Equilibration rates in a strongly coupled nonconformal quark-gluon plasma

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Based on arXiv:

1503.07114 with M.Heller and R.Myers; also on earlier work on quantum quenches with L.Lehner and R.Myers

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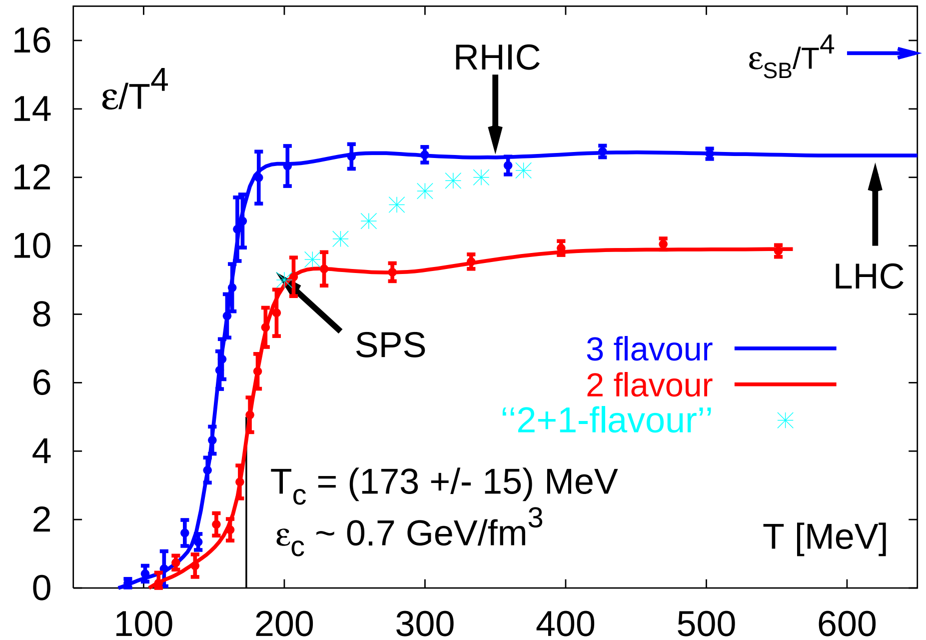
$\Longrightarrow$ So, we like to refer to AdS/CFT correspondence, and use large-N SYM as a proxy for a real QGP

The motivations:

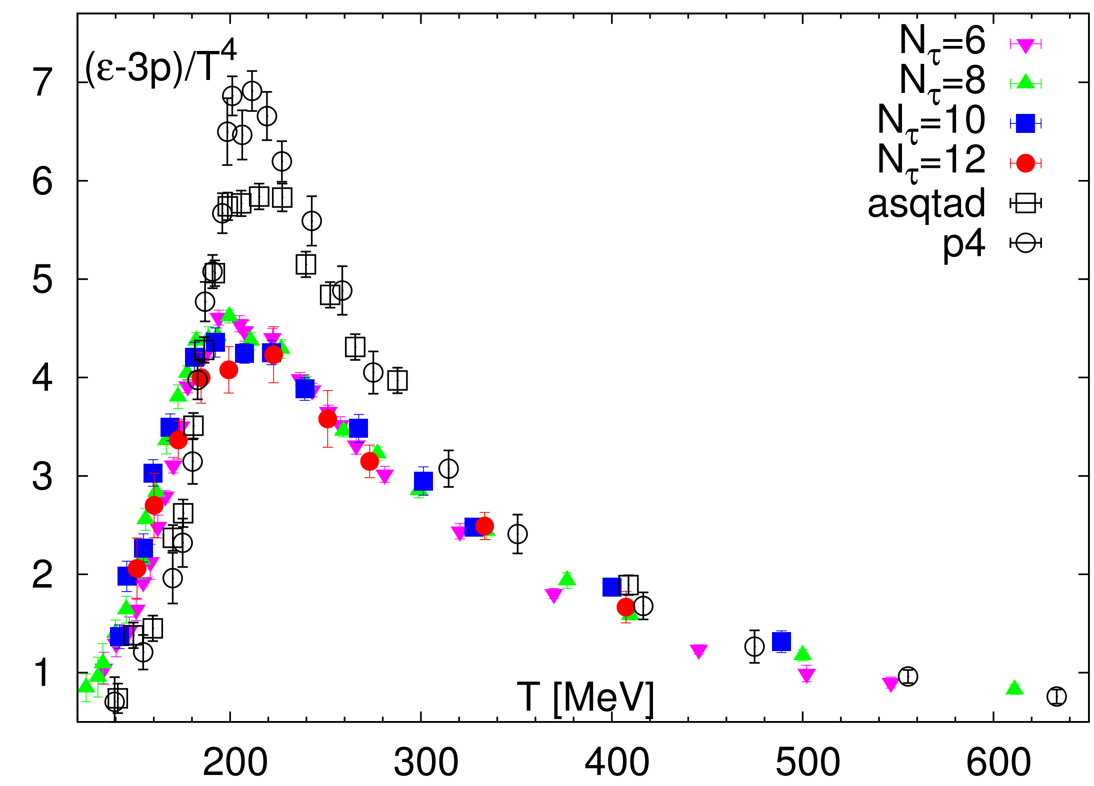
- QCD thermodynamics from lattice; (Karsch, Laermann, hep-lat/0305025). The plateau is $\sim 80\%$ of the SB result — close to 3/4 in SYM thermodynamics



- The small shear viscosity ratio (Policastro,Son,Starinets, hep-th/0104066)

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

$\Longrightarrow$ From A.Bazarov et.al (HotQCD Collaboration), arXiv:1407.6387:



$\Longrightarrow$ The violation of the conformality,

$$\frac{\epsilon - 3p}{\epsilon} \sim 50\%$$

at the maximum

So, should be use nonconformal modes of gauge/gravity correspondence to model equilibration of QGP?

