Holographic Kondo Defects

and

Universality in Holographic Superconductors with Broken Translation Symmetry

Johanna Erdmenger

Max–Planck–Institut für Physik, München

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Outline

1. Kondo models from holography

- Model — J.E., Hoyos, O'Bannon, Wu 1310.3271
- Quantum quenches — J.E., Flory, Newrzella, Strydom, Wu in progress
- Entanglement entropy — J.E., Flory, Newrzella 1410.7811
  J.E., Flory, Hoyos, Newrzella, O'Bannon, Wu in progress
- Two-point functions — J.E., Hoyos, O'Bannon, Papadimitriou, Probst, Wu in progress

Outline

1. Kondo models from holography

- Model — J.E., Hoyos, O'Bannon, Wu 1310.3271
- Quantum quenches — J.E., Flory, Newrzella, Strydom, Wu in progress
- Entanglement entropy — J.E., Flory, Newrzella 1410.7811
  J.E., Flory, Hoyos, Newrzella, O'Bannon, Wu in progress
- Two-point functions — J.E., Hoyos, O'Bannon, Papadimitriou, Probst, Wu in progress

2. S-Wave Superconductivity in Anisotropic Holographic Insulators

J.E., Herwerth, Klug, Meyer, Schalm 1501.07615

- Scalar condenses in helical Bianchi VII background
- Homes' Law

Kondo models from gauge/gravity duality

Kondo effect:

Screening of a magnetic impurity by conduction electrons at low temperatures

Kondo effect:

Screening of a magnetic impurity by conduction electrons at low temperatures

Motivation for study within gauge/gravity duality:

Kondo models from gauge/gravity duality

Kondo effect:

Screening of a magnetic impurity by conduction electrons at low temperatures

Motivation for study within gauge/gravity duality:

1. Kondo model: Simple model for a RG flow with dynamical scale generation

Kondo effect:

Screening of a magnetic impurity by conduction electrons at low temperatures

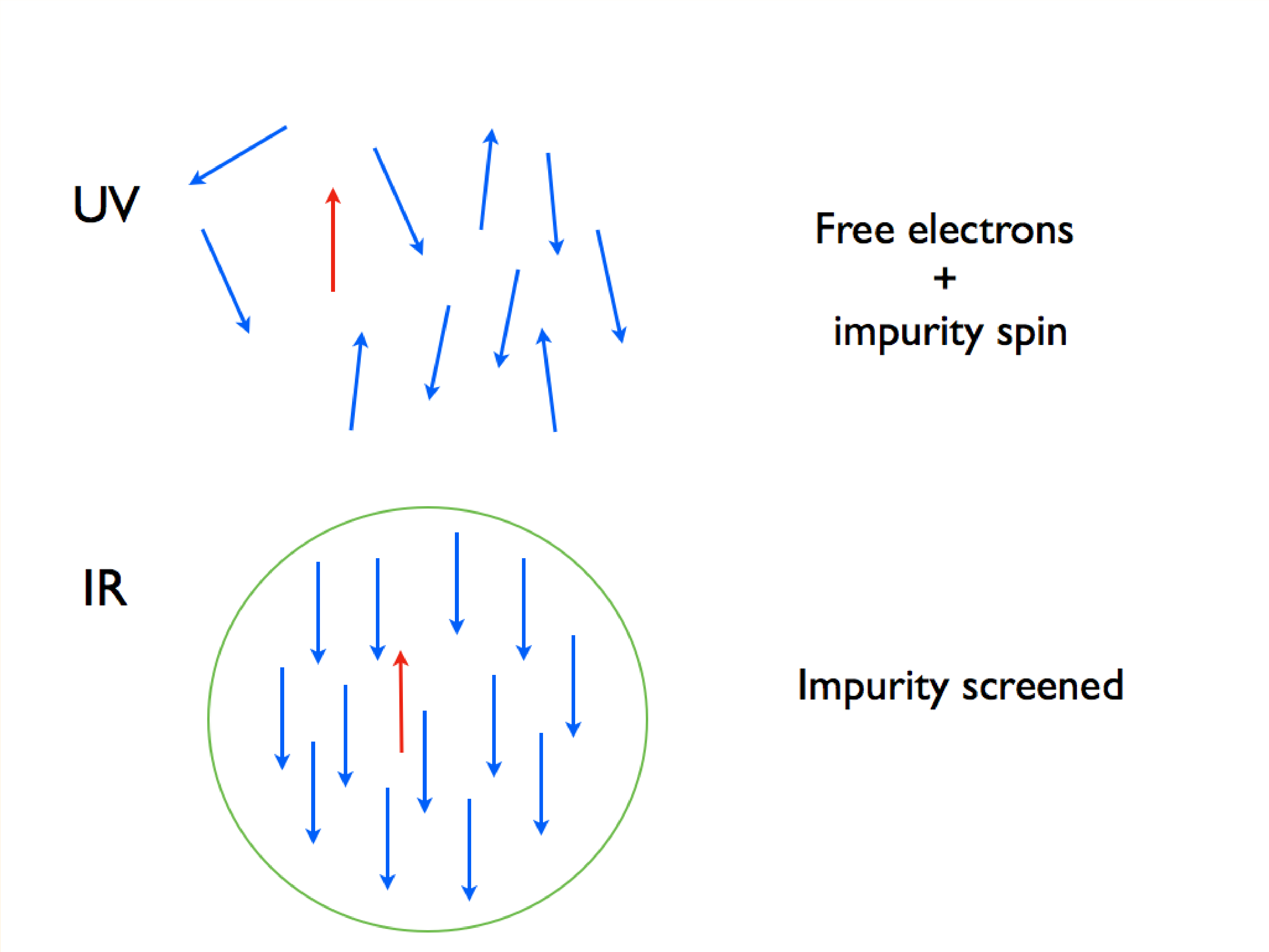
Motivation for study within gauge/gravity duality:

1. Kondo model: Simple model for a RG flow with dynamical scale generation

2. New applications of gauge/gravity duality to condensed matter physics:

- Magnetic impurity coupled to strongly correlated electron system
- Entanglement entropy
- Quantum quench
- Kondo lattice

Kondo effect



Kondo model

Kondo model

Original Kondo model (Kondo 1964):
Magnetic impurity interacting with free electron gas

Kondo model

Original Kondo model (Kondo 1964):
Magnetic impurity interacting with free electron gas

Impurity screened at low temperatures:
Logarithmic rise of resistivity at low temperatures

Kondo model

Original Kondo model (Kondo 1964):
Magnetic impurity interacting with free electron gas

Impurity screened at low temperatures:
Logarithmic rise of resistivity at low temperatures

Due to symmetries: Model effectively $(1+1)$-dimensional

Hamiltonian:

$$H = \frac{v_F}{2\pi}\psi^\dagger i\partial_x\psi + \lambda_K v_F \delta(x)\vec{S}\cdot\vec{J}, \quad \vec{J} = \psi^\dagger \frac{1}{2}\vec{T}\psi$$

Decisive in development of renormalization group
IR fixed point, CFT approach Affleck, Ludwig '90's

Gauge/gravity requires large N: Spin group $SU(N)$

Gauge/gravity requires large N: Spin group $SU(N)$

In this case, interaction term simplifies introducing slave fermions:

$S^a = \chi^\dagger T^a \chi$

Totally antisymmetric representation: Young tableau with Q boxes

Constraint: $\chi^\dagger \chi = q, \quad q = Q/N$

Kondo models from gauge/gravity duality

Gauge/gravity requires large N: Spin group $SU(N)$

In this case, interaction term simplifies introducing slave fermions:

$S^a = \chi^\dagger T^a \chi$

Totally antisymmetric representation: Young tableau with Q boxes

Constraint: $\chi^\dagger \chi = q, \quad q = Q/N$

Interaction: $J^a S^a = (\psi^\dagger T^a \psi)(\chi^\dagger T^a \chi) = \mathcal{O}\mathcal{O}^\dagger$, where $\mathcal{O} = \psi^\dagger \chi$

Gauge/gravity requires large N: Spin group $SU(N)$

In this case, interaction term simplifies introducing slave fermions:

$S^a = \chi^\dagger T^a \chi$

Totally antisymmetric representation: Young tableau with Q boxes

Constraint: $\chi^\dagger \chi = q, \quad q = Q/N$

Interaction: $J^a S^a = (\psi^\dagger T^a \psi)(\chi^\dagger T^a \chi) = \mathcal{O}\mathcal{O}^\dagger$, where $\mathcal{O} = \psi^\dagger \chi$

Screened phase has condensate $\langle \mathcal{O} \rangle$

Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192
Senthil, Sachdev, Vojta cond-mat/0209144

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

Results:

- Top-down realization in terms of probe branes
- RG flow from perturbation by 'double-trace' operator
- Dynamical scale generation
- Holographic superconductor: Condensate forms in AdS_2
- Screening

Kondo models from gauge/gravity duality

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Top-down brane realization

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

- 3-7 strings: Chiral fermions ψ in 1+1 dimensions
- 3-5 strings: Slave fermions χ in 0+1 dimensions
- 5-7 strings: Scalar (tachyon)

D3: $AdS_5 \times S^5$

D7: $AdS_3 \times S^5 \to$ Chern-Simons A_μ dual to $J^\mu = \psi^\dagger \sigma^\mu \psi$

D5: $AdS_2 \times S^4 \to \begin{cases} \text{YM } a_t \text{ dual to } \chi^\dagger \chi = q \\ \text{Scalar dual to } \psi^\dagger \chi \end{cases}$

Operator		Gravity field
Electron current J	$\Leftrightarrow$	Chern-Simons gauge field A in AdS_3
Charge $q = \chi^\dagger \chi$	$\Leftrightarrow$	2d gauge field a in AdS_2
Operator $\mathcal{O} = \psi^\dagger \chi$	$\Leftrightarrow$	2d complex scalar Φ

Bottom-up gravity dual for Kondo model

Action:

$$S = S_{CS} + S_{AdS_2},$$

$$S_{CS} = -\frac{N}{4\pi} \int\limits_{AdS_3} \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right),$$

$$S_{AdS_2} = -N \int d^3x \, \delta(x) \sqrt{-g} \left[\frac{1}{4} \mathrm{Tr} f^{mn} f_{mn} + g^{mn} \left(D_m \Phi \right)^\dagger D_n \Phi + V(\Phi^\dagger \Phi) \right]$$

Metric:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{z^2} \left(\frac{dz^2}{h(z)} - h(z) \, dt^2 + dx^2 \right),$$

$$h(z) = 1 - z^2/z_H^2 \,, \qquad T = 1/(2\pi z_H)$$

‘Double-trace’ deformation by $\mathcal{O}\mathcal{O}^\dagger$

Boundary expansion

$$\Phi = z^{1/2}(\alpha \ln z + \beta)$$

$$\alpha = \kappa\beta$$

κ dual to double-trace deformation

Witten hep-th/0112258

‘Double-trace’ deformation by $\mathcal{O}\mathcal{O}^\dagger$

Boundary expansion

$$\Phi = z^{1/2}(\alpha \ln z + \beta)$$

$$\alpha = \kappa\beta$$

κ dual to double-trace deformation

Witten hep-th/0112258

Φ invariant under renormalization $\Rightarrow$ Running coupling

$$\kappa_T = \frac{\kappa_0}{1 + \kappa_0 \ln\left(\frac{\Lambda}{2\pi T}\right)}$$

‘Double-trace’ deformation by $\mathcal{O}\mathcal{O}^\dagger$

Boundary expansion

$$\Phi = z^{1/2}(\alpha \ln z + \beta)$$

$$\alpha = \kappa\beta$$

κ dual to double-trace deformation

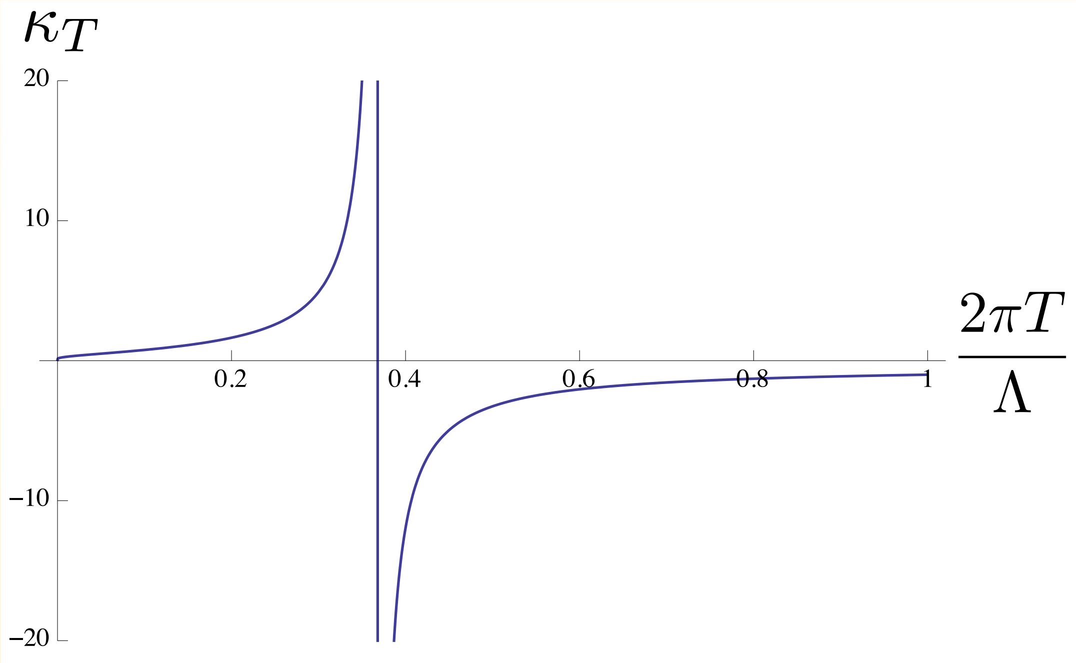
Witten hep-th/0112258

Φ invariant under renormalization $\Rightarrow$ Running coupling

$$\kappa_T = \frac{\kappa_0}{1 + \kappa_0 \ln\left(\frac{\Lambda}{2\pi T}\right)}$$

