Is the composite fermion a Dirac particle?

Dam T. Son

GGI conference “Gauge/gravity duality 2015”

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Plan

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- Fractional quantum Hall effect

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- Composite fermion orthodoxy

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- The old puzzle of particle-hole symmetry

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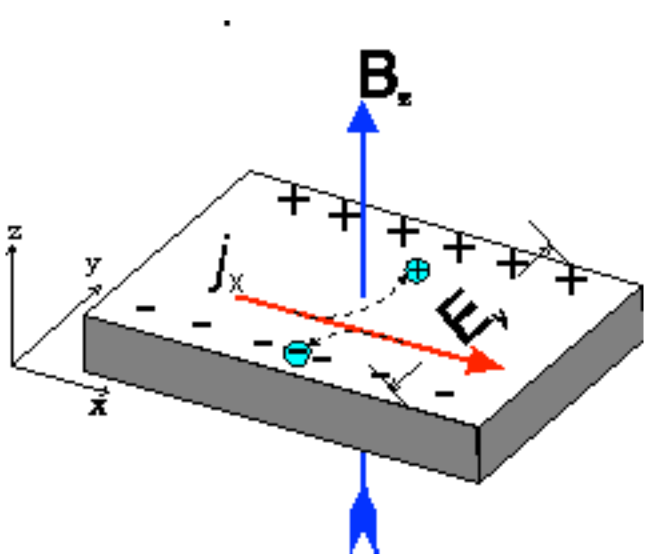
- Fractional quantum Hall effect
- Composite fermion orthodoxy
- The old puzzle of particle-hole symmetry
- The solution to the puzzle

Hall conductivity/resistivity

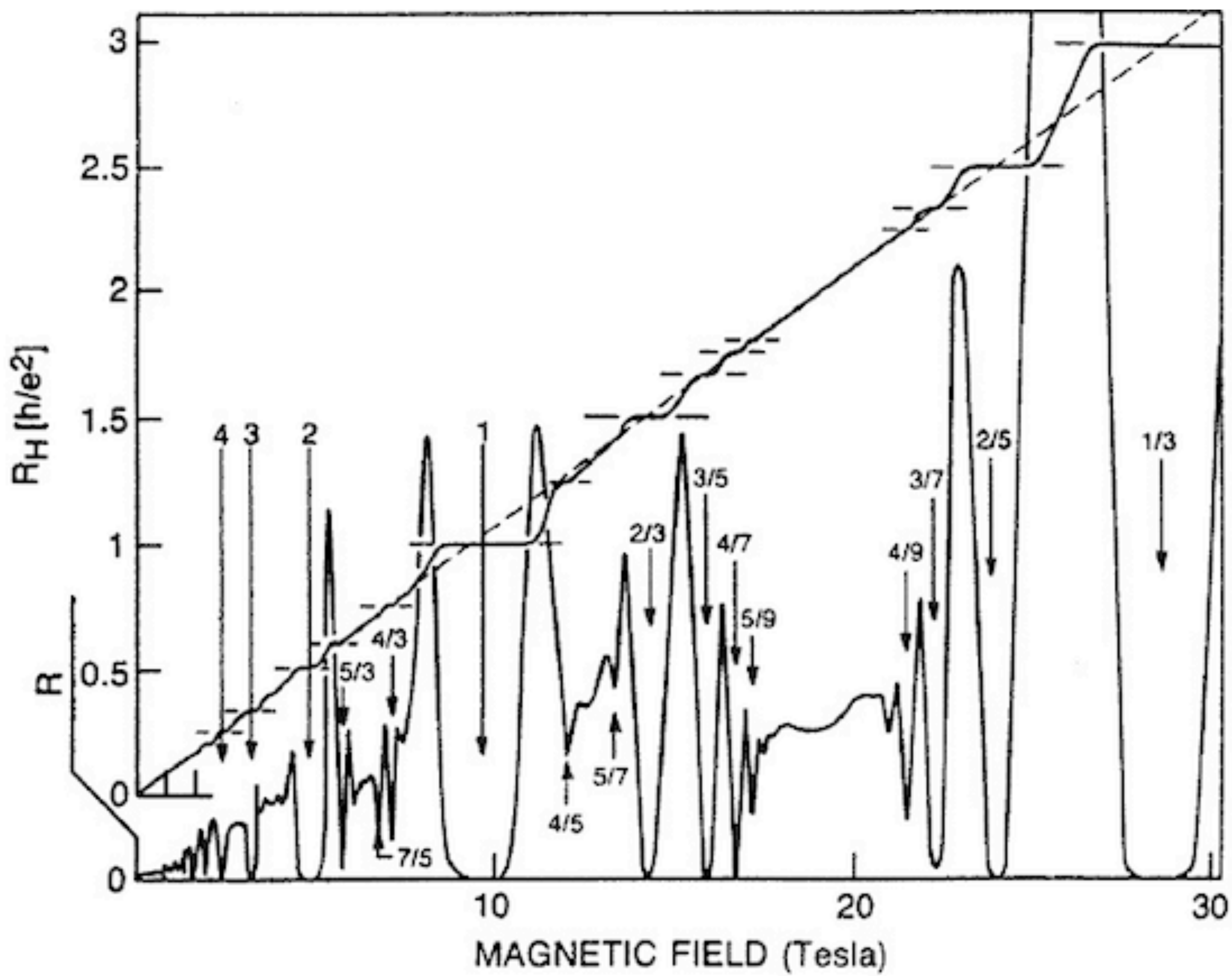


$$j_i = \sigma_{ij} E_j$$

$$E_i = \rho_{ij} j_j$$

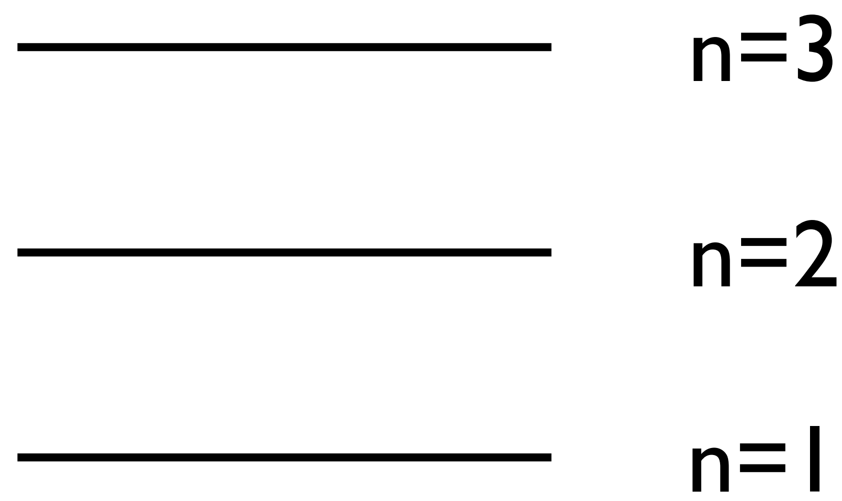
$$i, j = x, y$$

Fractional QH effect



Fractional quantum Hall effect

Landau levels of 2D electron in B field



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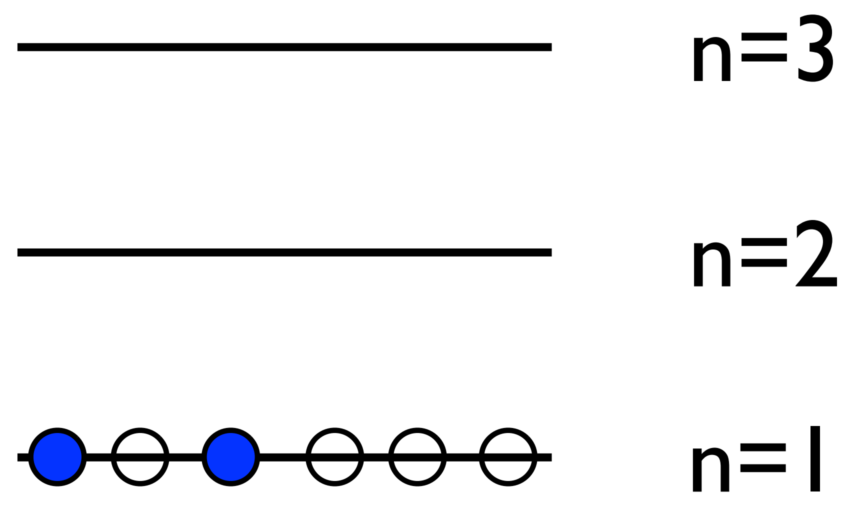

