gauge/Gravity Duality Workshop

Perturbations in global AdS



Fluctuation with energy ε^2

- can't dissipate
- gravity is attractive

Does it collapse into a black hole?

Outline

1. Holographic thermalization

2. Evidence for and against collapse

3. Position-space perturbation analysis

Holographic thermalization

BH formation in AdS ⟷ Thermalization in CFT

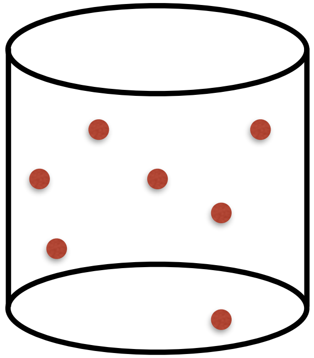
Poincare AdS ↔ CFT on $\mathcal{R}^{d,1}$

- energy falls though horizon
- forms planar BH

Global AdS ↔ CFT on $S^d \times \mathcal{R}$

- energy bounces through origin
- may form spherical BH

Black Hole vs. Thermal Gas



$$S_{BH} \sim \left(\frac{R_{BH}}{L_{Planck}}\right)^2$$

$$S_{gas} \sim \left(\frac{R_{BH} L_{AdS}}{L_{Planck}}\right)^{3/4}$$

Two limits:

1. classical limit $$\frac{R_{BH}}{L_{Planck}} \equiv \gamma \gg 1$$

2. small energy $$\frac{L_{AdS}}{R_{BH}} \equiv \beta \gg 1$$

BH formation is dual to…

$S_{BH} > S_{gas}$	$\gamma^2 > \beta^3$	thermalization
$S_{BH} < S_{gas}$	$\gamma^2 < \beta^3$	pre-thermalization

Initial perturbation → (collapse) → Black Hole → (evaporation) → Thermal Gas

BH formation is dual to…

$S_{BH} > S_{gas}$	$\gamma^2 > \beta^3$	thermalization
$S_{BH} < S_{gas}$	$\gamma^2 < \beta^3$	pre-thermalization

Initial perturbation → Black Hole → Thermal Gas

collapse

evaporation

collapse → classical if $\beta \ll \gamma^2$

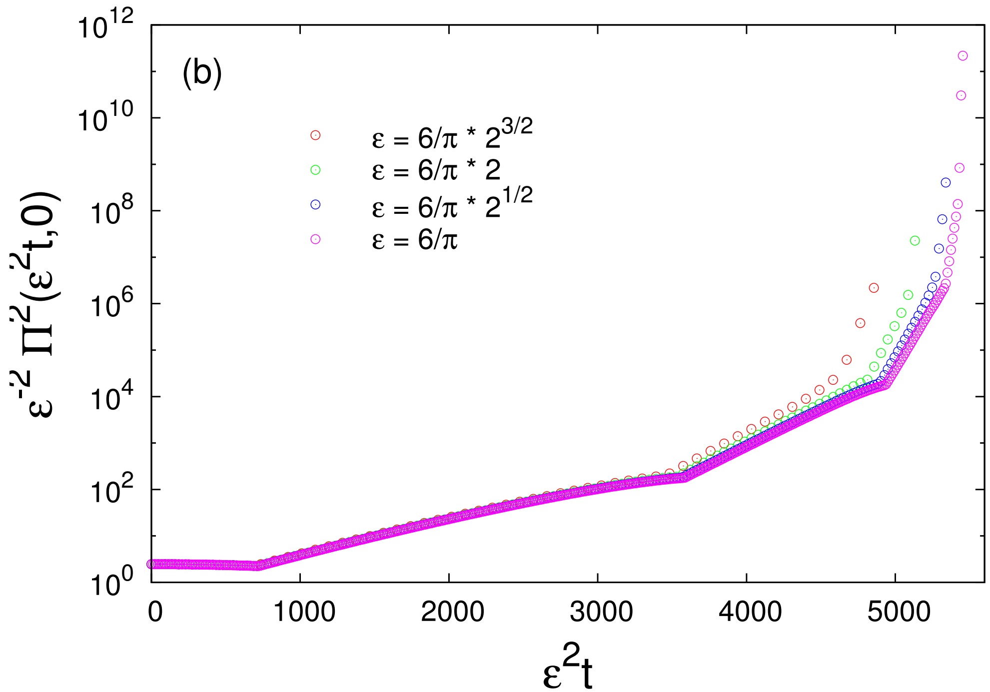
Evidence for collapse

Black hole forms

- Time scale $t_{BH} \sim \epsilon^{-2}$
- Turbulent instability → BH

Bizon & Rostworowski 2011
Buchel, Lerner, Liebling 2012
Dias, Horowitz, Santos 2011
…

