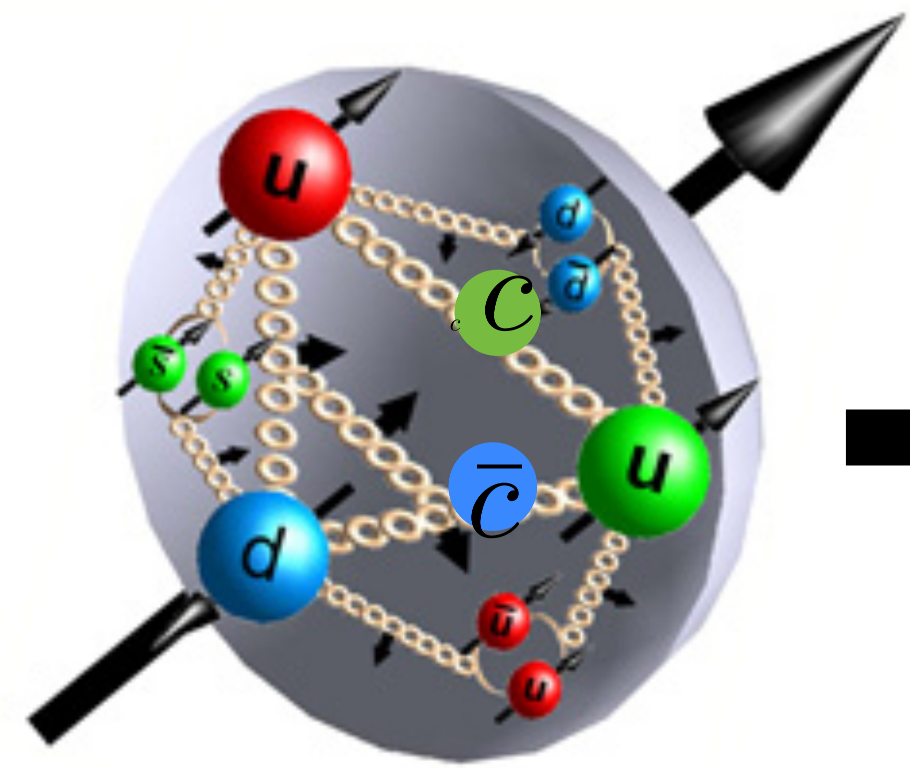
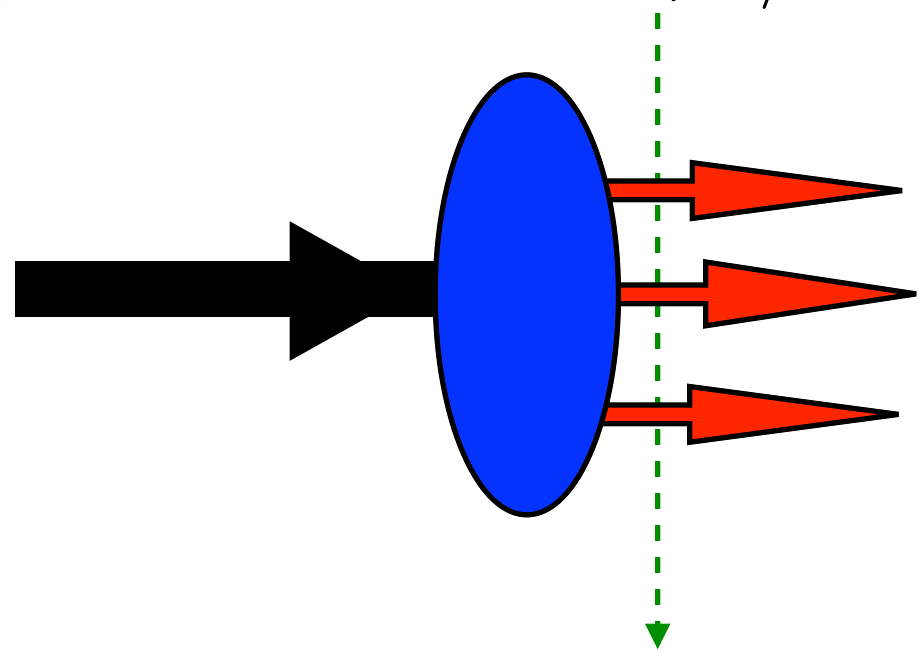
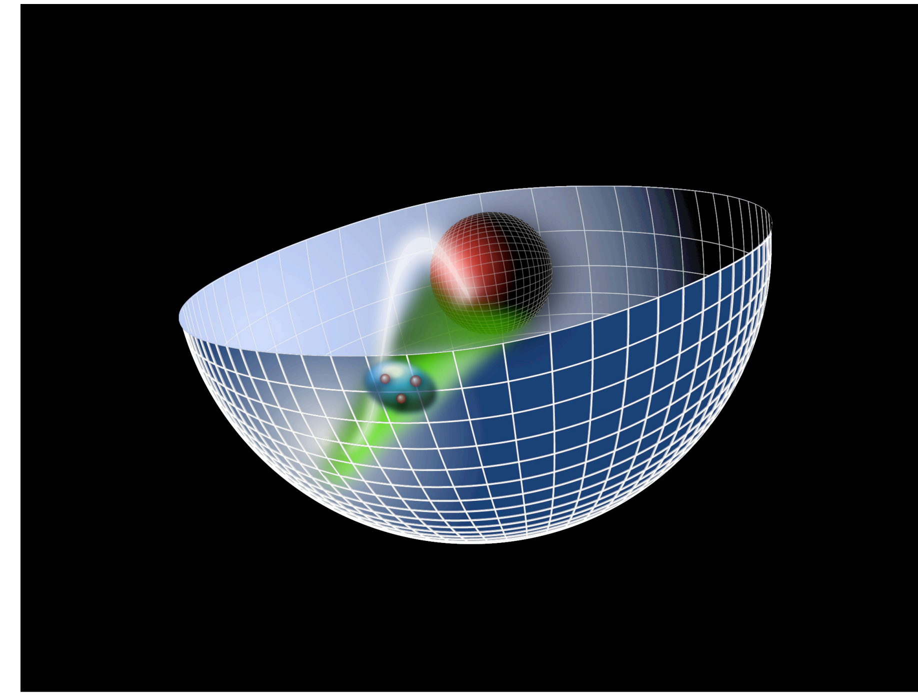
Light-Front Holography and New Advances in Nonperturbative QCD



Fixed $\tau = t + z/c$





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April 13, 2015

Goal: An analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- Confinement in QCD -- What is the analytic form of the confining interaction?
- What sets the QCD mass scale?
- QCD Running Coupling at all scales
- Hadron Spectroscopy-Regge Trajectories
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- Constituent Counting Rules
- Hadronization at the Amplitude Level
- Insights into QCD Condensates
- Chiral Symmetry
- Systematically improvable

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics

quark kinetic energy + quark-gluon dynamics

quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classically Conformal if m_q=0

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement

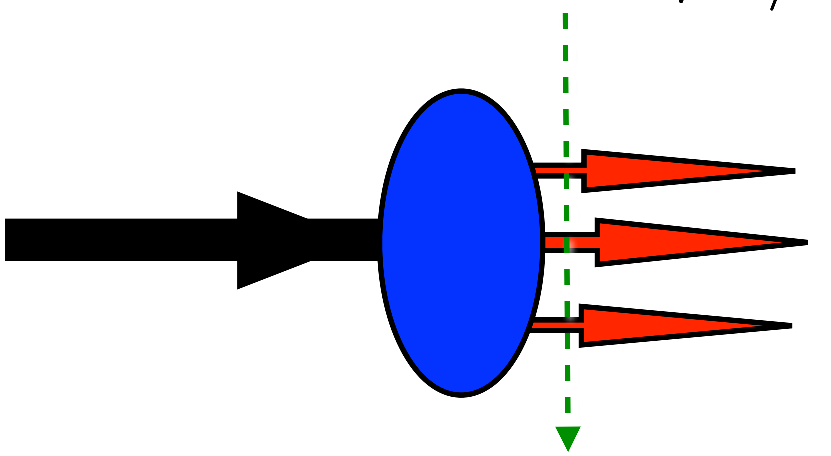
QCD Mass Scale from Confinement not Explicit

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

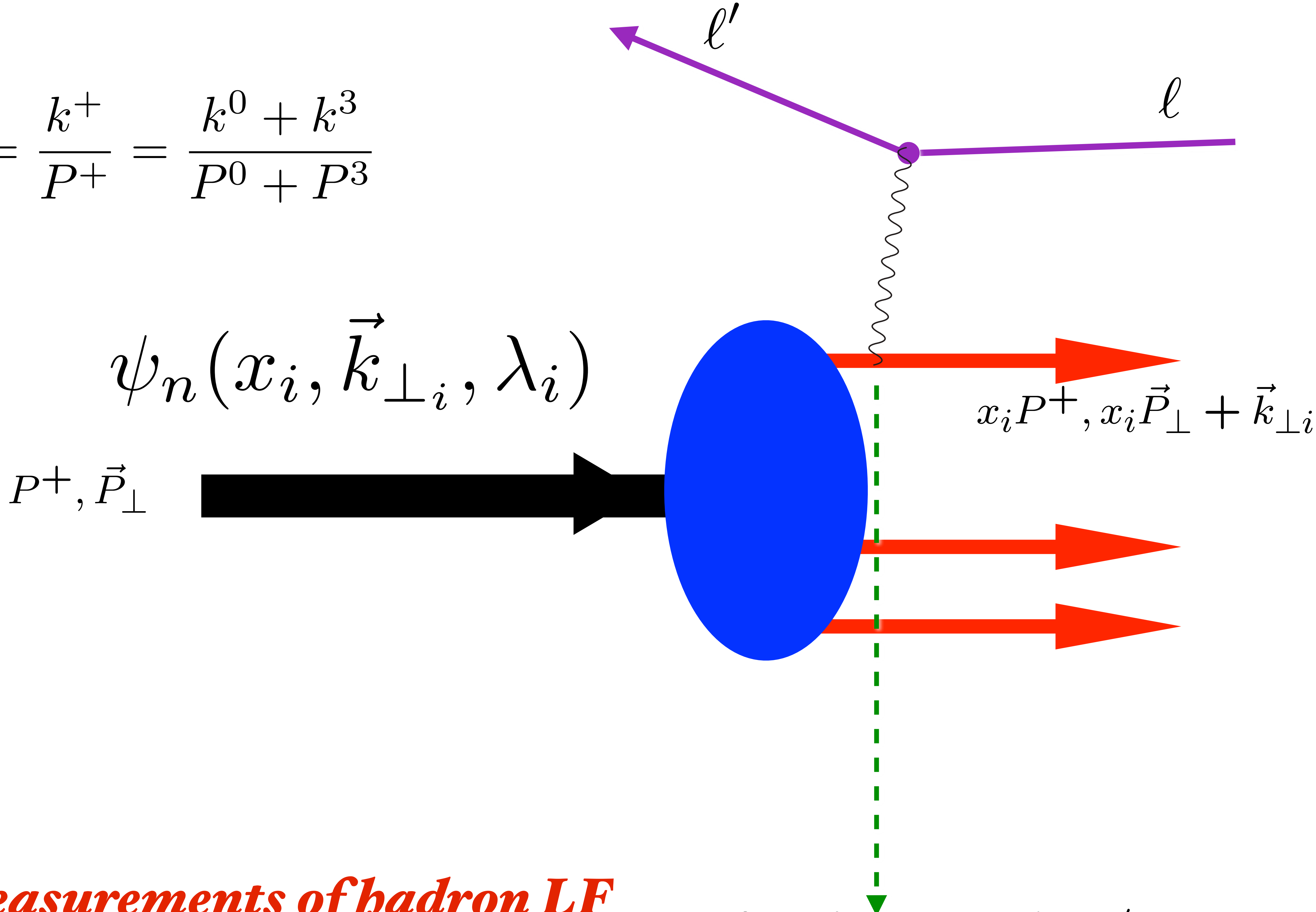
$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



Fixed $\tau = t + z/c$

Measurements of hadron LF wavefunction are at fixed LF time

$$x_{bj} = x = \frac{k^+}{P^+}$$

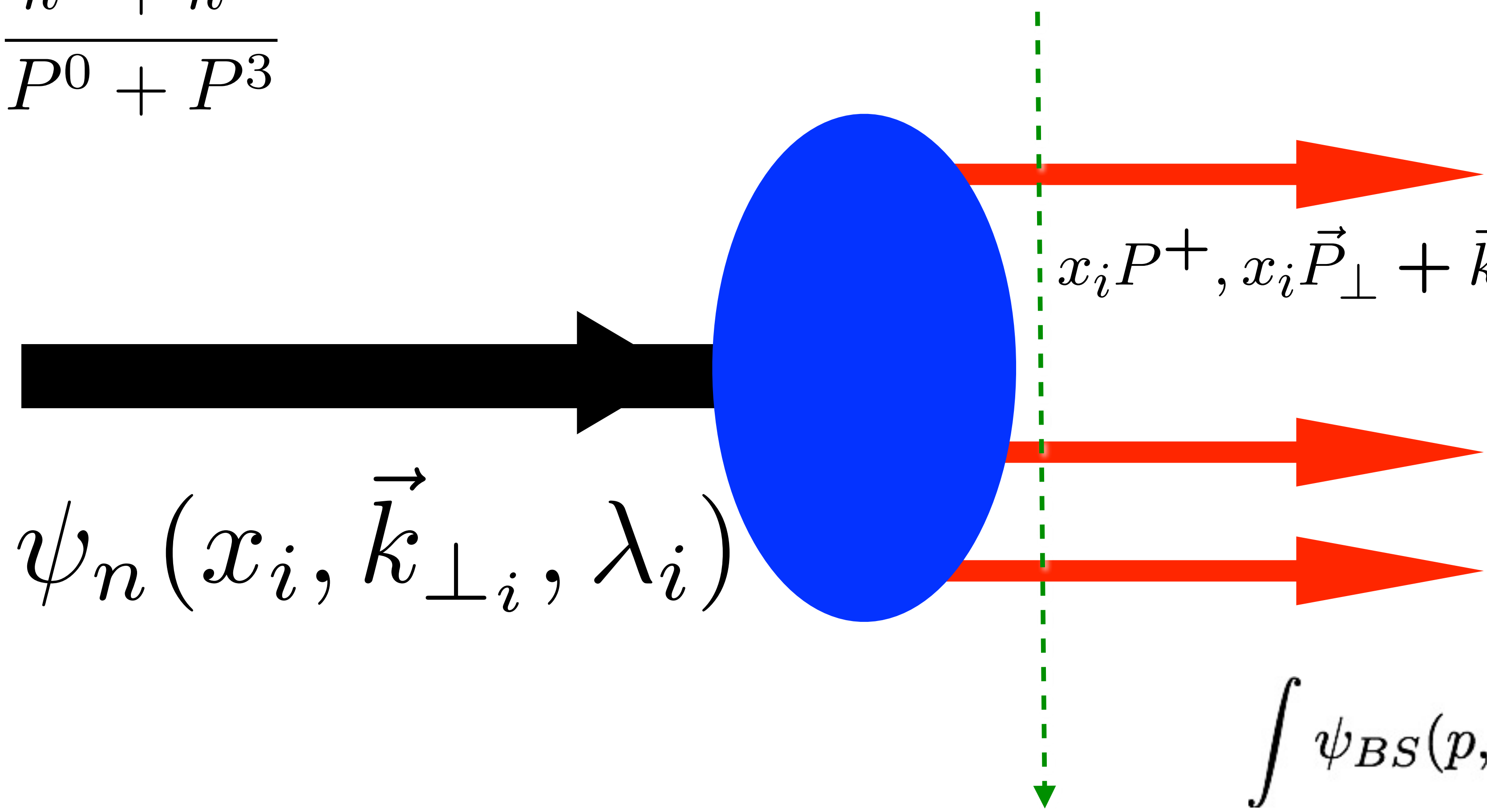
Like a flash photograph

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$P^+, \vec{P}_\perp$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

$$\int \psi_{BS}(p,k) dk^- \rightarrow \psi_{LF}$$

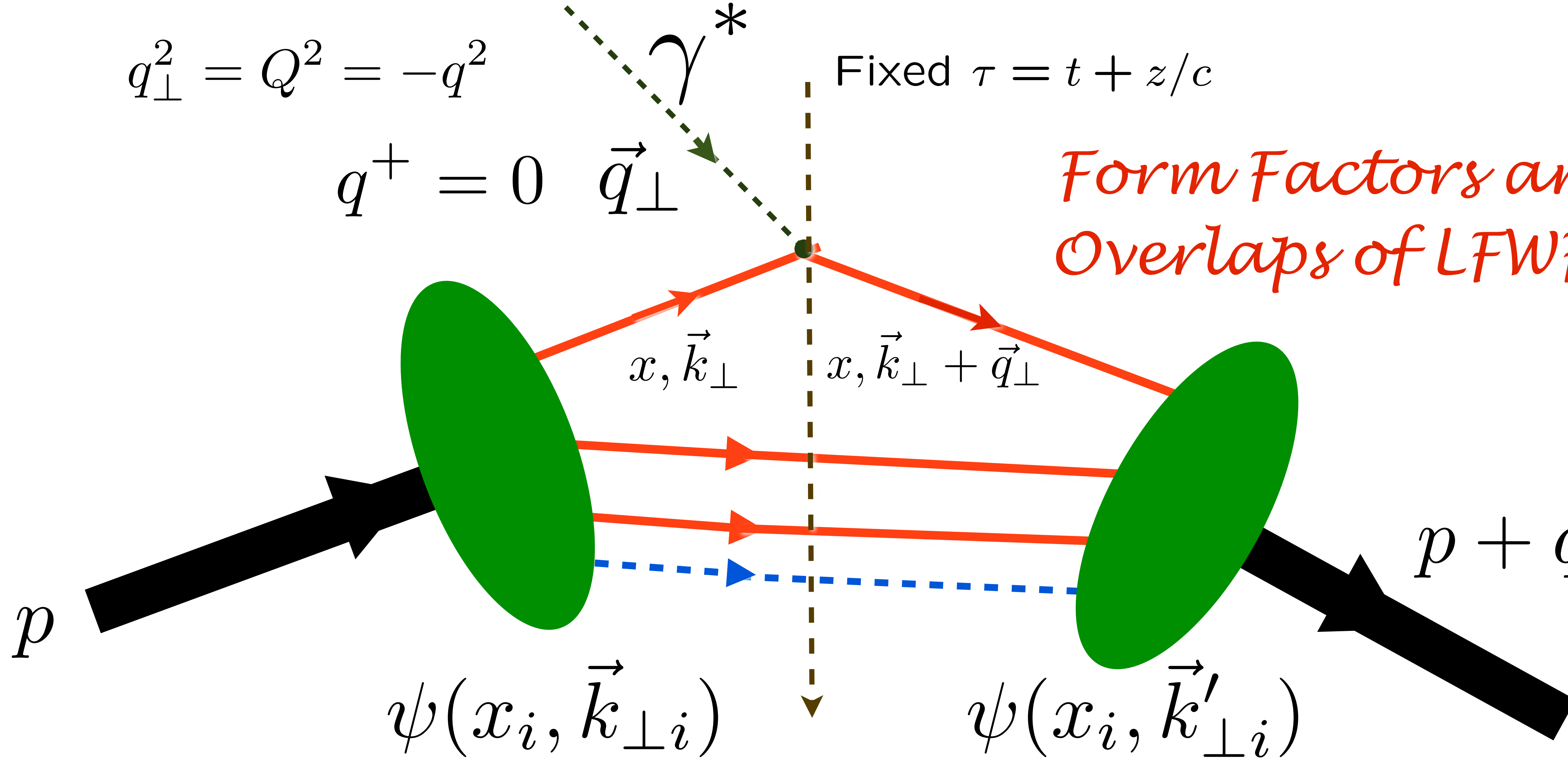
$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; x_i, \vec{k}_{\perp i}, \lambda_i >$$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

$$< p+q|j^+(0)|p > = 2p^+F(q^2)$$

Interaction picture



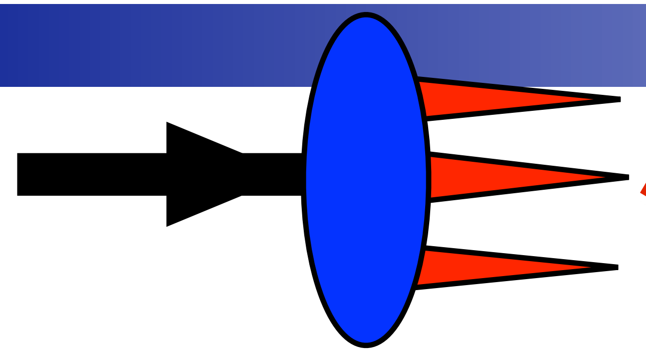
struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

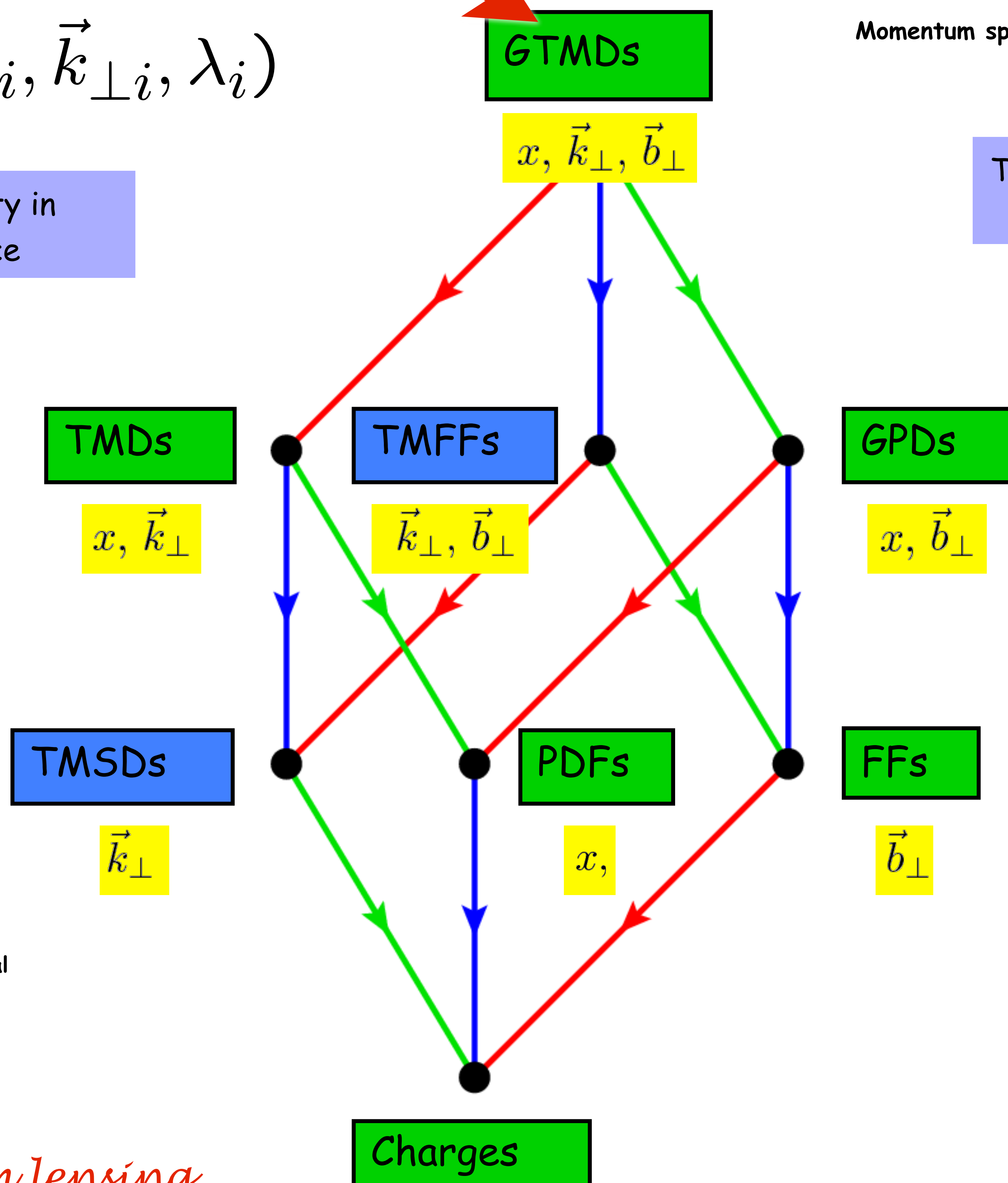
No comparable formula in instant form

- *Light Front Wavefunctions:*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

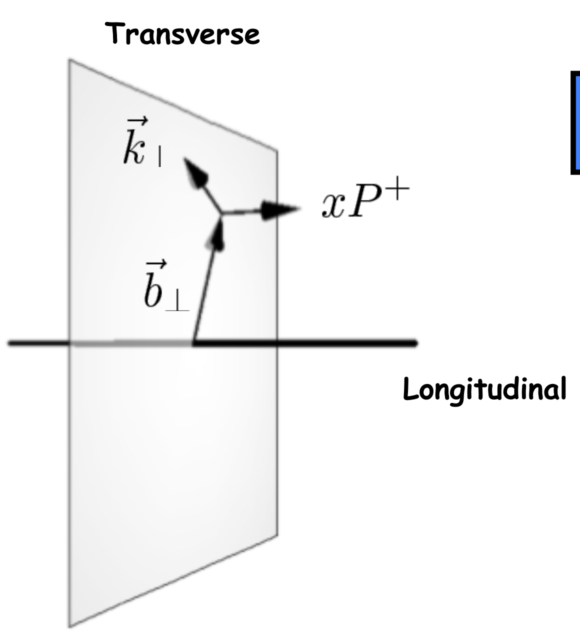


Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

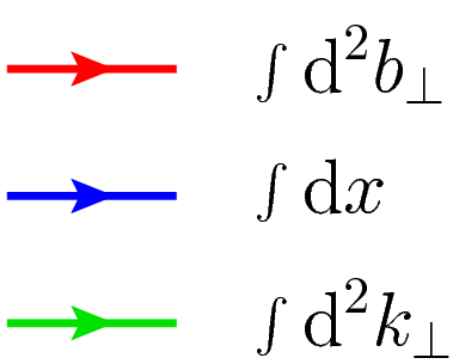
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in position space

Lorce, Pasquini



Sivers, T-odd from lensing



Advantages of the Dirac's Front Form for Hadron Physics

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent -- no boosts!
- No dependence on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no condensates!
- Profound implications for Cosmological Constant