Dynamical scale generation

Scale generation

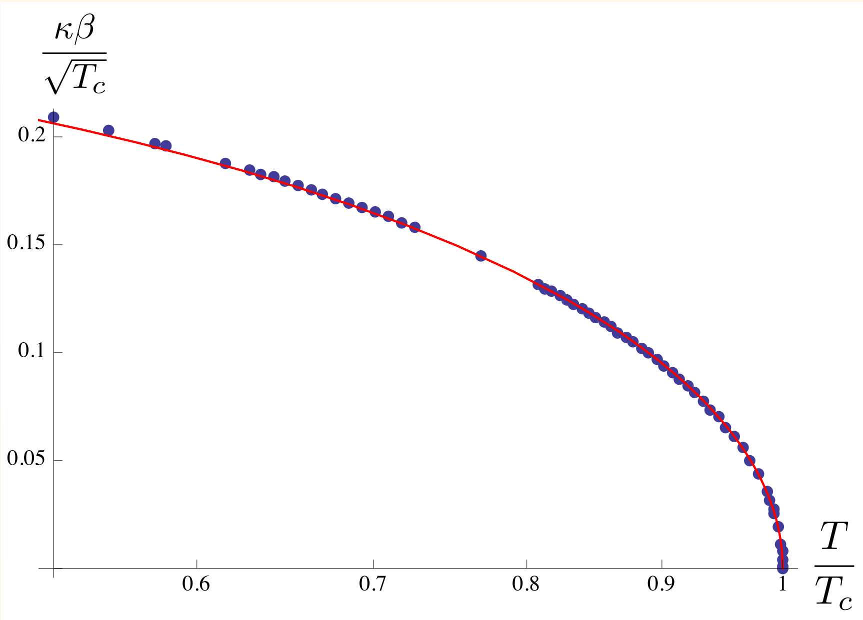


Divergence of Kondo coupling determines Kondo temperature T_K

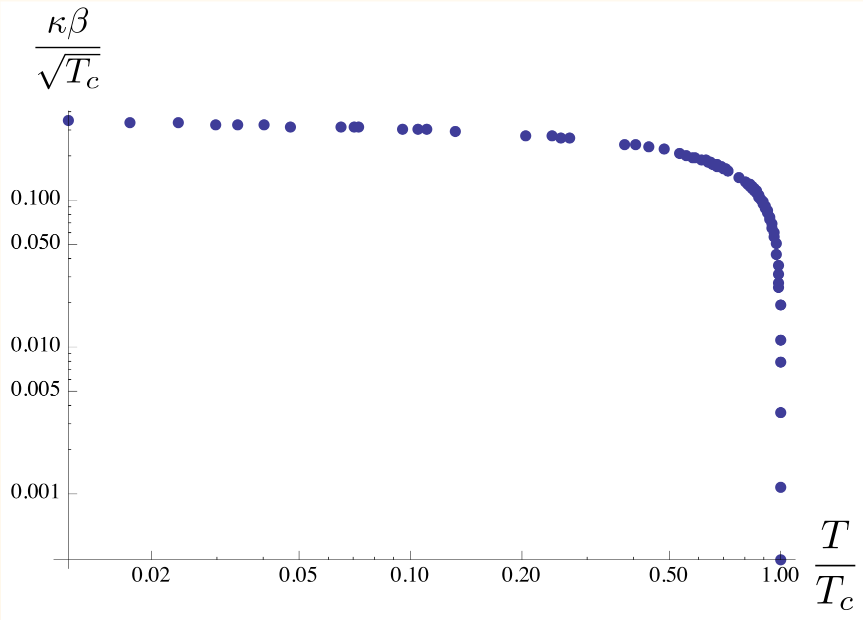
Transition temperature to phase with condensed scalar: T_c

$T_c < T_K$

Normalized condensate $\langle\mathcal{O}\rangle \equiv \kappa\beta$ as function of the temperature



(a)

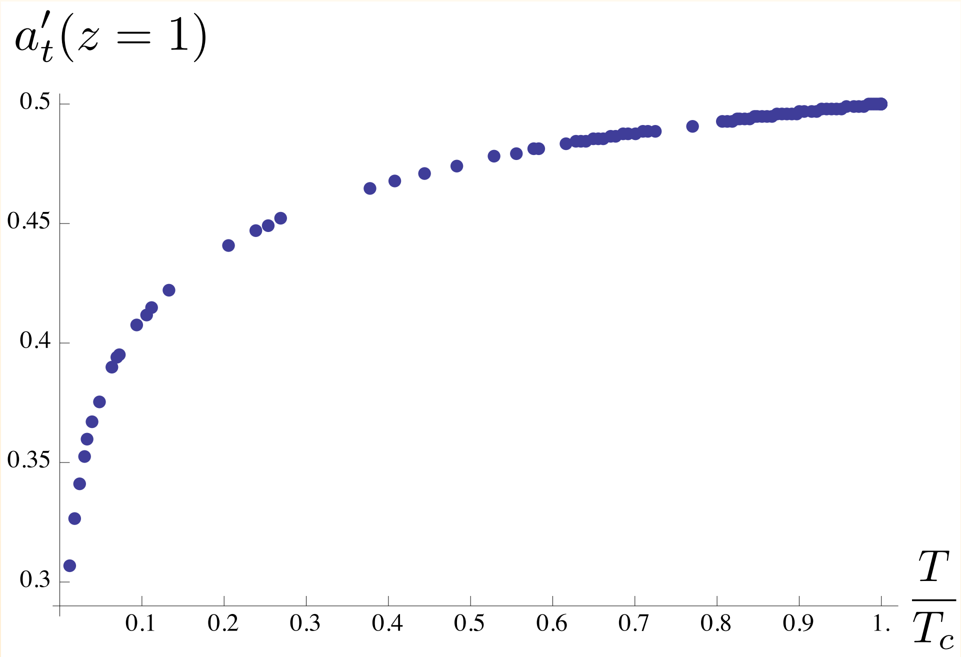


(b)

Mean field transition

$\langle\mathcal{O}\rangle$ approaches constant for $T \to 0$

Electric flux at horizon



$$\sqrt{-g}f^{tr}\Big|_{\partial AdS_2} = q = \chi^\dagger \chi$$

Impurity is screened

Time dependence

Allow for time dependence of the Kondo coupling and study response of the condensate

Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

Time dependence

Allow for time dependence of the Kondo coupling and study response of the condensate

Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

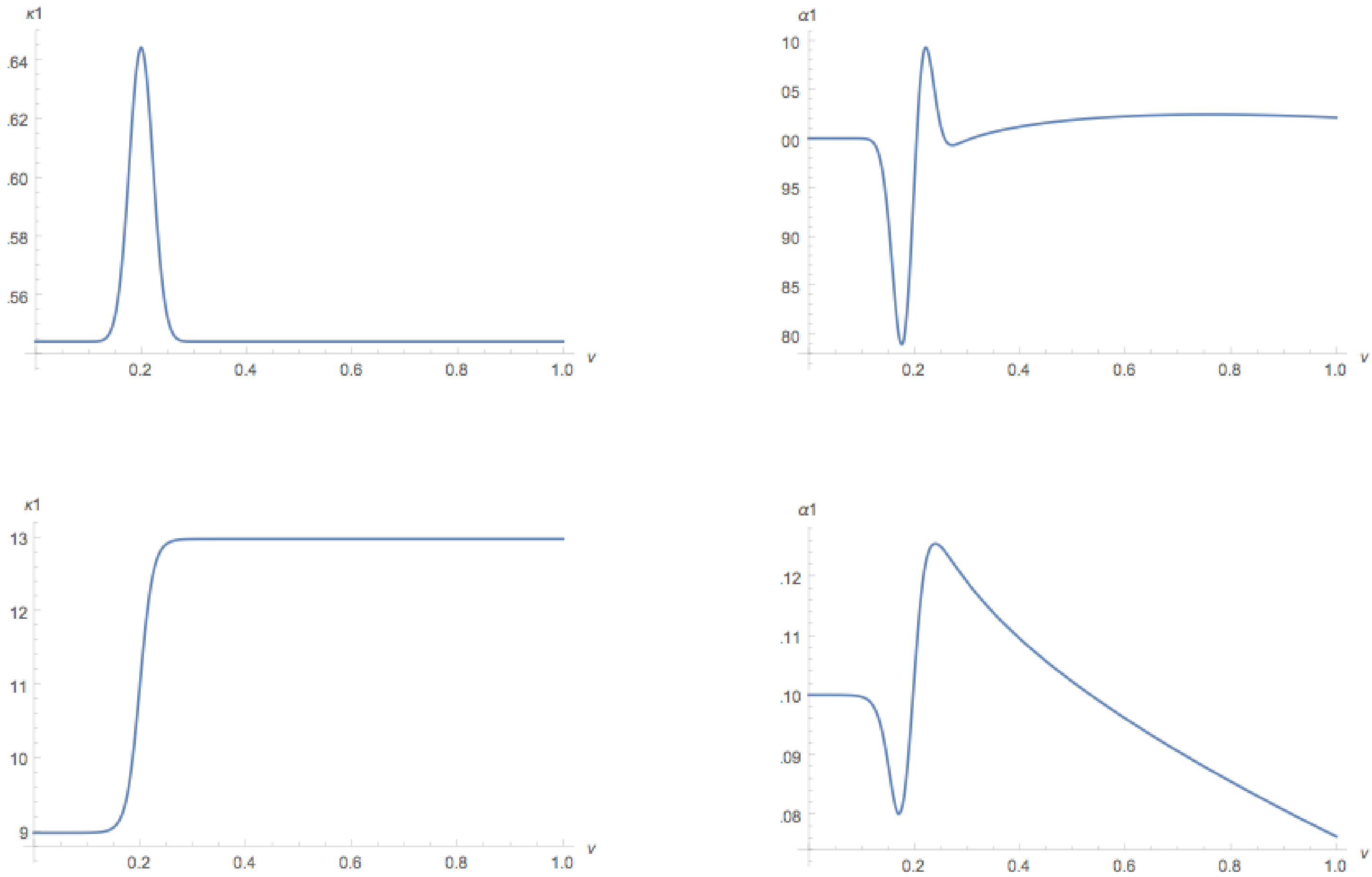
Observations:

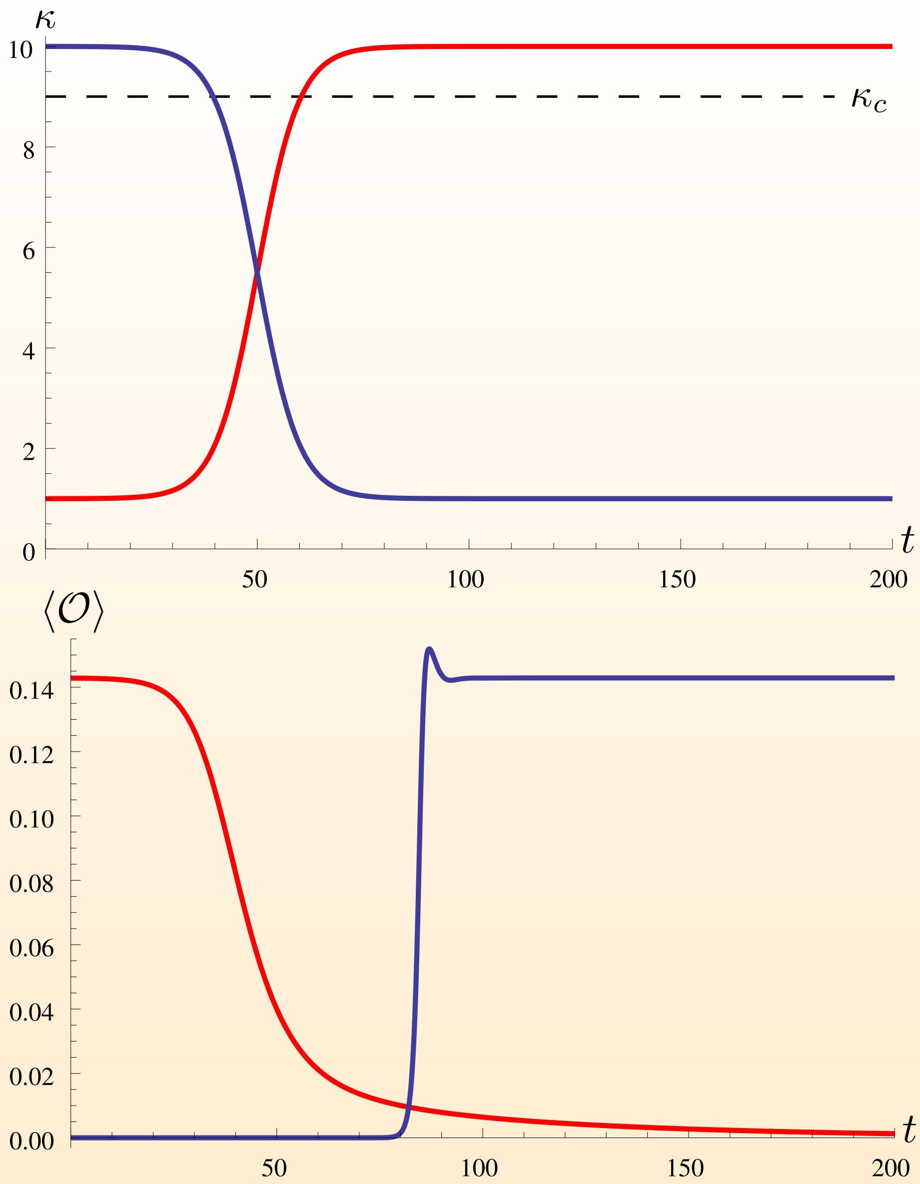
Different timescales depending on whether the condensate is asymptotically small or large

Anderson orthogonality catastrophe? $\tau \sim 1/\langle \text{initial} | \text{final} \rangle$