$\Longrightarrow$ I am going to use top-down holographic model to address this question

Outline of the talk:

- A toy model for holographic equilibration
  - where the relaxation time(s) is(are) encoded?
- $\mathcal{N} = 2^*$ gauge theory/supergravity holography
  - Gauge theory perspective
  - Holographic Pilch-Warner RG flow
  - Matrix model and localization results
- Spectra of quasinormal modes
  - Relaxation rates for homogeneous/isotropic perturbations: $\{\mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, T_{\mu\nu}, \cdots\}$
  - Relaxation rates for generic transverse/traceless perturbations of $T_{\mu\nu}$
- Conclusion — beyond $\mathcal{N} = 2^*$ holography
  - relaxation with chemical potential in $\mathcal{N} = 4$ SYM
  - relaxation in bottom-up holographic models

- Consider $\mathcal{N}=4$ large-N $SU(N)$ SYM theory at strong coupling in thermal state:

$$\epsilon=\frac{3}{8}\pi^2N^2T^4\,,\qquad p=\frac{1}{8}\pi^2N^2T^4\,,\qquad s=\frac{1}{2}\pi^2N^2T^3$$

- The holographic dual to this state is a Schwarzschild black hole in Poincare-slice AdS_5. It has the thermodynamic properties (Hawking temperature, Bekenstein-Hawking entropy,...) as above

- SYM has gauge invariant fermion bi-linear operators of dimension $\Delta=3$: $\mathcal{O}_3$, and gauge invariant scalar bi-linears of dimension $\Delta=2$: $\mathcal{O}_2$. In thermal equilibrium,

$$\langle\mathcal{O}_3\rangle_{T\neq0}=0\,,\qquad \langle\mathcal{O}_2\rangle_{T\neq0}=0$$

- We can 'prepare' a non-equilibrium states of the $\mathcal{N}=4$ plasma (thus inducing a non-trivial time-dependence of $\mathcal{O}_\Delta$) by quenching the coupling constants of these relevant operators:

$$H_{SYM}\ \to H_{SYM}+\lambda_\Delta\mathcal{O}_\Delta$$

$$\lambda_\Delta=\lambda_\Delta(t)\,,\qquad \lambda_\Delta(-\infty)=0$$

- Specifically, we assume

$$\lambda_\Delta(t) = \lambda_\Delta^0 \left(\frac{1}{2} + \frac{1}{2}\tanh\frac{t}{\mathcal{T}}\right), \qquad \mathcal{T} = \frac{\alpha}{T_i},$$

where:

- ■ T_i is the temperature of the thermal state at $t \to -\infty$,
- ■ λ_Δ^0 is the amplitude of the quench, taken to be small compare to the initial temperature,

$$\frac{|\lambda_\Delta|}{T_i^{4-\Delta}} \ll 1$$

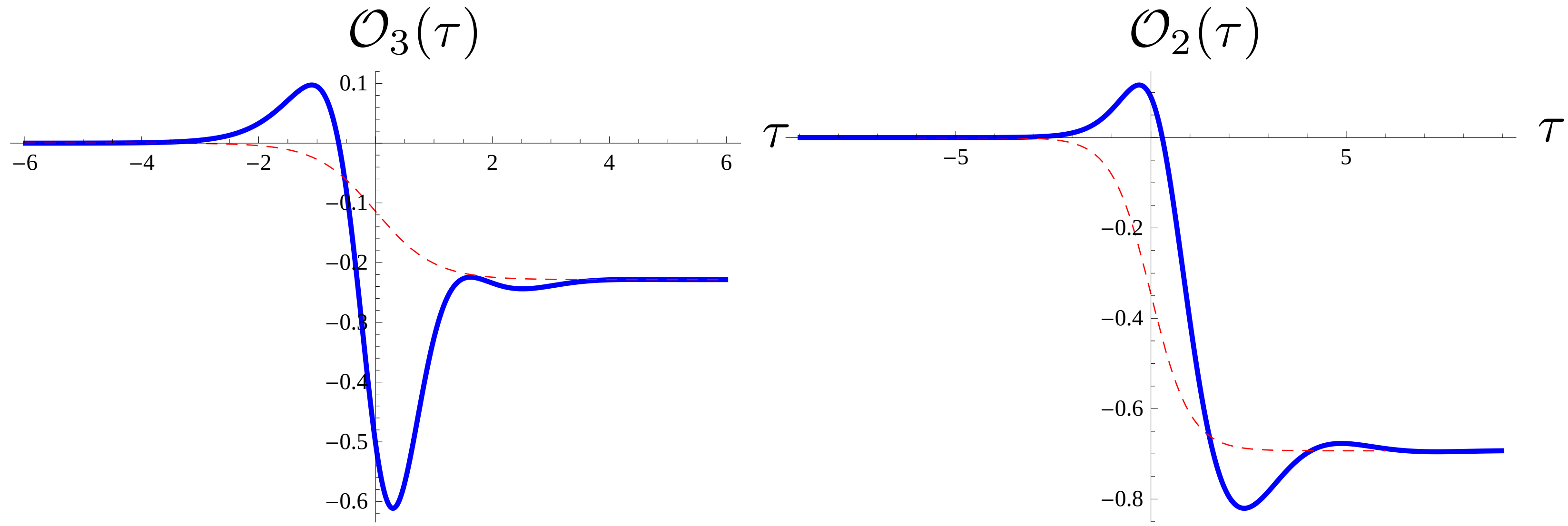
- ■ α is the rate of quench, measure in units of inverse temperature.
- ■ Note that α can be arbitrarily small/large, corresponding to abrupt/adiabatic quenches

- $\alpha \to 0$ limit (infinitely sharp – step-function — quench) can be thought as preparing a system in an excited state at $t = 0$ and allowing it to relax

$\Longrightarrow$ I am not going to explain how to set up about quench holographically, and rather move to to discuss the results