energy density at origin



from Bizon & Rostworowski 2011

Evidence for stability

Maybe black hole does not form

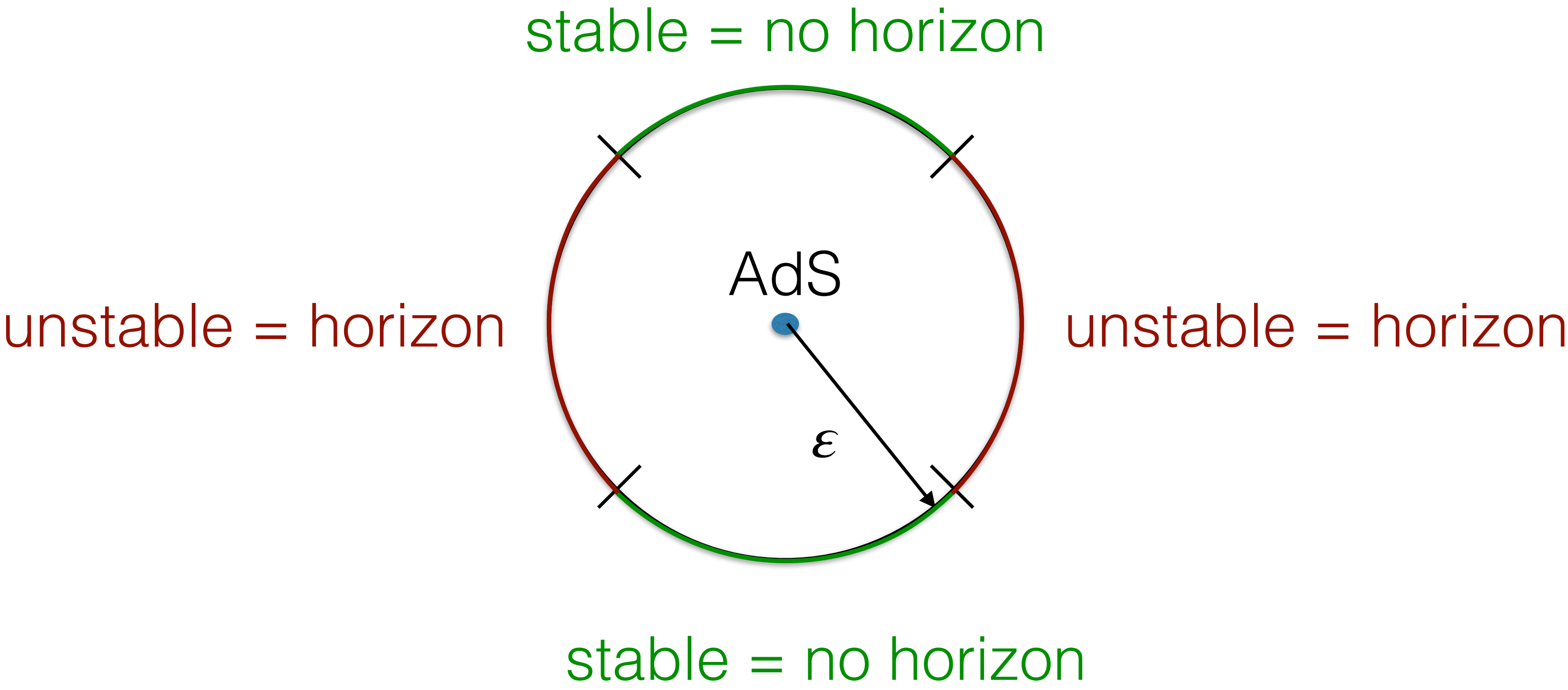
- islands of stability stay near AdS
- quasi-periodic solutions

  e.g geons, boson stars
- perturbative conservation laws

Bizon & Rostworowski 2011
Buchel et al 2013, 2015
Dias et al 2011, 2012
Craps et al 2014, 2015
…

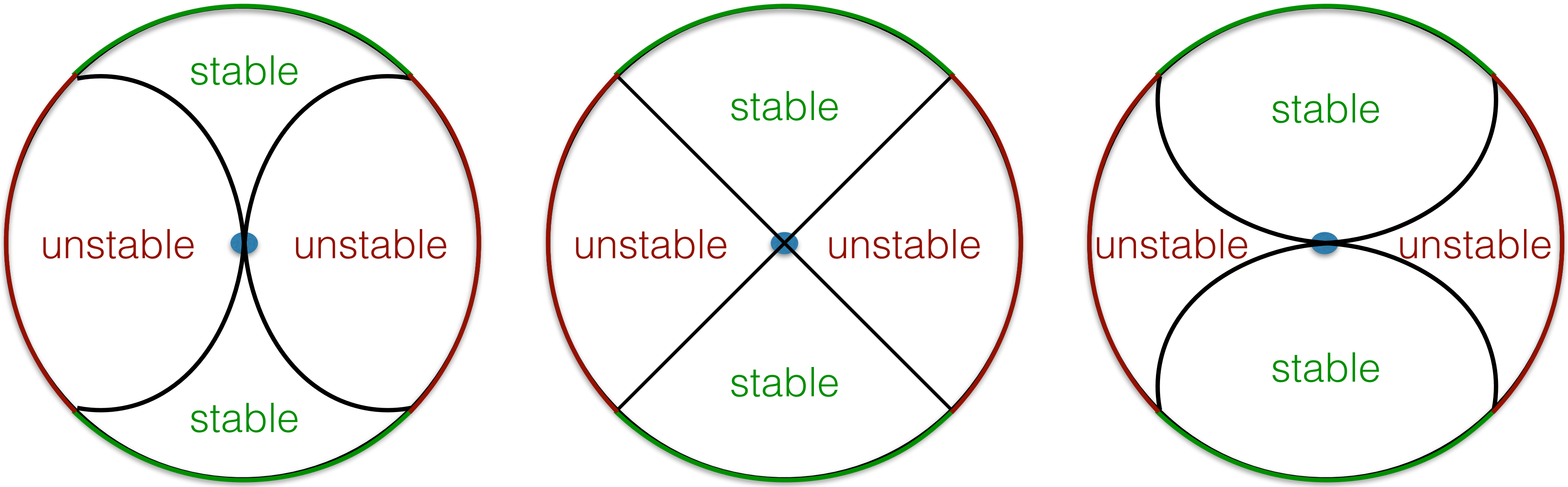
How generic is black hole formation?