Time dependence

Kondo coupling → Condensate





Quantum quenches in holographic Kondo model

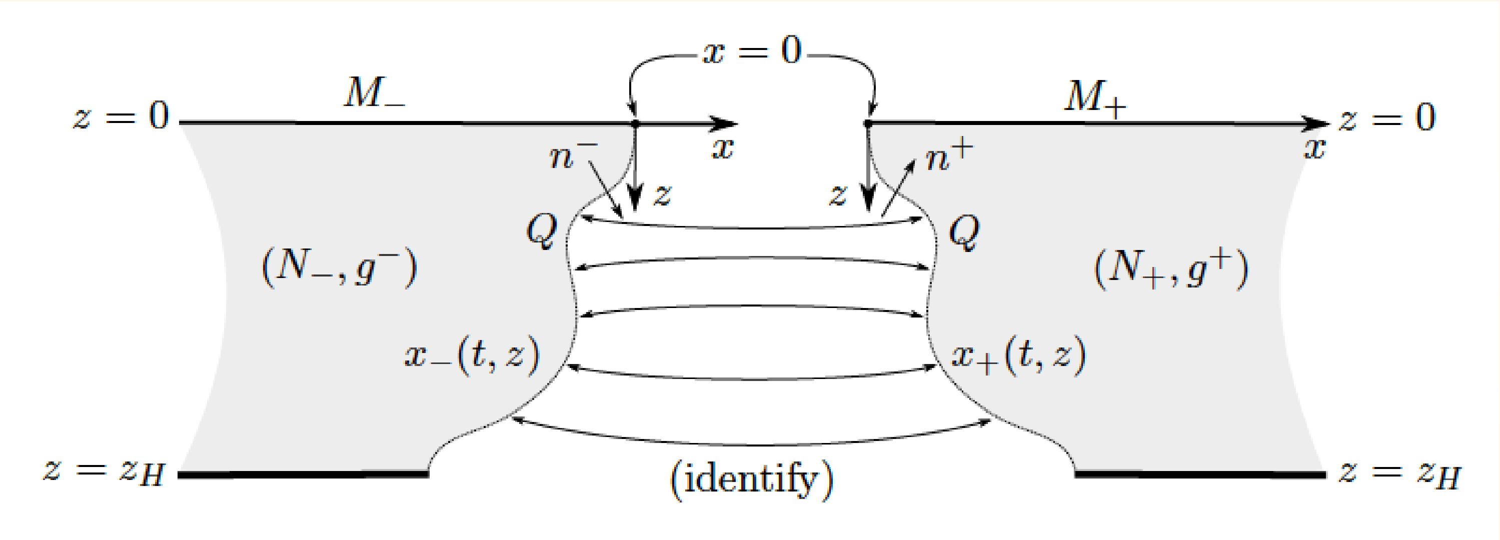
To and from condensed phase

J.E., Flory, Newrzella, Strydom, Wu

Entanglement entropy for magnetic impurity

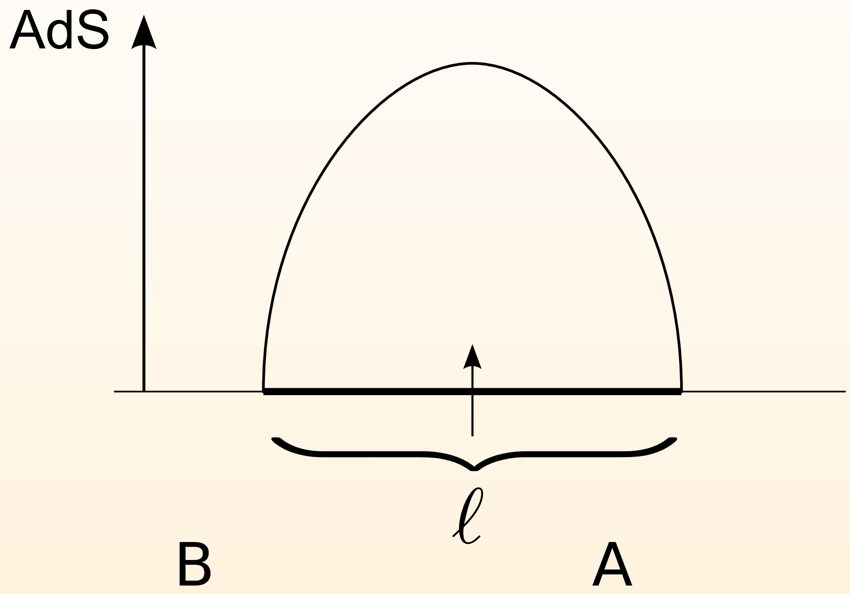
(see talk by Mario Flory on Wednesday)

Including the backreaction using a thin brane and Israel junction conditions



J.E., Flory, Newrzella 1410.7811

Entanglement entropy for magnetic impurity



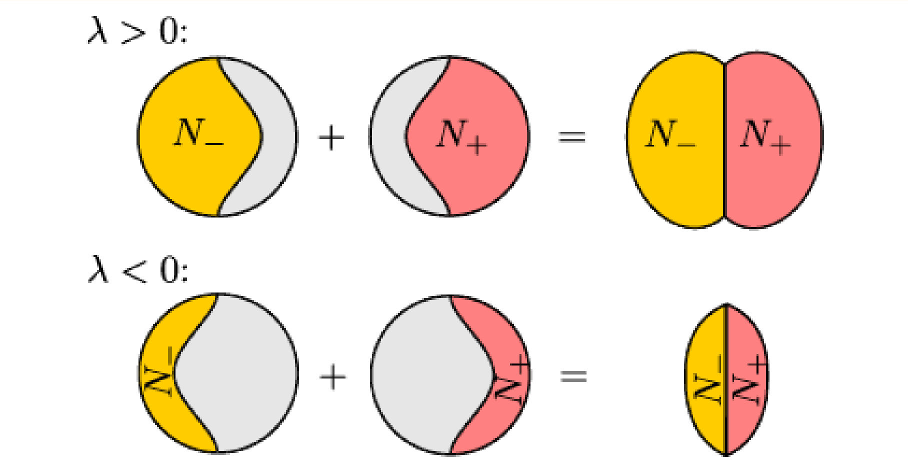
Impurity entropy:

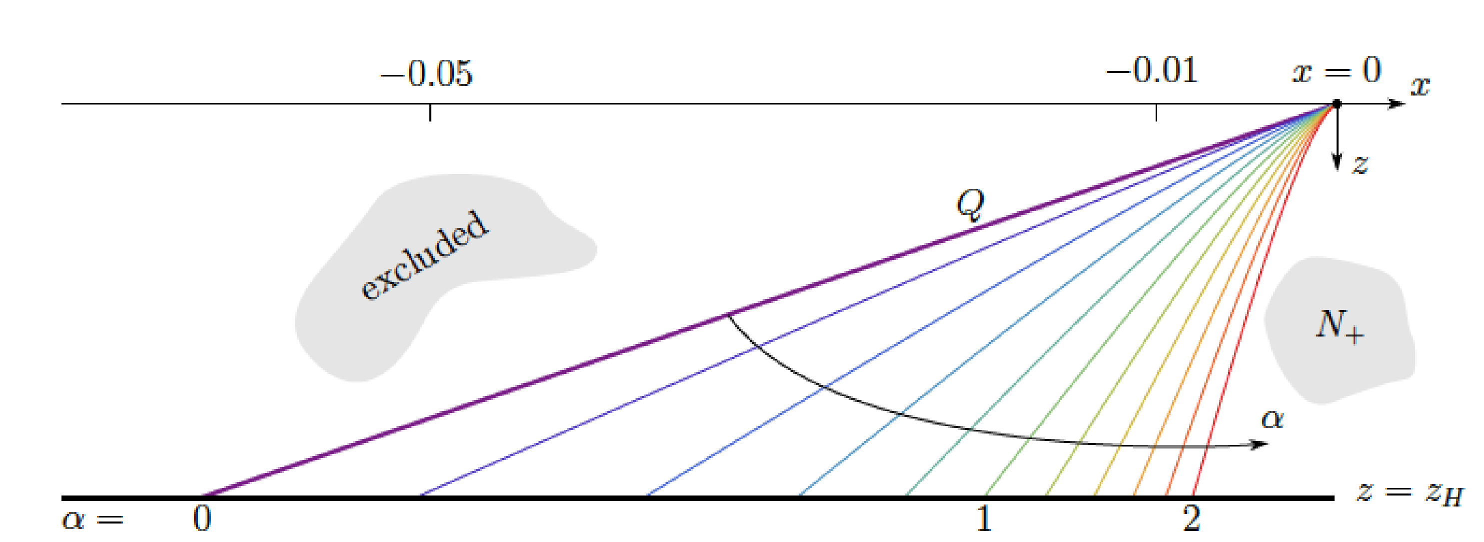
$$S_{\rm imp} = S_{\rm condensed\ phase} - S_{\rm normal\ phase}$$