$\Longrightarrow$ Our primary observable is the expectation value of the quenching operator:

$$\mathcal{O}_\Delta = \mathcal{O}_\Delta(t)$$

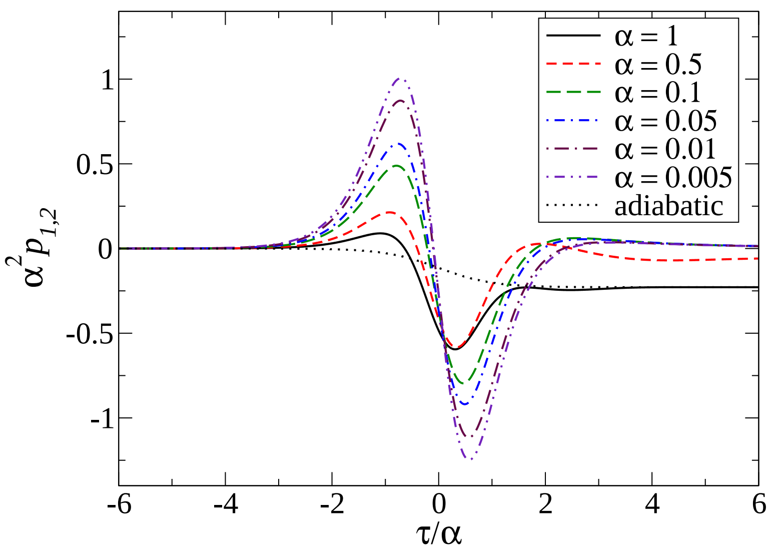


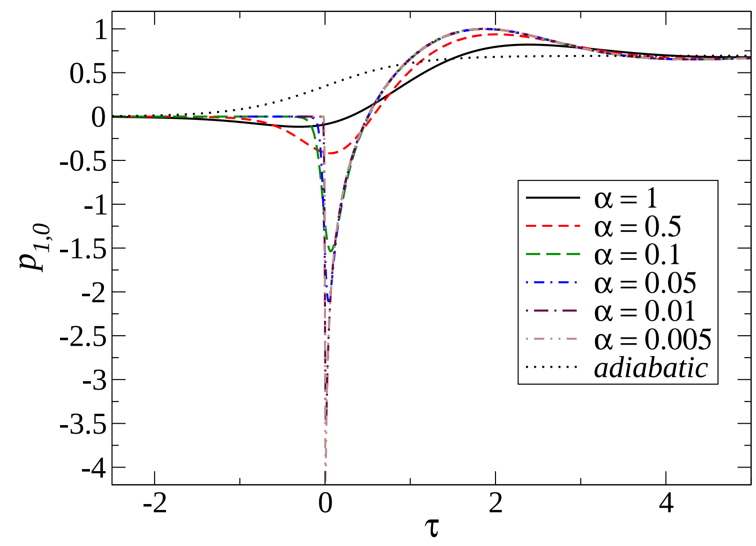
- Evolution of the normalizable component $\mathcal{O}_3$ (left panel) and $\mathcal{O}_2$ (right panel) during the quenches with $\alpha = 1$. The dashed red lines represent the adiabatic response.

- As $\tau \to +\infty$ the expectation values approach their equilibrium values in a damped-oscillatory manner (More on this later).

The response of $\mathcal{O}_\Delta$ depends on Δ:

- for fast quenches, α is small,





$\Uparrow$ $\Uparrow$

$\alpha^2\mathcal{O}_3(t)$ $\mathcal{O}_2(t)$

$\Longrightarrow$ The response is quite different!

$\Longrightarrow$ How do we characterize equilibration time?

- Introduce

$$\delta_{neq}(\tau) \equiv \left| \frac{\mathcal{O}_\Delta(\tau) - [\mathcal{O}_\Delta(\tau)]_{adiabatic}}{[\mathcal{O}_\Delta(\tau)]_{equilibrium}} \right| ,$$

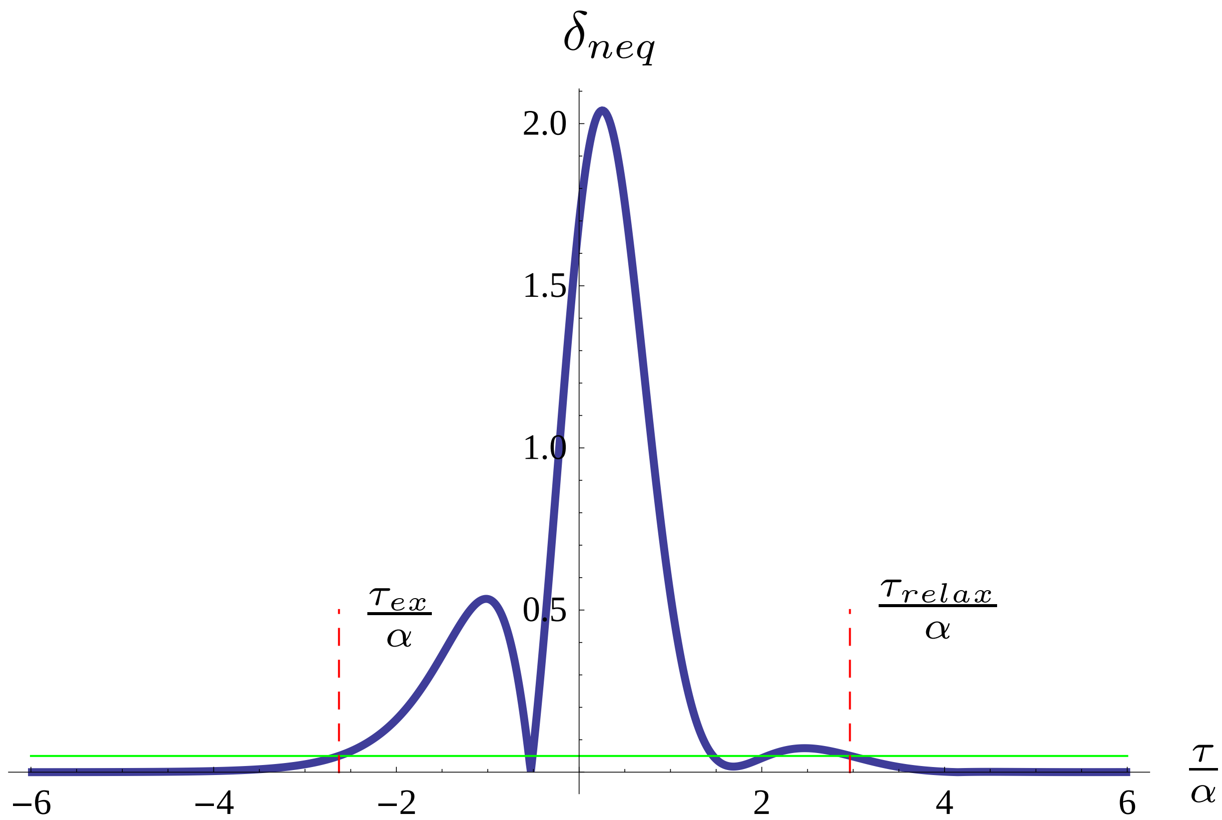
where $[\mathcal{O}_\Delta(\tau)]_{equilibrium}$ is the adiabatic response that can be computed analytically.