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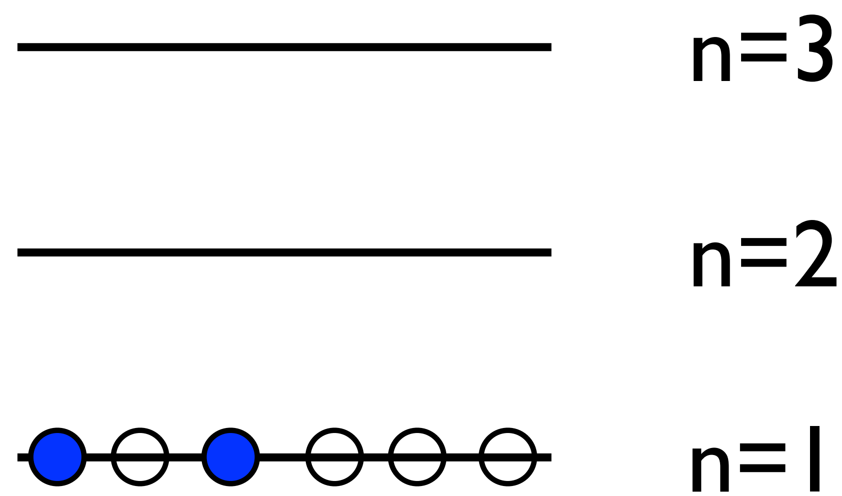
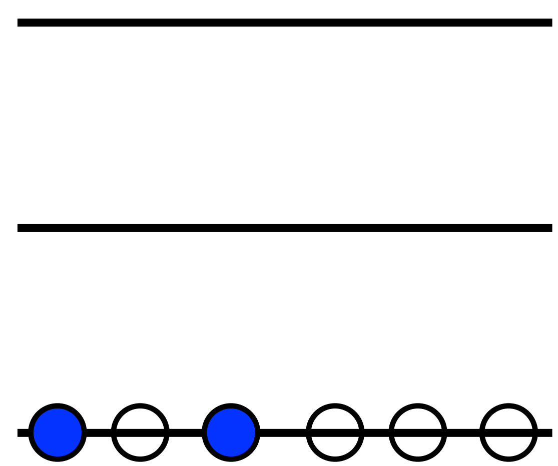


Filling factor $\nu = \dfrac{n}{B/2\pi}$

Fractional quantum Hall effect

Landau levels of 2D electron in B field



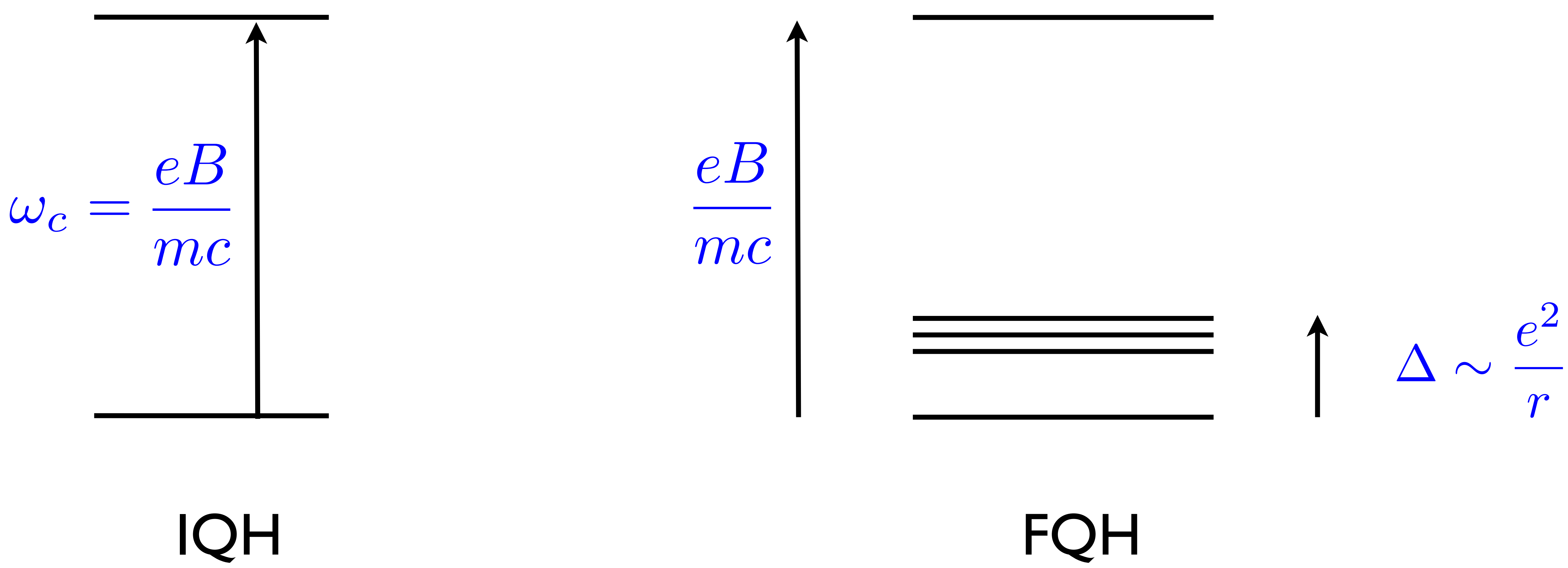
n=3

n=2

n=1 $\nu = \frac{1}{3}$

Filling factor $\nu = \frac{n}{B/2\pi}$

Energy scales



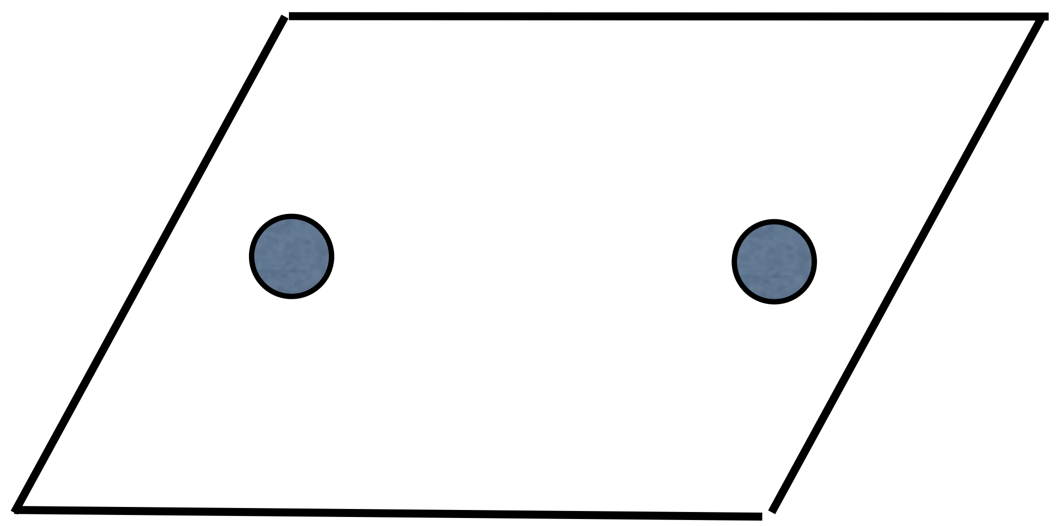
Interesting limit: eB/mc >> Δ (m→0)
only lowest Landau level (LLL) states survives

No small parameter

Flux attachment

(Wilczek 1982)

Attaching flux changes statistics



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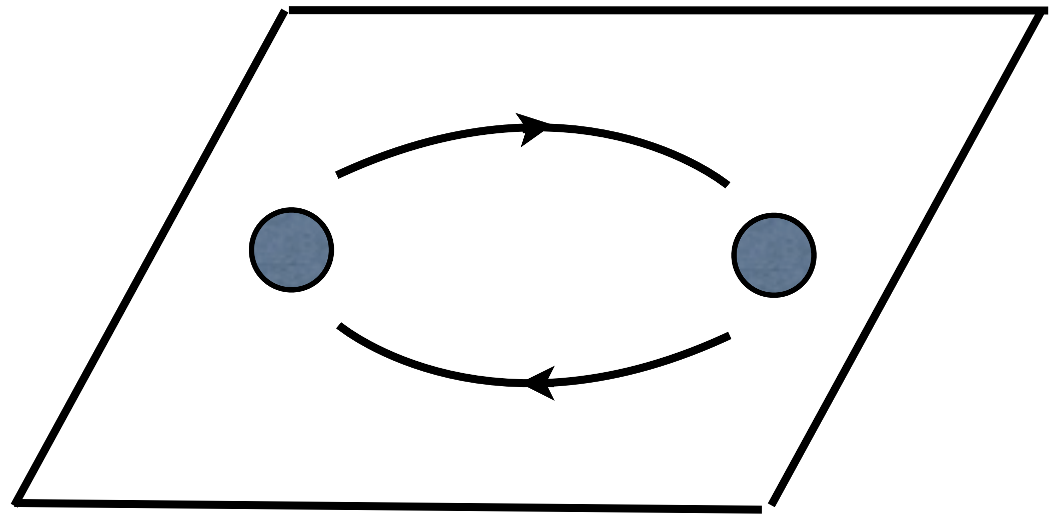