Subtraction also guarantees UV regularity

Entanglement entropy for magnetic impurity

J.E., Flory, Newrzella 1410.7811

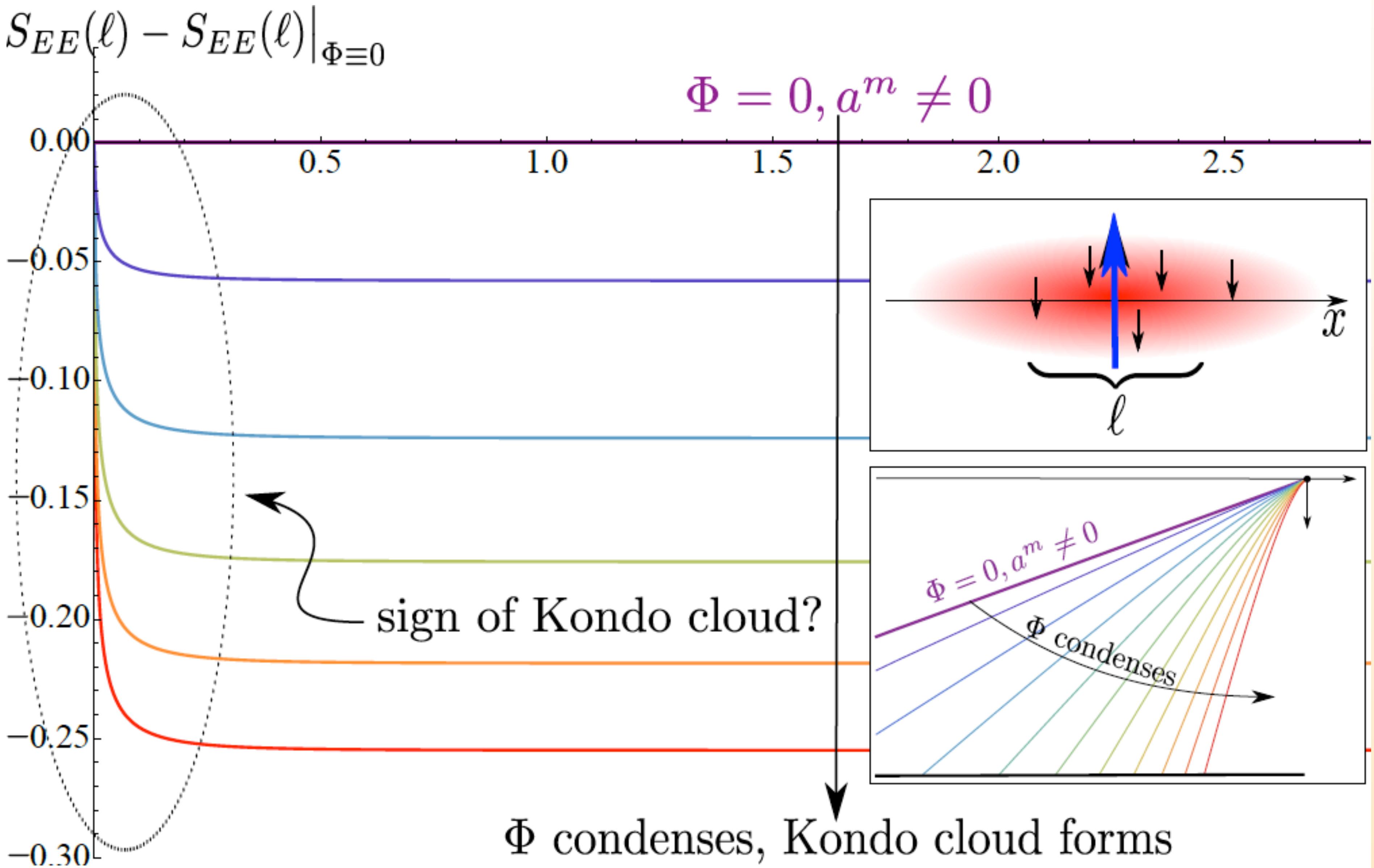




The larger the condensate, the shorter the geodesic

Entanglement entropy

J.E., Flory, Hoyos, Newrzella, O'Bannon, Wu in progress



Application example II: Universal properties of superconductors

Universality: IR fixed point determines physical properties
Macroscopic properties do not depend on microscopic degrees of freedom

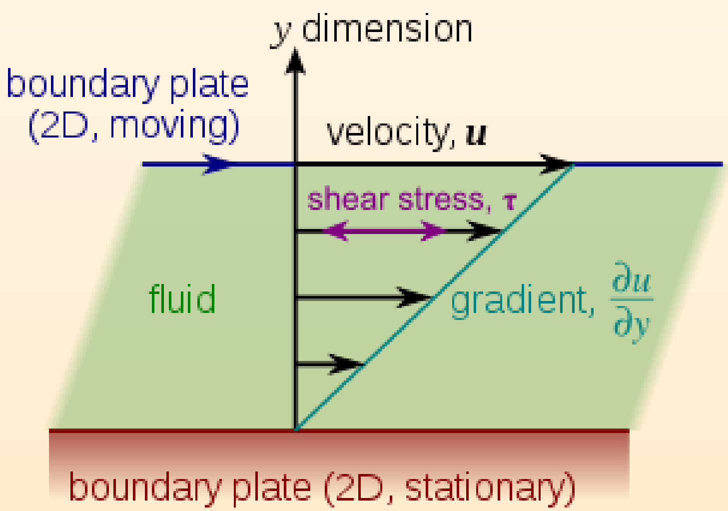
Application example II: Universal properties of superconductors

Universality: IR fixed point determines physical properties
Macroscopic properties do not depend on microscopic degrees of freedom

Example: Universal result from gauge/gravity duality:

Shear viscosity over entropy density:

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$



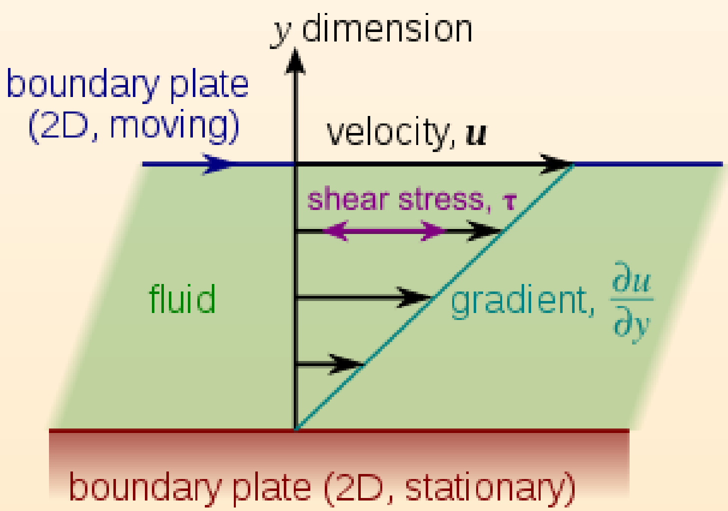
Application example II: Universal properties of superconductors

Universality: IR fixed point determines physical properties
Macroscopic properties do not depend on microscopic degrees of freedom

Example: Universal result from gauge/gravity duality:

Shear viscosity over entropy density:

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$



Planckian dissipator: relaxation time $\tau = \frac{\hbar}{k_B T}$

Damle, Sachdev 1997

Application example II: Universal properties of superconductors

Is there a similiar universal result for applications of the duality within condensed matter physics?