- Note,

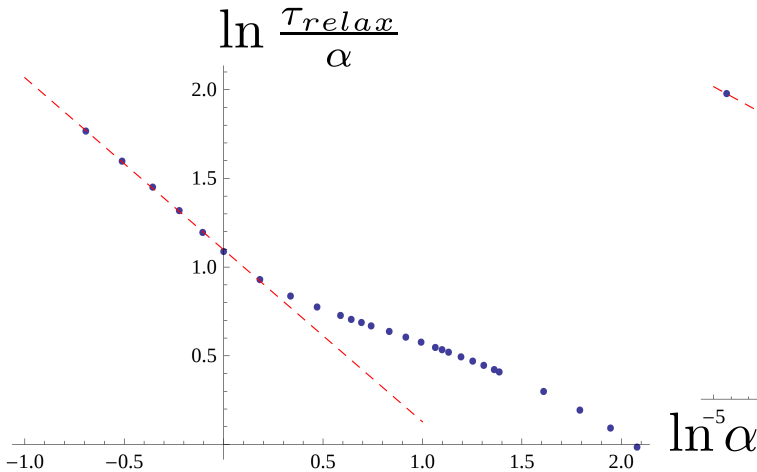
$$\lim_{\tau \to \pm\infty} \delta_{neq}(\tau) \to 0$$

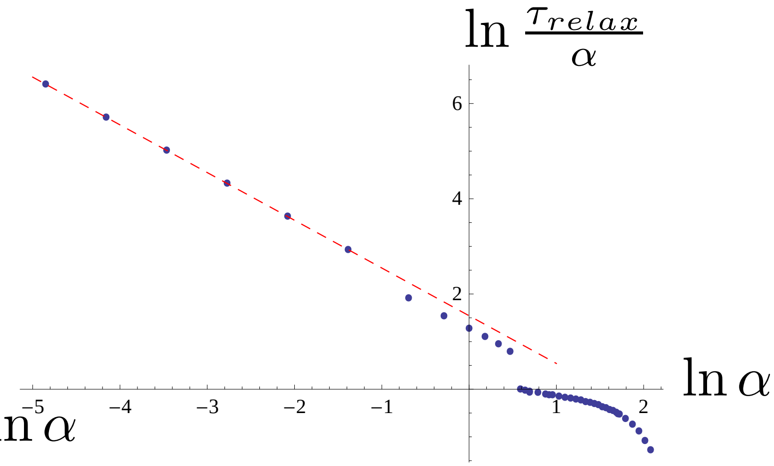
as at early/late times the system is in equilibrium.

$\Longrightarrow$ In practice,



Extraction of the excitation/equilibration rates for $\alpha = 1$ quench. The horizontal green line is the threshold for excitation/equilibration which we define to be 5% away from local equilibrium as determined by δ_{neq}. The dashed red lines indicate the earliest and latest times of crossing this threshold, which we denote as τ_{ex} (for excitation time) and τ_{relax} (for equilibration time), respectively.





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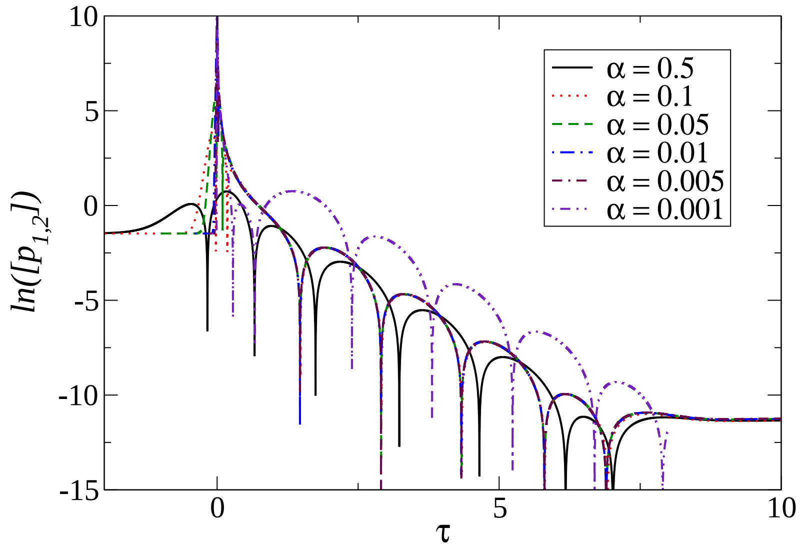
$\mathcal{O}_3(t)$ $\mathcal{O}_2(t)$