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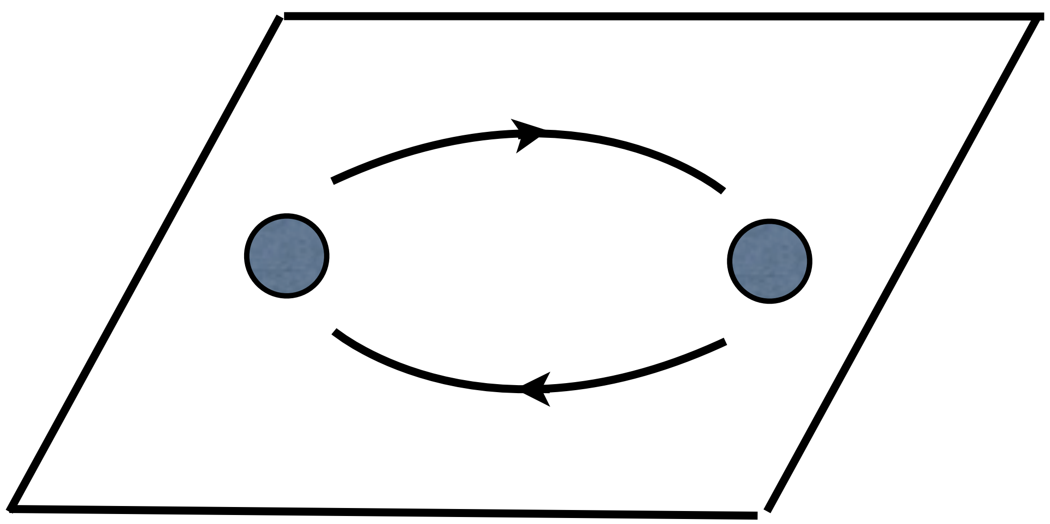
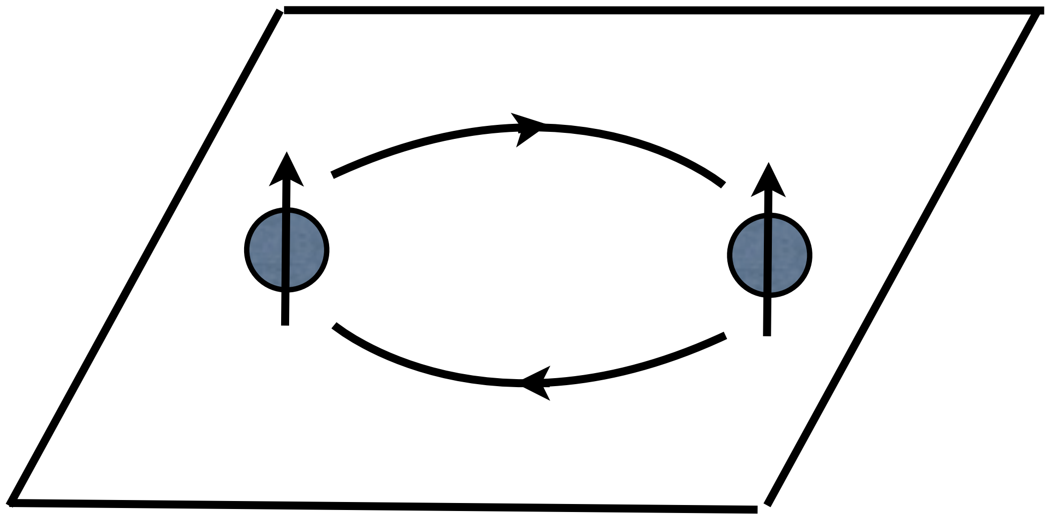


(-1)

Flux attachment

(Wilczek 1982)

Attaching flux changes statistics



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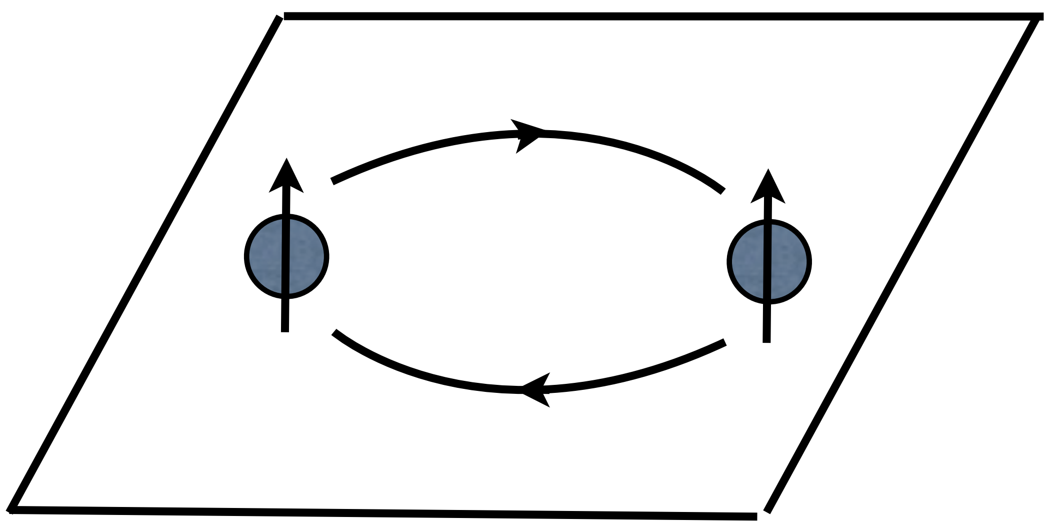

$$(-1)\,\exp(i\pi) = (+1)$$

Flux attachment

(Wilczek 1982)

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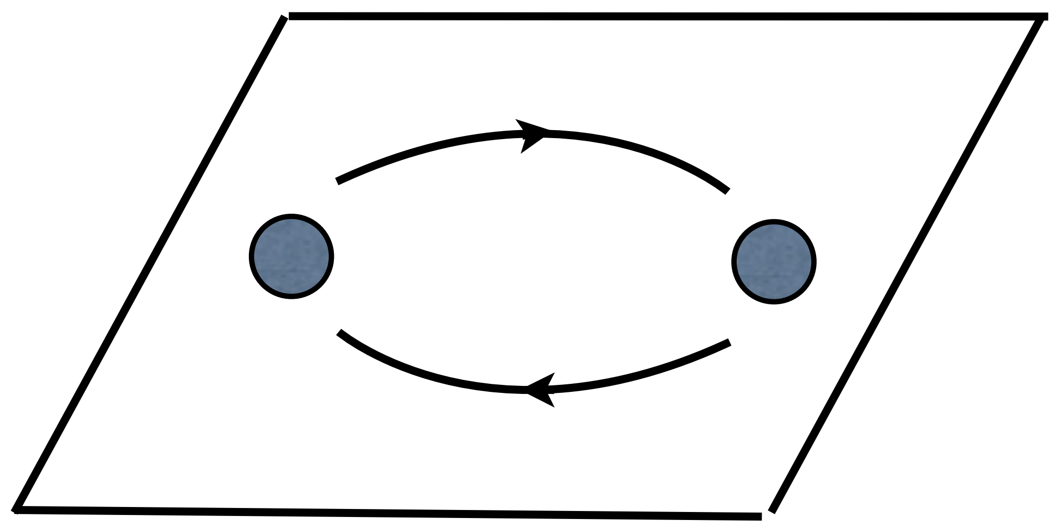

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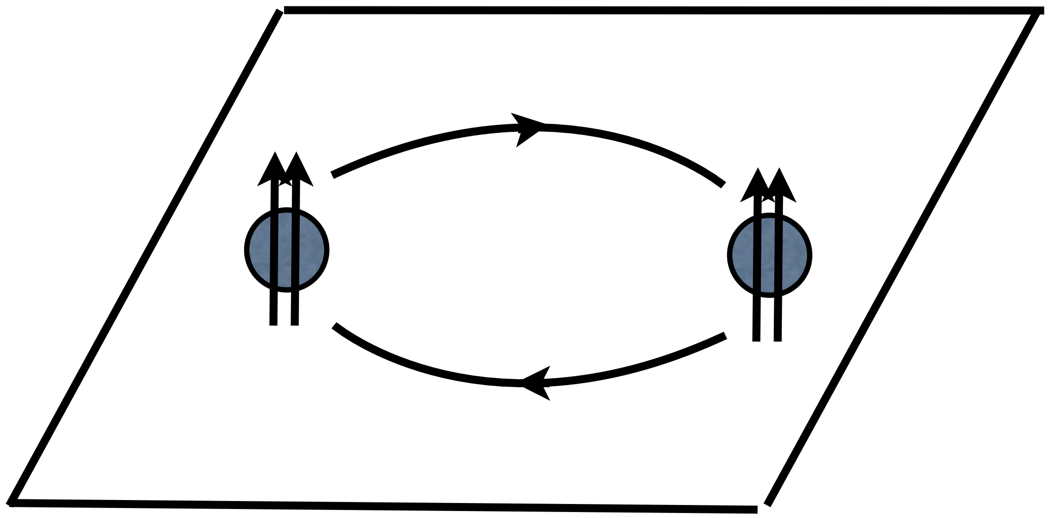

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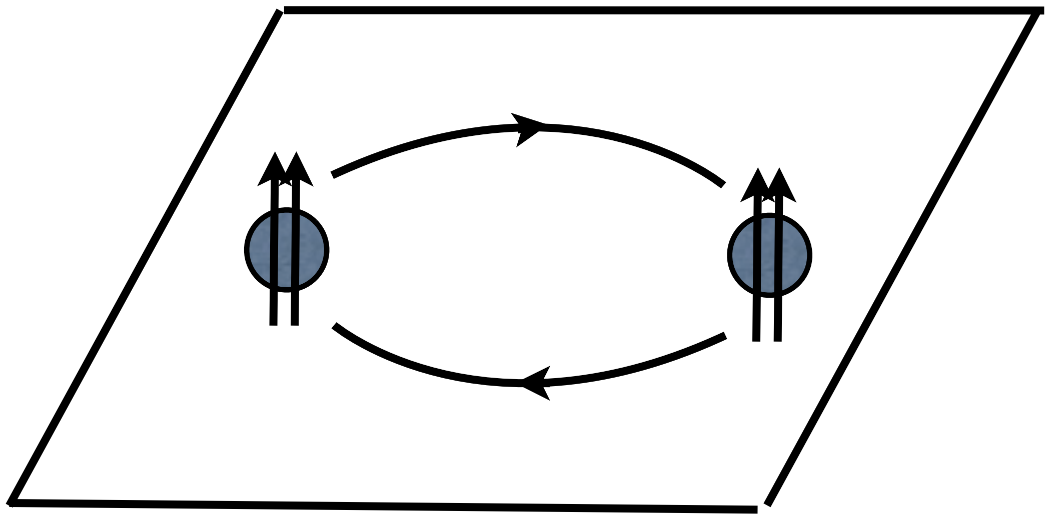

$$(-1)\exp(2i\pi) = (-1)$$

Flux attachment

(Wilczek 1982)