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

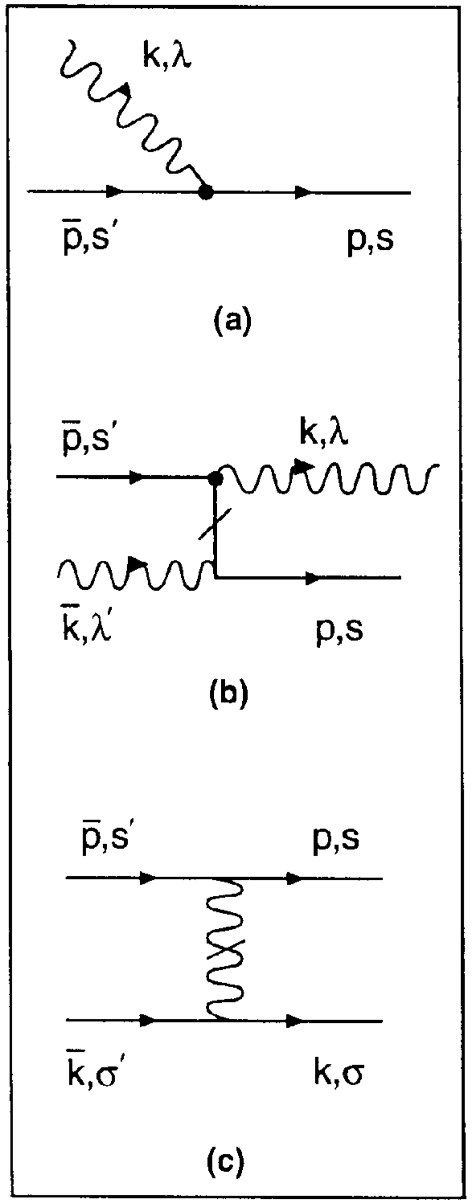
$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H_{LF}^{int}$$

H_{LF}^{int}: Matrix in Fock Space

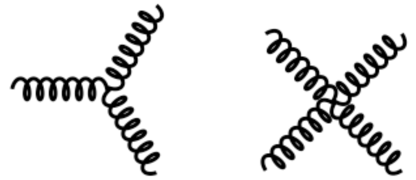
$$H_{LF}^{QCD} |\Psi_h> = \mathcal{M}_h^2 |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$



Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

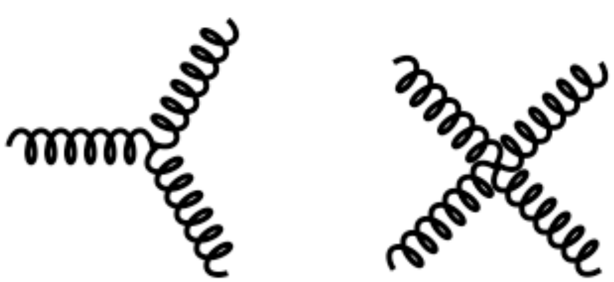
LFWFs: Off-shell in P- and invariant mass

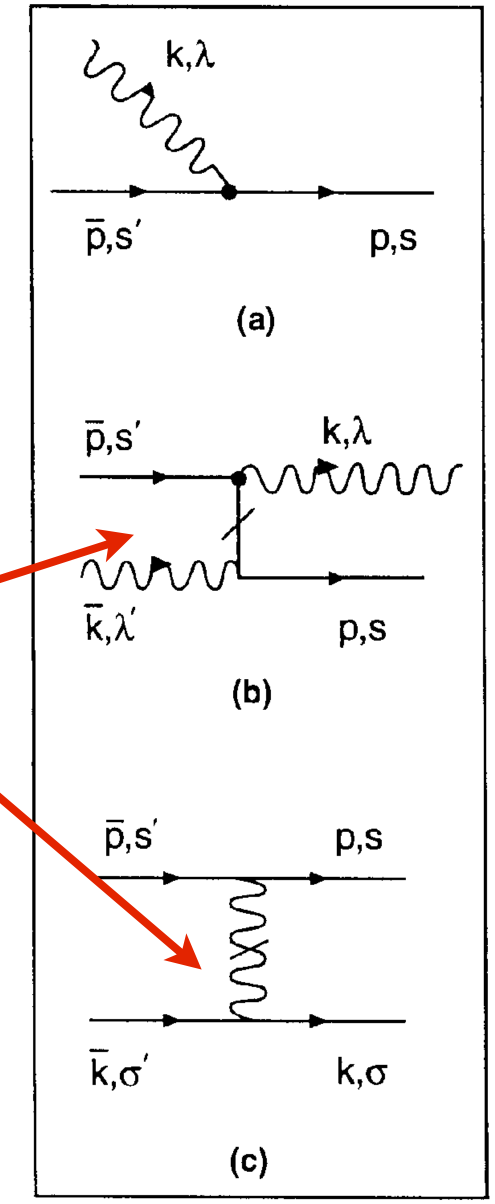


H_{LF}^{int}

$$\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$\begin{aligned}
H^{LF}_{QCD} &= \frac{1}{2}\int d^3x \bar{\tilde{\psi}}\gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+}\tilde{\psi} - A^i_a (i\partial^\perp)^2 A_{ia} \\
&- \frac{1}{2} g^2 \int d^3x \mathbf{Tr}\left[\tilde{A}^\mu, \tilde{A}^\nu\right]\left[\tilde{A}_\mu, \tilde{A}_\nu\right] \\
&+ \frac{1}{2} g^2 \int d^3x \bar{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \frac{1}{(i\partial^+)^2} \bar{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \\
&- g^2 \int d^3x \bar{\tilde{\psi}}\gamma^+ \left(\frac{1}{(i\partial^+)^2}\left[i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right)\tilde{\psi} \\
&+ g^2 \int d^3x \mathbf{Tr}\left(\left[i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\frac{1}{(i\partial^+)^2}\left[i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right) \\
&+ \frac{1}{2} g^2 \int d^3x \bar{\tilde{\psi}} \tilde{\not{A}} \frac{\gamma^+}{i\partial^+} \tilde{\not{A}} \tilde{\psi} \\
&+ g \int d^3x \bar{\tilde{\psi}} \tilde{\not{A}} \tilde{\psi} \\
&+ 2g \int d^3x \mathbf{Tr}\left(i\partial^\mu \tilde{A}^\nu \left[\tilde{A}_\mu, \tilde{A}_\nu\right]\right)
\end{aligned}$$





Physical gauge: $A^+ = 0$

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; \vec{k}_{\perp_i}, \lambda_i >$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

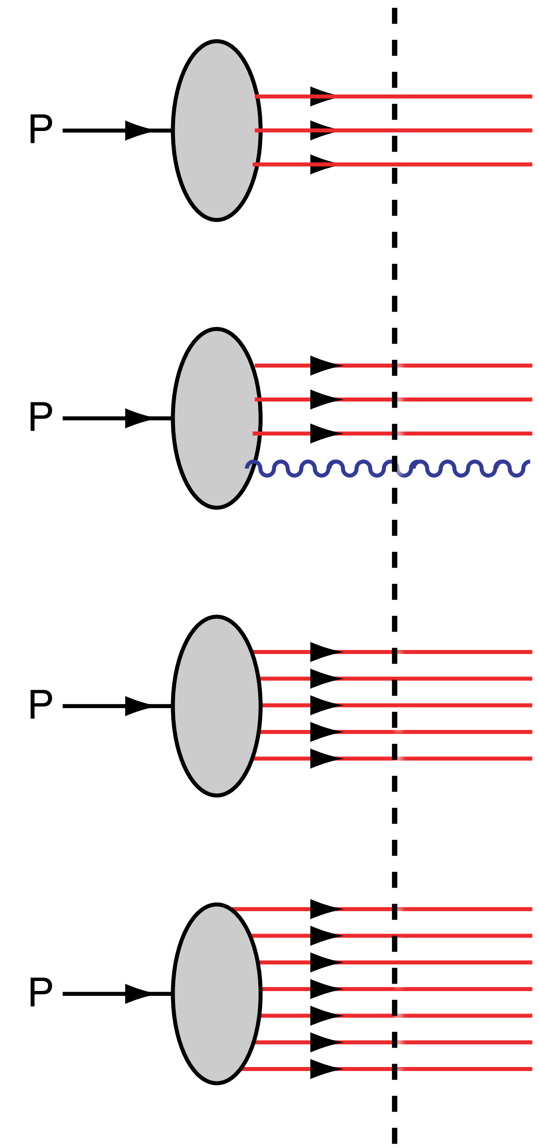
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

Mueller: gluon Fock states **BFKL Pomeron**

Hidden Color

Atomic Physics from First Principles

$\mathcal{L}_{QED} \rightarrow$

$$H_{QED}$$

QED atoms: positronium and muonium

$\downarrow$

$$(H_0 + H_{int}) \; |\Psi >= E \; |\Psi >$$

Coupled Fock states

$\downarrow$

Eliminate higher Fock states and retarded interactions

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})] \; \psi(\vec{r}) = E \; \psi(\vec{r})$$

Effective two-particle equation

$\downarrow$

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)] \; \psi(r) = E \; \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

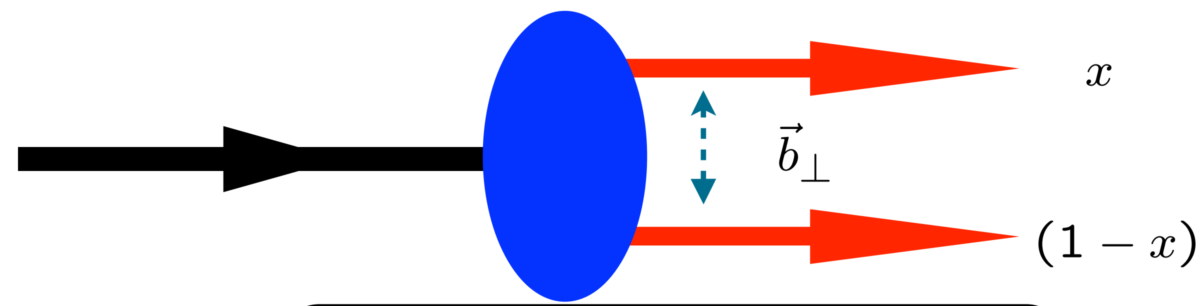
Coulomb potential

Semiclassical first approximation to QED --> **Bohr Spectrum**

Light-Front QCD

Fixed $\tau = t + z/c$

$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$



$$\zeta^2 = x(1-x)b^2_\perp$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Azimuthal Basis

$$\zeta, \phi$$

$$m_q = 0$$

AdS/QCD:

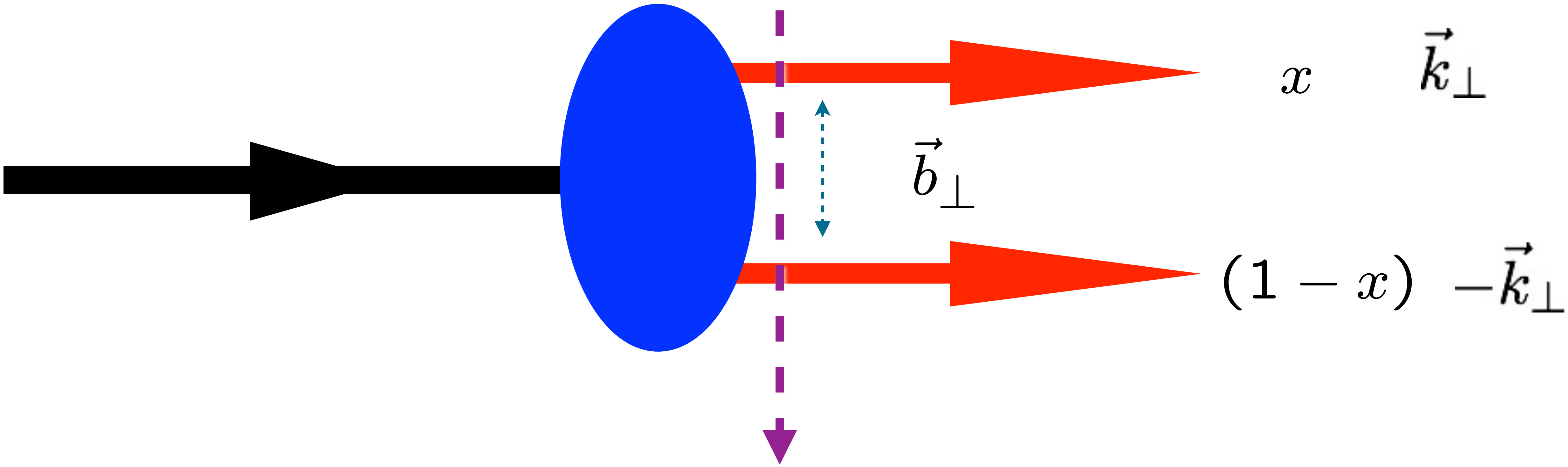
$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD

Fixed $\tau = t + z/c$



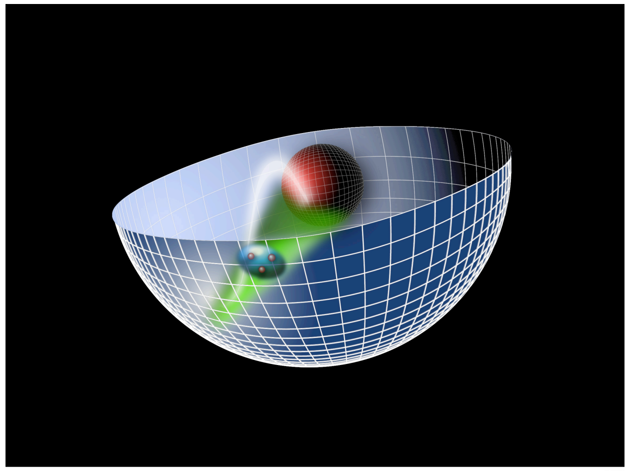
$$\zeta^2 \equiv b_\perp^2 x(1-x)$$

Invariant transverse separation

$$\zeta^2 \text{ conjugate to } \frac{k_\perp^2}{x(1-x)} = (p_q + p_{\bar{q}})^2 = \mathcal{M}_{q+\bar{q}}^2$$

$$\int dk^- \Psi_{BS}(P,k) \to \psi_{LF}(x, \vec{k}_\perp)$$

de Tèramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

Light-Front Holography

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6 \; GeV$$

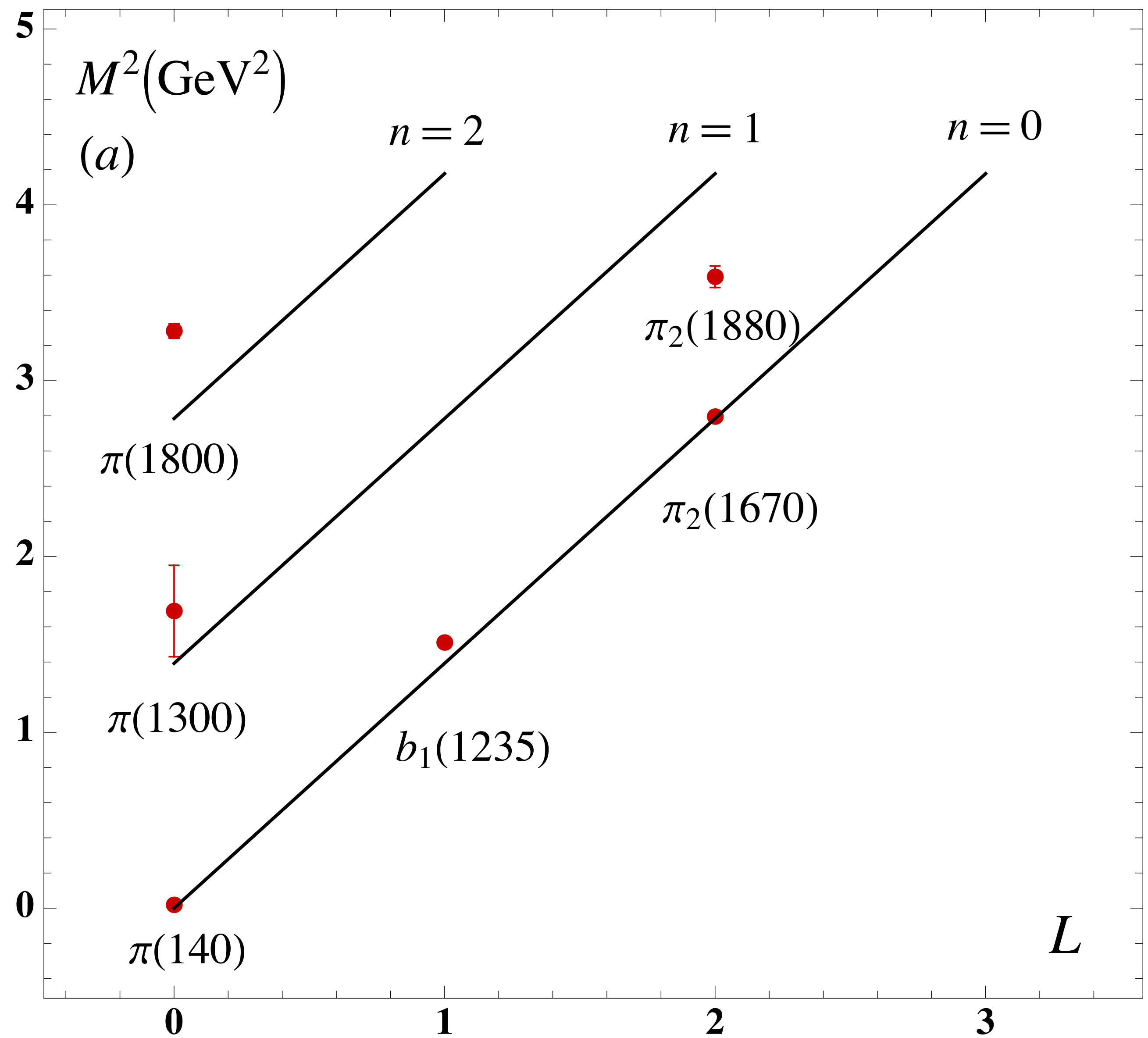
$$1/\kappa \simeq 1/3 \; fm$$

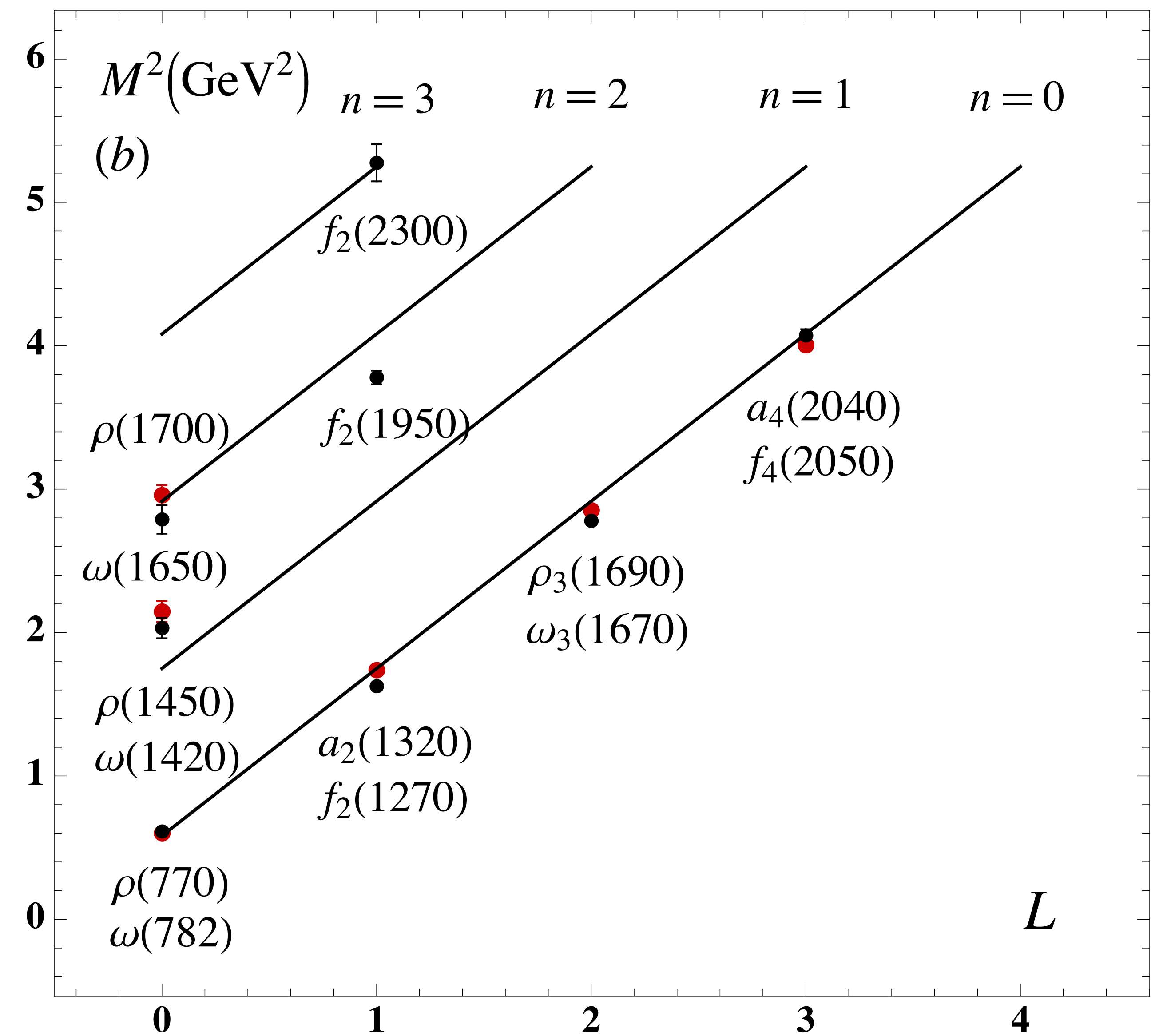
- **de Alfaro, Fubini, Furlan:**
  - **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

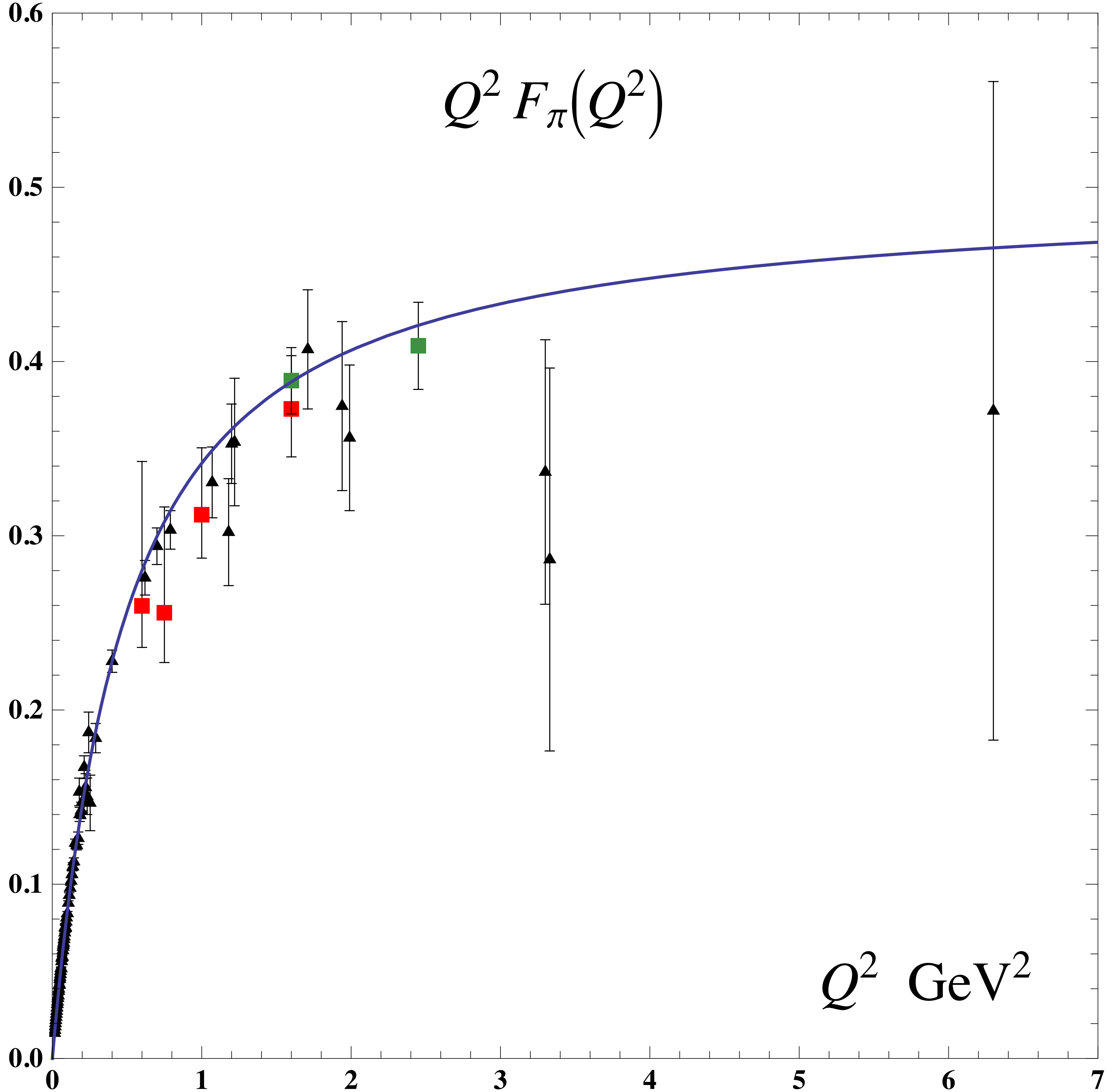
$$m_u = m_d = 0$$

Preview

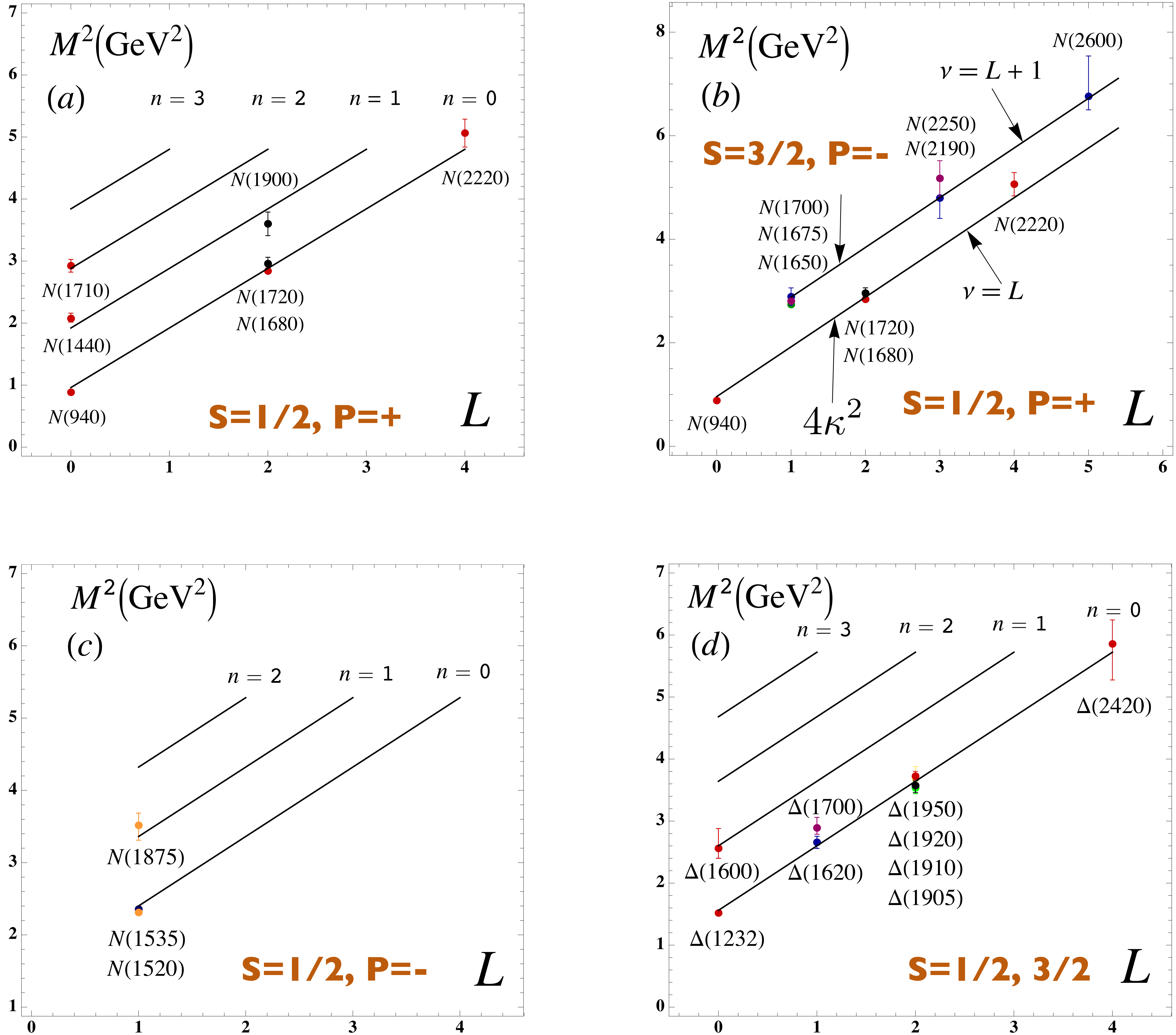




$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$



$Q^2\,F_\pi(Q^2)$
Q^2 GeV^2
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7