Candidate: Homes' relation

$\rho_s(T=0) \; = \; C \, \sigma_{\mathrm{DC}}(T_c) \, T_c$

Application example II: Universal properties of superconductors

Homes' relation $\rho_s(T=0) \,=\, C\,\sigma_{\rm DC}\,T_c$

general form may be deduced from Planckian dissipation   Zaanen 2004

Application example II: Universal properties of superconductors

Homes' relation $\rho_s(T=0) \;=\; C\,\sigma_{\rm DC}\,T_c$

general form may be deduced from Planckian dissipation   Zaanen 2004

J.E., Herwerth, Klug, Meyer, Schalm arXiv:1501.07615:

Talk by René Meyer on Tuesday

Investigation of C in a family of gauge/gravity duality models

In particular region of parameter space:

$$C \approx 6.2$$

BCS superconductor in 'dirty limit': $C = 8.1$,
High-T_c superconductors: $C = 4.4$

Application example II: Universal properties of superconductors

Condition for identifying ρ_s:
Translation symmetry broken

Application example II: Universal properties of superconductors

Condition for identifying ρ_s:
Translation symmetry broken

All normal state degrees of freedom have to condense at $T = 0$

Application example II: Universal properties of superconductors

Condition for identifying ρ_s:
Translation symmetry broken

All normal state degrees of freedom have to condense at $T = 0$

Holography: J.E., Kerner Müller 2012

Condition for identifying ρ_s:
Translation symmetry broken

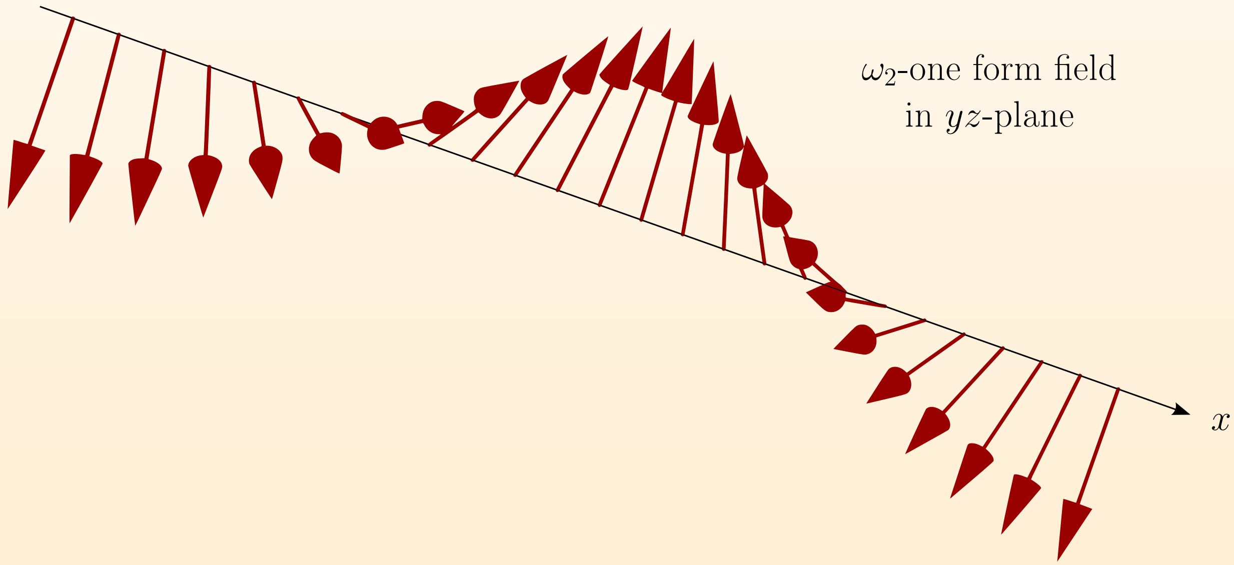
All normal state degrees of freedom have to condense at $T = 0$

Holography: J.E., Kerner Müller 2012

Weak momentum relaxation is not enough
Horowitz, Santos 2013

Background: Helical Bianchi VII symmetry   Donos, Gauntlett; Donos, Hartnoll

Model with broken translation symmetry:



Background: (Hartnoll, Donos)

$$S_{\text{helix}} = \int d^{4+1}x \sqrt{-g}\Big[R + 12 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}W^{\mu\nu}W_{\mu\nu} - m^2 B_\mu B^\mu\Big] - \frac{\kappa}{2}\int B \wedge F \wedge W.$$

$$B = w(r)\omega_2, \qquad w(\infty) = \lambda$$

$$\omega_1 = dx\,,$$
$$\omega_2 = \cos(px)\,dy - \sin(px)\,dz$$
$$\omega_3 = \sin(px)\,dy + \cos(px)\,dz$$

S-wave superconductivity in helical symmetry background

S-wave superconductivity in helical symmetry background

Add charged scalar:

$$S_{\text{total}} = S_{\text{helix}} + \int \mathrm{d}^{4+1}x\,\sqrt{-g}\left[-\left|\partial\rho - \mathrm{i}qA\rho\right|^2 - m_\rho^2|\rho|^2\right]$$