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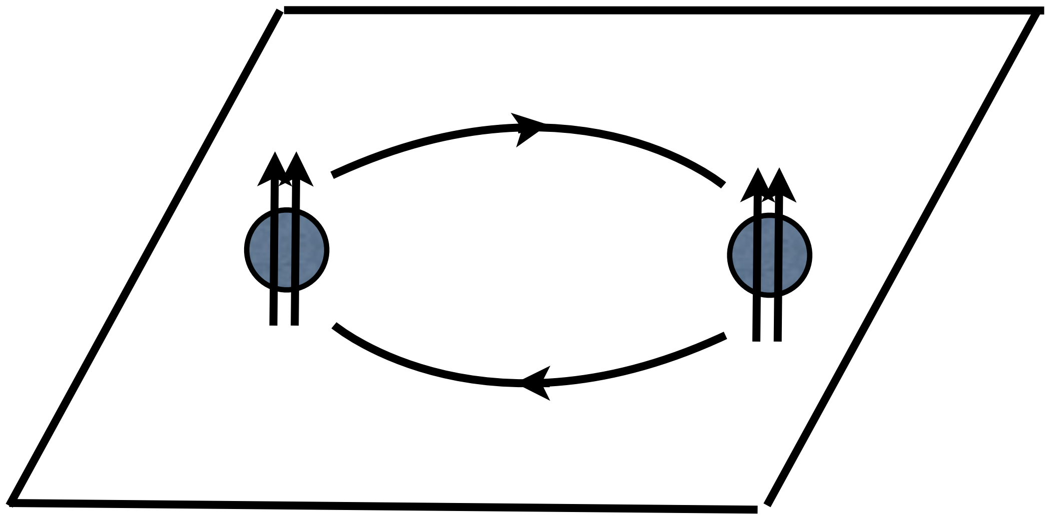

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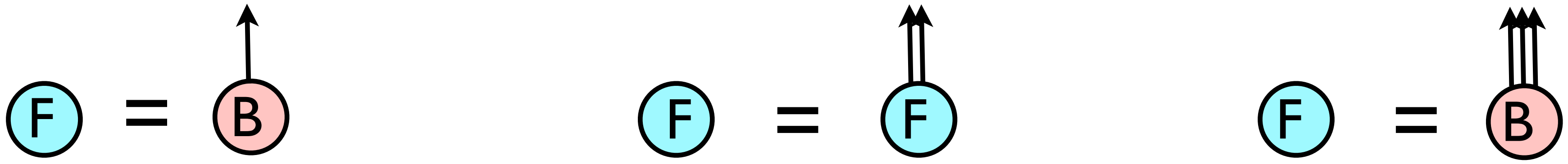
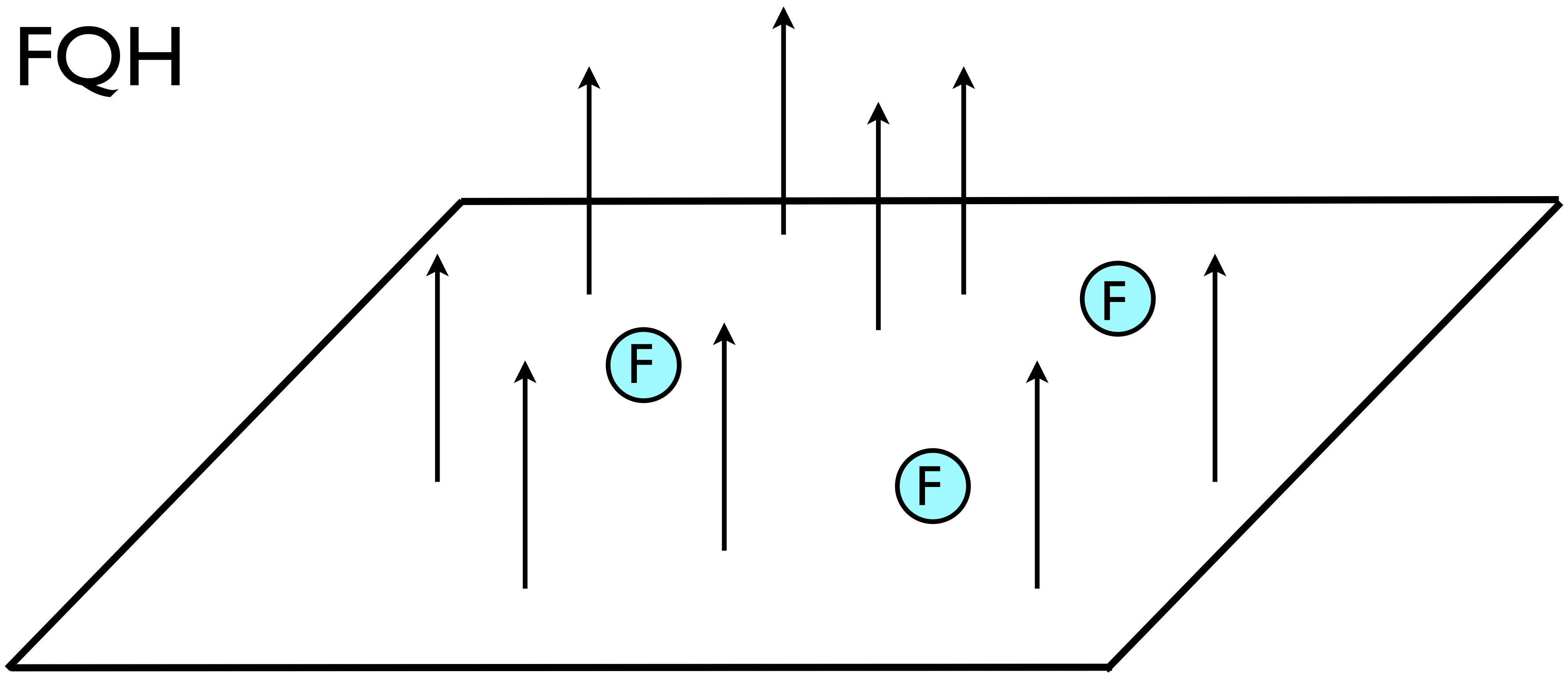
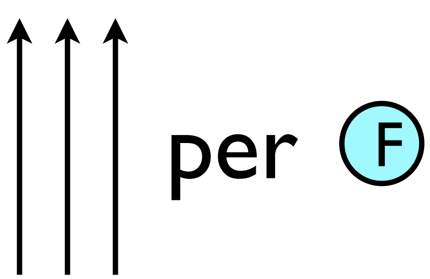


Composite fermion

$\nu = 1/3$ FQH





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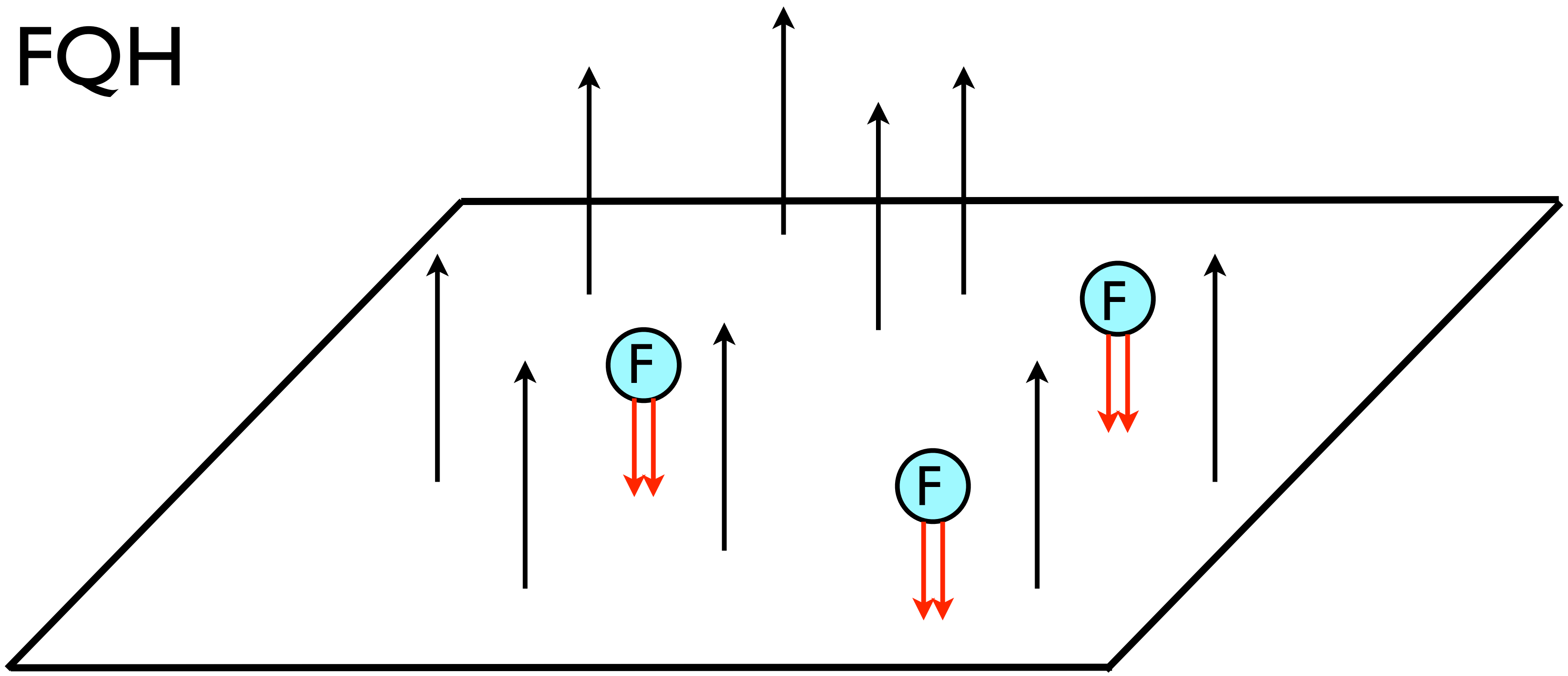