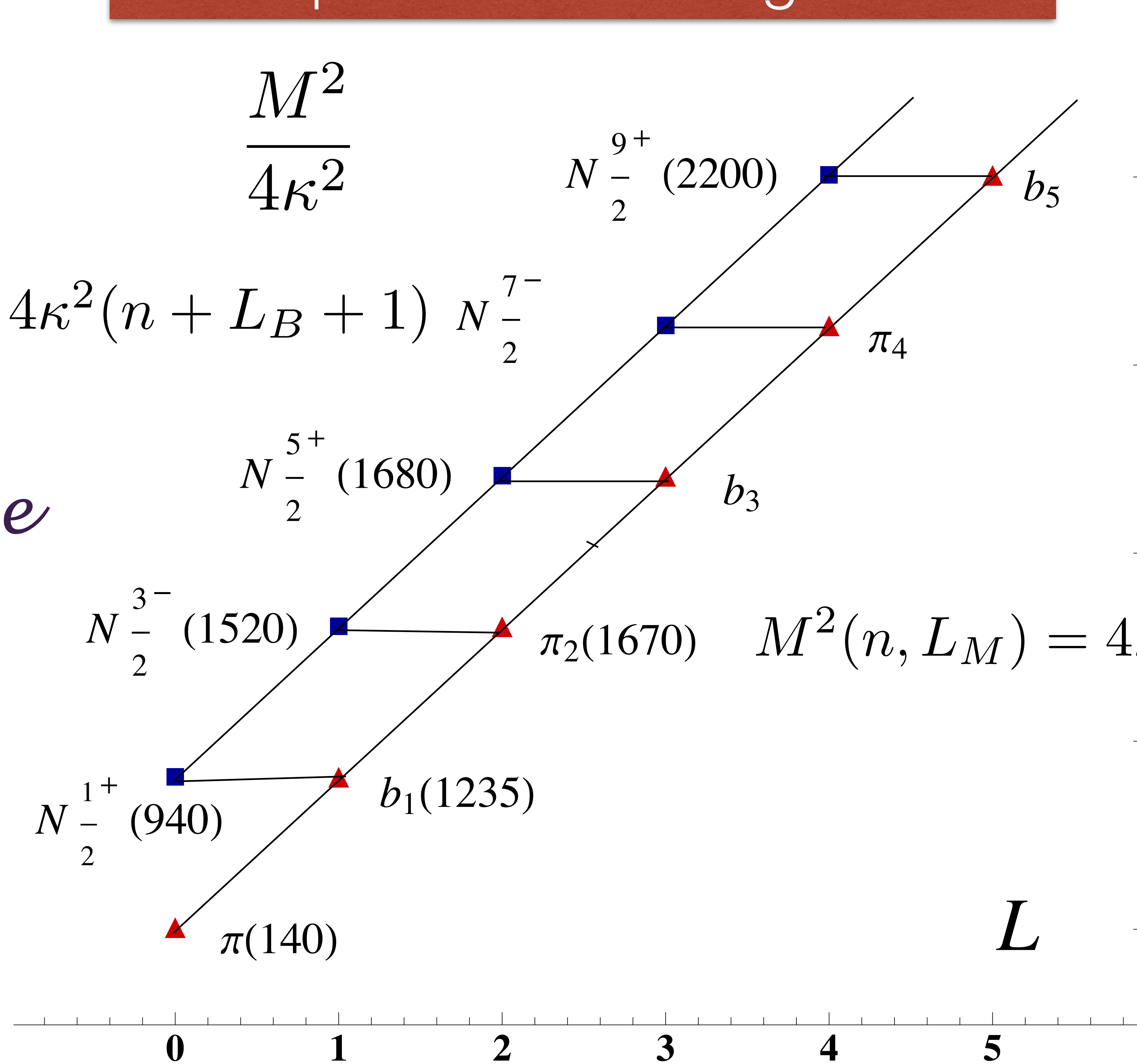
(a)
$M^2(\mathrm{GeV}^2)$
$n=3$ $n=2$ $n=1$ $n=0$
N(1900)
N(2220)
N(1710)
N(1440)
N(940)
N(1720)
N(1680)
S=1/2, P=+
L
(b)
$M^2(\mathrm{GeV}^2)$
$\nu = L+1$
$\nu = L$
N(2600)
N(2250)
N(2190)
S=3/2, P=-
N(1700)
N(1675)
N(1650)
N(2220)
N(1720)
N(1680)
N(940)
$4\kappa^2$
S=1/2, P=+
L
(c)
$M^2(\mathrm{GeV}^2)$
$n=2$ $n=1$ $n=0$
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
(d)
$M^2(\mathrm{GeV}^2)$
$n=3$ $n=2$ $n=1$ $n=0$
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L

Superconformal Algebra

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

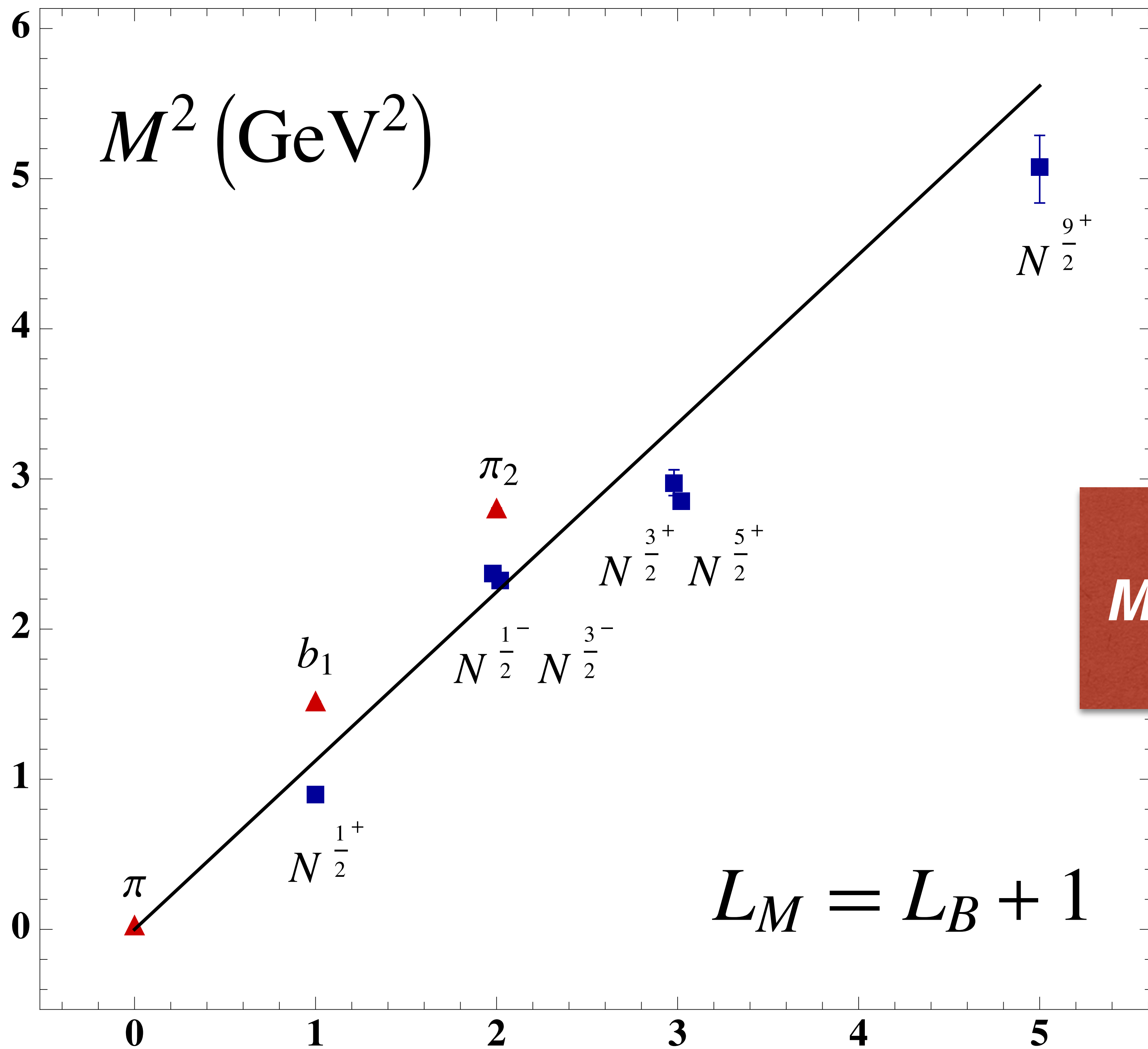
Same slope

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$



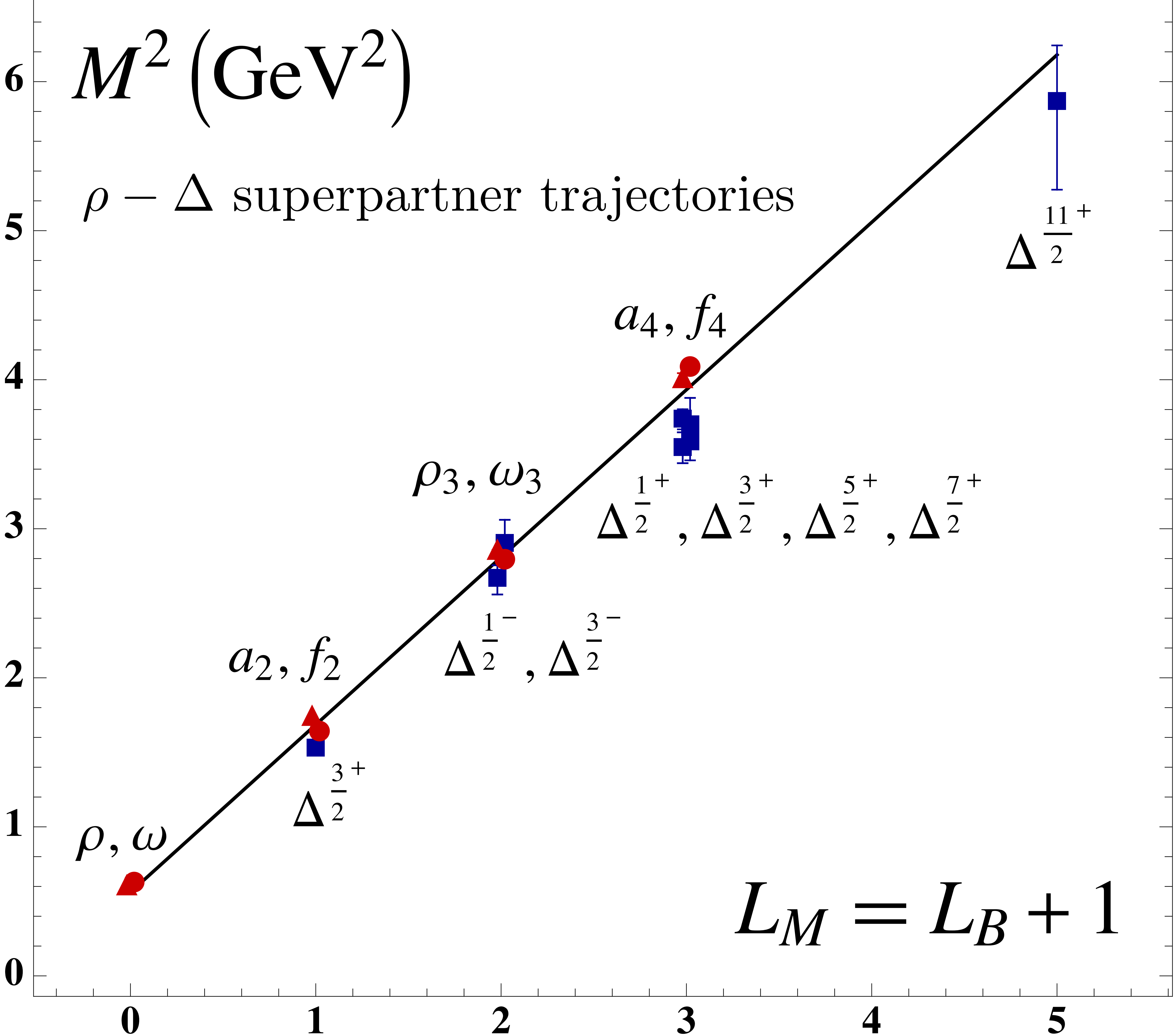
Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!



Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon



Dosch, de Teramond, sjb

Some Features of AdS/QCD

- ***Regge spectroscopy—same slope in n,L for mesons,***
- ***Chiral features for m_q=0: m_π =0, chiral-invariant proton***
- ***Hadronic LFWFs***
- ***Counting Rules***
- ***Connection between hadron masses and $\Lambda_{\overline{MS}}$***

Superconformal AdS Light-Front Holographic QCD (LFHQCD)

Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

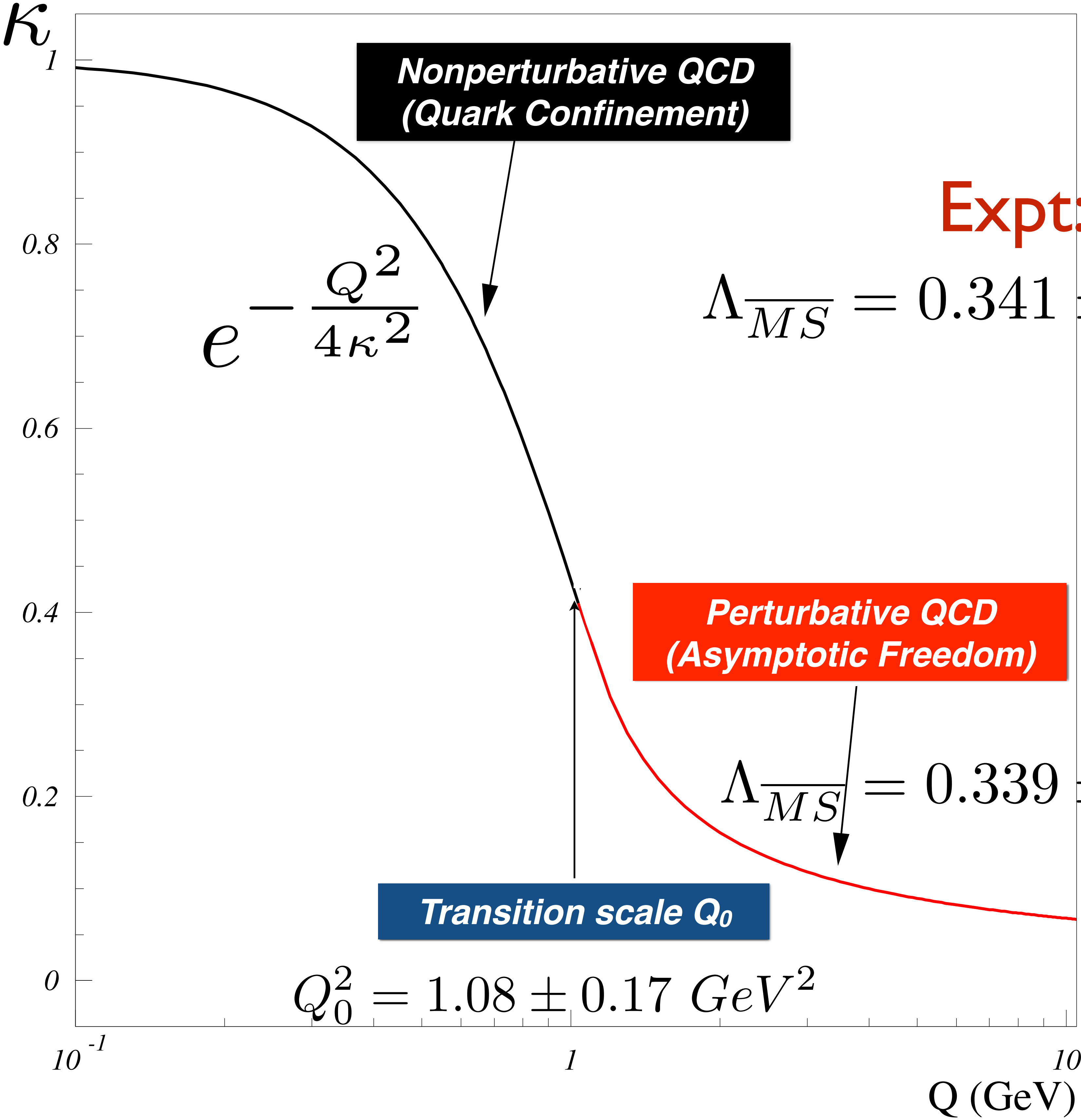
$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

All-Scale QCD Coupling

Deur, de Tèramond, sjb

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



Expt:

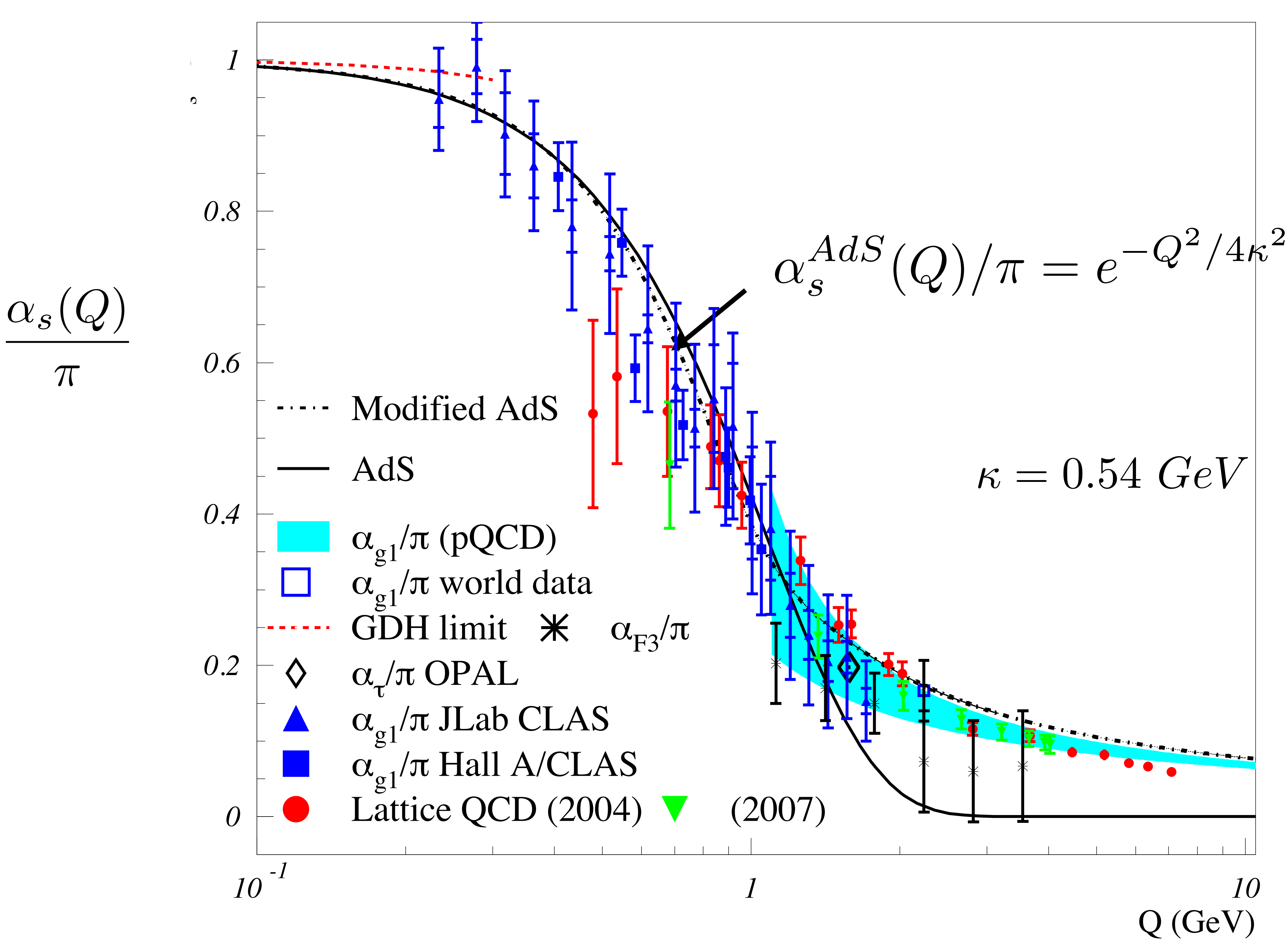
$$\Lambda_{\overline{MS}} = 0.341 \pm 0.024\ GeV$$

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016\ GeV$$

$$Q_0^2 = 1.08 \pm 0.17\ GeV^2$$

$$\lambda \equiv \kappa^2$$

Analytic, defined at all scales, IR Fixed Point



$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Tèramond, sjb

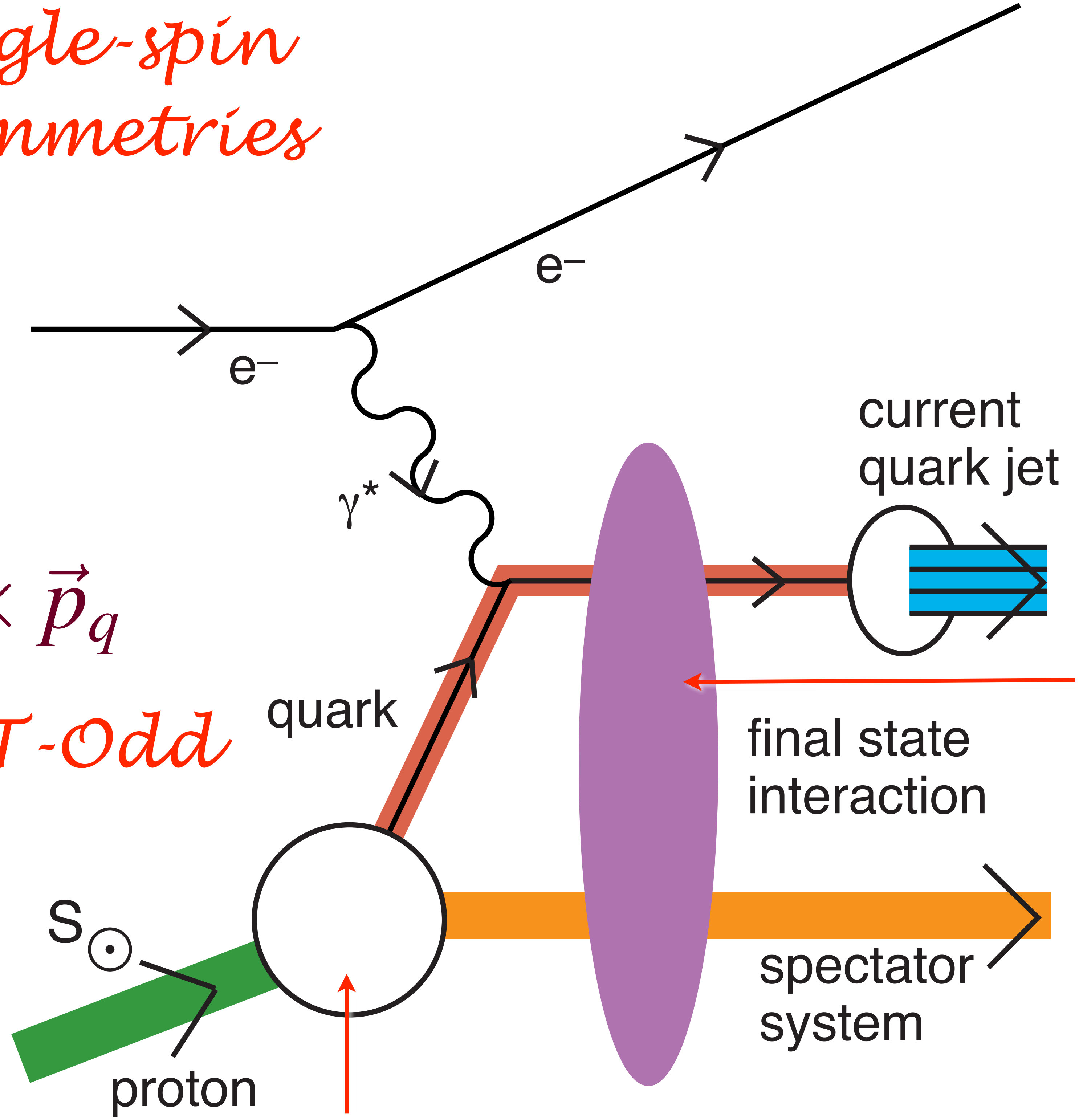
Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd



Light-Front Wavefunction S and P- Waves!