Phase space


stable = no horizon
unstable = horizon
AdS
unstable = horizon
ε
stable = no horizon

Phase space

generic unstable | mixed | generic stable



$$\left.\frac{V_{stable}}{V_{unstable}}\right|_{\epsilon\to 0} \to 0$$

constant

∞

Weak-Gravity Perturbations

The Model

Classical gravity + massless scalar

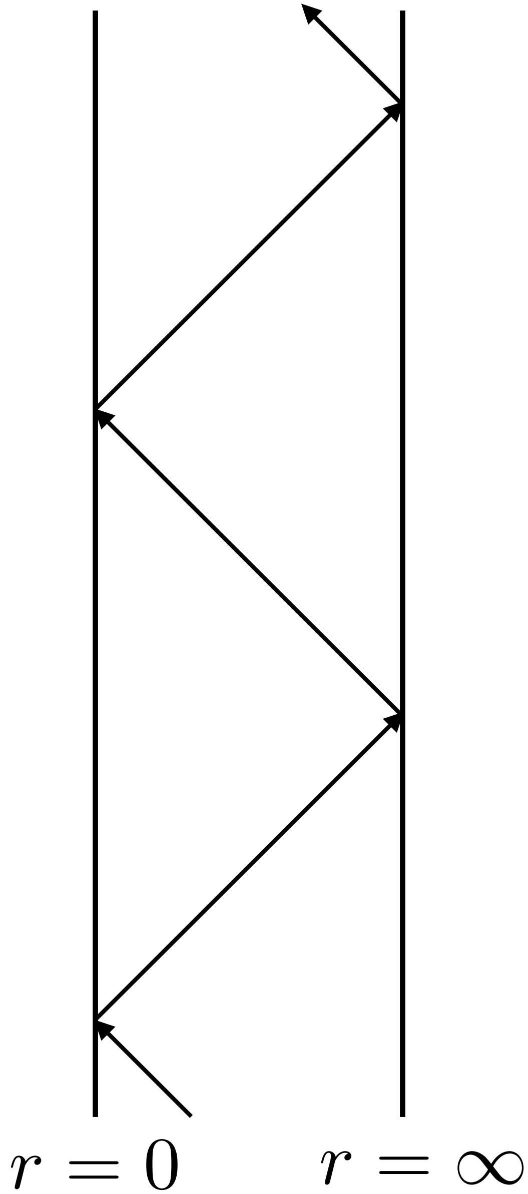
$$S = \int d^4x \sqrt{g} \left(\frac{R}{16\pi^2} + \frac{6}{L_{AdS}^2} - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi \right)$$

Spherically symmetric

Expansion

$$\phi = \epsilon \phi_{(1)} + \epsilon^3 \phi_{(3)} + \mathcal{O}(\epsilon^5)$$

Periodicity



free field profile periodic

$$\phi(t) = \phi(t + 2\pi L_{AdS})$$

radial mode expansion:

$$\phi(t, r) = \sum_n \alpha_n e^{i\omega_n t} e_n(r)$$

$$\omega_n = (2n + 3)/L_{AdS}$$

interactions → resonances

Energy cascade

Resonances

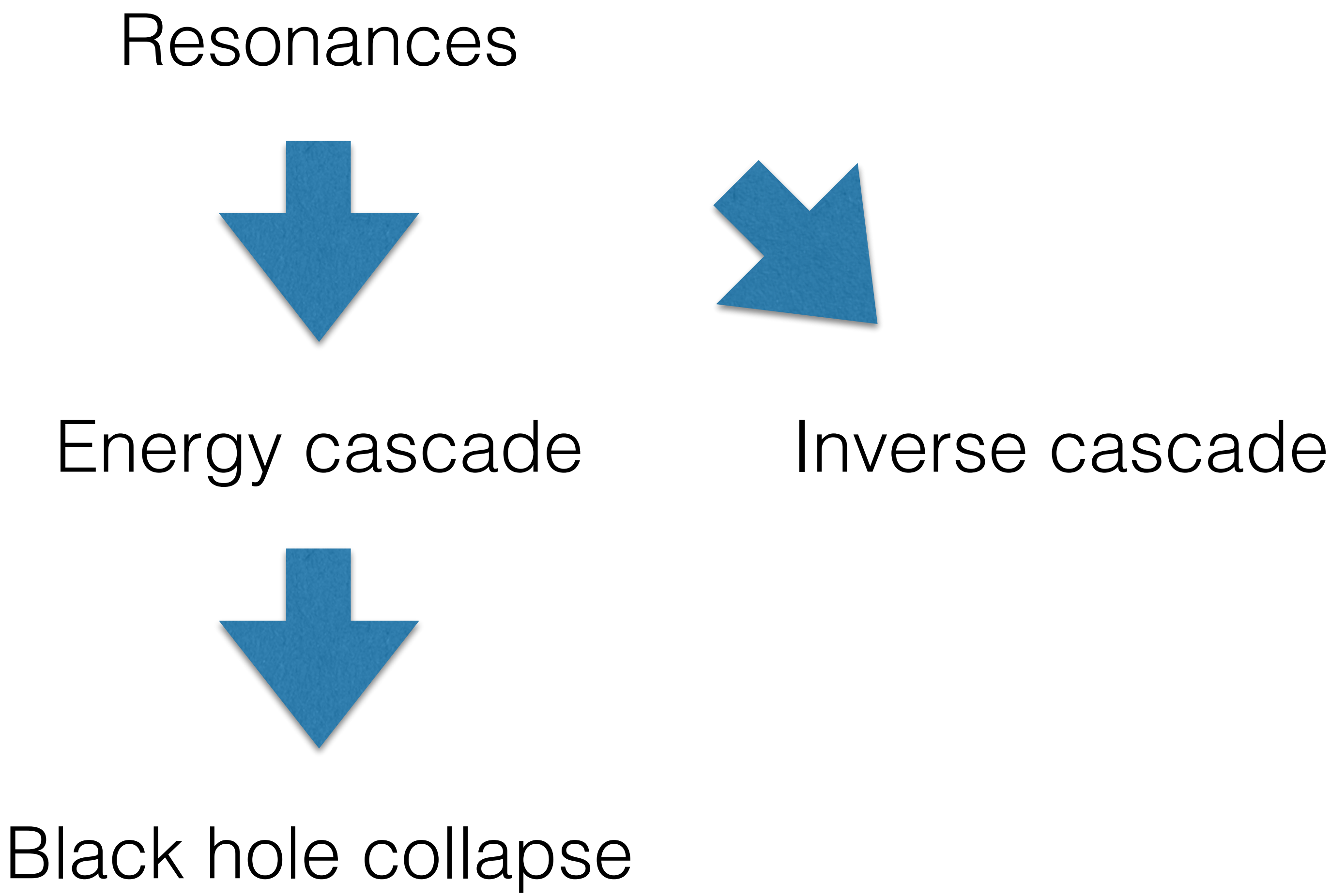
↓

Energy cascade

↓

Black hole collapse

Energy cascade



Energy cascade

Resonances

Energy cascade | Inverse cascade

Black hole collapse | Energy not focused

$$\Delta x \geq 1/\Delta k$$

Position space perturbations

Number of bounces $N = \frac{t}{2\pi L_{AdS}}$

Free: $\phi(t + 2\pi_{AdS} N) \equiv \phi_N = \phi_0$

Weak-gravitational self-interaction:

$$\phi_{N+1} = \phi_N + F_3[\phi_N, \partial\phi_N] + F_5[\phi_N, \partial\phi_N] + ...$$