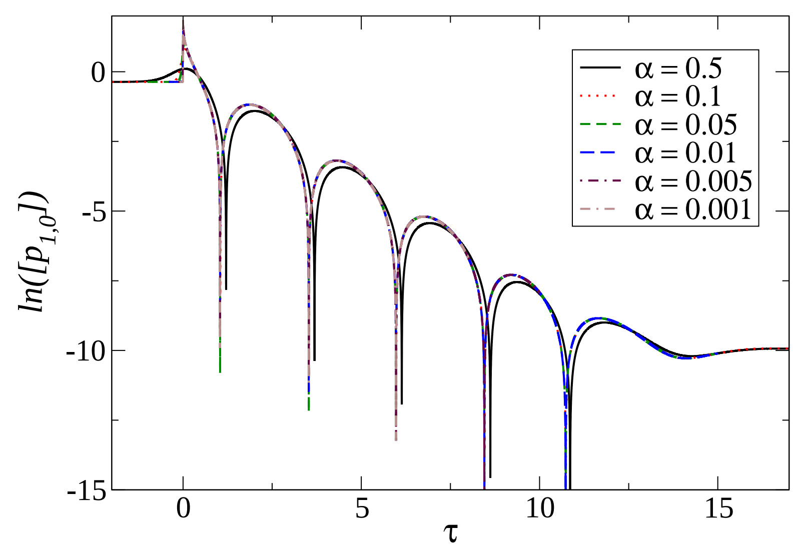
$\Longrightarrow$ Going to small α ($\ln \alpha \to -\infty$) corresponds to preparing the state with an abrupt quench of a dim-Δ operator. The dashed scaling line translates into a **universal relaxation time**:

$$t_{relax} \sim \frac{1}{T}$$

independent of α!

$\Longrightarrow$ We can do more:





Behavior of the response coefficients versus time for representative fast quenches. As is evident in the picture, the same quasinormal mode governs the dynamics very quickly after the quench:

$$\Delta = 3: \qquad \frac{\omega}{2\pi T}\bigg|_{fit} \simeq (1.095 - i\,0.87)\,, \qquad \frac{\omega}{2\pi T}\bigg|_{BH} \simeq (1.099 - i\,0.879)$$

$$\Delta = 2: \qquad \frac{\omega}{2\pi T}\bigg|_{fit} \simeq (0.64 - i\,0.4)\,, \qquad \frac{\omega}{2\pi T}\bigg|_{BH} \simeq (0.644 - i\,0.411)$$

$\Longrightarrow$ Moral of the story:

Lowest quasinormal modes of the black hole in the gravitational dual control the relaxation in strongly coupled gauge theory plasma

- Such feature was also observed in various other holographic examples
- It probably should not be a surprise that the relaxation rate is $\frac{1}{T}$, as after preparing the state in a high-temperature plasma after an abrupt quench temperature is **the only scale**

$\Longrightarrow$ This also motivates to look at non-conformal examples of gauge/gravity correspondence.

$\mathcal{N}=2^*$ gauge theory (a QFT story)

$\Longrightarrow$ Start with $\mathcal{N}=4$ $SU(N)$ SYM. In $\mathcal{N}=1$ 4d susy language, it is a gauge theory of a vector multiplet V, an adjoint chiral superfield Φ (related by $\mathcal{N}=2$ susy to V) and an adjoint pair $\{Q,\tilde{Q}\}$ of chiral multiplets, forming an $\mathcal{N}=2$ hypermultiplet. The theory has a superpotential:

$$W=\frac{2\sqrt{2}}{g_{YM}^2}\mathrm{Tr}\left(\left[Q,\tilde{Q}\right]\Phi\right)$$

We can break susy down to $\mathcal{N}=2$, by giving a mass for $\mathcal{N}=2$ hypermultiplet:

$$W=\frac{2\sqrt{2}}{g_{YM}^2}\mathrm{Tr}\left(\left[Q,\tilde{Q}\right]\Phi\right)+\frac{m}{g_{YM}^2}\left(\mathrm{Tr}Q^2+\mathrm{Tr}\tilde{Q}^2\right)$$

This theory is known as $\mathcal{N}=2^*$ gauge theory

When $m \neq 0$, the mass deformation lifts the $\{Q, \tilde{Q}\}$ hypermultiplet moduli directions; we are left with the $(N-1)$ complex dimensional Coulomb branch, parametrized by

$$\Phi = \mathrm{diag}\,(a_1, a_2, \cdots, a_N)\ , \qquad \sum_i a_i = 0$$

We will study $\mathcal{N} = 2^*$ gauge theory at a particular point on the Coulomb branch moduli space:

$$a_i \in [-a_0, a_0]\,, \qquad a_0^2 = \frac{m^2 g_{YM}^2 N}{\pi}$$

with the (continuous in the large N-limit) linear number density

$$\rho(a) = \frac{2}{m^2 g_{YM}^2}\sqrt{a_0^2 - a^2}\,, \qquad \int_{-a_0}^{a_0} da\ \rho(a) = N$$

Reason: we understand the dual supergravity solution only at this point on the moduli space.