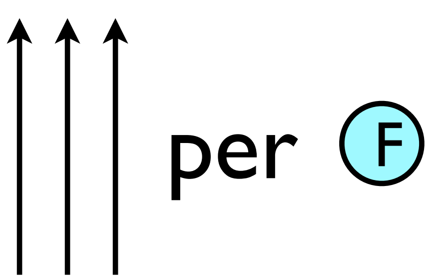

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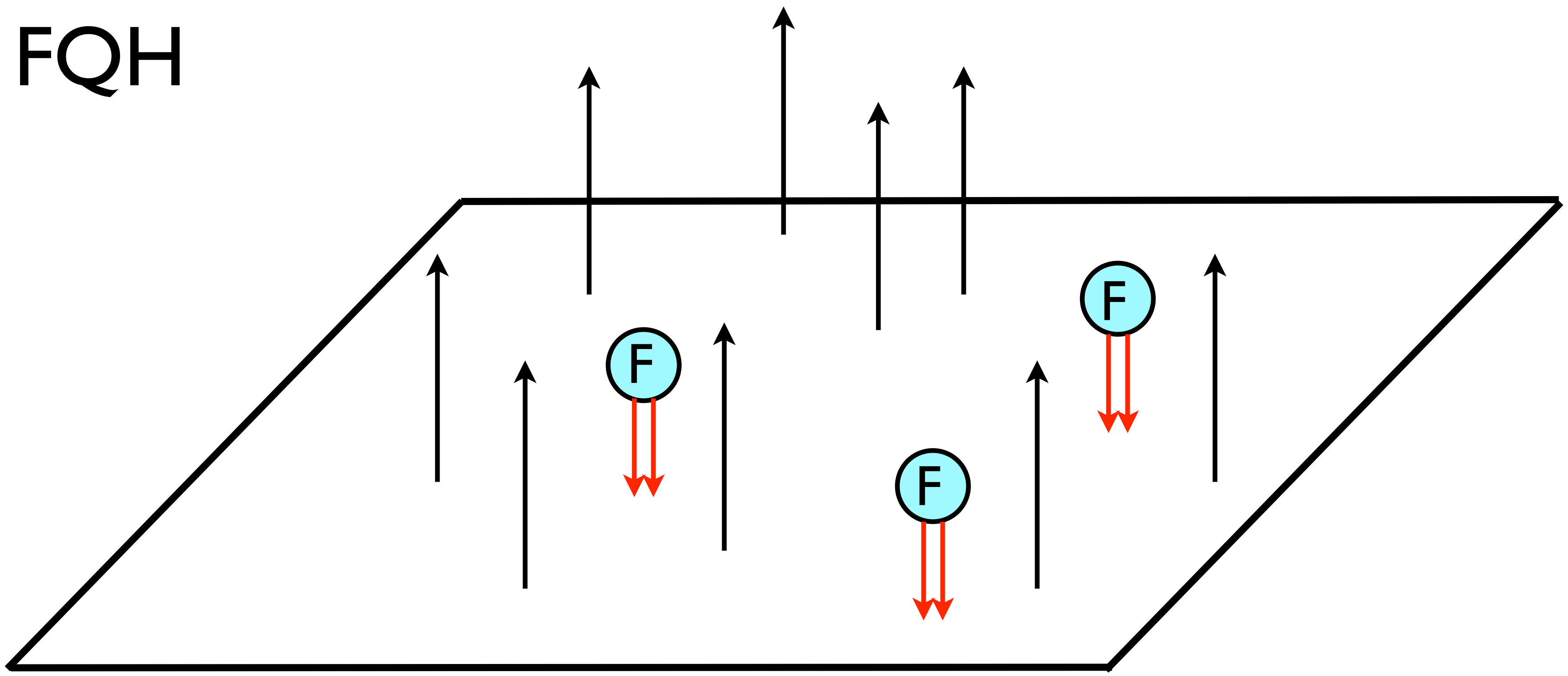

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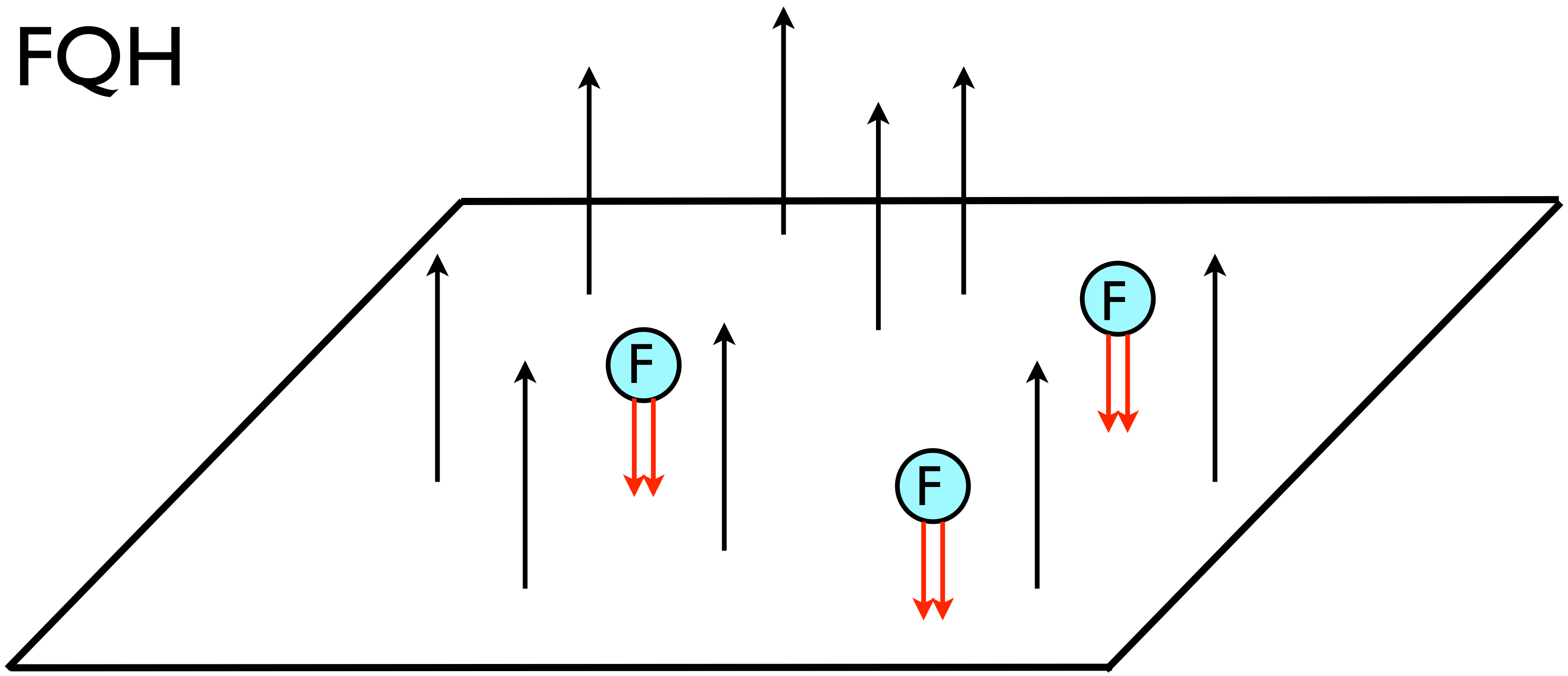
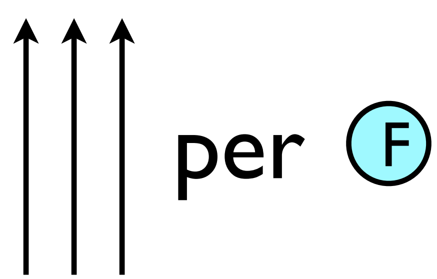
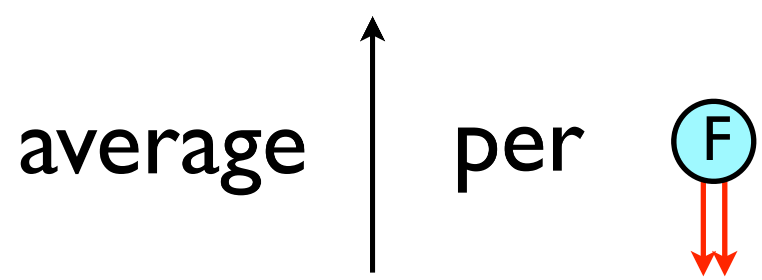