Rescale: $\tilde{\phi} = \phi/\epsilon$

$$\tilde{\phi}_{N+1} = \tilde{\phi}_N + \epsilon^2 F_3[\tilde{\phi}_N, \partial\tilde{\phi}_N] + \mathcal{O}(\epsilon^4)$$

Asymptotic Scaling Symmetry

Slow timescale $\tau = \epsilon^2 N$

Evolution equation

$$\frac{d\tilde{\phi}}{d\tau} \equiv \lim_{\epsilon \to 0} \frac{\tilde{\phi}_{N+1} - \tilde{\phi}_N}{\epsilon^2} = F_3[\tilde{\phi}, \partial\tilde{\phi}]$$

Stable solutions

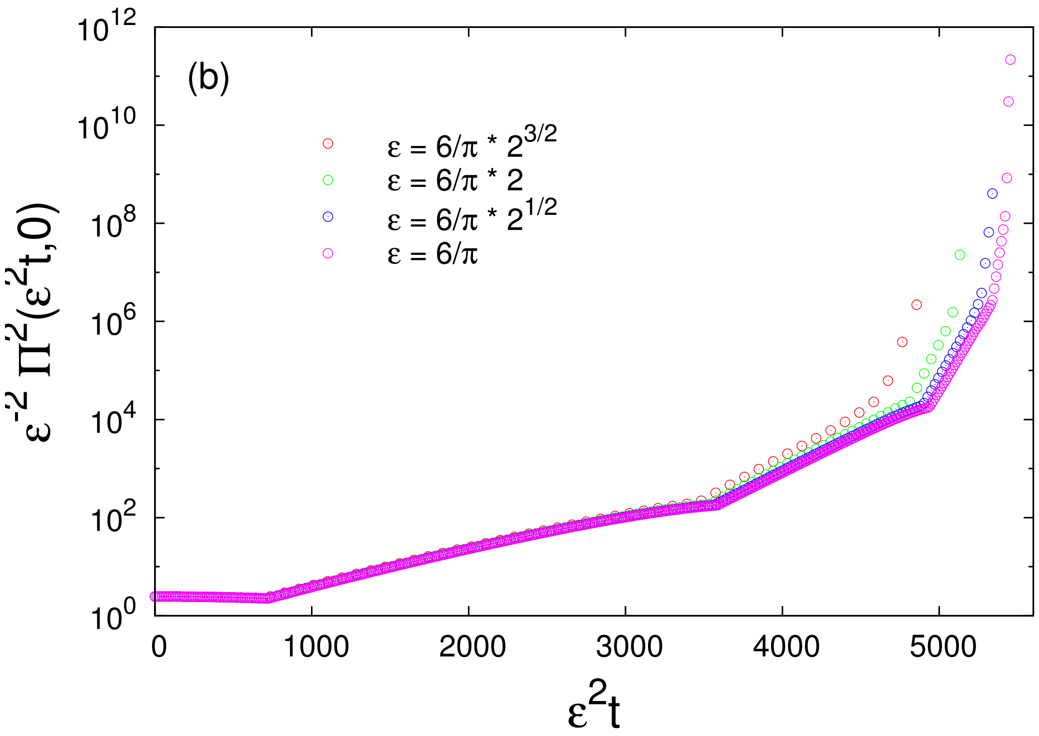
solution at $\varepsilon > 0$ remains stable as $\varepsilon \to 0$