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Reason: This moduli space point is a large-N saddle point obtained from localization (in 2 transparencies)

$\mathcal{N} = 2^*$ gauge theory (a supergravity story — a.k.a Pilch-Warner flow)

Consider 5d gauged supergravity, dual to $\mathcal{N} = 2^*$ gauge theory. The effective five-dimensional action is

$$S = \frac{1}{4\pi G_5} \int_{\mathcal{M}_5} d\xi^5 \sqrt{-g} \left(\tfrac{1}{4} R - (\partial\alpha)^2 - (\partial\chi)^2 - \mathcal{P}\right) ,$$

where the potential $\mathcal{P}$ is

$$\mathcal{P} = \frac{1}{16} \left[\left(\frac{\partial W}{\partial \alpha}\right)^2 + \left(\frac{\partial W}{\partial \chi}\right)^2 \right] - \frac{1}{3} W^2 ,$$

with the superpotential

$$W = -\frac{1}{\rho^2} - \frac{1}{2}\rho^4 \cosh(2\chi) , \qquad \alpha \equiv \sqrt{3} \ln \rho$$

$\Longrightarrow$ The 2 supergravity scalars $\{\alpha, \chi\}$ are holographic dual to dim-2 and dim-3 operators which are nothing but (correspondingly) the bosonic and the fermionic mass terms of the $\mathcal{N} = 4 \to \mathcal{N} = 2$ SYM mass deformation.

PW geometry ansatz:

$$ds_5^2 = e^{2A}\left(-dt^2 + d\vec{x}^{\,2}\right) + dr^2$$

solving the Killing spinor equations, we find a susy flow:

$$\frac{dA}{dr} = -\frac{1}{3}W\,, \qquad \frac{d\alpha}{dr} = \frac{1}{4}\frac{\partial W}{\partial\alpha}\,, \qquad \frac{d\chi}{dr} = \frac{1}{4}\frac{\partial W}{\partial\chi}$$

Solutions to above are characterized by a single parameter k:

$$e^A = \frac{k\rho^2}{\sinh(2\chi)}\,, \qquad \rho^6 = \cosh(2\chi) + \sinh^2(2\chi)\,\ln\frac{\sinh(\chi)}{\cosh(\chi)}$$

In was found (Polchinski,Peet,AB) that

$$k = 2m$$

$\Longrightarrow$ Precision test on $\mathcal{N}=2^*$ holography from Pestun's localization

- Supersymmetrically compactify $\mathcal{N}=2^*$ gauge theory on S^4
- Moduli of the theory are conformally coupled scalars, so they will all be lifted via coupling to S^4 curvature
- The exact partition function of the compactified theory is known due to Pestun's localization (reduces to a matrix model):

$$Z_{\mathcal{N}=2^*} = \int d^{N-1}\hat{a} \prod_{i<j} \frac{(\hat{a}_i-\hat{a}_j)^2 H^2(\hat{a}_i-\hat{a}_j)}{H(\hat{a}_i-\hat{a}_j-mR)H(\hat{a}_i-\hat{a}_j+mR)}$$

$$\times\ e^{-\frac{8\pi^2 N}{\lambda}\sum_j \hat{a}_j^2}\ \left|\mathcal{Z}_{\text{inst}}\right|^2$$

where

$$H(x) \equiv \prod_{n=1}^{\infty}\left(1+\frac{x^2}{n^2}\right)^n\ e^{-\frac{x^2}{n}}$$

- In the large-N limit the partition function is dominated by the saddle point, that can be computed analytically (AB, J.G.Russo and K.Zarembo, 1301.1597)

$\Longrightarrow$ One recovers:

- the moduli space point picked out by supergravity (as a matrix model saddle point)
- the susy Wilson loops agree both in matrix model and in holographic dual
- the matrix model free energy agree with the holographic free energy (Bobev et.al, arXiv:1311.1508)

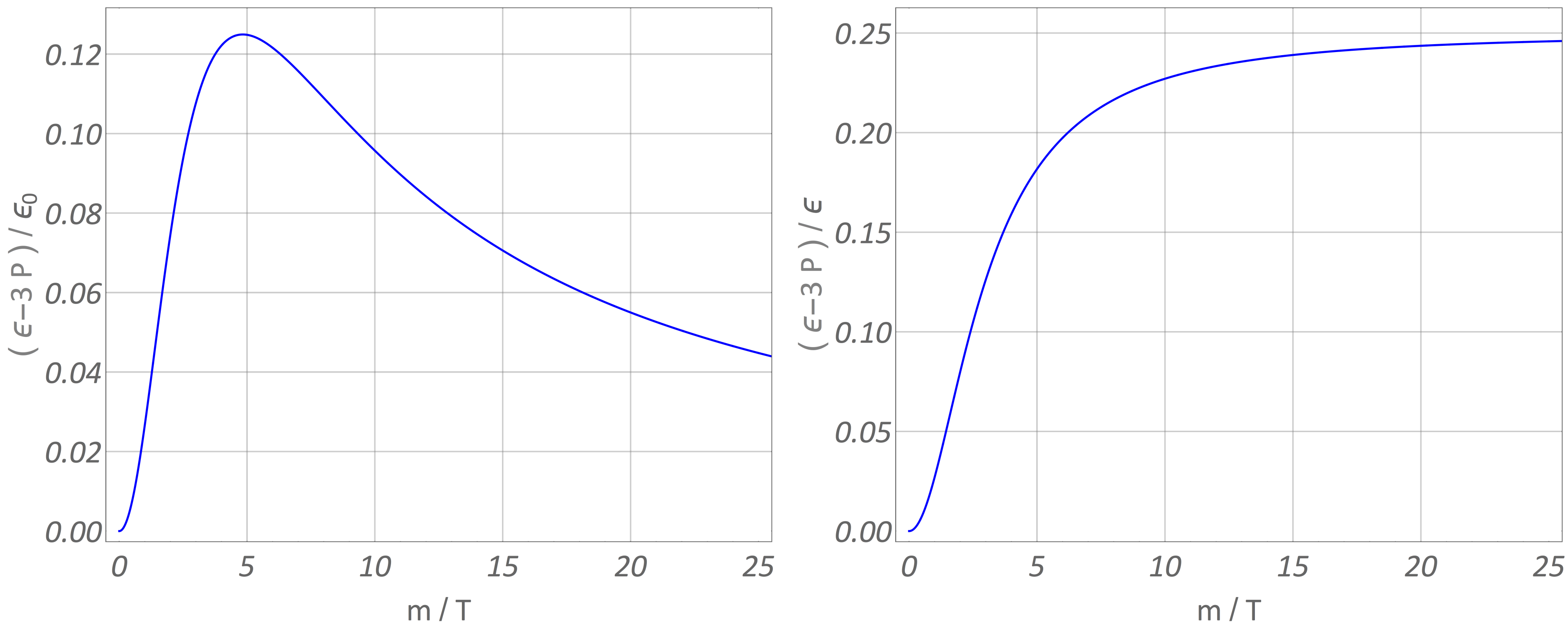
$\Longrightarrow$ All these checked are in addition to earlier agreement with the metric on the moduli space computed either in supergravity or from QFT using Seiberg-Witten techniques