Collins, Burkardt, Ji, Yuan. Pasquini, ...

QCD S- and P- Coulomb Phases --Wilson Line

"Lensing Effect"

"Lensing" involves soft scales

Sign reversal in DY!

Leading-Twist Rescattering Violates pQCD Factorization!

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu,\ z \to \lambda z$, maps scale transformations into the holographic coordinate z.

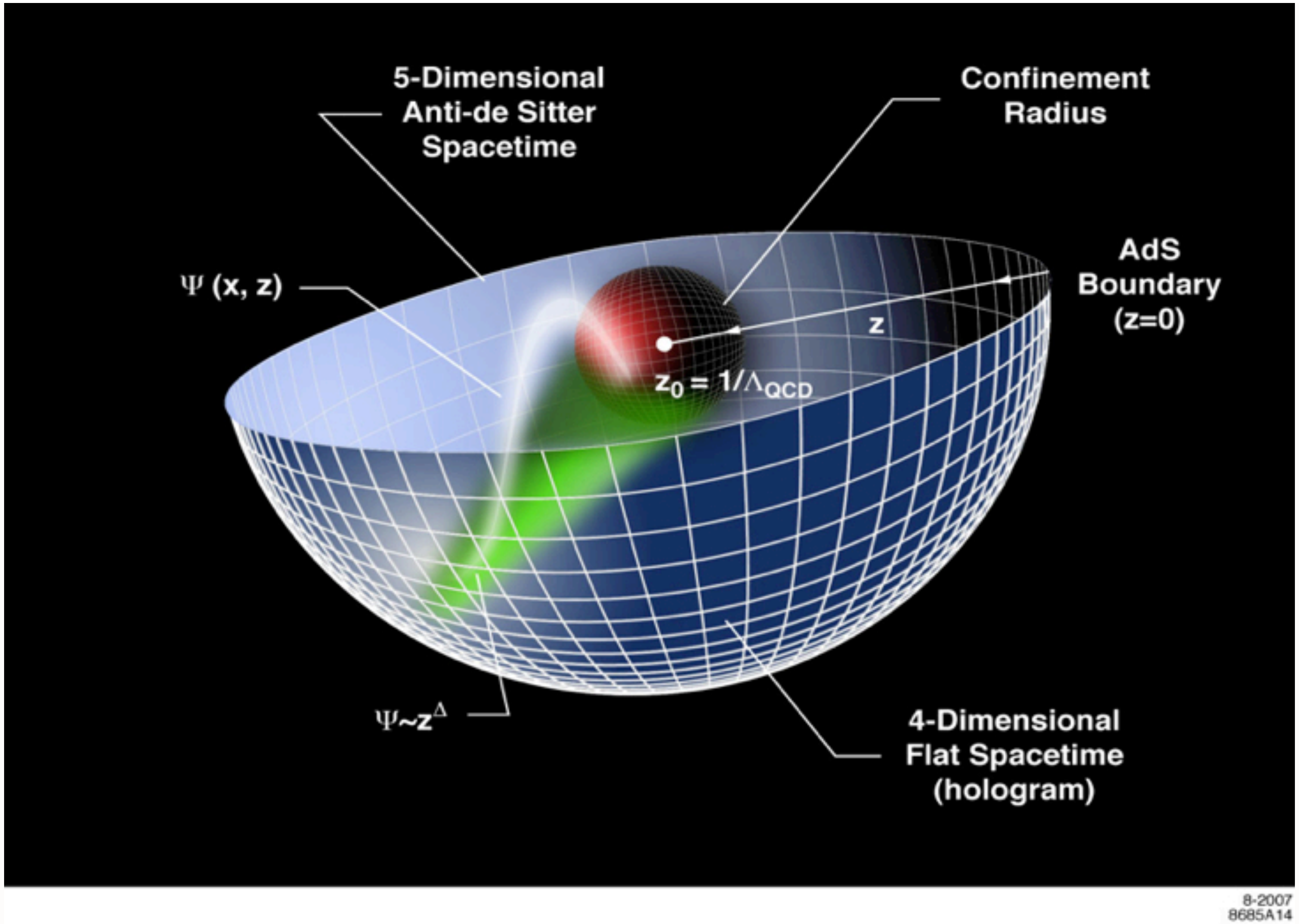
- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.



Changes in physical length scale mapped to evolution in the 5th dimension z

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale** κ
- **Uses AdS_5 as template for conformal theory**

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

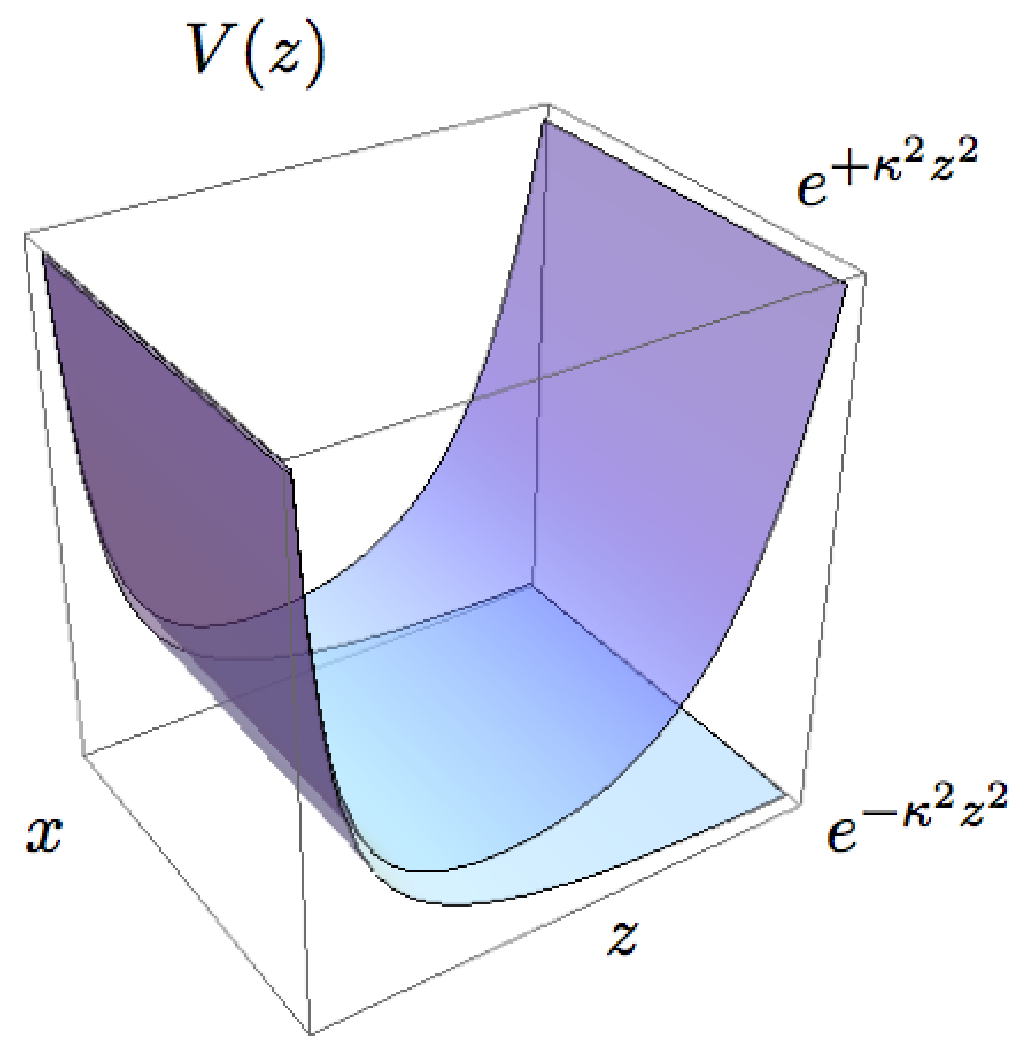
where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$

- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1} x \sqrt{g}\, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g}\, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x, z) = e^{-iP \cdot x}\, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z \mathcal{M}) \qquad \Delta = \tfrac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Light-Front Bound State Equation!

$$z \qquad \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

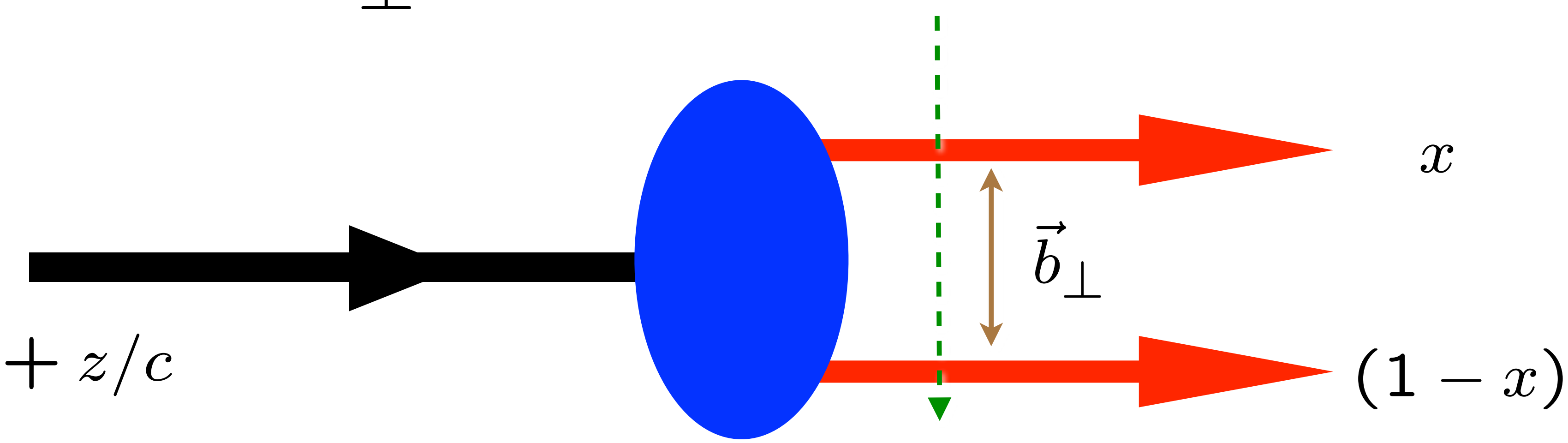
de Teramond, sjb

LF(3+1) ⟷ AdS_5

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



Fixed $\tau = t + z/c$

$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

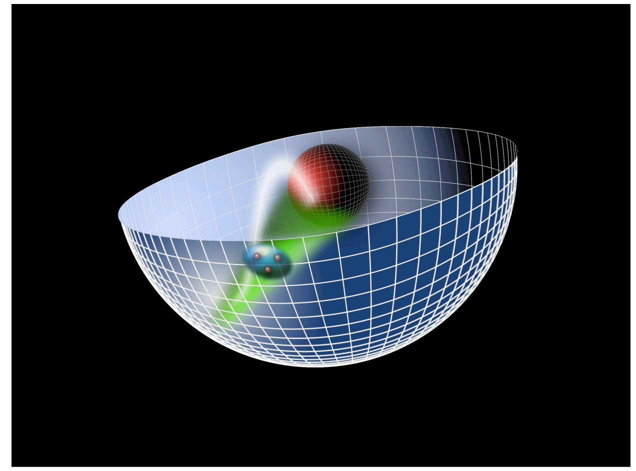
$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

de Tèramond, Dosch, sjb

AdS/QCD Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:**
  - **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

de Tèramond, Dosch, sjb

General-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

we find the LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L+S-1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}$$

with $(\mu R)^2 = -(2-J)^2 + L^2$

Meson Spectrum in Soft Wall Model

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta \, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

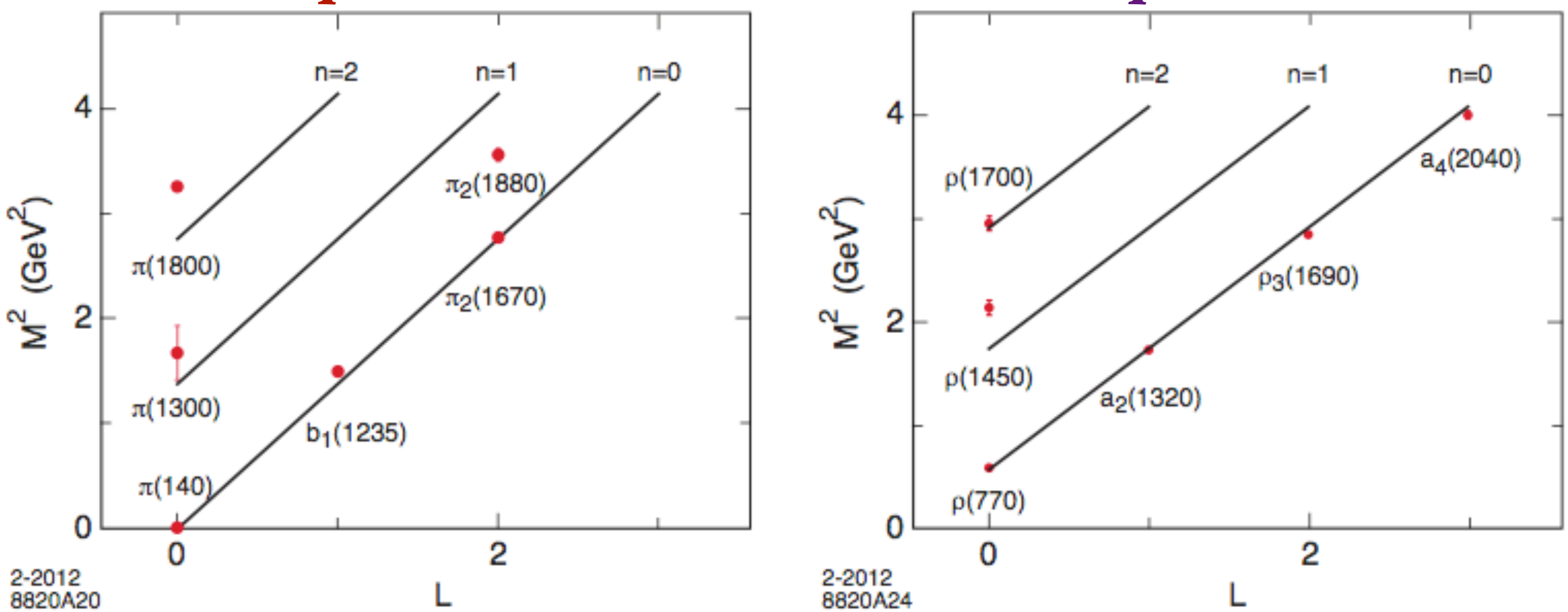
G. de Teramond, H. G. Dosch, sjb

- $J = L + S, I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2\,(n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$$m_q = 0$$

Massless pion in Chiral Limit! ***Same slope in n and L!***



I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

G. de Teramond, H. G. Dosch, sjb

Quark separation increases with L

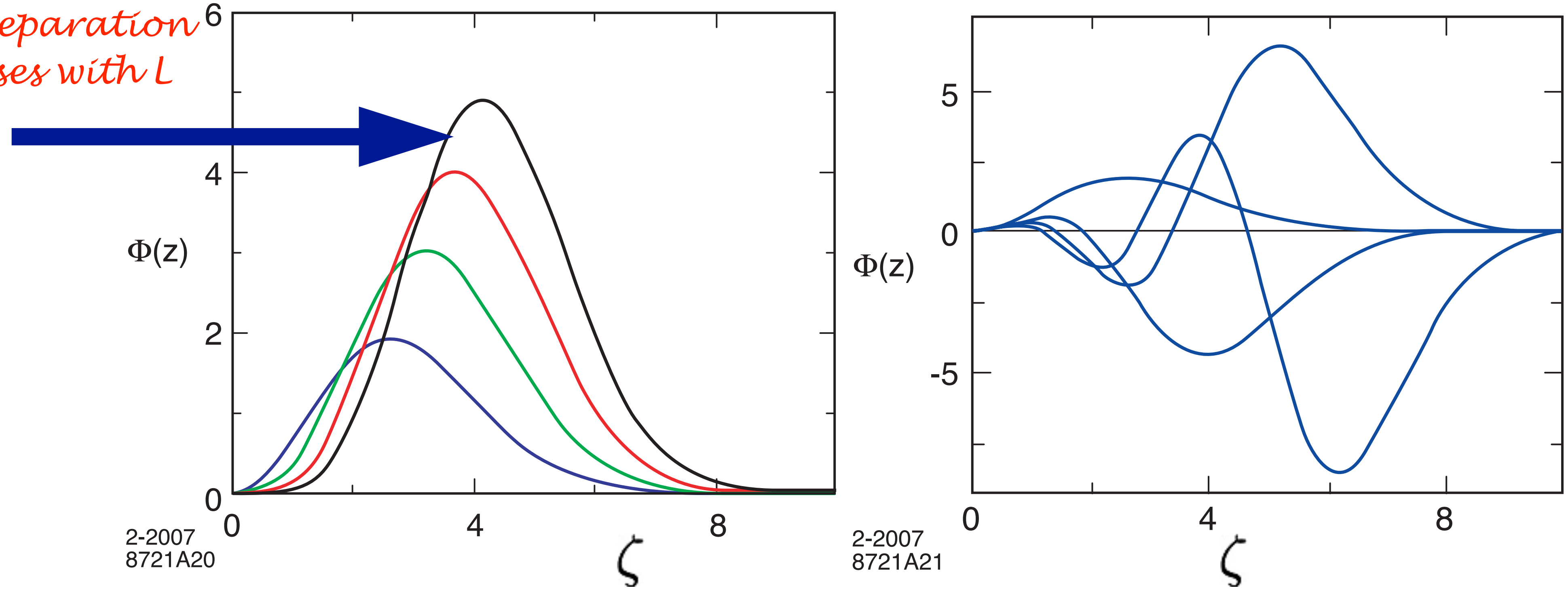
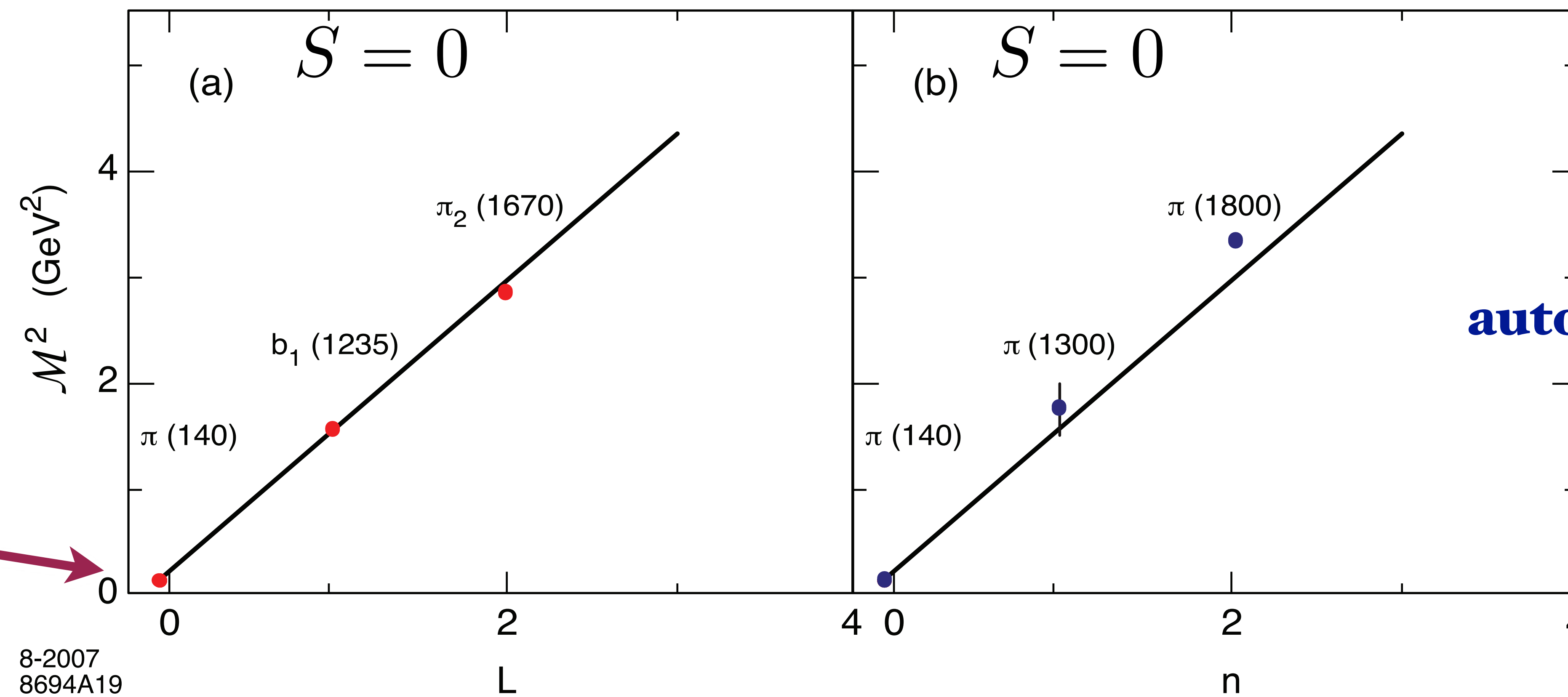


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Same slope in n and L!

Soft Wall Model

Pion has zero mass!



Pion mass automatically zero!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

De Teramond, Dosch, sjb

- Results easily extended to light quarks masses (Ex: K-mesons)

- First order perturbation in the quark masses

$$\Delta M^2 = \langle\psi| \sum_a m_a^2/x_a |\psi\rangle$$

- Holographic LFWF with quark masses

$$\lambda \equiv \kappa^2$$

$$\psi(x,\zeta) \sim \sqrt{x(1-x)}\, e^{-\frac{1}{2\lambda}\left(\frac{m_q^2}{x}+\frac{m_q^2}{1-x}\right)} e^{-\frac{1}{2}\lambda\zeta^2}$$

- Ex: Description of diffractive vector meson production at HERA

  [J. R. Forshaw and R. Sandapen, PRL **109**, 081601 (2012)]

- For the K^*

$$M^2_{n,L,S} = M^2_{K^\pm} + 4\lambda\left(n + \frac{J+L}{2}\right)$$

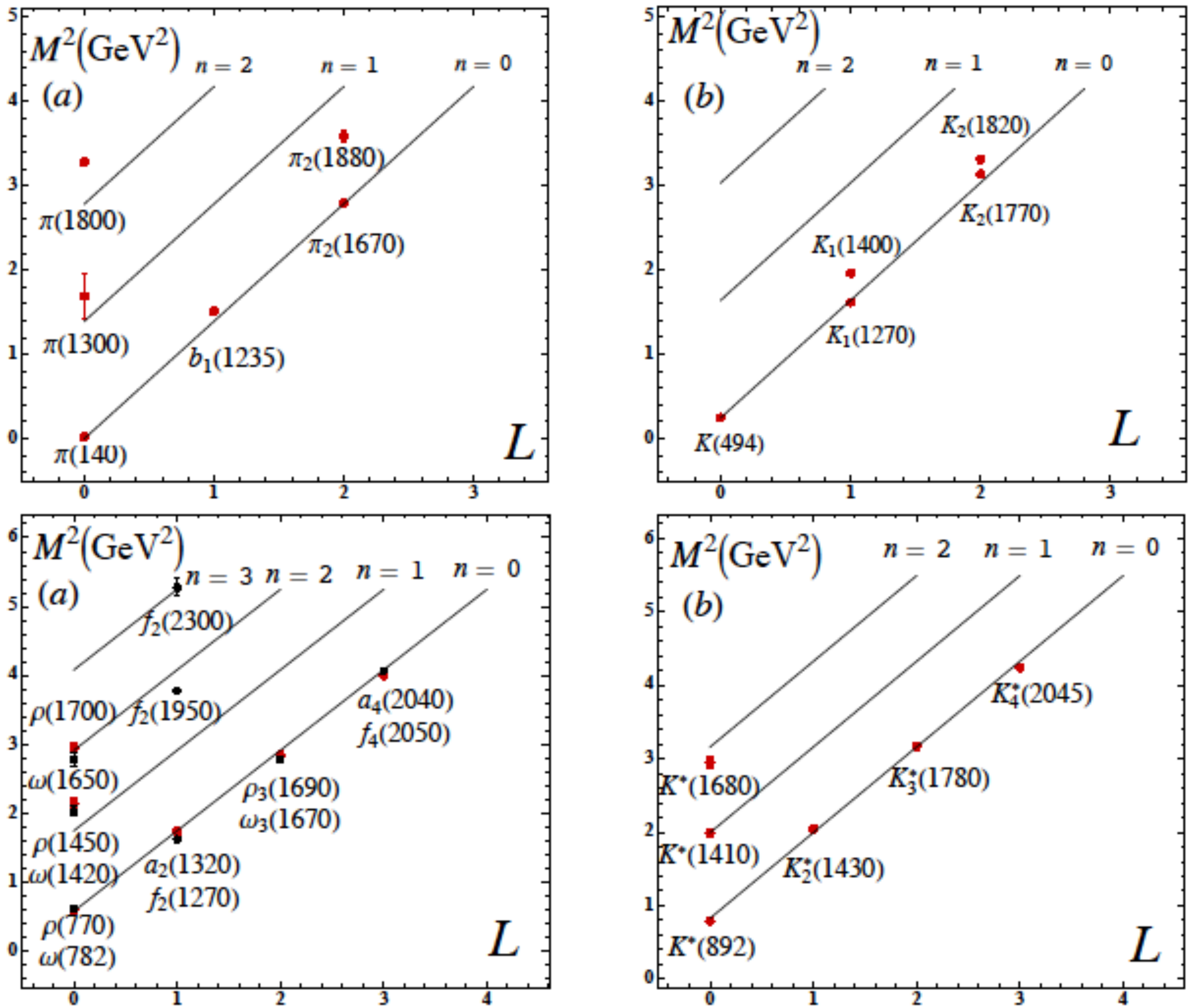
- Effective quark masses from reduction of higher Fock states as functionals of the valence state:

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$

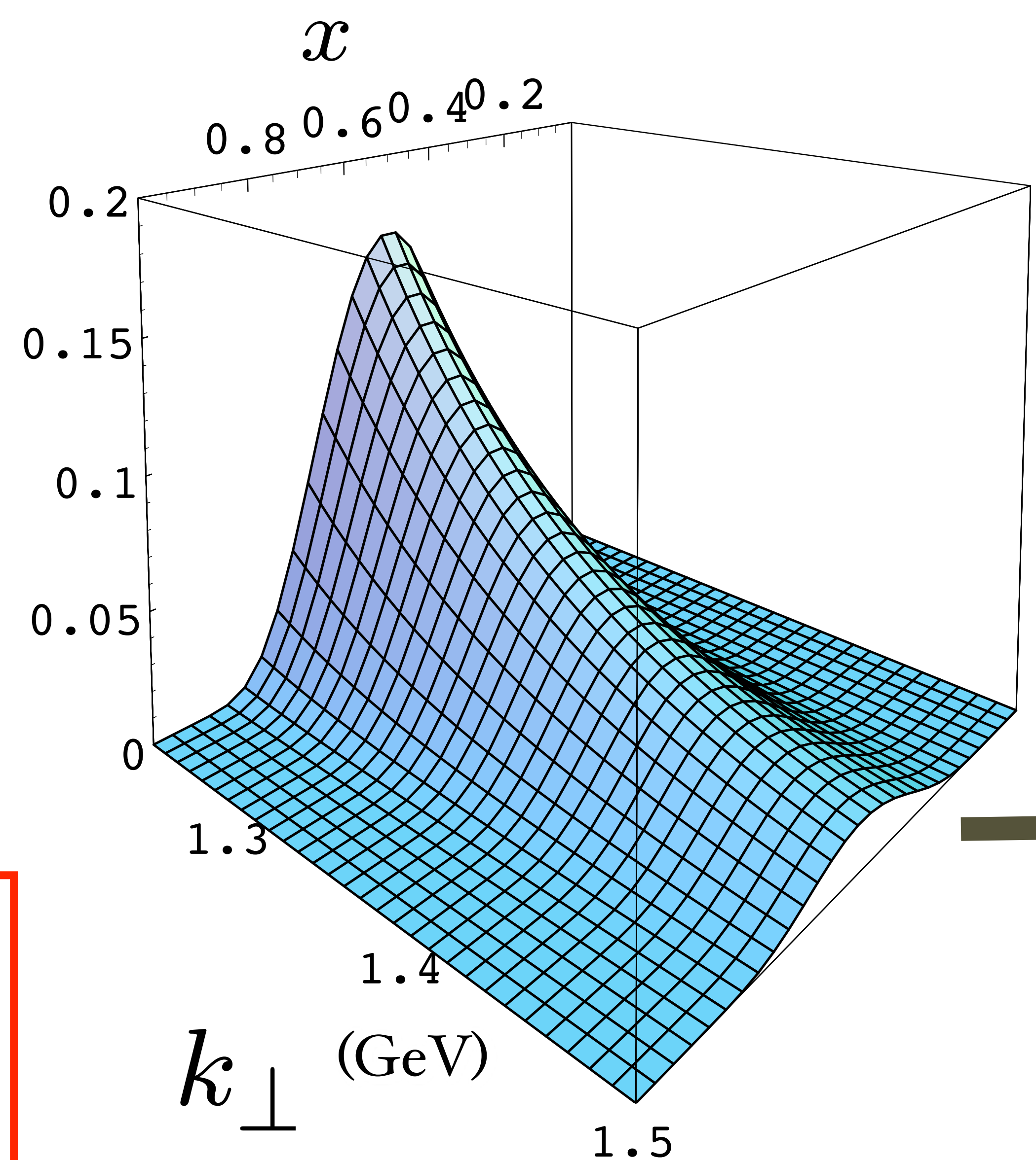


Prediction from AdS/QCD: Meson LFWF

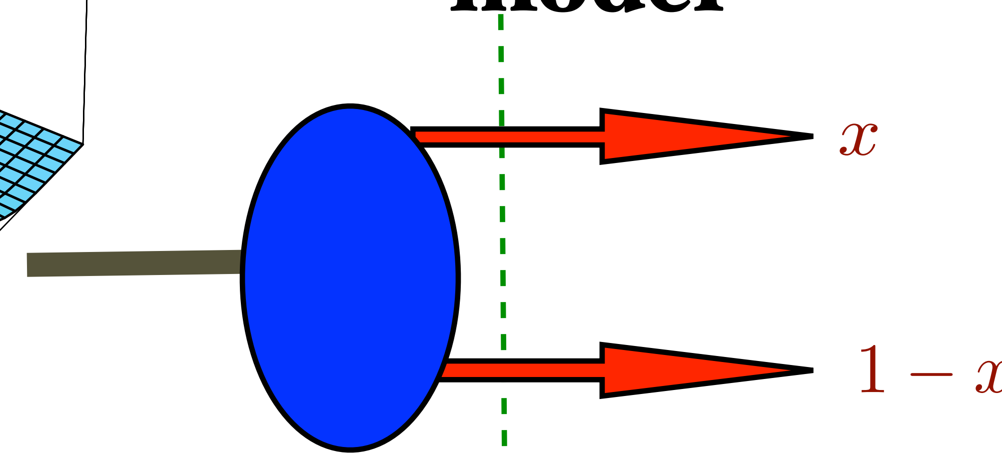
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

de Teramond, Cao, sjb

$$\psi_M(x, k_\perp^2)$$



"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Same as DSE!

Provides Connection of Confinement to Hadron Structure

PRL **109,** 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

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(Received 5 April 2012; published 20 August 2012)

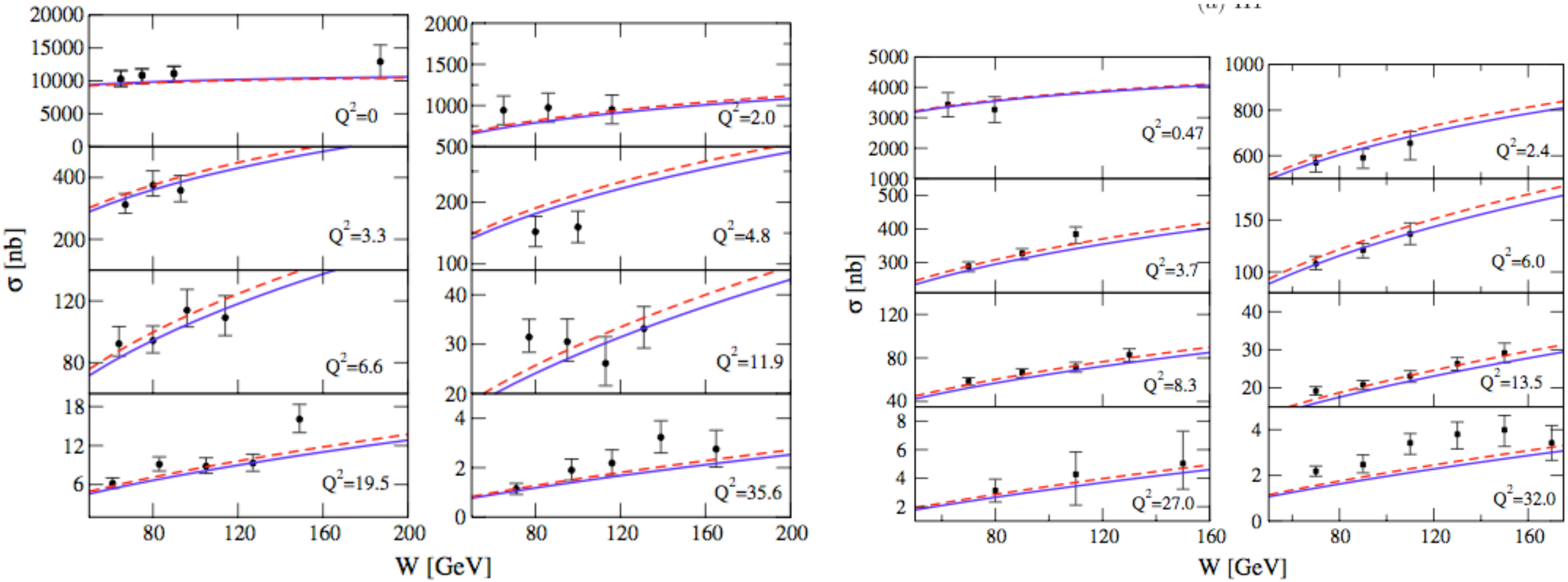
We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

See also Ferreira and Dosch

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

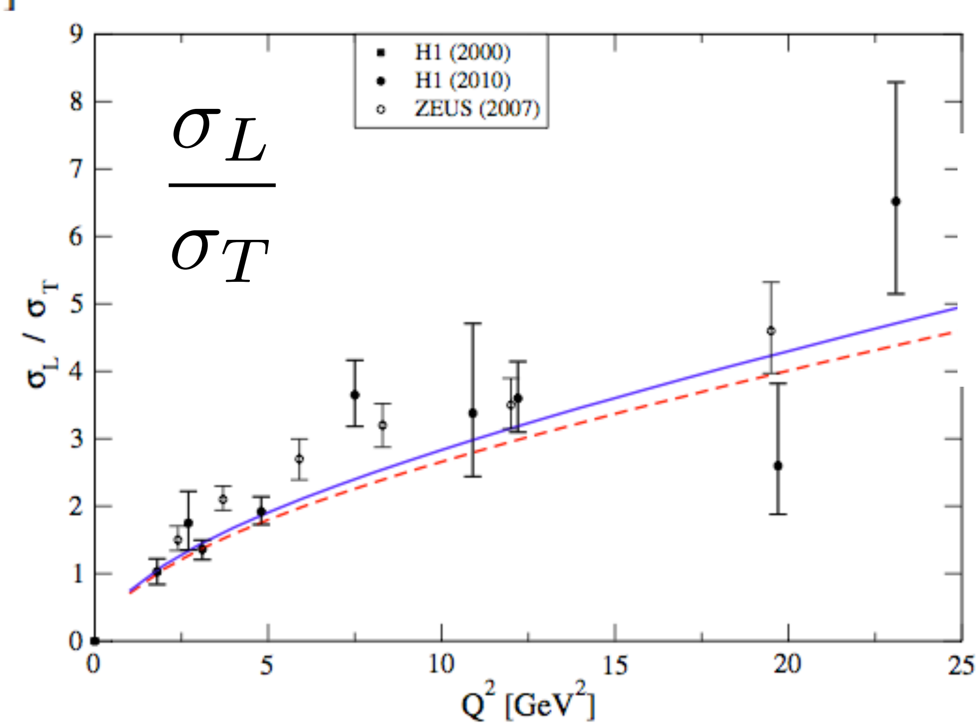


(a) H

(b) ZEUS

J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

See also Ferreira and Dosch

Uniqueness

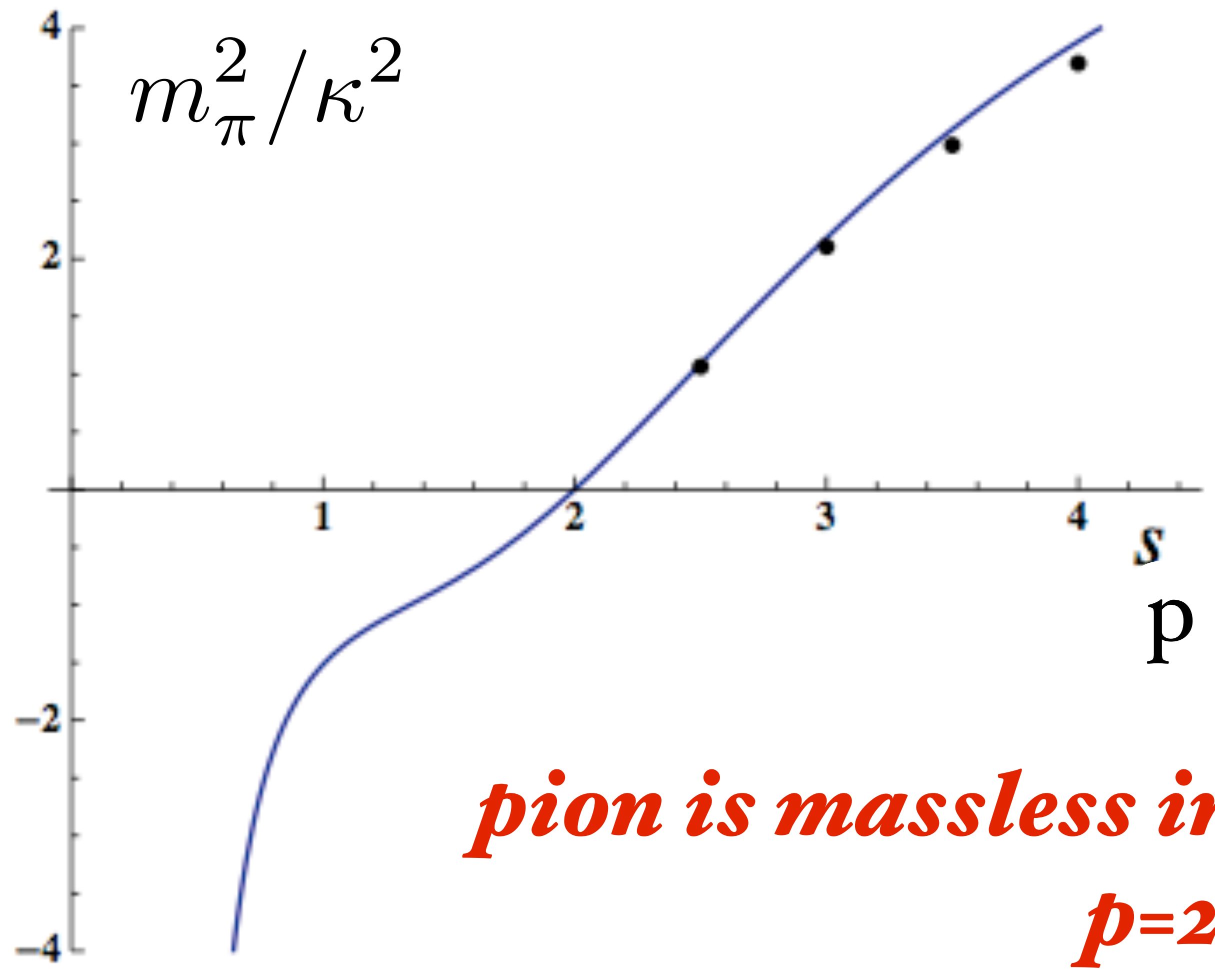
de Tèramond, Dosch, sjb

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1) \qquad e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **ζ^2 confinement potential and dilaton profile unique!**
- **Linear Regge trajectories in n and L: same slope!**
- **Massless pion in chiral limit! No vacuum condensate!**
- **Conformally invariant action for massless quarks retained despite mass scale**
- **Same principle, equation of motion as de Alfaro, Furlan, Fubini, <u>Conformal Invariance in Quantum Mechanics</u> Nuovo Cim. A34 (1976) 569**

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **Dosch, de Tèramond, sjb**

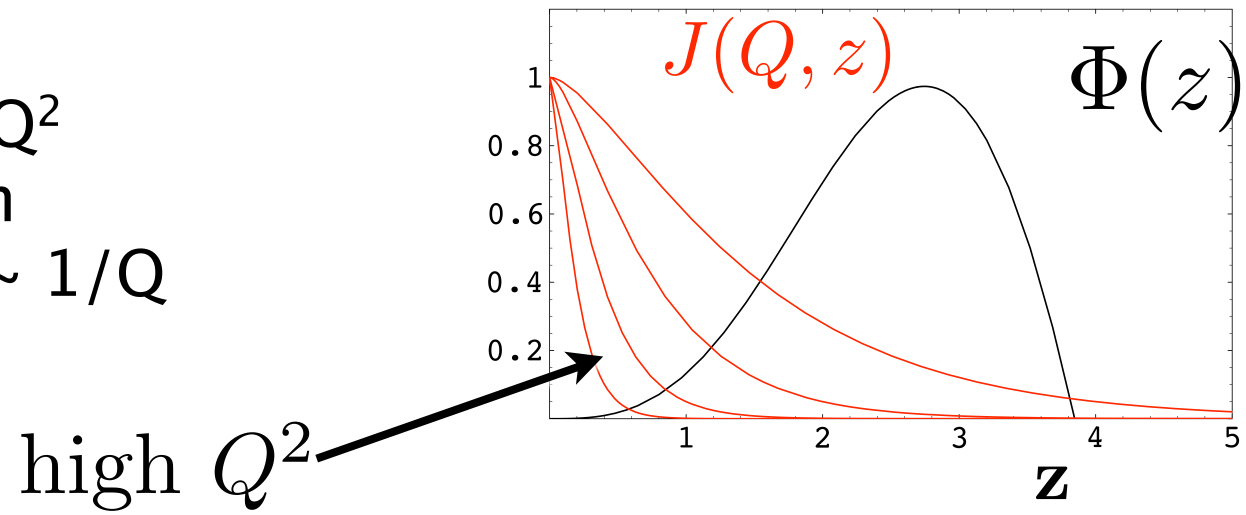
Hadron Form Factors from AdS/QCD

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$.

Twist $\tau = n + L$

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

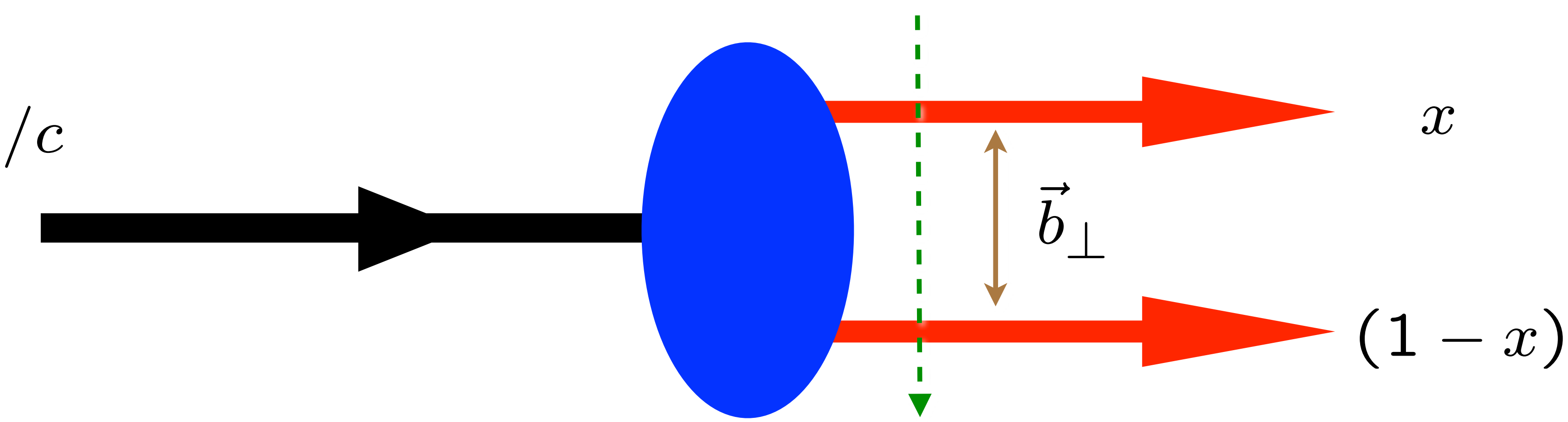
Identical to Polchinski-Strassler Convolution of AdS Amplitudes de Teramond, sjb

$LF(3+1) \longleftrightarrow AdS_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

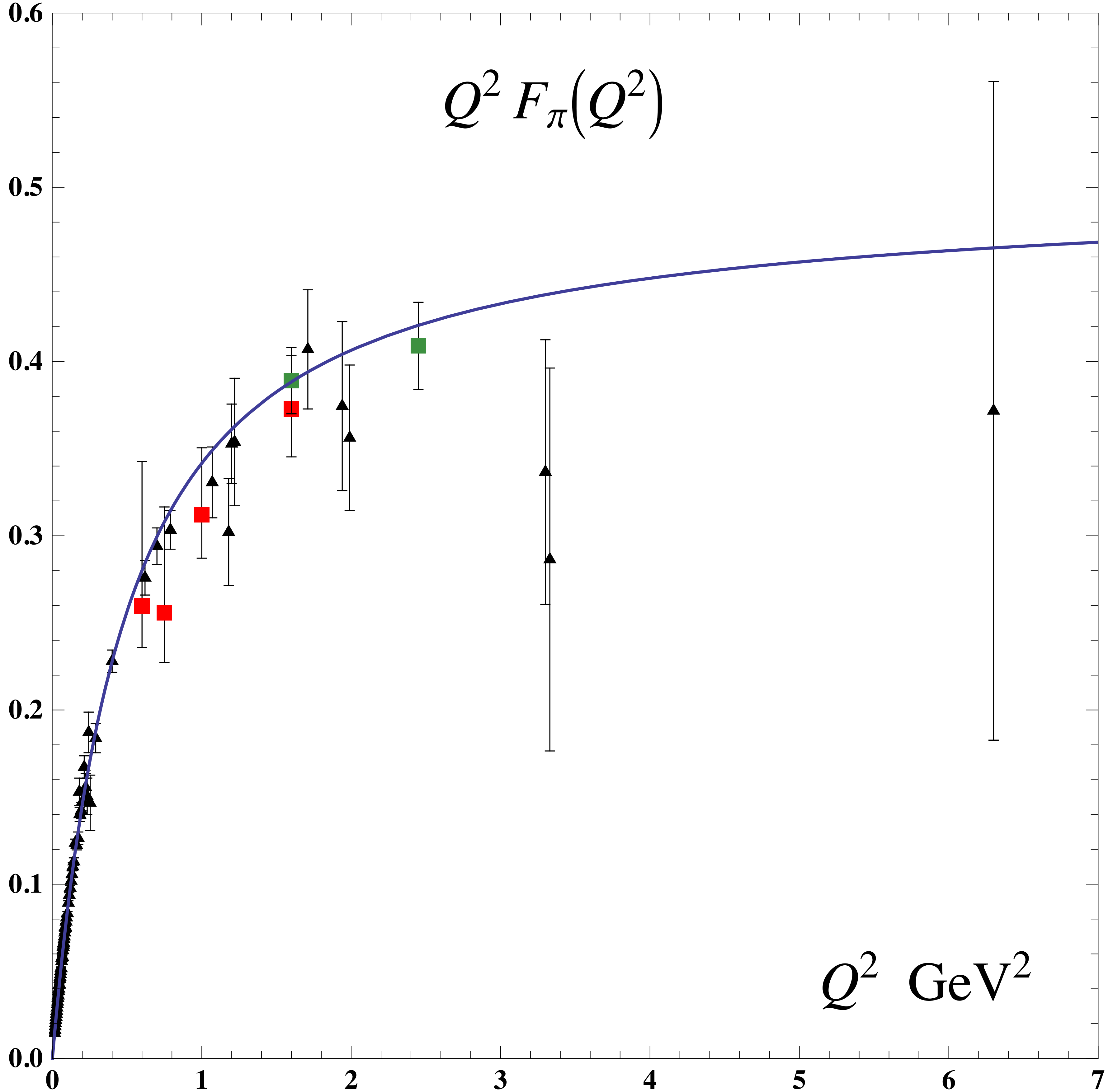
$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$

Fixed $\tau = t + z/c$



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Current Matrix Elements in AdS Space (SW)

$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

Dressed Current in Soft-Wall Model

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2}\, \Phi(z) J_\kappa(Q,z) \Phi(z).$$

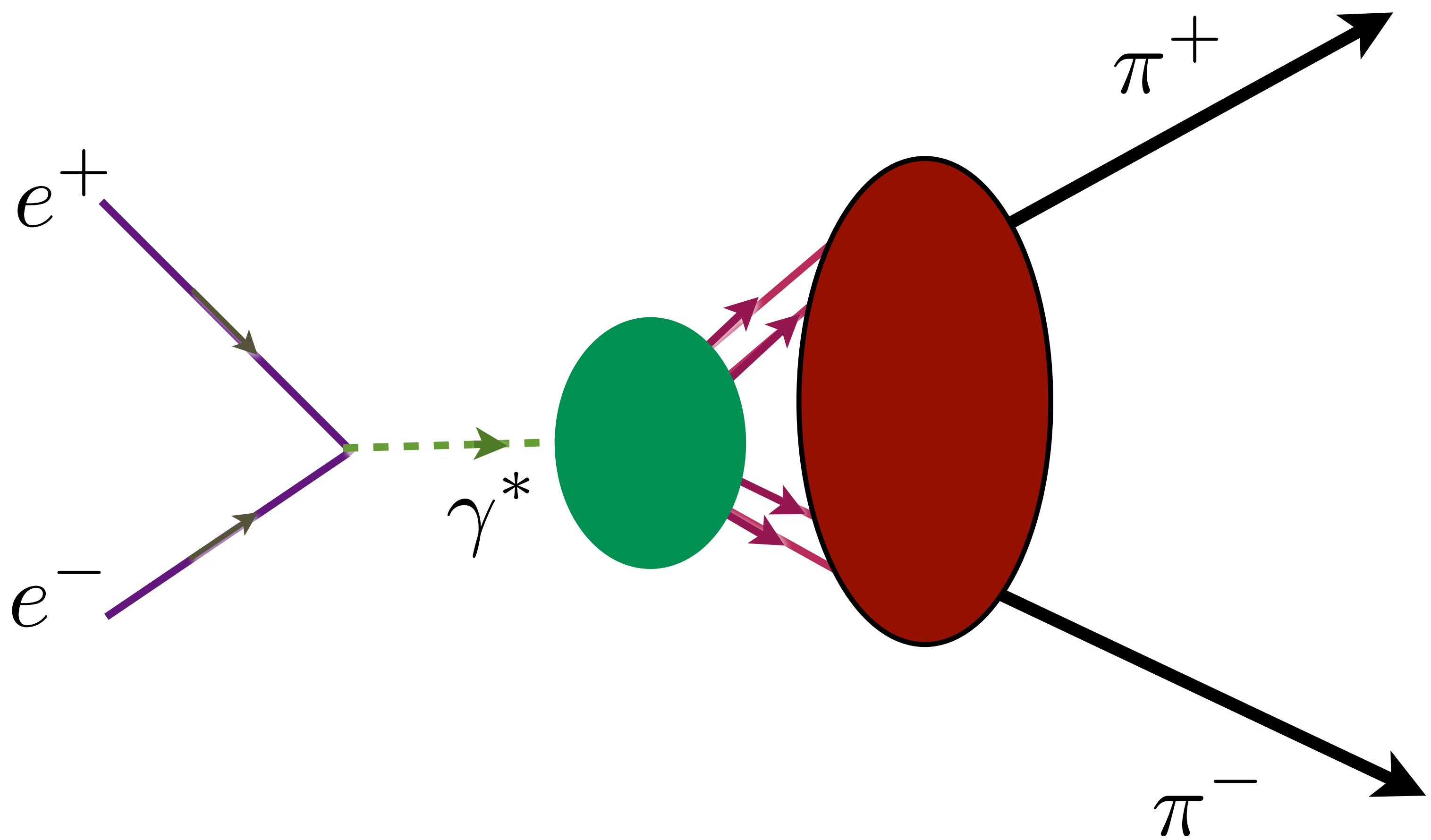
- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