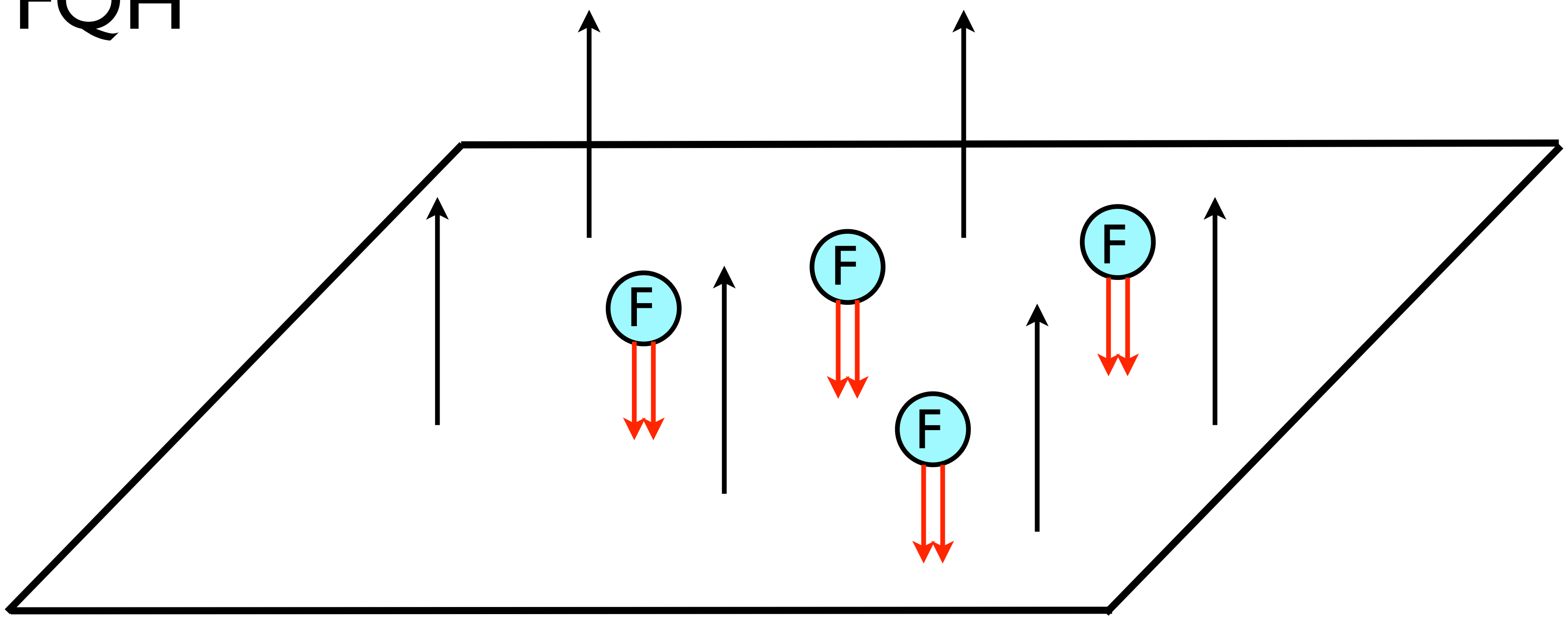




FQHE for original fermions = IQHE for composite fermions

Composite fermion

$\nu = 2/3$ FQH



FQHE for
original fermions

=

IQHE for
composite fermions (n=2)

Mathematically

Lopez, Fradkin
Halperin, Lee, Read

$$\mathcal{L} = i\psi^\dagger(\partial_0 - iA_0 + ia_0)\psi - \frac{1}{2m}|(\partial_i - iA_i + ia_i)\psi|^2 + \frac{1}{4\pi p}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + \cdots$$

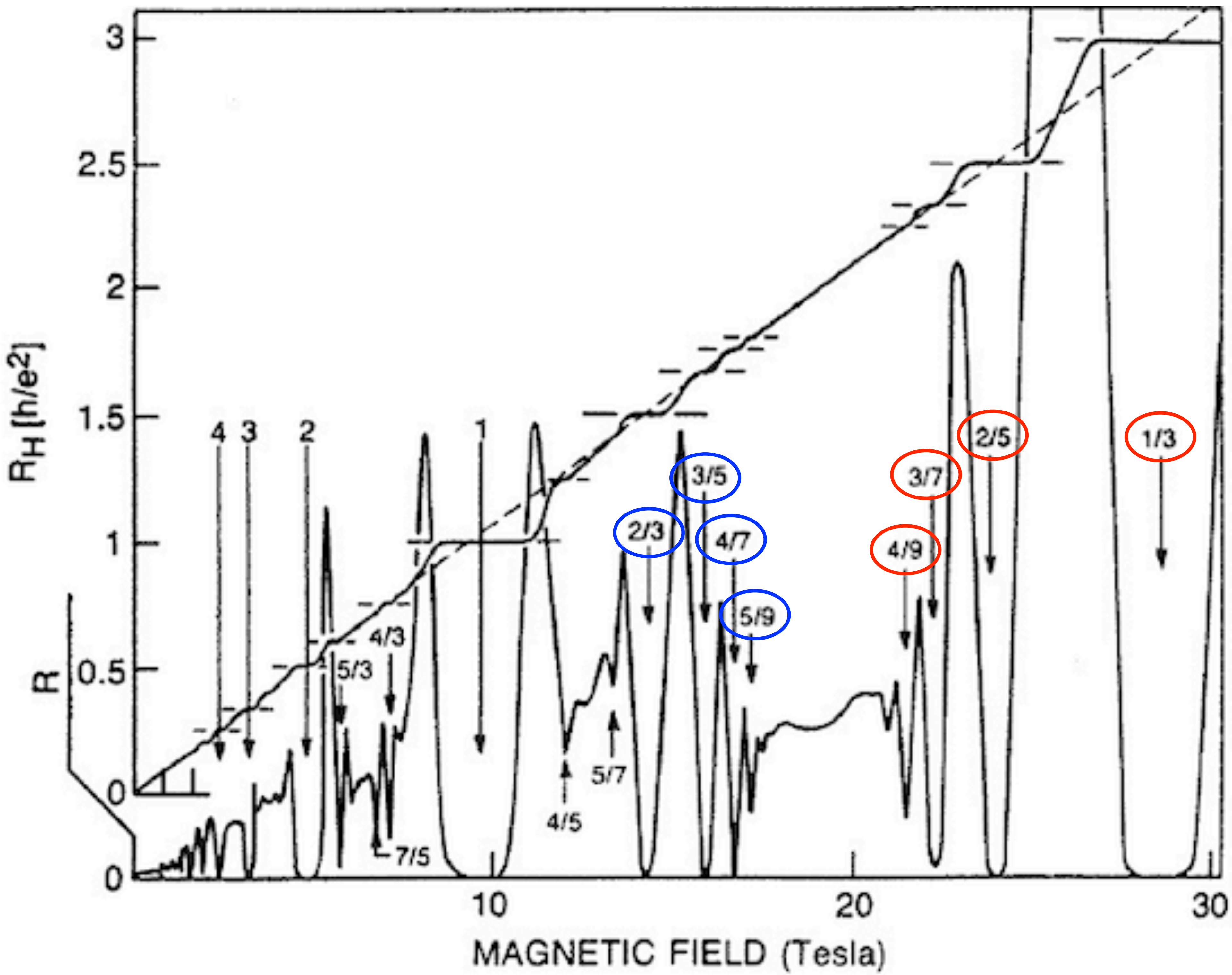
# of attached
flux quanta

$$\boldsymbol{\nabla} \times \mathbf{a} = 2\pi p\,\psi^\dagger\psi$$

Comments on flux attachment

- No small expansion parameter: $p \sim 1$
- Difficulty with energy scales, especially in the limit $m \to 0$
- Nevertheless, explains a number of facts
  - Jain sequences
  - Gapless $\nu = 1/2$ state