Broken scaling symmetry

Unstable solutions

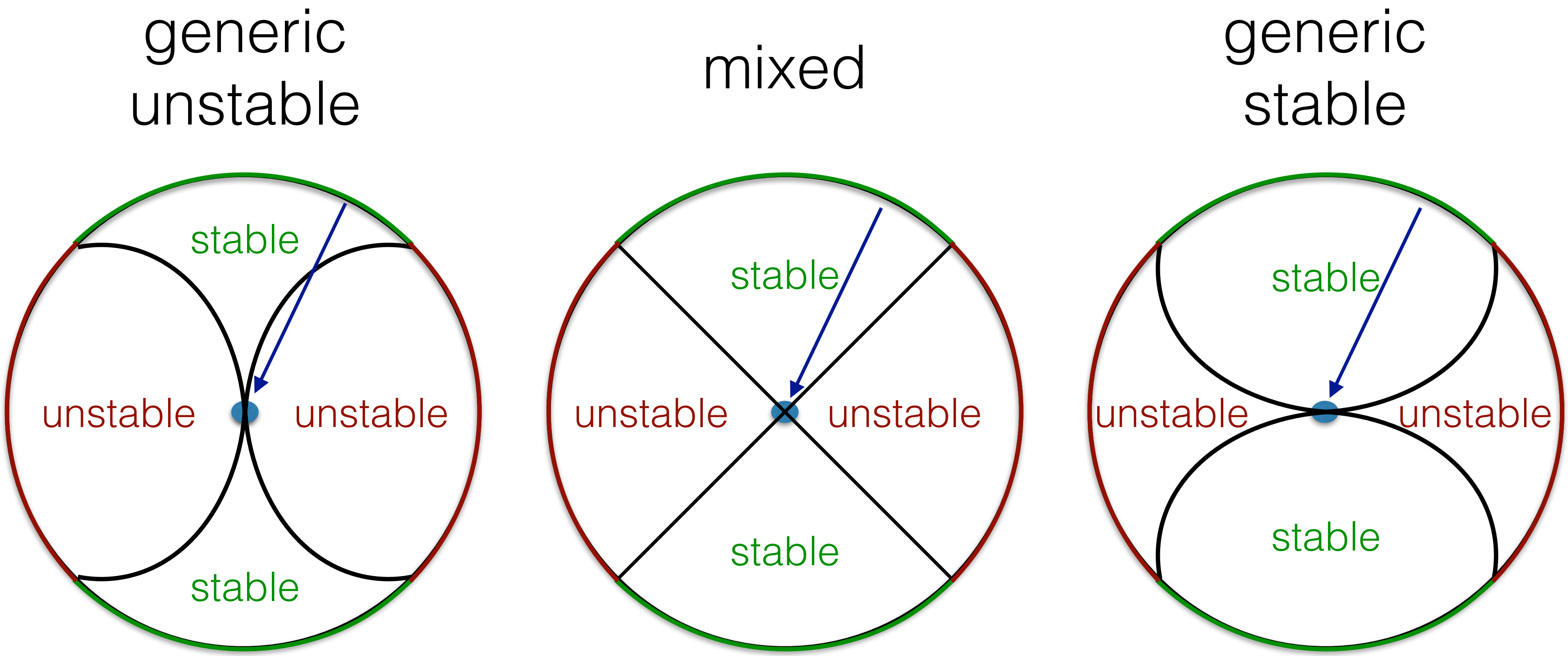
perturbation expansion breaks down

no $\varepsilon \to 0$ scaling → not protected

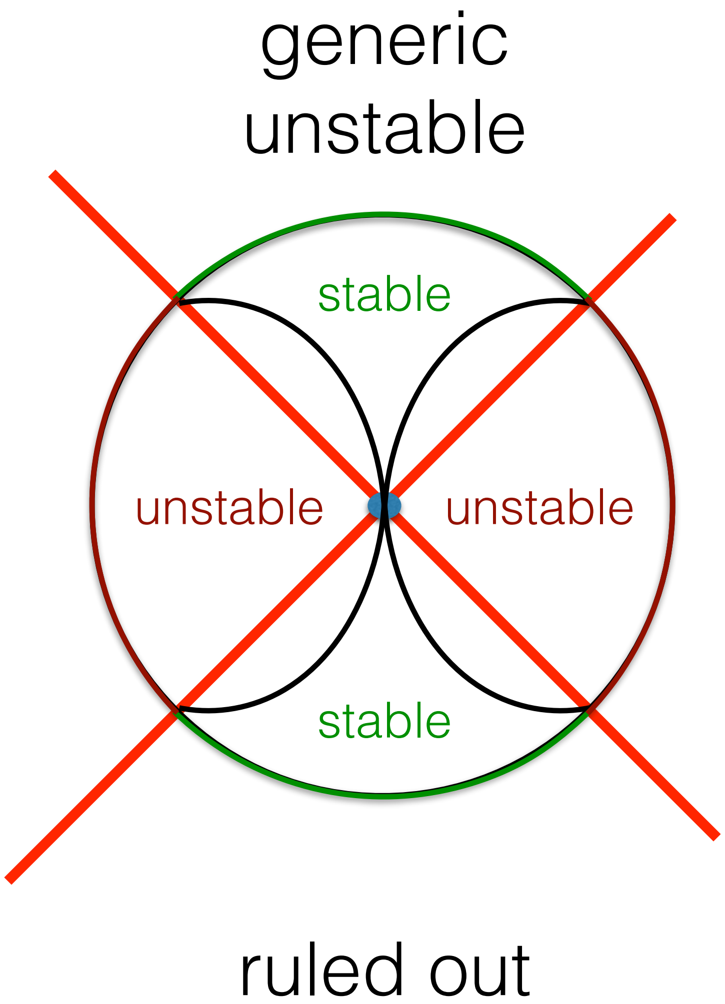


from Bizon & Rostworowski, 1104.3702

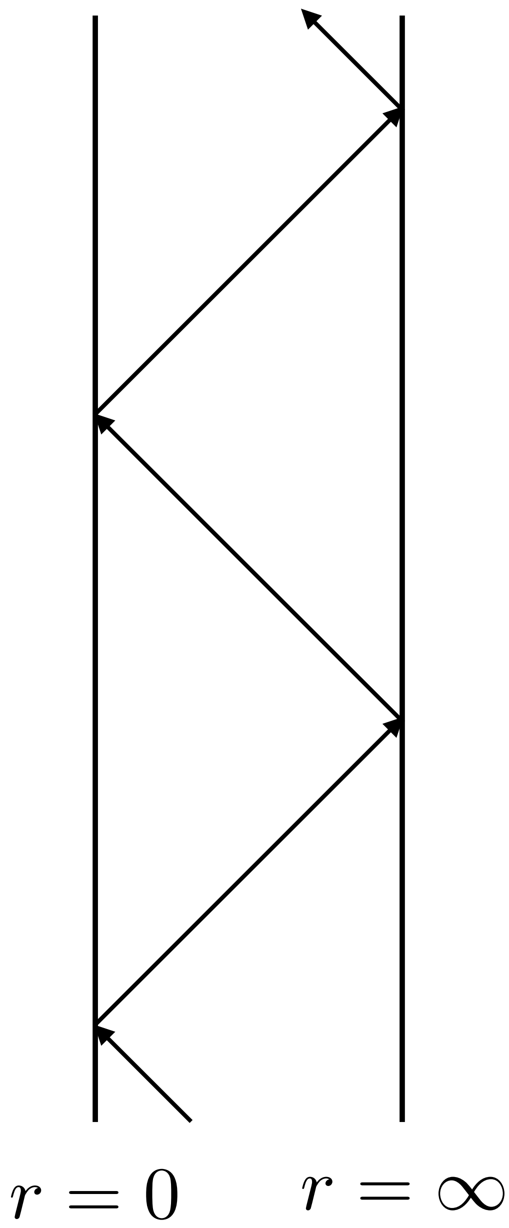
Phase space


generic
unstable
mixed
generic
stable
stable
unstable
unstable
stable
stable
unstable
unstable
stable
stable
unstable
unstable
stable