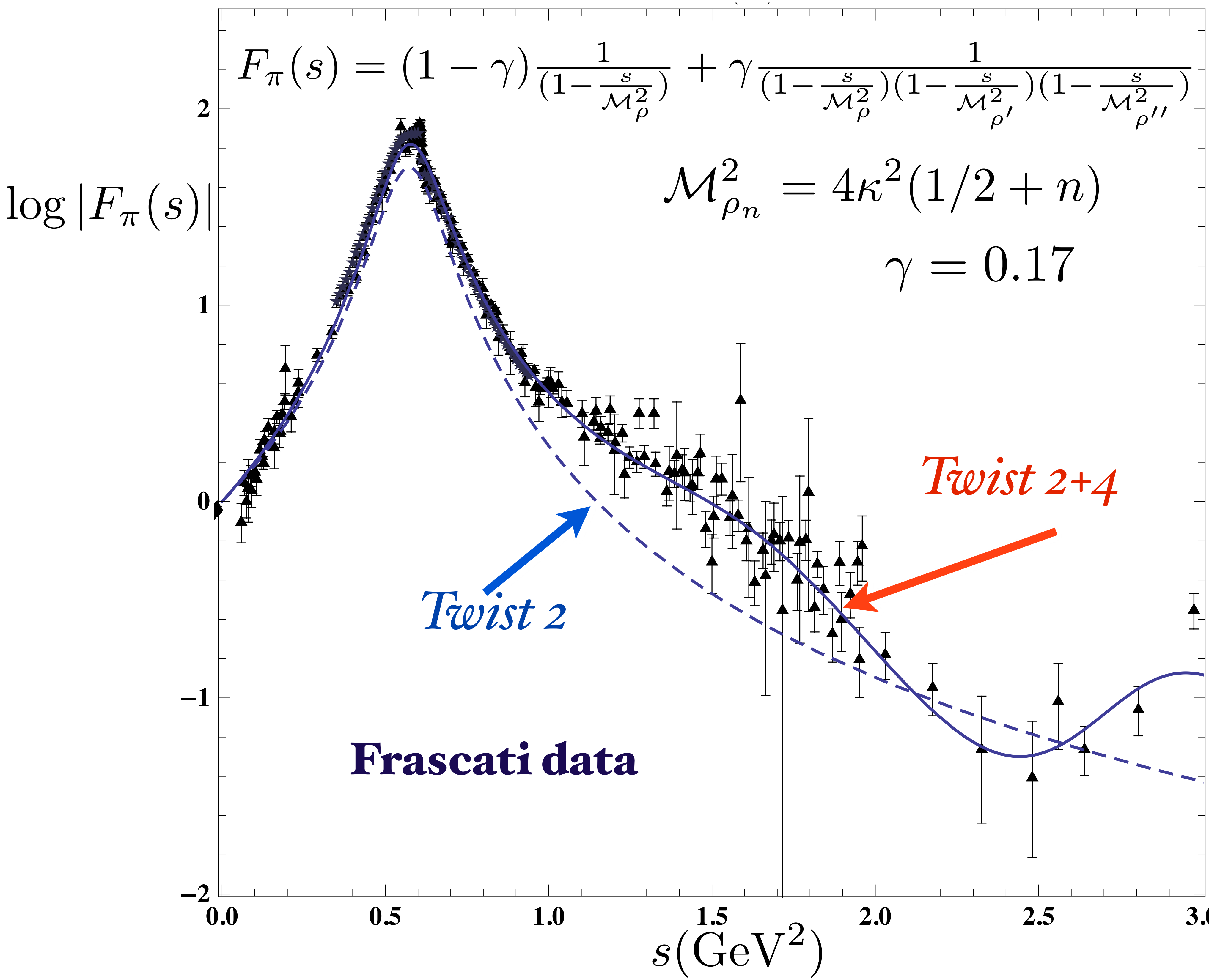
the external current decouples from the dilaton field.

de Tèramond & sjb
Grigoryan and Radyushkin

Dressed soft-wall current brings in higher Fock states and more vector meson poles



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



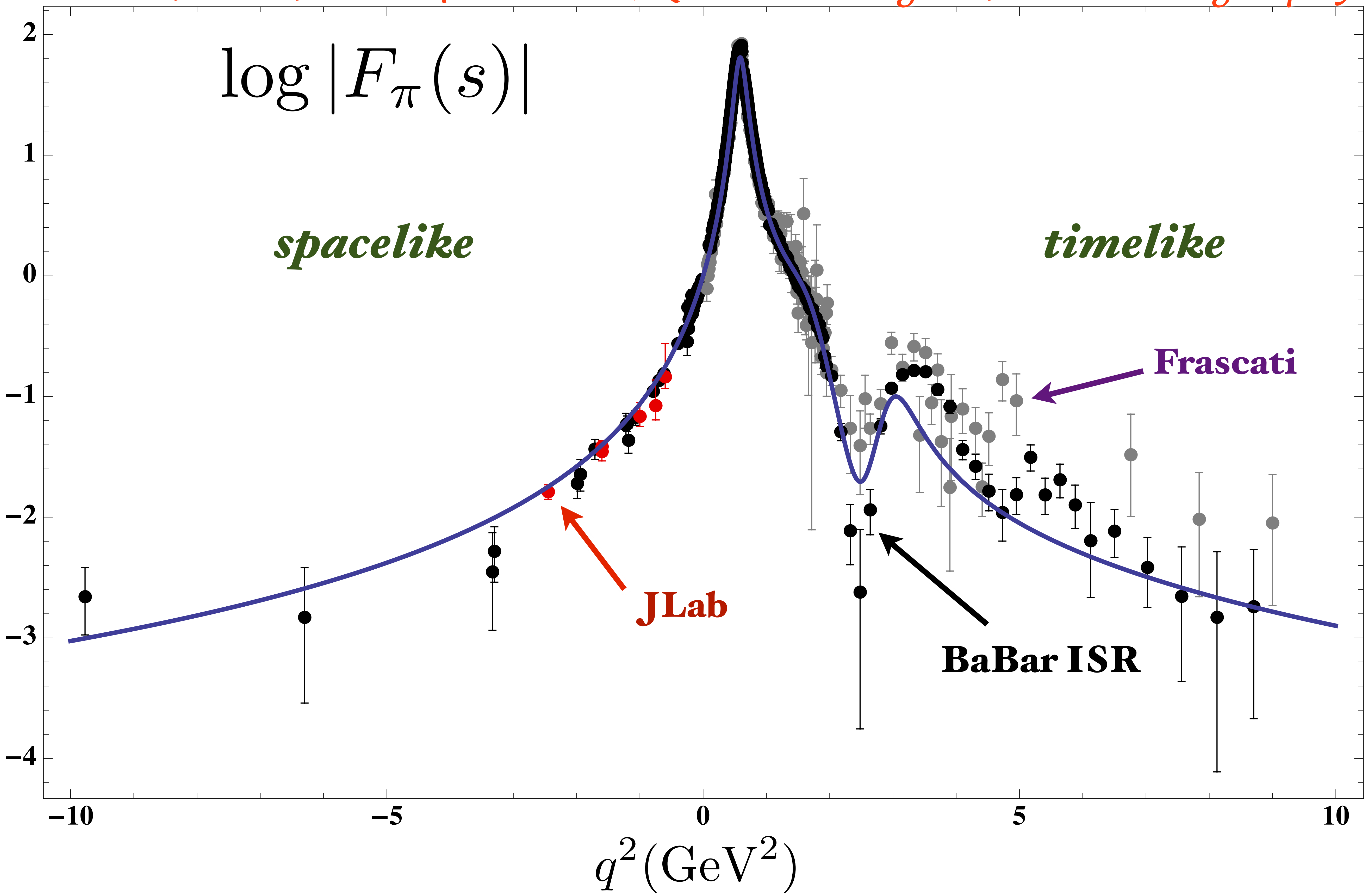
Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

G. de Teramond & sjb

Pion Form Factor from AdS/QCD and Light-Front Holography



Remarkable Features of Light-Front Schrödinger Equation

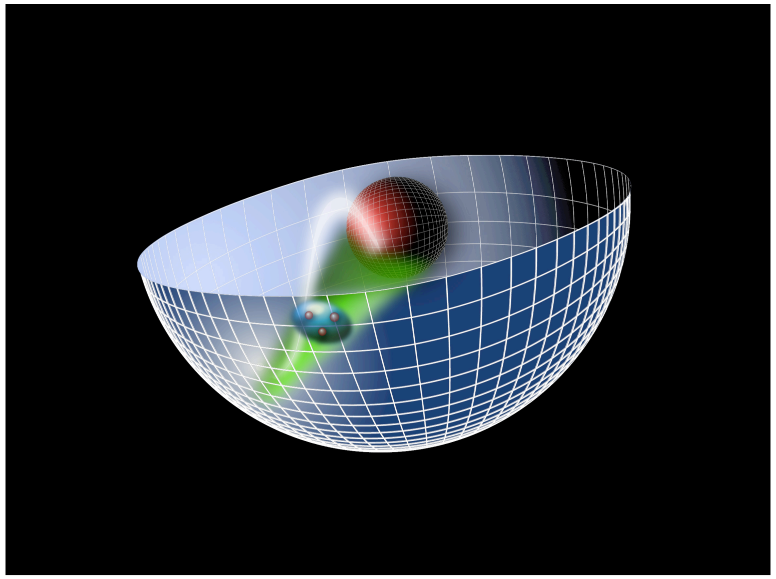
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for n and L -- not usual HO**
- **Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

$\phi(z)$

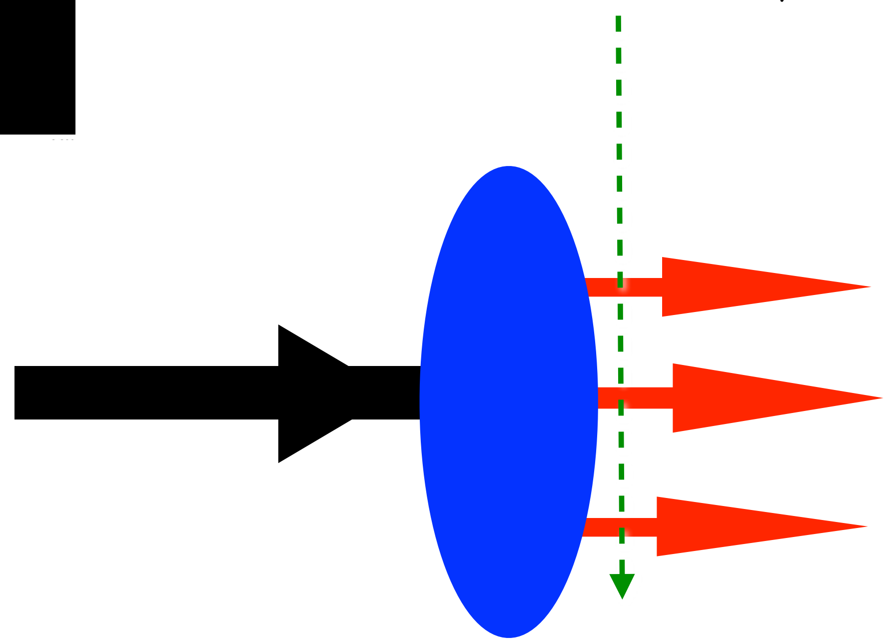
AdS5: Conformal Template for QCD



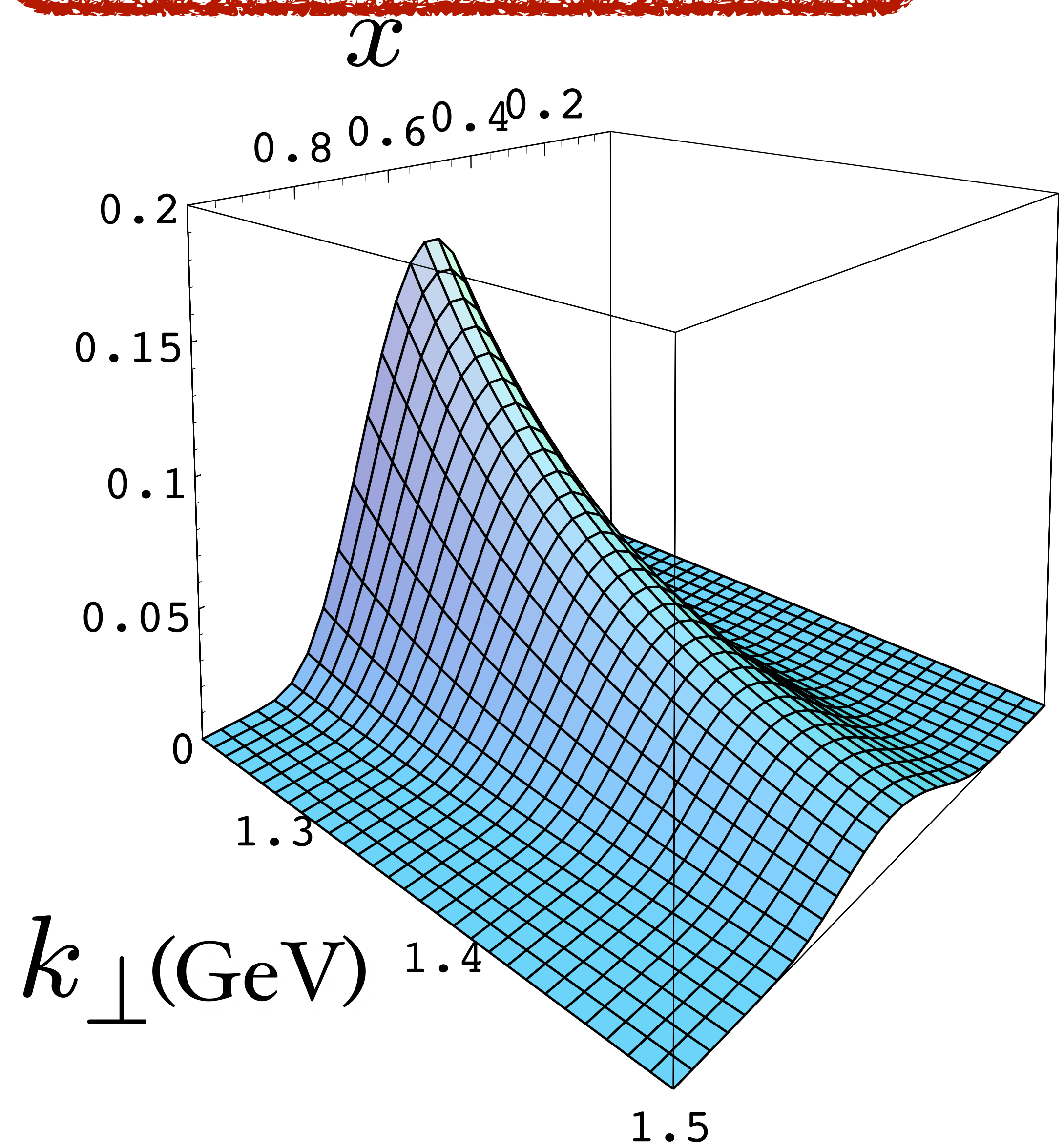
- *Light-Front Holography*

Duality of AdS$_5$ with LF Hamiltonian Theory

Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



- *Light Front Wavefunctions:*

Light-Front Schrödinger Equation
Spectroscopy and Dynamics

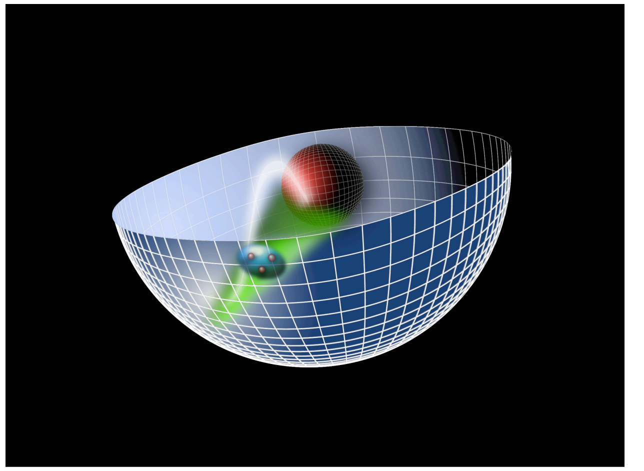
de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Single scheme-independent fundamental mass scale

κ



Light-Front Holography

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale:

(m_q=0)

$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \cancel{\sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale Λ_{QCD} come from?

How does color confinement arise?

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

- **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau)>= i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

New term

$$G = uH + vD + wK$$

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

- **Dosch, de Teramond, sjb**

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

What determines the QCD mass scale Λ_{QCD}?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)
- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$
- dAFF: Confinement Scale κ appears spontaneously via the Hamiltonian: $G = uH + vD + wK$ $\quad 4uw - v^2 = \kappa^4 = [M]^4$
- The confinement scale regulates infrared divergences, connects Λ_{QCD} to the confinement scale κ
- Only dimensionless mass ratios (and M times R) predicted
- Mass and time units [GeV] and [sec] from physics external to QCD
- New feature: bounded frame-independent relative time between constituents

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**

Interpretation of Mass Scale κ

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of
- Value of κ itself not determined -- place holder
- Need external constraint such as f_π

Baryon Spectrum in Soft-Wall Model

- Upon substitution $z \longrightarrow \zeta$ and

$$\Psi_J(x,z) = e^{-iP\cdot x} z^2 \psi^J(z) u(P),$$

find LFWE for $d = 4$

$$\begin{aligned} \frac{d}{d\zeta}\psi_+^J + \frac{\nu+\frac{1}{2}}{\zeta}\psi_+^J + U(\zeta)\psi_+^J &= \mathcal{M}\psi_-^J, \\ -\frac{d}{d\zeta}\psi_-^J + \frac{\nu+\frac{1}{2}}{\zeta}\psi_-^J + U(\zeta)\psi_-^J &= \mathcal{M}\psi_+^J, \end{aligned}$$

$$U = \kappa^2 \zeta$$

- Eigenfunctions

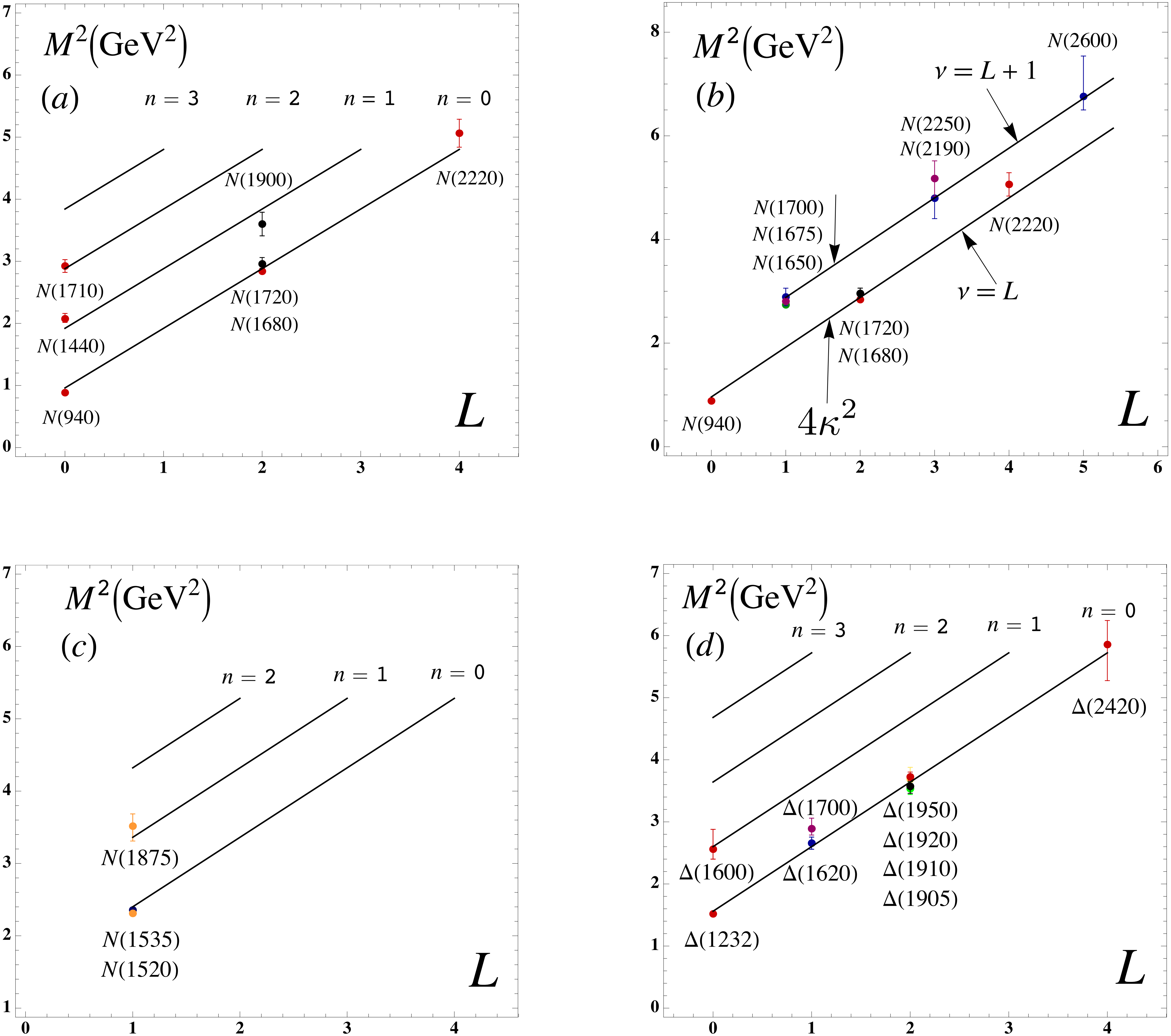
$$\psi_+^J(\zeta) \sim \zeta^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2), \qquad \psi_-^J(\zeta) \sim \zeta^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1), \quad \nu = L+1 \quad (\tau = 3)$$

Independent of J

- Full $J-L$ degeneracy (different J for same L) for baryons along given trajectory !



M^2(GeV^2)
(a)
n = 3
n = 2
n = 1
n = 0
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
L
M^2(GeV^2)
(b)
ν = L + 1
N(2600)
N(2250)
N(2190)
N(1700)
N(1675)
N(1650)
N(2220)
ν = L
N(1720)
N(1680)
N(940)
4κ^2
L
M^2(GeV^2)
(c)
n = 2
n = 1
n = 0
N(1875)
N(1535)
N(1520)
L
M^2(GeV^2)
(d)
n = 3
n = 2
n = 1
n = 0
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
L

Table 1: $SU(6)$ classification of confirmed baryons listed by the PDG. The labels S, L and n refer to the internal spin, orbital angular momentum and radial quantum number respectively. The $\Delta\frac{5}{2}^-(1930)$ does not fit the $SU(6)$ classification since its mass is too low compared to other members **70**-multiplet for $n = 0$, $L = 3$.

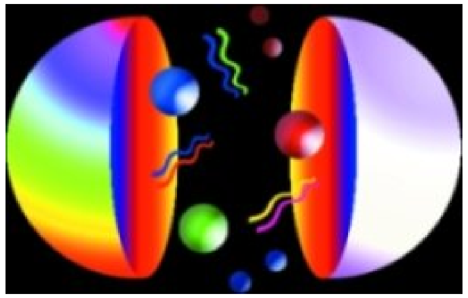
$SU(6)$	S	L	n	Baryon State
56	$\frac{1}{2}$	0	0	$N\frac{1}{2}^+(940)$
	$\frac{1}{2}$	0	1	$N\frac{1}{2}^+(1440)$
	$\frac{1}{2}$	0	2	$N\frac{1}{2}^+(1710)$
	$\frac{3}{2}$	0	0	$\Delta\frac{3}{2}^+(1232)$
	$\frac{3}{2}$	0	1	$\Delta\frac{3}{2}^+(1600)$
70	$\frac{1}{2}$	1	0	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	0	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{3}{2}$	1	1	$N\frac{1}{2}^-$ $N\frac{3}{2}^-(1875)$ $N\frac{5}{2}^-$
	$\frac{1}{2}$	1	0	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	0	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{1}{2}$	2	1	$N\frac{3}{2}^+(1900)$ $N\frac{5}{2}^+$
	$\frac{3}{2}$	2	0	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	0	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	0	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	0	$\Delta\frac{5}{2}^-$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	0	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	0	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	0	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	0	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

PDG 2012

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta) = 1$$

Chiral Symmetry of Eigenstate!

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- "Chiral partners"

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S_q^z >= 0$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

No mass -degenerate parity partners!