Jain's sequences

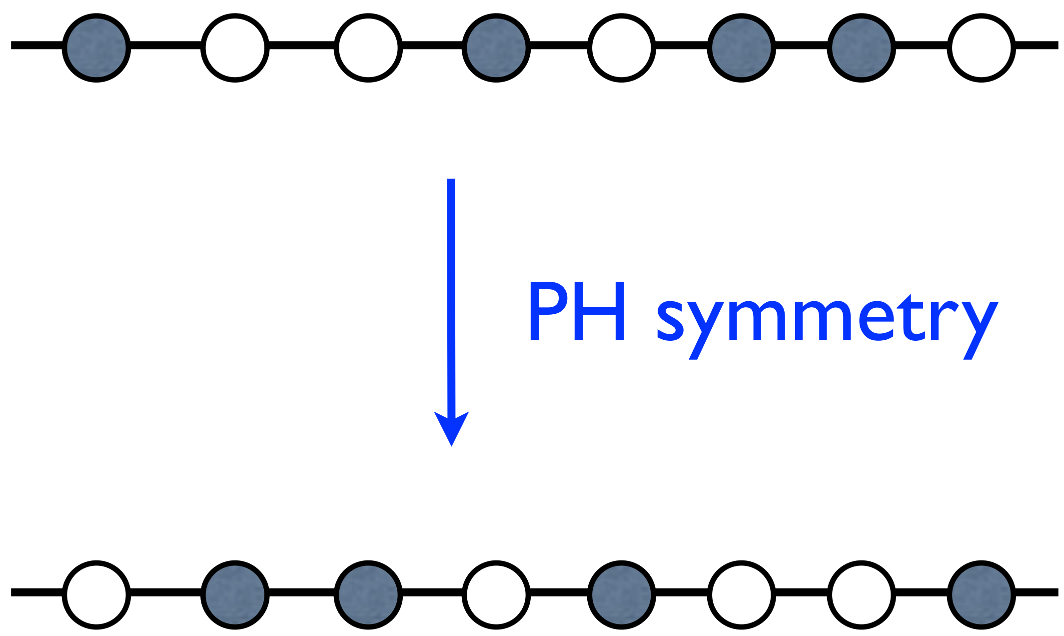


ν=1/2 state

- After flux attachment: average magnetic field=0
- Ground state: a gapless Fermi liquid
  - described by Halperin-Lee-Read (HLR) field theory
- Substantial experimental evidence

Particle-hole symmetry

Girvin 1984



$$\nu \to 1 - \nu$$

Formalized as an anti-unitary transformation

exact symmetry on the LLL, when mixing of higher LLs negligible

CF and particle-hole symmetry

- Comparing states on two Jain sequences

ν=1/3 ν=2/3

CF and particle-hole symmetry

- Comparing states on two Jain sequences

$\nu=2/5$ $\nu=3/5$

CF and particle-hole symmetry

- Comparing states on two Jain sequences

ν=3/7 ν=4/7

CF and particle-hole symmetry

- Comparing states on two Jain sequences

$\nu=3/7$ $\nu=4/7$

CF picture does not respect PH symmetry

PH symmetry of ν=1/2 Fermi liquid

How can the inside of a circle be equivalent to the outside?

PH symmetric CFs?

$$\nu = \frac{n}{2n+1} \longrightarrow \nu_{\mathrm{CF}} = n$$

$$\nu = \frac{n+1}{2n+1} \longrightarrow \nu_{\mathrm{CF}} = n+1$$

PH symmetric CFs?

$$\nu = \frac{n}{2n+1}$$

$$\nu_{\mathrm{CF}} = n + \frac{1}{2} \ ?$$

$$\nu = \frac{n+1}{2n+1}$$

PH symmetric CFs?

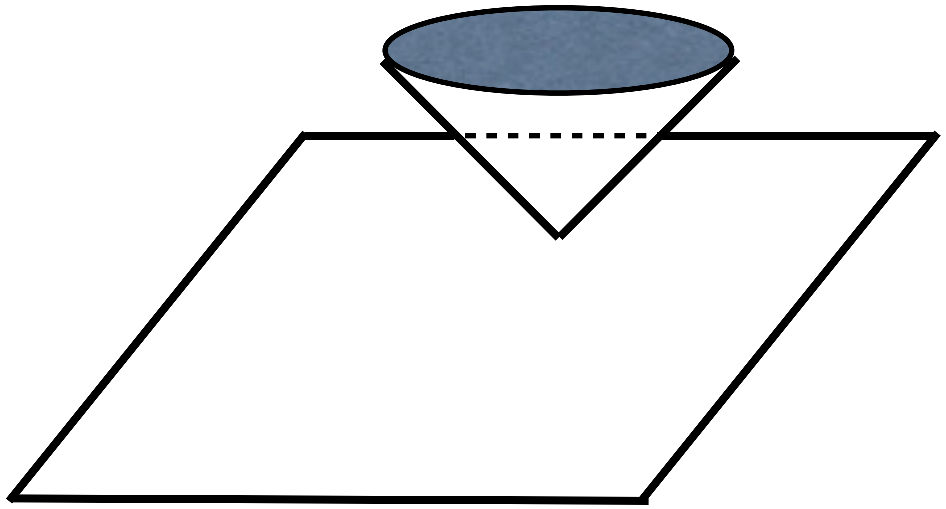
$$\nu = \frac{n}{2n+1}$$

$$\nu_{\mathrm{CF}} = n + \frac{1}{2} \; ?$$

$$\nu = \frac{n+1}{2n+1}$$

Can the filling factor of an IQH state be half-integer?

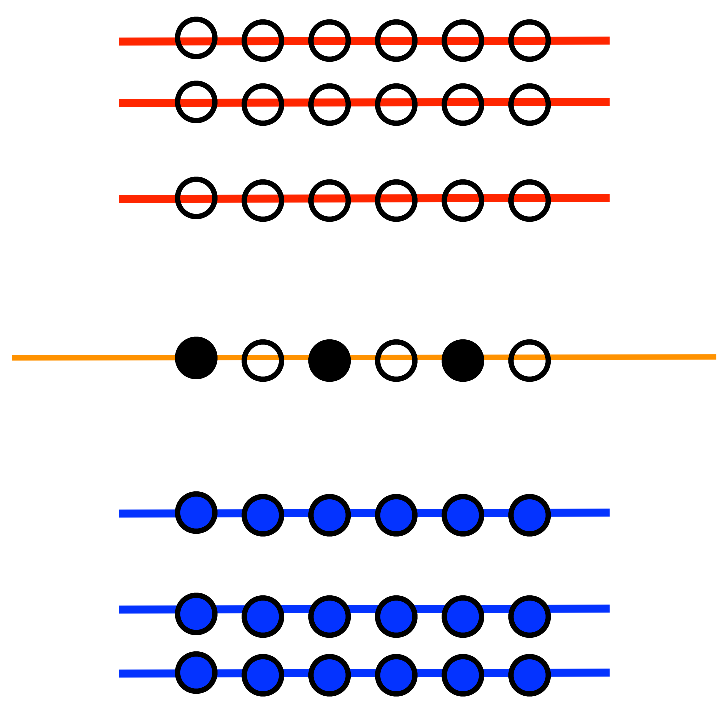
Relativistic model with FQHE



$$S = \int d^3x \, i\bar{\psi}\gamma^{\mu}(\partial_{\mu} - iA_{\mu})\psi - \frac{1}{4e^2}\int d^4x \, F_{\mu\nu}^2$$

(Graphene: 4 types of fermions, electrons and photons have different speeds)

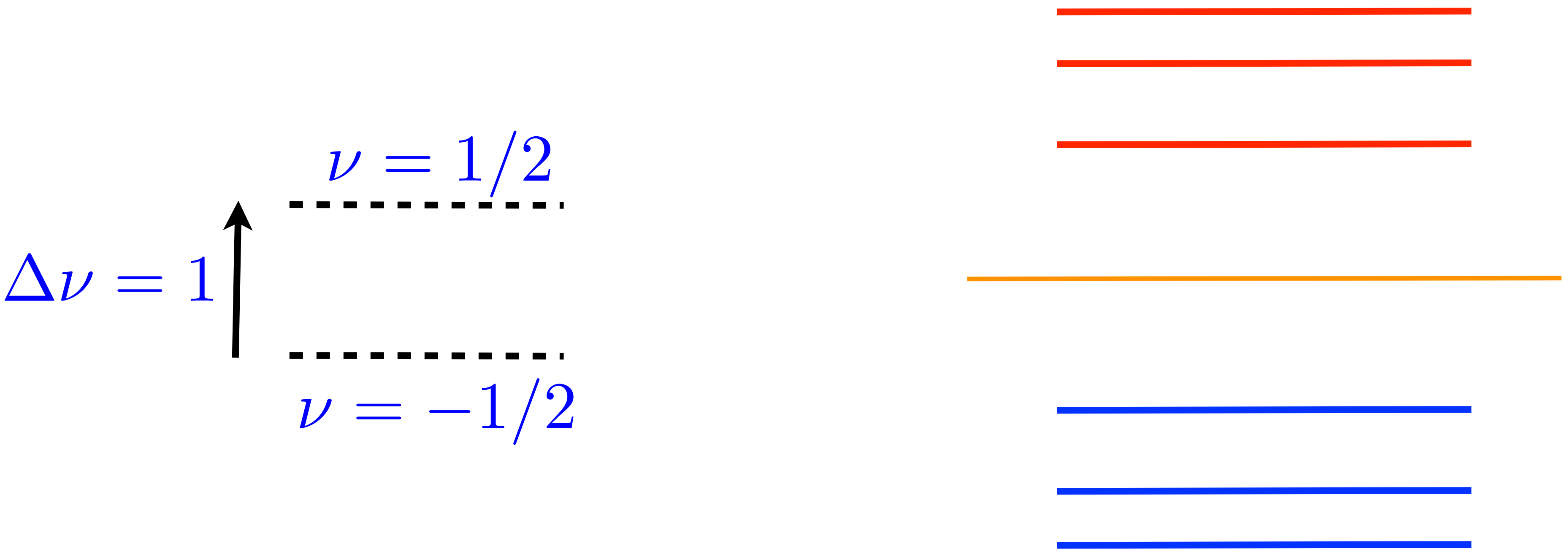
Ground state in finite magnetic field



Ground state is not determined without interaction
When $e^2 \ll 1$: exactly the same FQH problem