Phase space


generic
unstable
stable
unstable
unstable
stable
ruled out
mixed
stable
unstable
unstable
stable
generic
stable
stable
unstable
unstable
stable

Recursive solution



Goal: solve $\dfrac{d\tilde{\phi}}{d\tau} = F_3$ up to time τ

Method:

Solve one bounce:

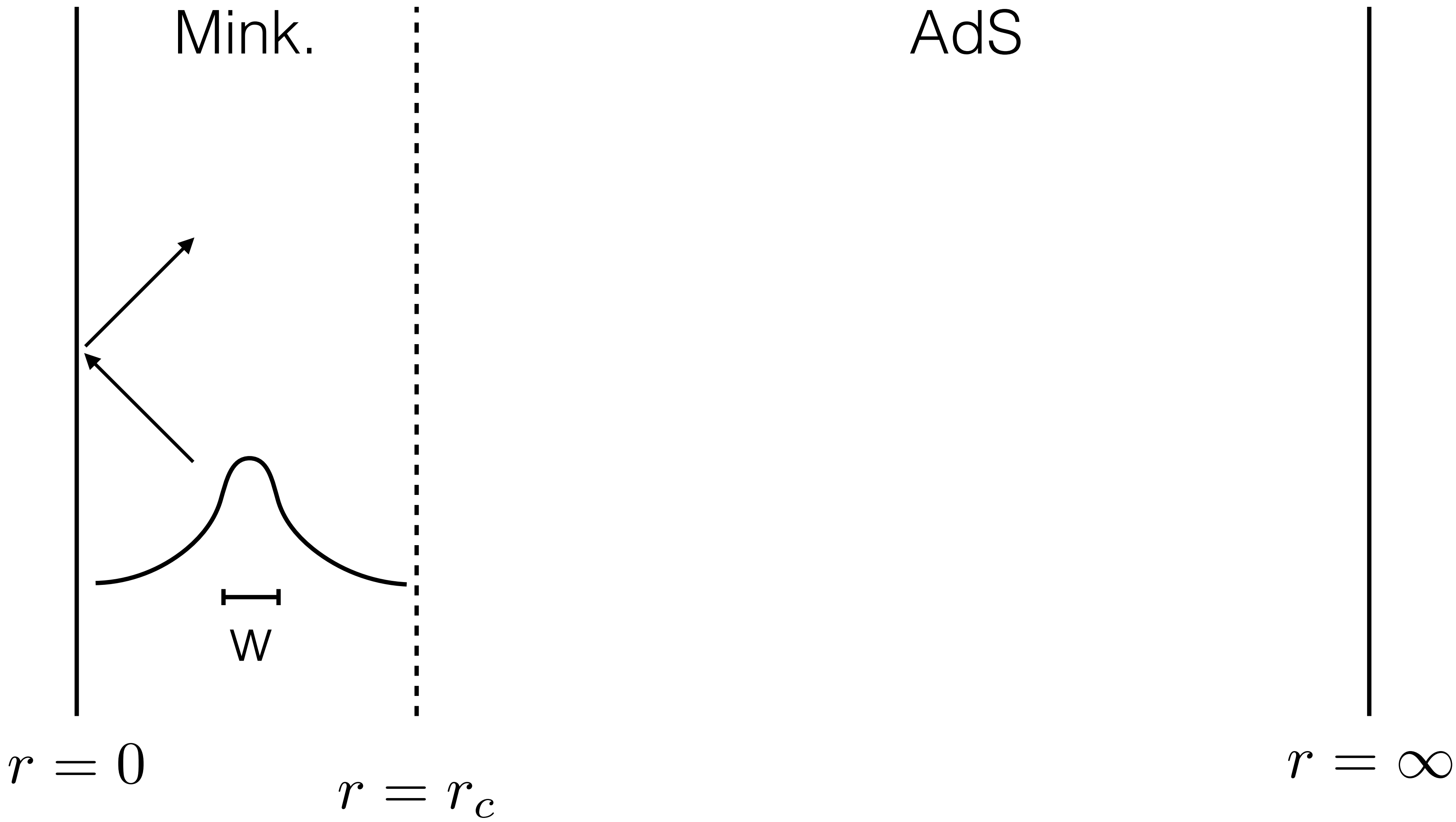
$$\tilde{\phi}_{N+1} = \tilde{\phi}_N + \epsilon^2 F_3$$

Harder Method:

Iterate for τ/ε^2 bounces

Bouncing thin shell

separation of scales $\epsilon^2 \ll w \ll r_c \ll L_{Ads}$



Bouncing thin shell

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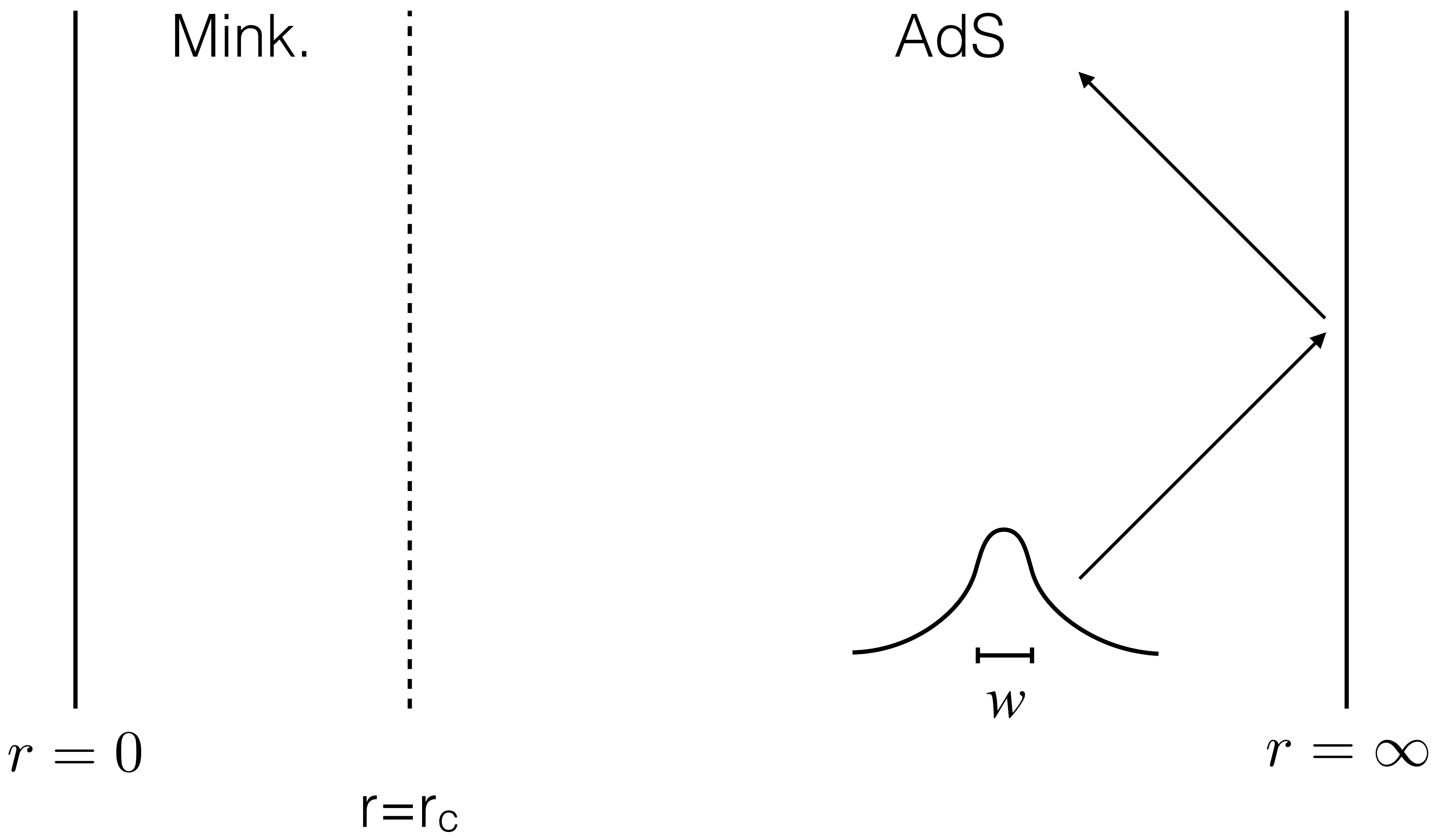
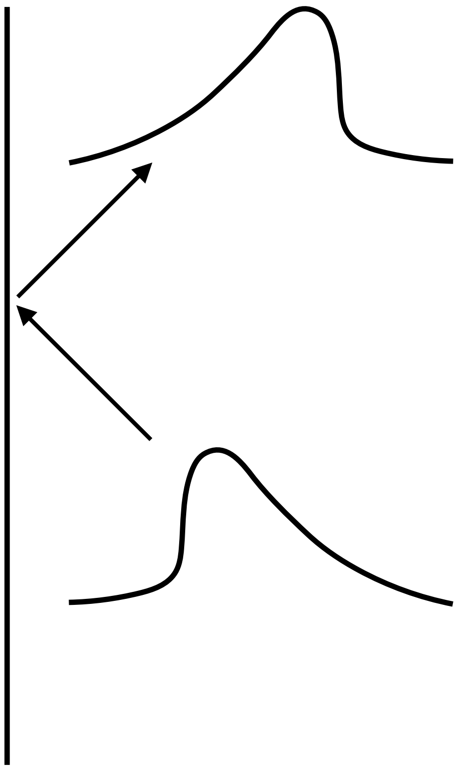