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4}\, V(Q,z)\Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2}\sqrt{\frac{2n!}{(n+L)!}}\, z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^2(z) = 1$$

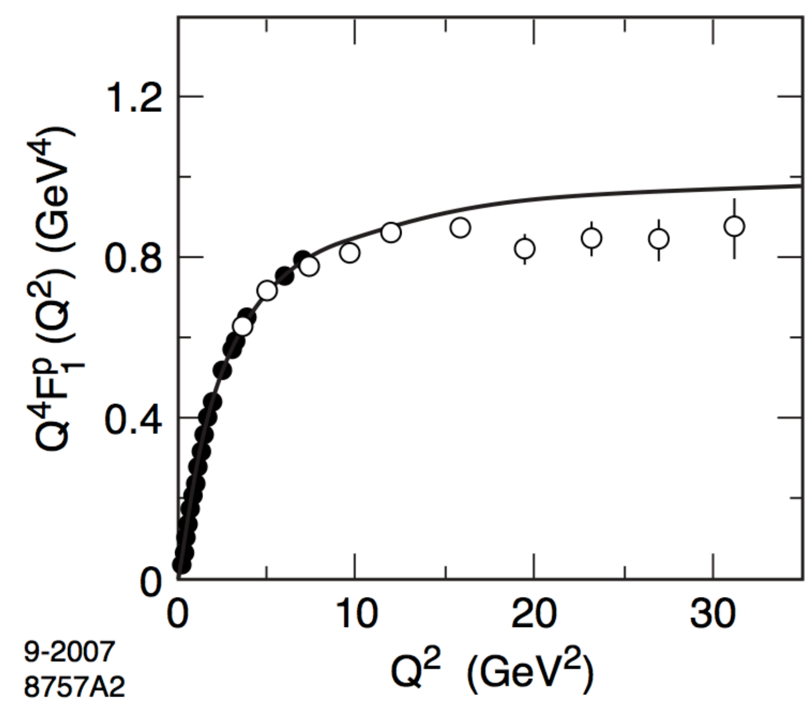
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}}\, e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

$$F_1^p(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}{}^2 \to 4\kappa^2(n+1/2)$



Fubini and Rabinovici

Superconformal Algebra

de Teramond Dosch and SJB

1+1

$$\{\psi, \psi^+\} = 1$$

two anti-commuting fermionic operators

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

Realization as Pauli Matrices

$$Q = \psi^+[-\partial_x + W(x)], \quad Q^+ = \psi[\partial_x + W(x)], \qquad W(x) = \frac{f}{x}$$

(Conformal)

$$S = \psi^+ x, \quad S^+ = \psi x$$

Introduce new spinor operators

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

$$\{Q, Q\} = \{Q^+, Q^+\} = 0, \quad [Q, H] = [Q^+, H] = 0$$

Superconformal Algebra

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates the conformal algebra

[H,D]= i H, [H, K] =2 i D, [K, D] = - i K

Superconformal Algebra

Baryon Equation

Consider $R_w = Q + wS;$ w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \Big(-\partial_x^2 + w^2x^2 + 2wf - w + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2}\Big)$$

$$G_{22} = \Big(-\partial_x^2 + w^2x^2 + 2wf + w + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2}\Big)$$

Identify $f - \frac{1}{2} = L_B$, $w = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

$$x \to \zeta$$

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\,\psi_J^+,$$

G22

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B\,L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\,\psi_J^-.$$

G11

$$M_B^2(n, L_B) = 4\lambda_B^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

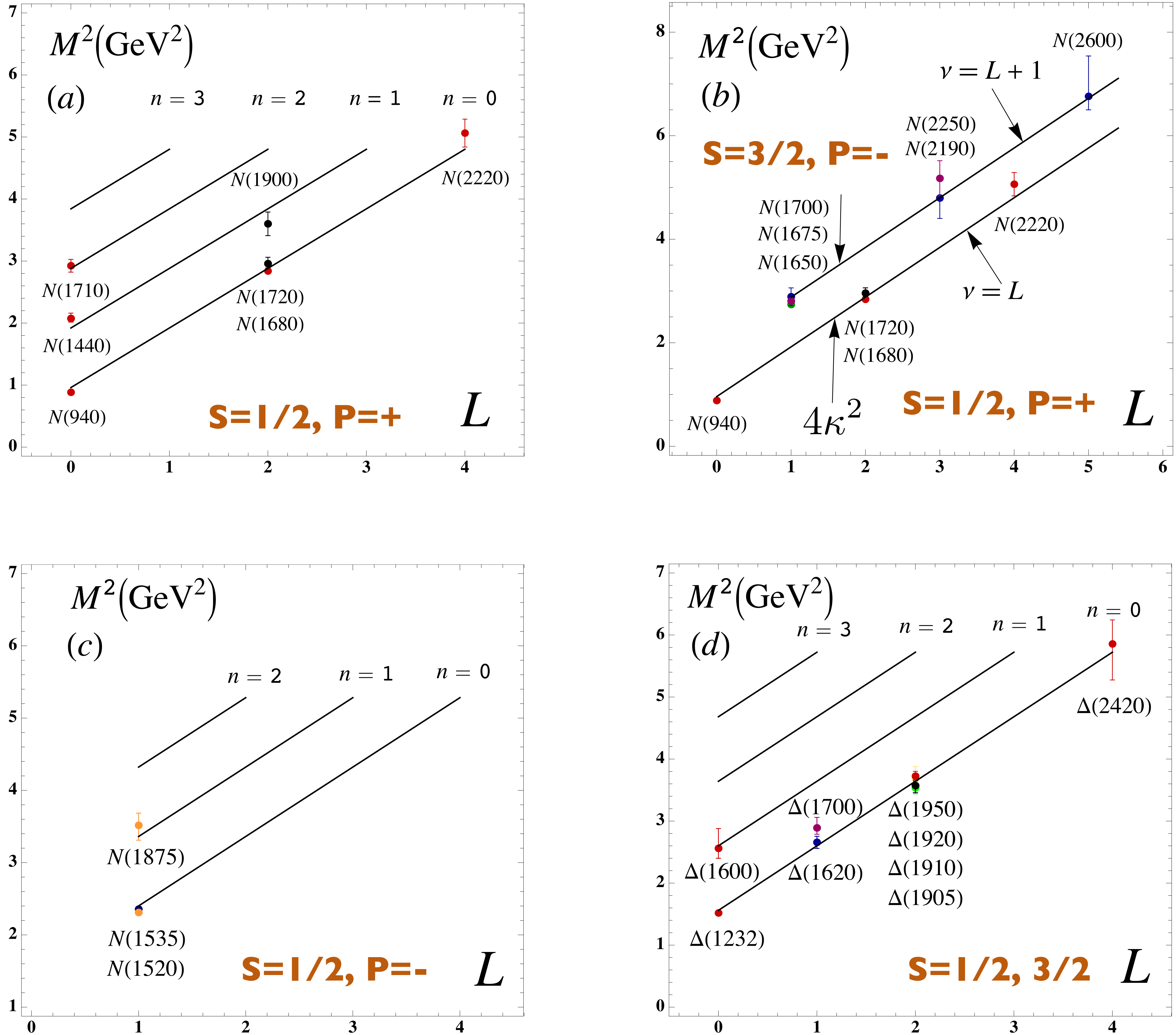
$$\left(-\frac{d^2}{d\zeta^2} + \lambda_M^2\,\zeta^2 + 2\lambda_M\,(J-1) + \frac{4\nu^2-1}{4\zeta^2}\right)\phi_J = M^2\,\phi_J,$$

G11

$$M_M^2(n, L_M, S=0) = 4\lambda_M^2(n + L_M) \qquad \nu = L_M$$

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$ $\lambda_M^2 = \lambda_B^2 = \kappa^4$



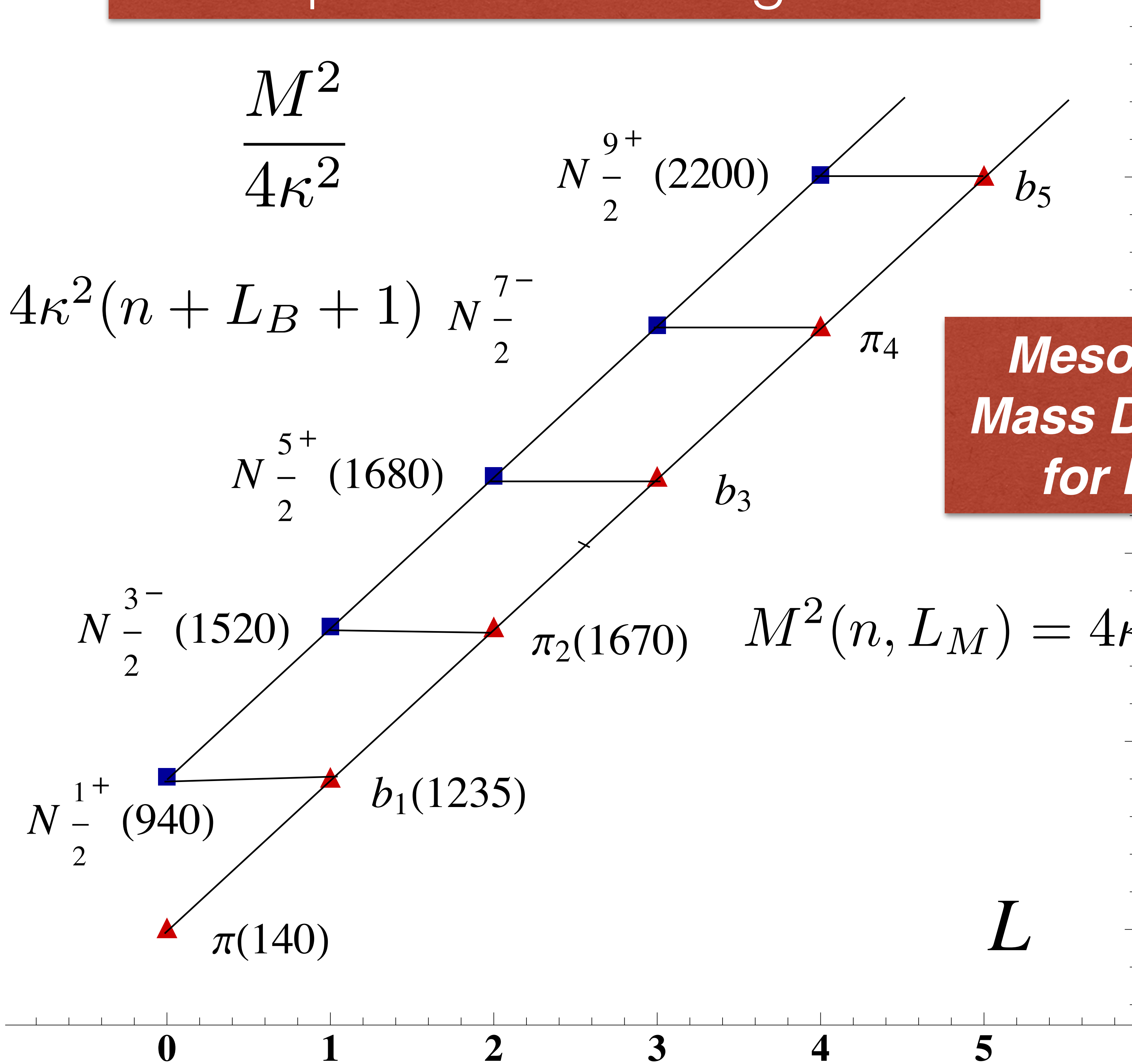
$M^2(\mathrm{GeV}^2)$
(a)
$n=3$ $n=2$ $n=1$ $n=0$
$N(1900)$
$N(2220)$
$N(1710)$
$N(1440)$
$N(1720)$
$N(1680)$
$N(940)$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
$N(2600)$
$\nu = L+1$
$N(2250)$
$N(2190)$
S=3/2, P=-
$N(1700)$
$N(1675)$
$N(1650)$
$N(2220)$
$\nu = L$
$N(1720)$
$N(1680)$
$N(940)$
$4\kappa^2$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(c)
$n=2$ $n=1$ $n=0$
$N(1875)$
$N(1535)$
$N(1520)$
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
$n=3$ $n=2$ $n=1$ $n=0$
$\Delta(2420)$
$\Delta(1700)$
$\Delta(1950)$
$\Delta(1920)$
$\Delta(1910)$
$\Delta(1905)$
$\Delta(1600)$
$\Delta(1620)$
$\Delta(1232)$
S=1/2, 3/2
L

Superconformal Algebra

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same κ

$$\lambda = \kappa^2$$



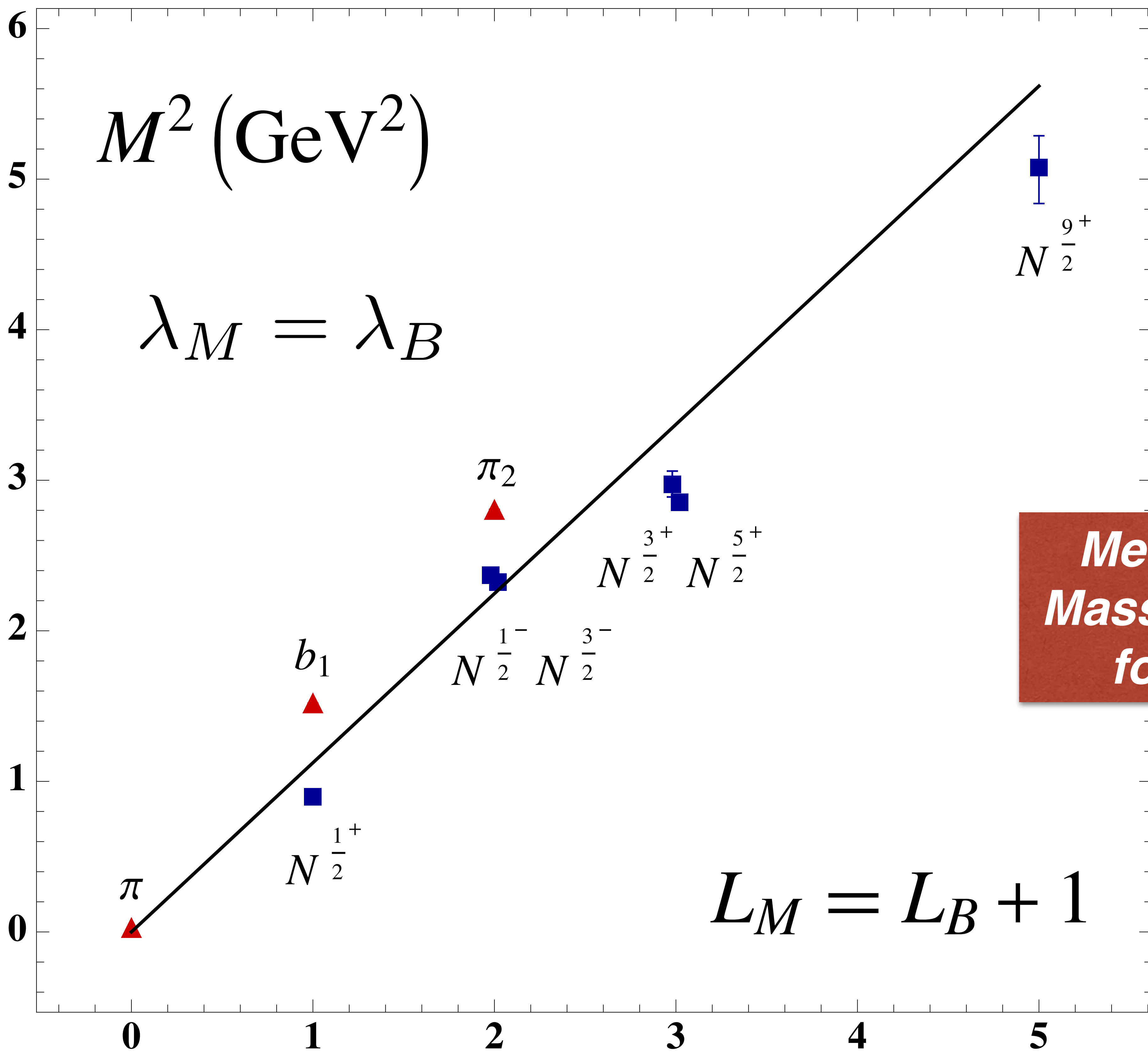
Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

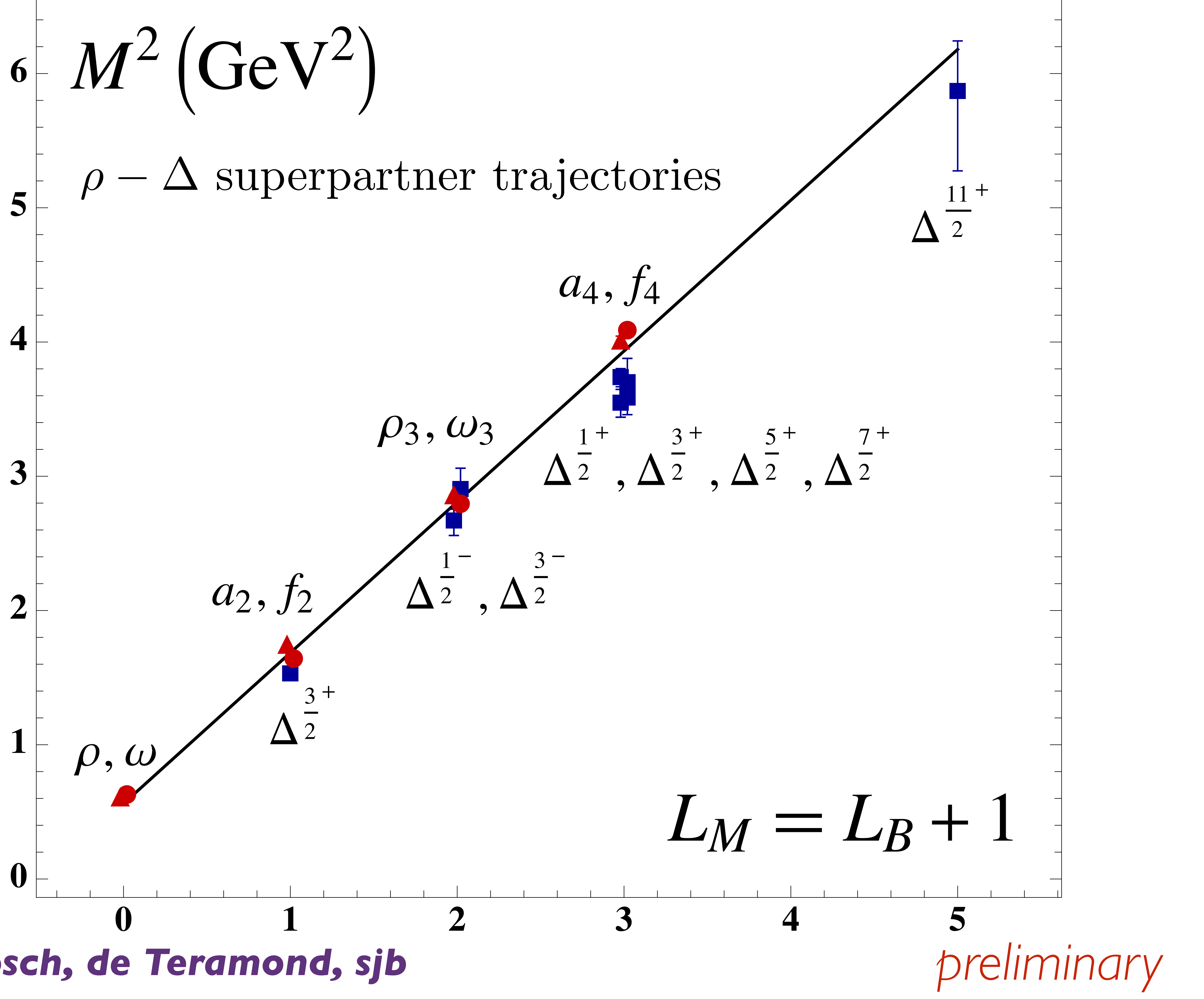
S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!

$\lambda = \kappa^2$



Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$



$M^2\,(\mathrm{GeV}^2)$
$\rho - \Delta$ superpartner trajectories
$\Delta_{\frac{11}{2}^+}$
a_4, f_4
ρ_3, ω_3
$\Delta_{\frac{1}{2}^+}, \Delta_{\frac{3}{2}^+}, \Delta_{\frac{5}{2}^+}, \Delta_{\frac{7}{2}^+}$
$\Delta_{\frac{1}{2}^-}, \Delta_{\frac{3}{2}^-}$
a_2, f_2
$\Delta_{\frac{3}{2}^+}$
ρ, ω
$L_M = L_B + 1$
0
1
2
3
4
5
6
Dosch, de Teramond, sjb
preliminary

Features of Supersymmetric Equations

- J =L+S baryon simultaneously satisfies both equations of G with L , L+1 for same mass eigenvalue
- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $\quad S^z = \pm 1/2$
- Baryon spin carried by quark orbital angular momentum: $\langle J^z \rangle = L^z + 1/2$
- Mass-degenerate meson "superpartner" with $L_M = L_B + 1$. *"Shifted meson-baryon Duality"*

Meson and baryon have same κ !

Baryon Spectrum

$$M^2 = 4\kappa^2(n + \nu + 1)$$

Table 1. Orbital assignment for baryon trajectories according to parity and internal spin.

$$\nu = |\mu R| - 1/2$$

	$S = \frac{1}{2}$	$S = \frac{3}{2}$
P = +	$\nu = L$	$\nu = L + \frac{1}{2}$
P = -	$\nu = L + \frac{1}{2}$	$\nu = L + 1$

$$M^{2\,(+)}_{n,L,S=\frac{3}{2}} = M^{2\,(-)}_{n,L,S=\frac{1}{2}}$$

No spin-orbit coupling

J=1/2 "Chiral partners", e.g. N(1535) and N(1400), with different L, non-degenerate

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$
\begin{aligned}
F_+(Q^2) &= g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_-(Q^2) &= g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,
\end{aligned}
$$

  where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

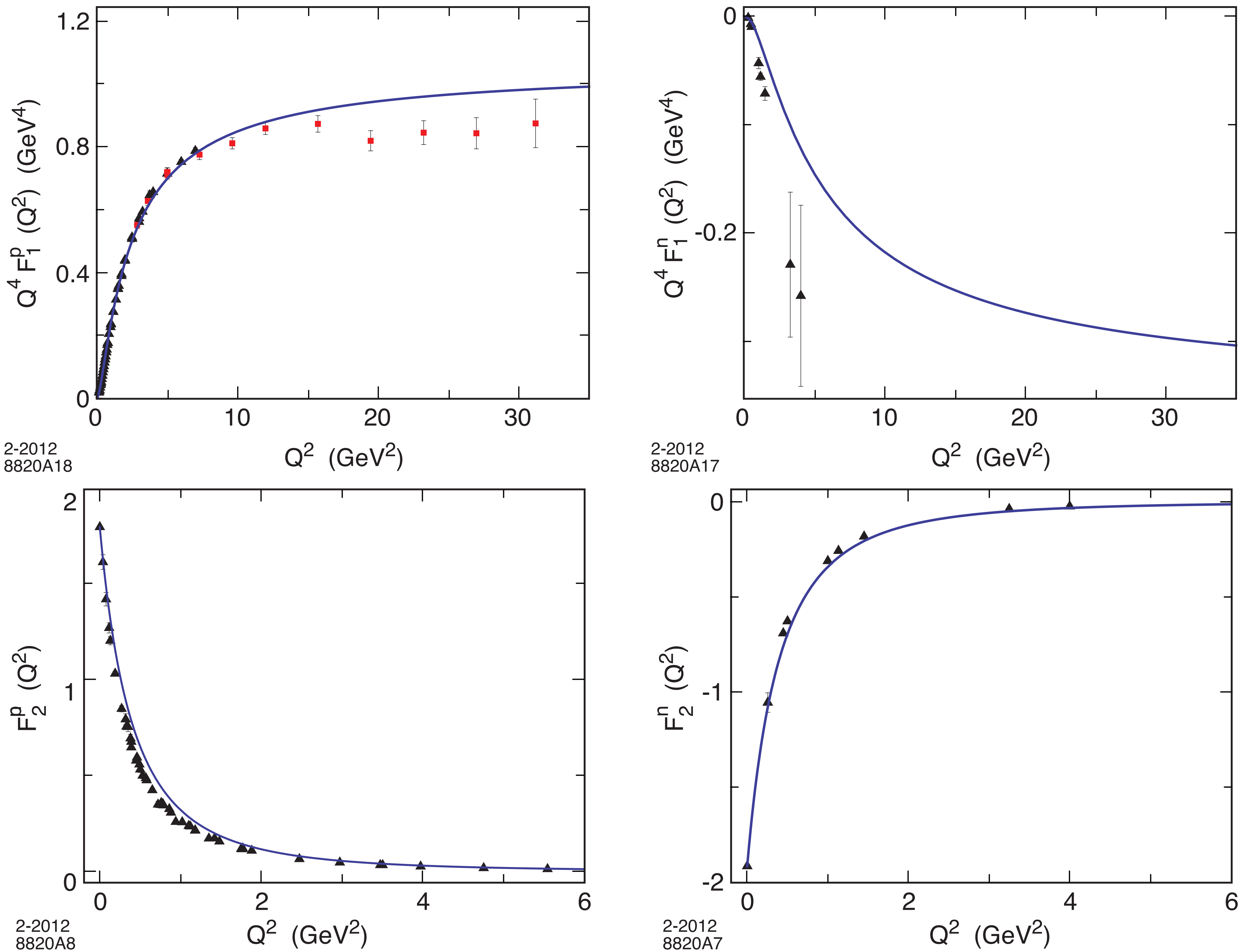
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$
\begin{aligned}
F_1^p(Q^2) &= \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_1^n(Q^2) &= -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],
\end{aligned}
$$

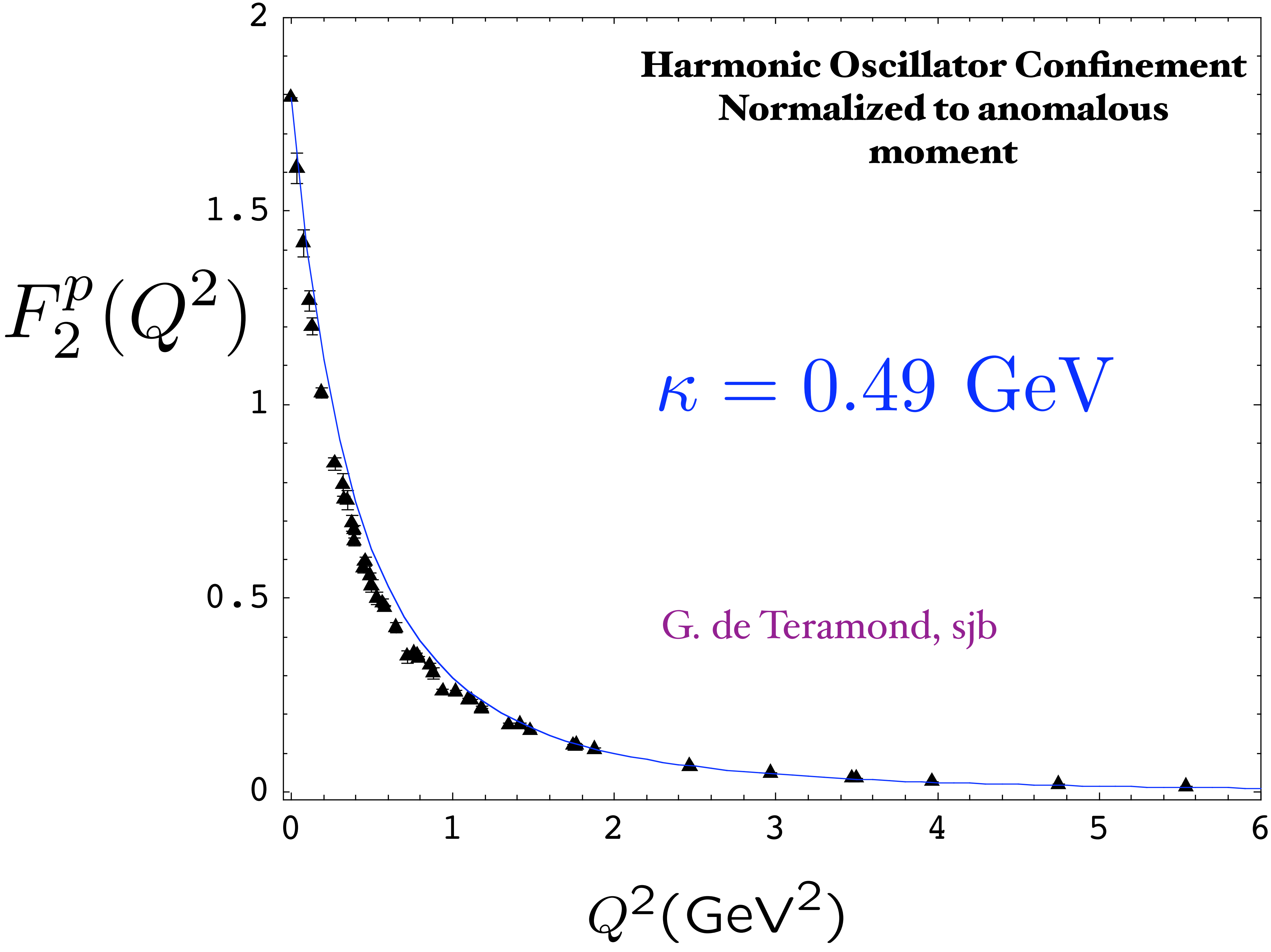
  where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