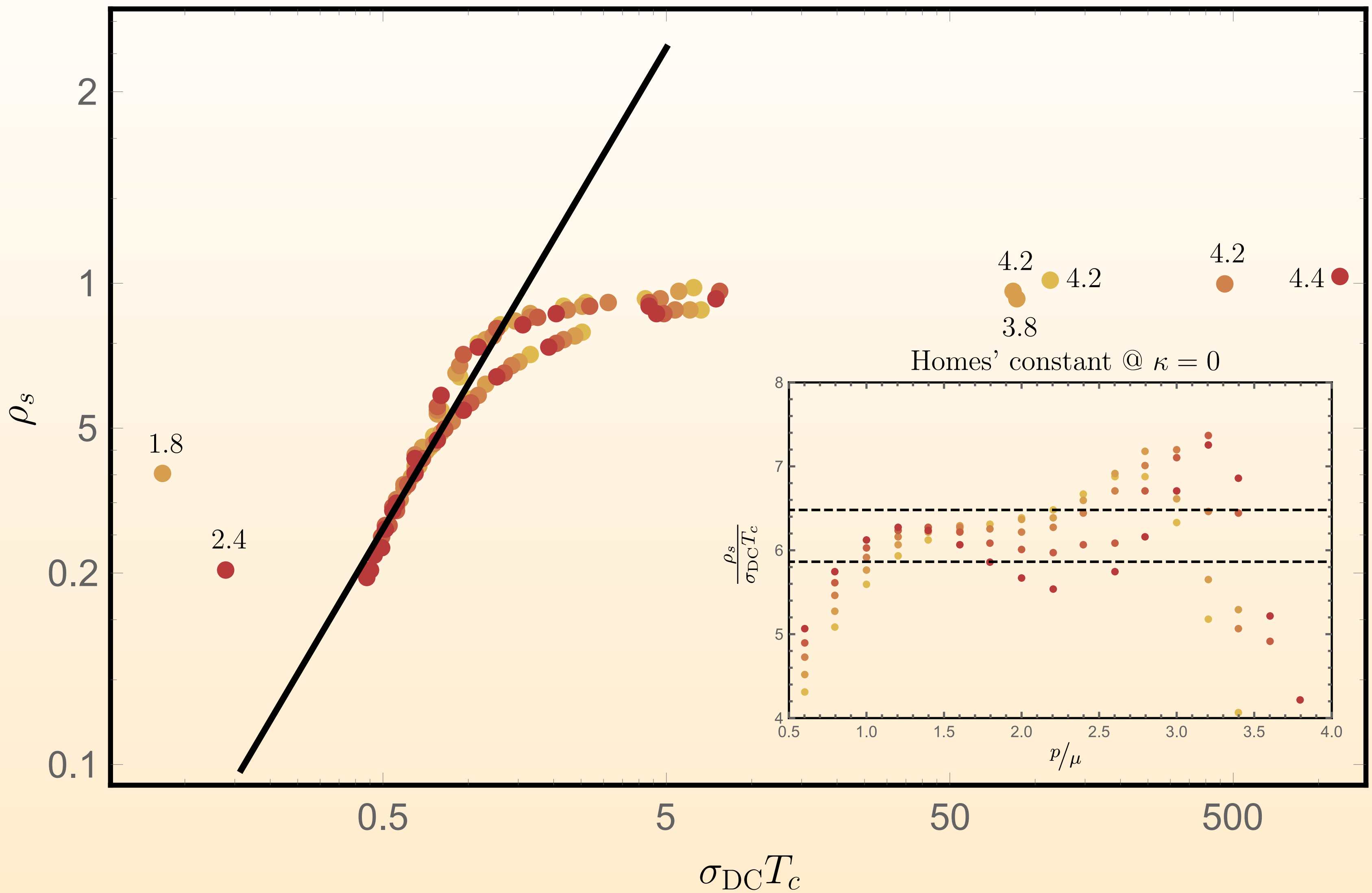
S-wave superconductivity in helical symmetry background

Add charged scalar:

$$S_{\text{total}} = S_{\text{helix}} + \int \mathrm{d}^{4+1}x\, \sqrt{-g}\Big[-|\partial\rho - \mathrm{i}qA\rho|^2 - m_\rho^2|\rho|^2 \Big]$$

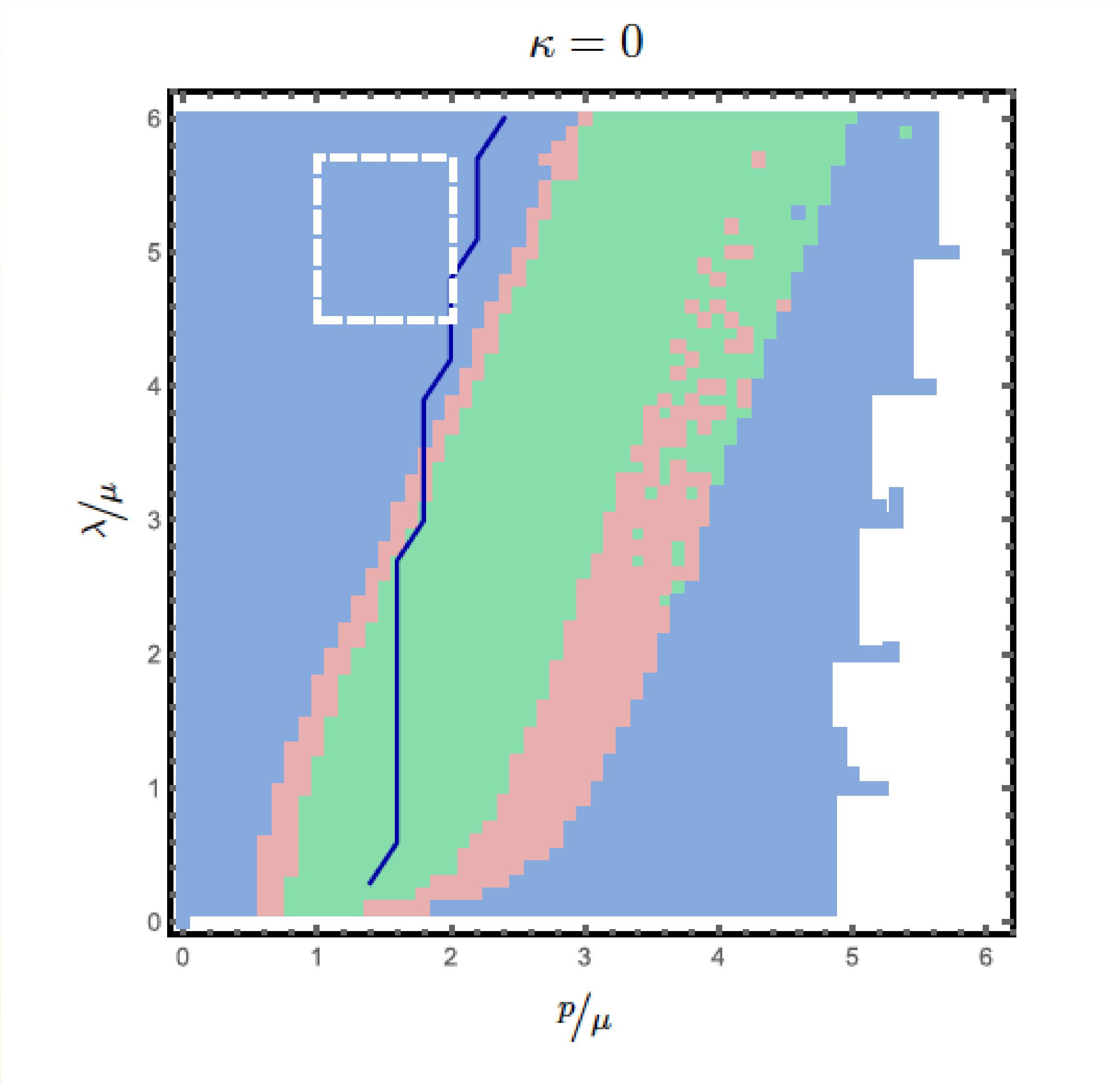
All charged degrees of freedom condense at $T = 0$

Homes' relation for $q = 6$ & $\kappa = 0$



J.E., Herwerth, Klug, Meyer, Schalm 1501.07615

Phase diagram



Conclusions and outlook

- Kondo model:
- Magnetic impurity coupled to strongly coupled system
- Quantum quench
- Entanglement entropy
- S-wave superconductor in Bianchi VII background:
- Homes' Law

Advertising our book