Asymmetric thin shells

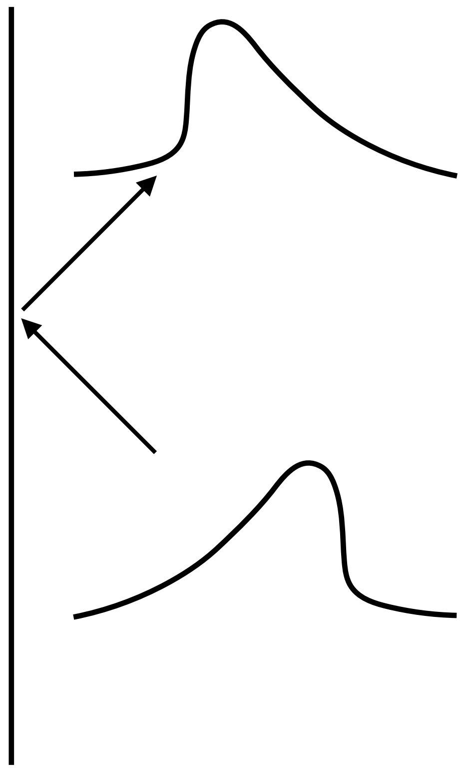
<u>front-heavy</u>



$r = 0$

$\Delta w < 0$ → focusing

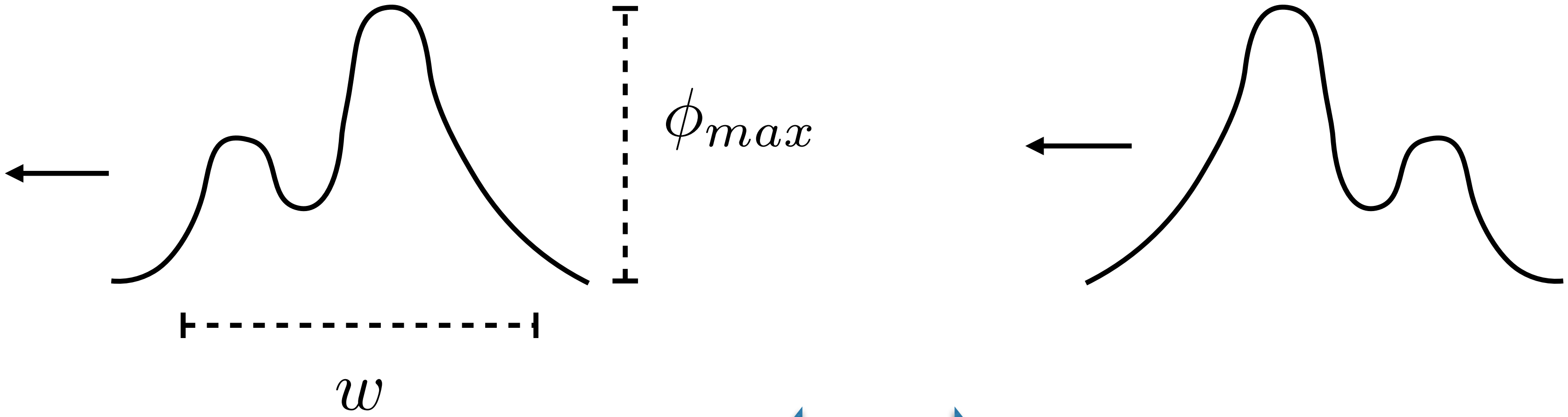
<u>back-heavy</u>



$r = 0$

$\Delta w > 0$ → defocusing

Profile reflection

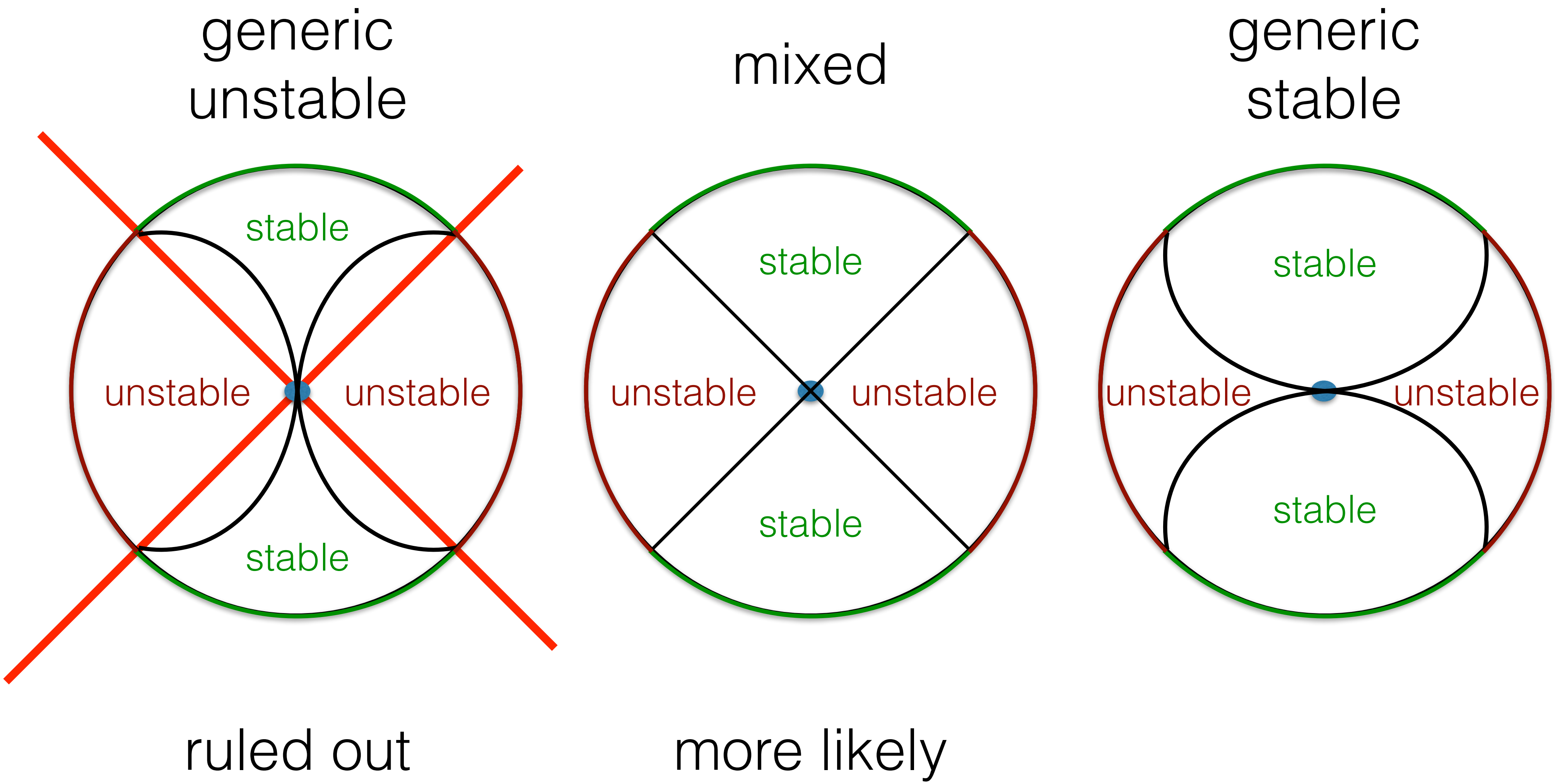


Δw ⟷ $-\Delta w$

$\Delta \phi_{max}$ ⟷ $-\Delta \phi_{max}$

$\Delta \rho_{max}$ ⟷ $-\Delta \rho_{max}$

Phase space



generic
unstable
stable
unstable
unstable
stable
ruled out
mixed
stable
unstable
unstable
stable
more likely
generic
stable
stable
unstable
unstable
stable

Summary

Generic perturbations do not lead to BH in $t \sim \epsilon^{-2}$

Position space perturbations

- spherical symmetric, thin shell
- self-gravity only near origin
- equally likely to focus or defocus energy

Open Questions:

- Breaking spherical symmetry
- Longer time behavior $t > \epsilon^{-2}$
- Other boundary topologies