$\mu=0$: half-filled Landau level

The offset of 1/2



$$\nu = \nu_{\mathrm{NR}} - \frac{1}{2}$$

Offset of 1/2 in graphene

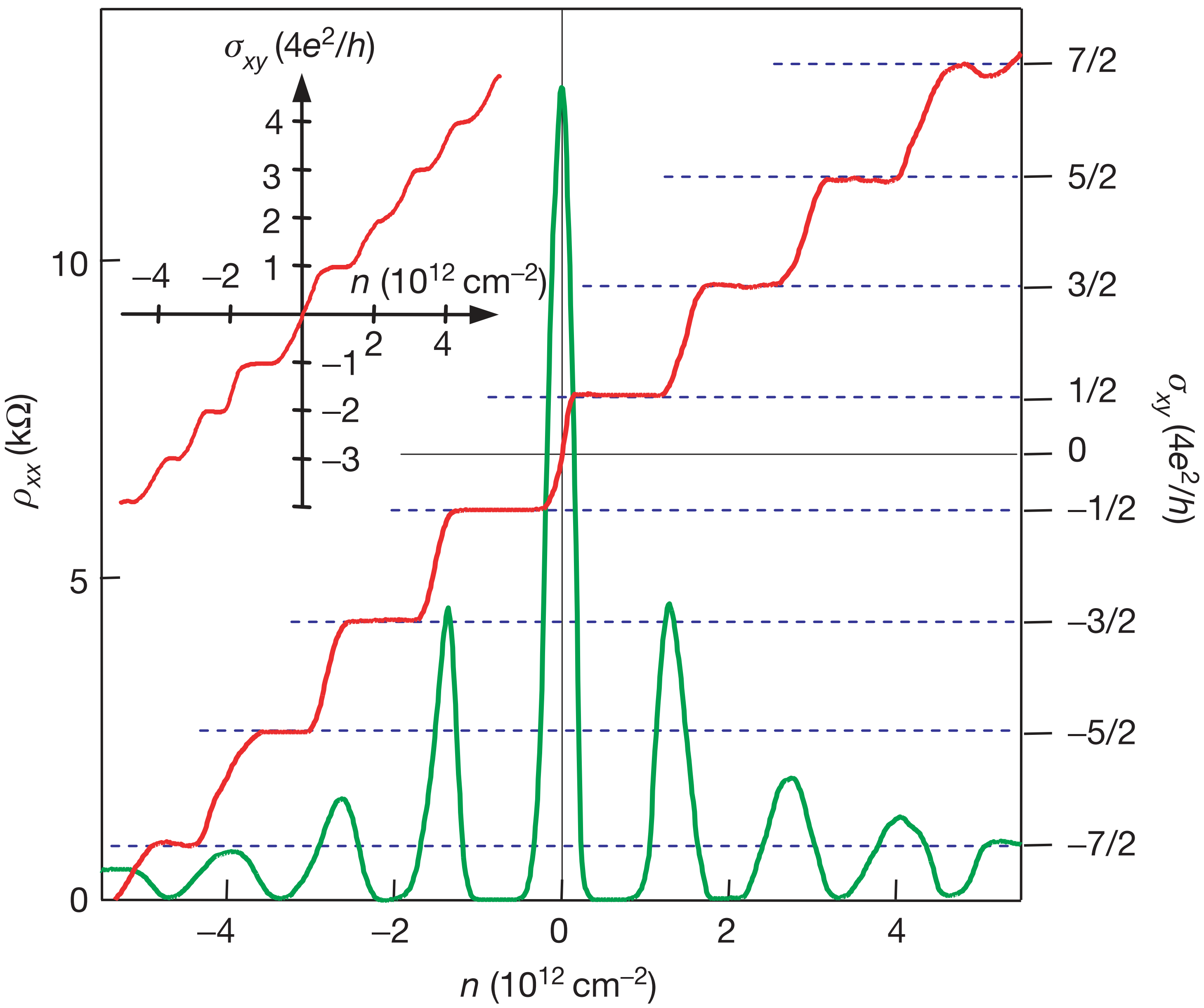


Figure 4 | QHE for massless Dirac fermions. Hall conductivity σ_{xy} and longitudinal resistivity ρ_{xx} of graphene as a function of their concentration at $B = 14$ T and $T = 4$ K. $\sigma_{xy} \equiv (4e^2/h)\nu$ is calculated from the measured

$$\sigma_{xy} = \left(n + \frac{1}{2}\right)\frac{e^2}{2\pi\hbar}$$

Discrete symmetries of (2+1)D Dirac fermion

	C	P $x \to x$, $y \to -y$	T	PT	CP
B	$-$	$-$	$-$	$+$	$+$
μ	$-$	$+$	$+$	$+$	$-$

CP and PT are symmetries at μ=0

PH symmetry = CT = (CP)(PT)
is a symmetry before projection to LLL

PH symmetry for Dirac fermion

Jain sequences

$$\nu_{\rm NR} = \frac{n}{2n+1} \longrightarrow \nu = -\frac{1}{2(2n+1)}$$

$$\nu_{\rm CF} = n + \frac{1}{2}$$

$$\nu_{\rm NR} = \frac{n+1}{2n+1} \longrightarrow \nu = \frac{1}{2(2n+1)}$$

$$2\nu = \frac{1}{2\nu_{\rm CF}} \ ?$$

Particle-vortex duality?

original fermion	composite fermion
magnetic field	density
density	magnetic field

Particle-vortex duality?

original fermion	composite fermion
magnetic field	density
density	magnetic field

This suggests the following effective action for CFs:

$$S = \int d^3x \left[i\bar{\psi}\gamma^\mu(\partial_\mu + 2ia_\mu)\psi + \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda + \cdots \right]$$

$$-\frac{1}{4e^2}\int d^4x\, F_{\mu\nu}^2$$

$$S = \int d^3x \left[i\bar{\psi}\gamma^\mu(\partial_\mu + 2ia_\mu)\psi + \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda + \cdots \right]$$

$$j^\mu = \frac{\delta S}{\delta A_\mu} = \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$$

$$\frac{\delta S}{\delta a_0} = 0 \longrightarrow \langle \psi\bar{\gamma}^0\psi \rangle = \frac{B}{4\pi}$$

Dirac composite fermions

- No Chern-Simons interaction *ada*
  - *ada* would break CP and CT
- conflict with flux attachment idea?
- composite fermions have Berry phase π around Fermi surface

A few related works

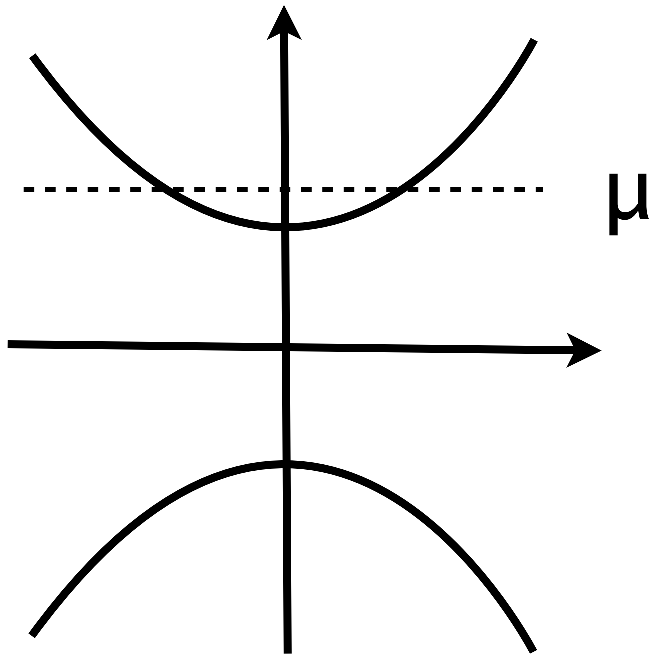
- “Composite Dirac liquid” on surface of topological insulators Mross, Essin, Alicea 2014
- Emergent Fermi surface from mirror symmetry
  Hook, Kachru, Torroba, Wang 2014

Consequences

- Exact particle hole symmetry in linear respose
  - at $\nu = \frac{1}{2}$, $\sigma_{xy} = \frac{1}{2}$ exactly
- New particle-hole symmetric gapped nonabelian state at ν=1/2:

$$\langle \epsilon^{\alpha\beta} \psi_\alpha \psi_\beta \rangle \neq 0$$

CS theory as the NR limit



ada

When CP is broken, CF has mass

In the NR limit: NR action for CF

Integrating out Dirac sea: Chern-Simons interaction between CF

Conclusion and open questions

- PH symmetry: a challenge for CF picture
- Proposal: Dirac CF with gauge, non-CS interaction
- a CP-invariant Pfaffian-like state
- Alternative: PH symmetry spontaneously broken
  Barkeshli Mulligan Fisher 2015
- Open questions:
  - derivation of the effective theory
  - experimental measurement of the Berry phase