$\Longrightarrow$ What do we do:

- Take PW gravitational dual to $\mathcal{N}=2^*$ gauge theory and construct black hole solutions. The thermodynamics of the black hole has a nontrivial dependence of 2 scales: T and m. The m dependence is quite profound:

$$s(T) \propto \begin{cases} N^2\ T^3\,, & \frac{m}{T} \ll 1 \\ N^2\ \frac{T^4}{m}\,, & \frac{m}{T} \gg 1 \end{cases}$$

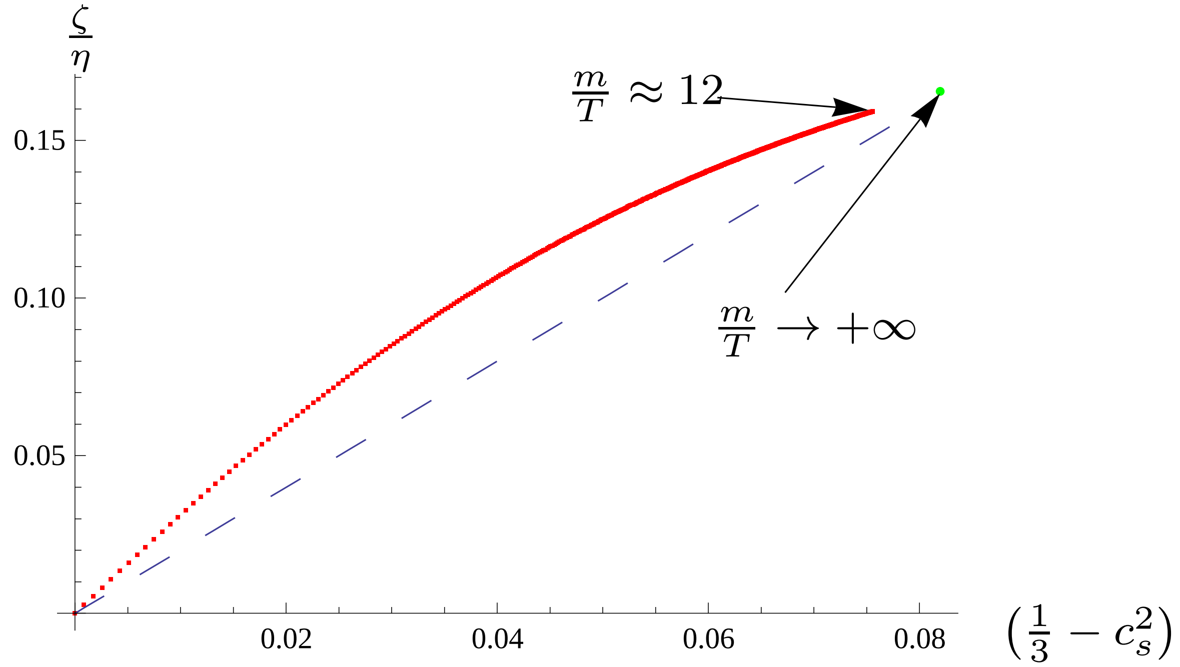
- Using the standard techniques we compute quasinormal modes of the BH corresponding to
  - the stress-energy tensor $T_{\mu\nu}$
  - operators $\{\mathcal{O}_2, \mathcal{O}_3\}$ inducing the RG flow
  - 'passive' operators $\{\mathcal{O}_2, \mathcal{O}_3\}$
  - also study the momentum dependence on the quasinormal frequencies to get an idea of relaxation of spatially inhomogeneous excitations



$\Longrightarrow$ (L) Trace of the energy-momentum tensor normalized to the energy density of $\mathcal{N}=4$ SYM ($\epsilon_0 = \frac{3}{8}\pi^2 N_c^2 T^4$ with N_c denoting the number of colors) as a function of m/T. The results indicate that, thermodynamically, the effects of the conformal symmetry breaking are the strongest at $m/T \approx 4.8$.

$\Longrightarrow$ (R) Trace anomaly in deep IR — approach to a CFT_5

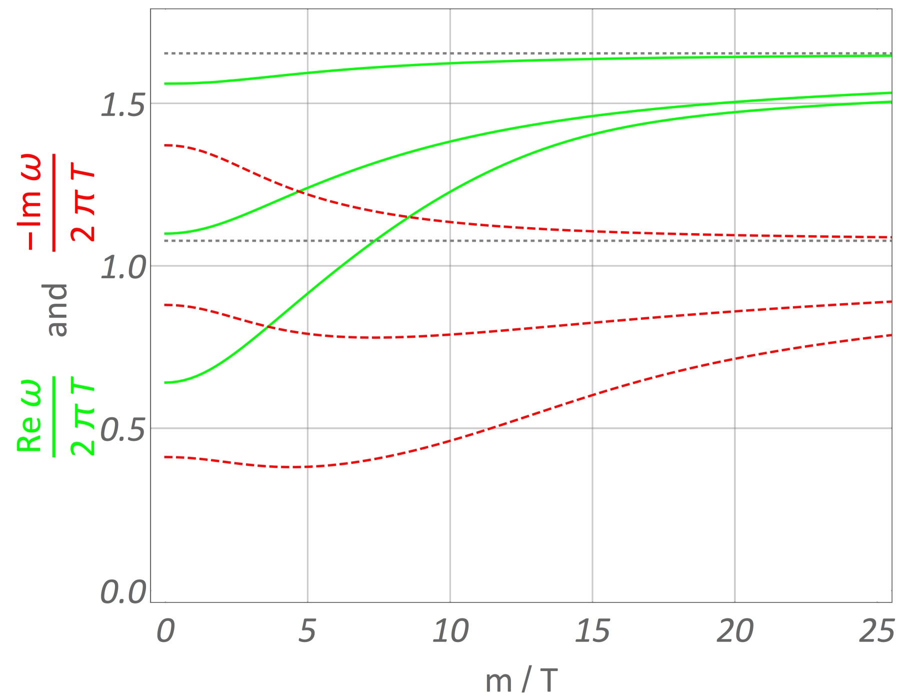
$\Longrightarrow$ There is an interesting story with the IR properties of the flow:



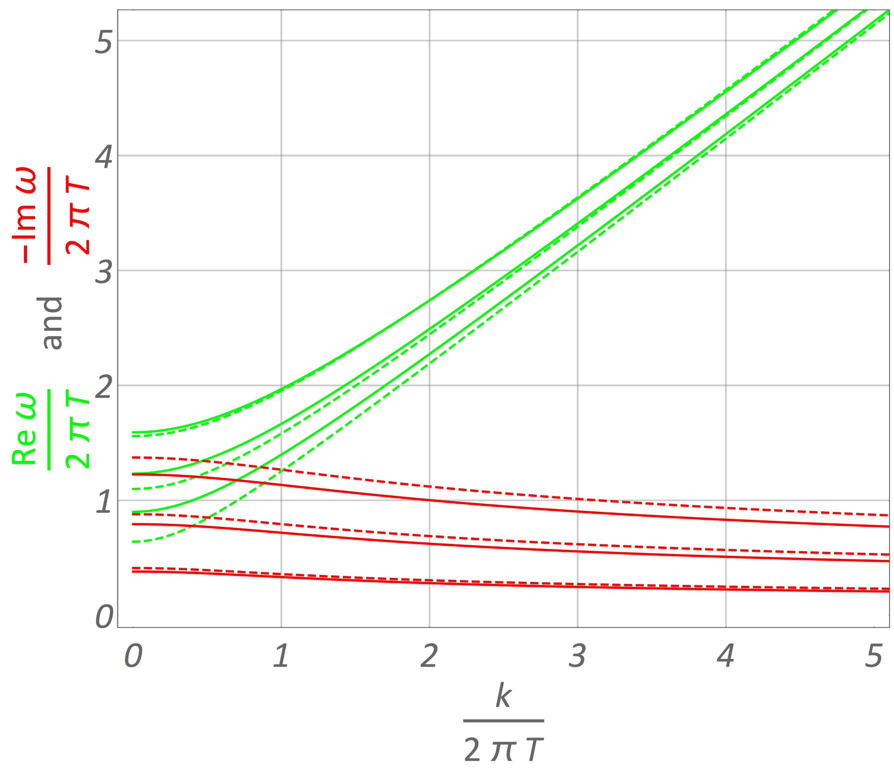
$\Longrightarrow$Ratio of viscosities $\frac{\zeta}{\eta}$ versus the speed of sound in $\mathcal{N} = 2^*$ gauge theory plasma (AB, arXiv:0708.3459). Dashed line is the bulk viscosity bound, $\frac{\zeta}{\eta} \geq 2\left(\frac{1}{3} - c_s^2\right)$. A single point represents extrapolation of the speed of sound and the viscosity ratio to $T \to +0$.

$\Longrightarrow$ Note, for a CFT_5, $\epsilon = 4p$, so $c^2_{s,CFT_5} - \frac{1}{4}$, and

$$\frac{1}{3} - c_s^2 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} = 0.08333\cdots$$



Real (green continuous) and minus imaginary (red dashed) parts of the lowest quasinormal mode frequencies for operators of dimensions $\Delta = 2, 3$ and 4 (from bottom to top). The frequencies do not change significantly as a function of m/T, which leads to universal equilibration in $1/T$. One can also infer from this plot that all the frequencies asymptote at low temperatures to the quasinormal mode of a massless scalar field living in the (1+5)-dimensional AdS-Schwarzschild geometry (dotted curves).



- Momentum dependence of the real (green) and minus imaginary part (red) of the QNM frequency of operators with $\Delta = 2, 3$ and 4 (from bottom to top) for $m/T = 0$ ($\mathcal{N} = 4$ SYM, dashed) and $m/T = 4.8$ (continuous). Surprisingly, corresponding curves are very close to each other despite of the fact that $m/T = 4.8$ matches the locus of the maximal deviation from conformal invariance in thermodynamics of $\mathcal{N} = 2^*$.
- There is **very weak** dependence on k in Imω

Conclusion:

- I argued that relaxation time in strongly coupled plasma is encoded in the spectrum of quasinormal modes of BH in the holographic dual
- One can have a controlled examples of the top-down holography where it is possible to systematically study effects of non-conformality on the relaxation time
- We found that

$$\tau_{relax} \propto \frac{1}{T}$$

  universally, even though are other microscopic scales in the plasma (masses, etc)
- The spatial relaxation is **ultralocal** — imaginary parts of the quasinormal modes are almost flat in momentum k
- Similar conclusions are reached in other models:
  - thermalization in $\mathcal{N} = 4$ plasma in the presence of charge densities/magnetic fields (J.Fuini, L.Yaffe, arXiv:1503.07148)
  - Bottom-up pheno models of holography (R.Janik et.al, arXiv:1503.07148; T.Ishii et.al, arXiv:1503.07766)