Using $SU(6)$ flavor symmetry and normalization to static quantities



Spacelike Pauli Form Factor

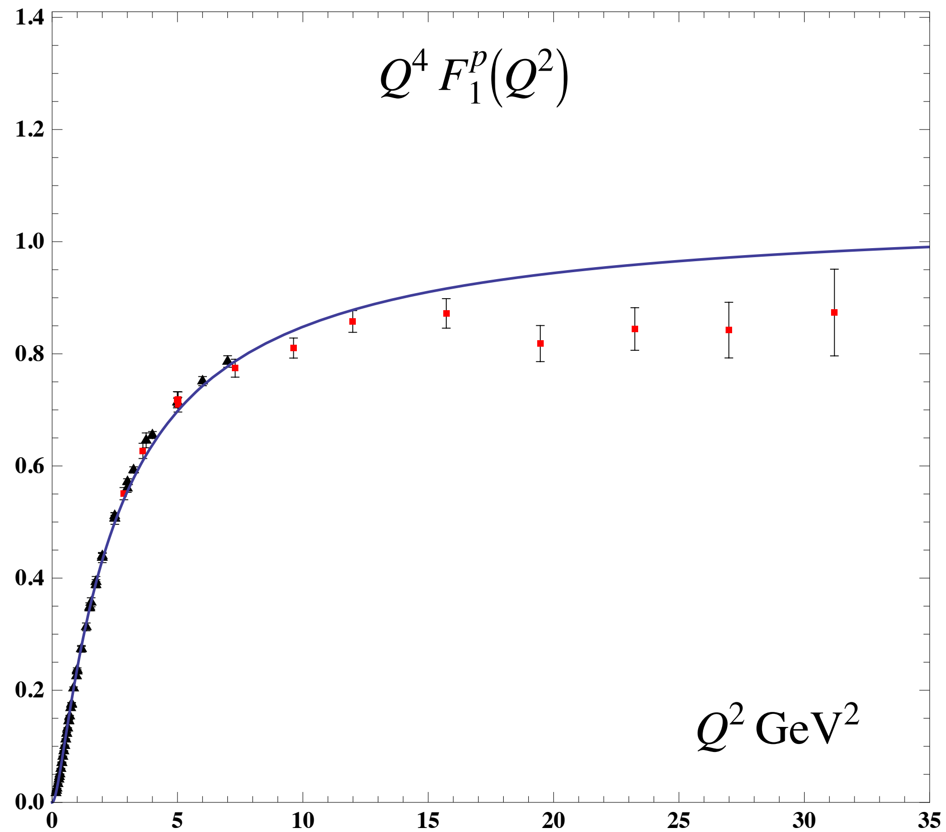
From overlap of $L = 1$ and $L = 0$ LFWFs

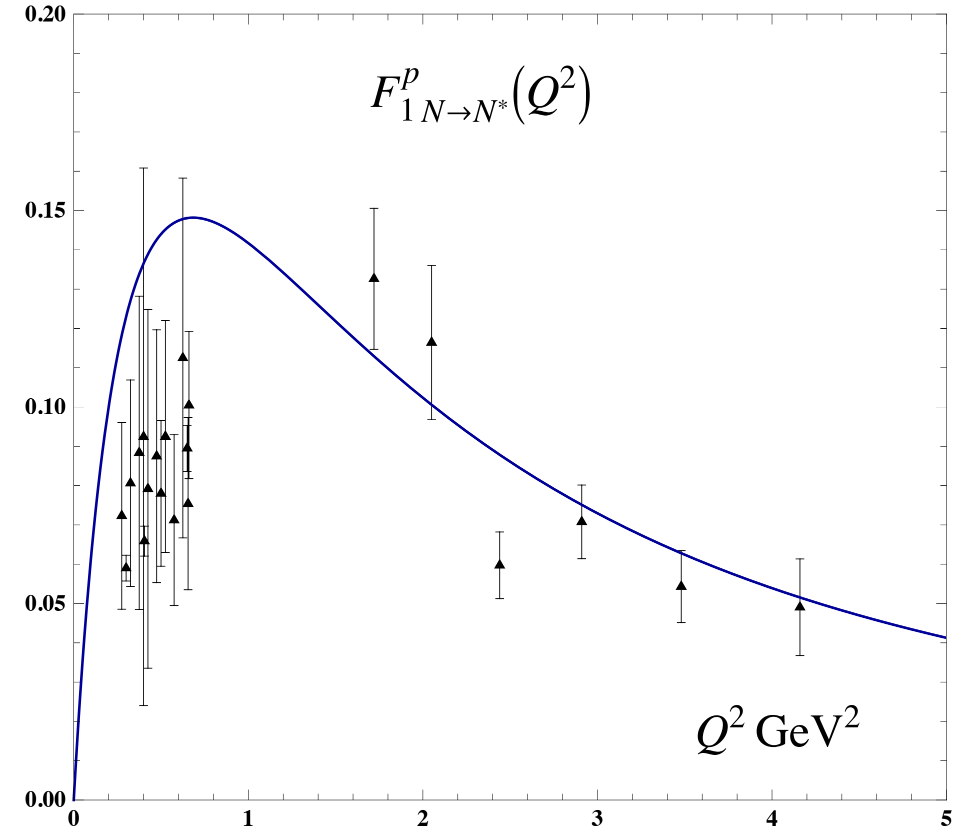


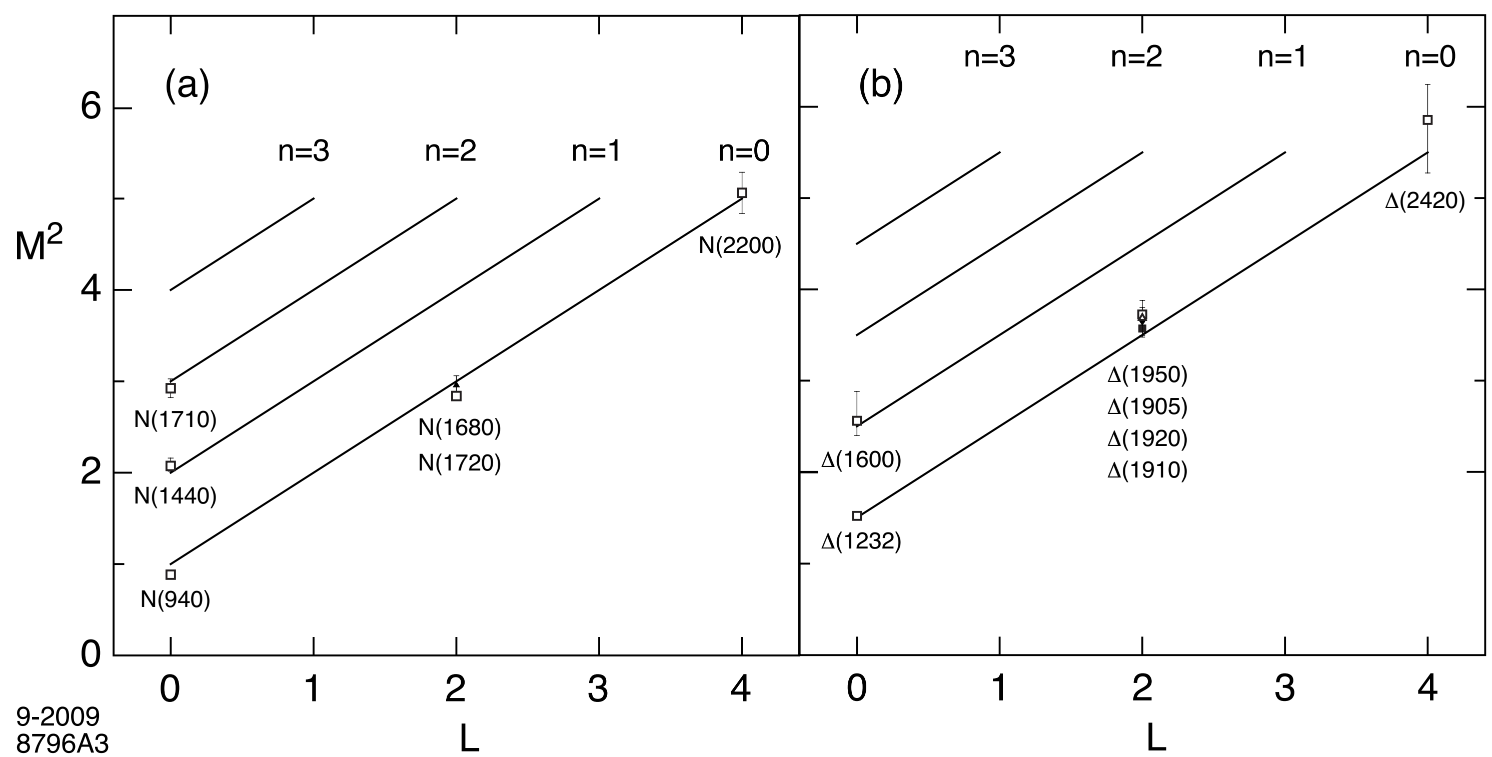
Predict hadron spectroscopy and dynamics

Excited Baryons in Holographic QCD

G. de Teramond & sjb







Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_{1\,N\to N^*}^{\,p}(Q^2) = R^4 \int \frac{dz}{z^4} \, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\left(F_{1\,N\to N^*}^{\,p}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4} \, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_{1\,N\to N^*}^{\,p}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

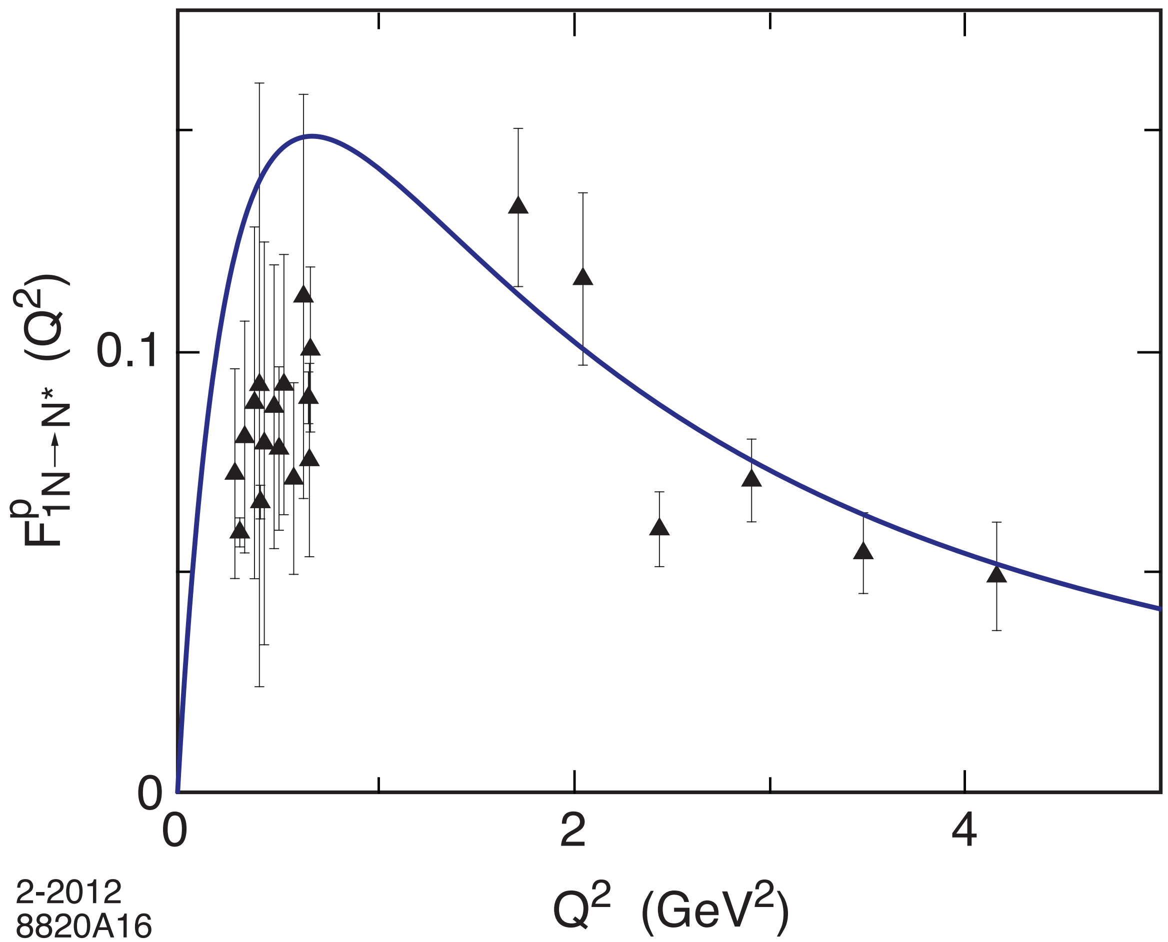
with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n + 1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

Nucleon Transition Form Factors

$$F^{p}_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3} \frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}.$$

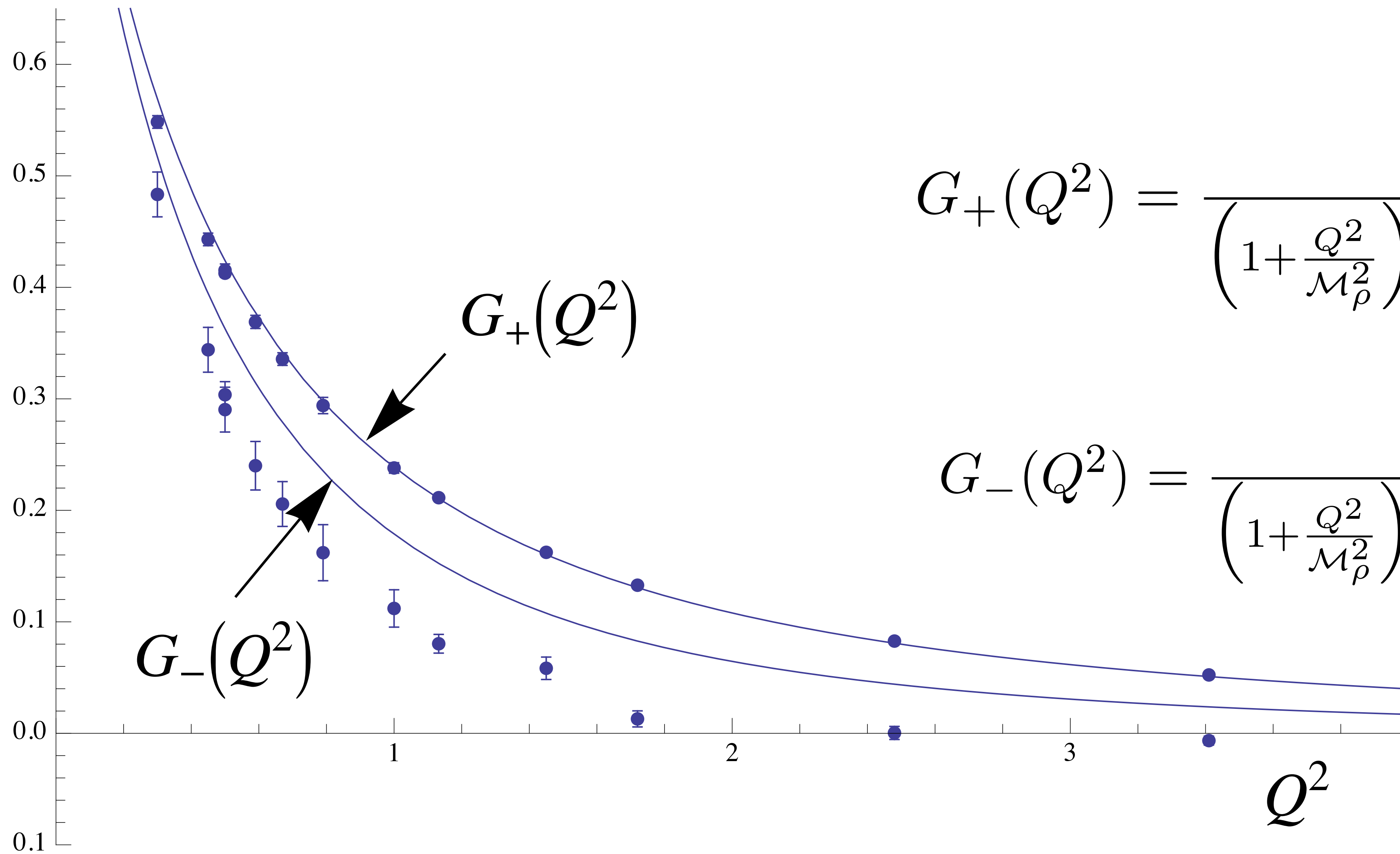


Proton transition form factor to the first radial excited state. Data from JLab

Flavor Decomposition of Elastic Nucleon Form Factors

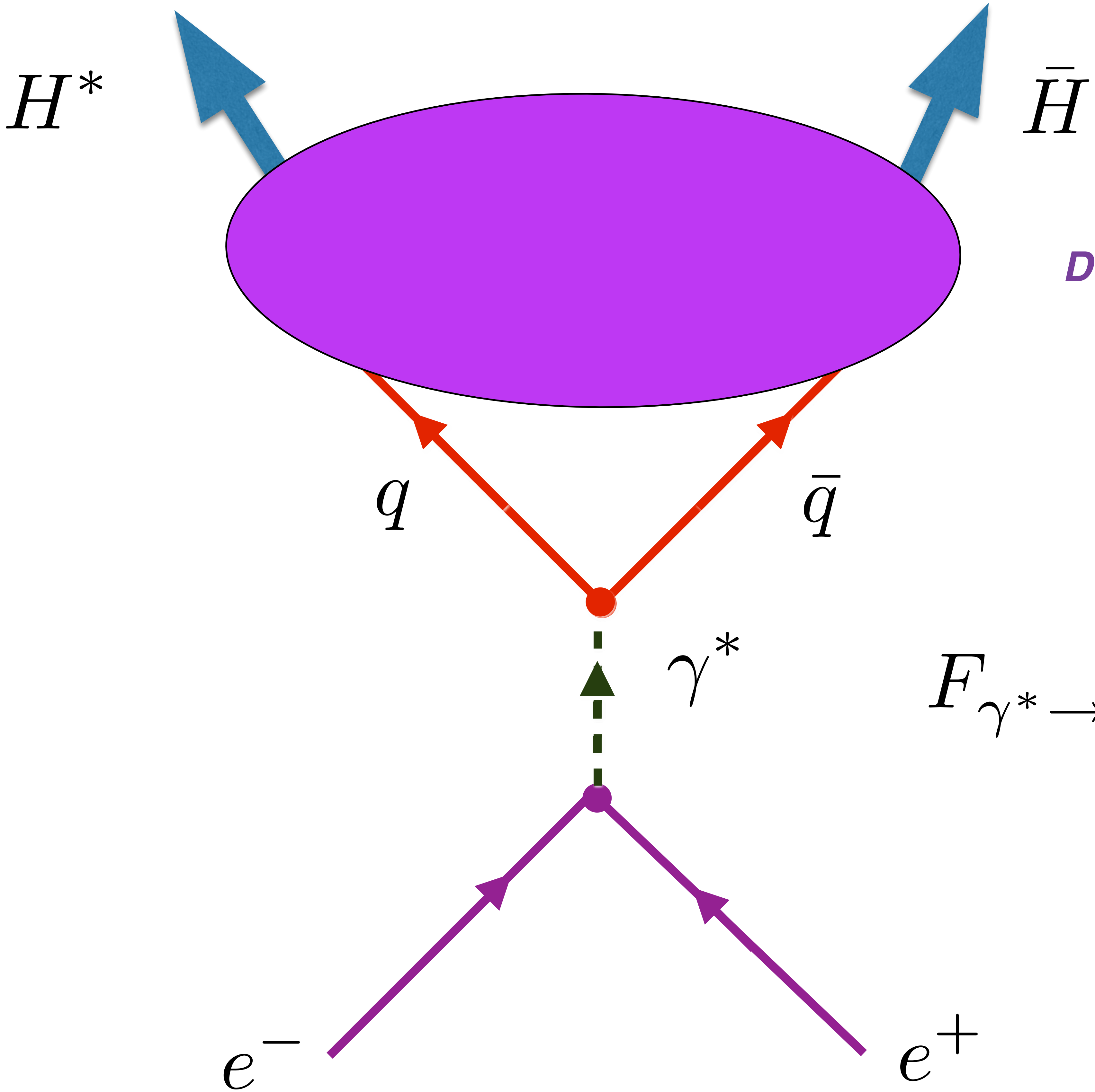
G. D. Cates *et al.* Phys. Rev. Lett. **106**, 252003 (2011)

- Proton SU(6) WF: $F^p_{u,1} = \frac{5}{3}G_+ + \frac{1}{3}G_-, \qquad F^p_{d,1} = \frac{1}{3}G_+ + \frac{2}{3}G_-$
- Neutron SU(6) WF: $F^n_{u,1} = \frac{1}{3}G_+ + \frac{2}{3}G_-, \qquad F^n_{d,1} = \frac{5}{3}G_+ + \frac{1}{3}G_-$



$$G_+(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)}$$

$$G_-(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho''}}\right)}$$



Dosch, de Teramond, sjb

$$F_{\gamma^* \to H^* \bar{H}}(s)$$

Prediction from Super Conformal AdS/QCD:
Same Form Factors for H= M and H=B if $L_M = L_B + 1$

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta \, J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$

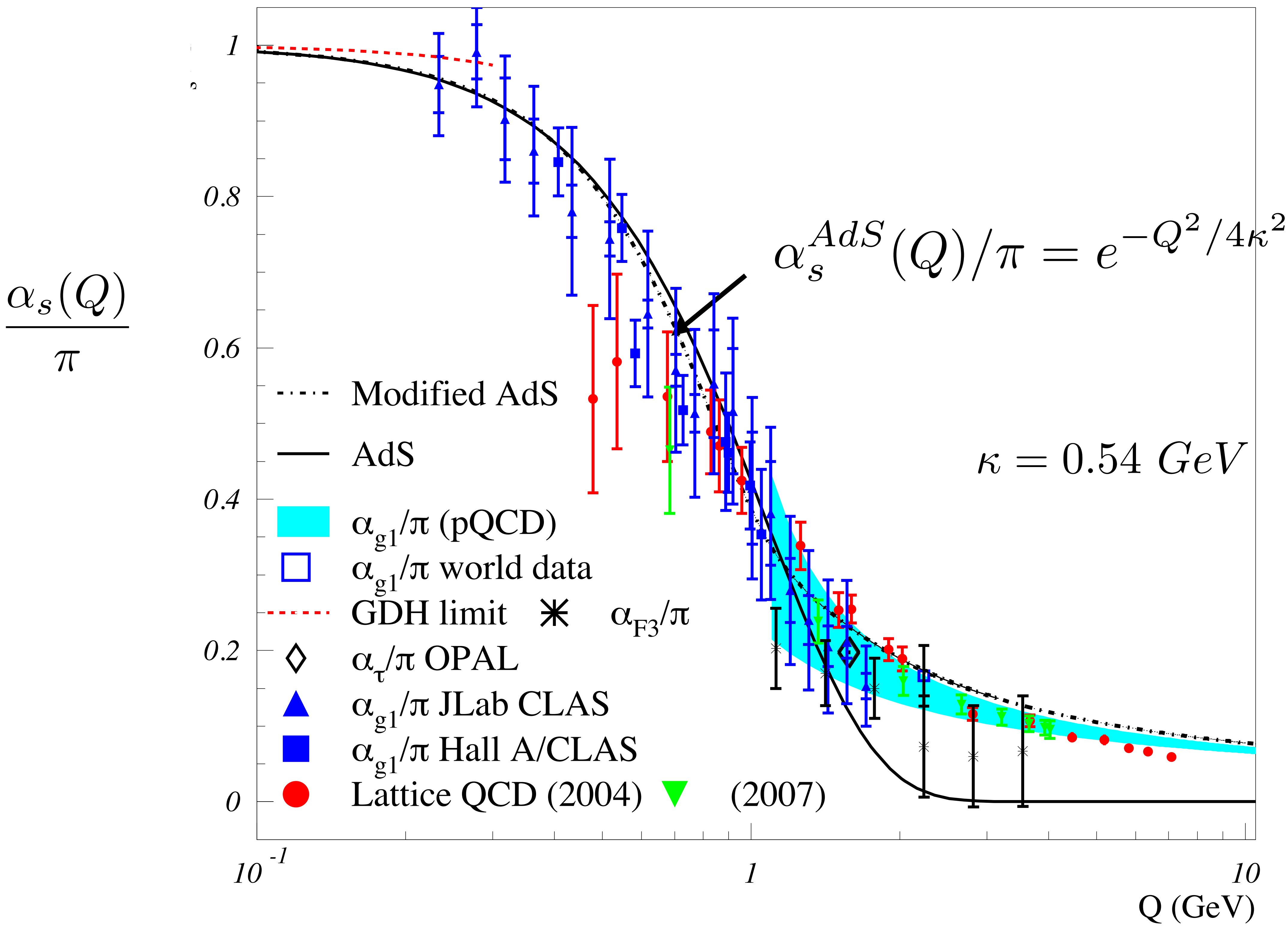
where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

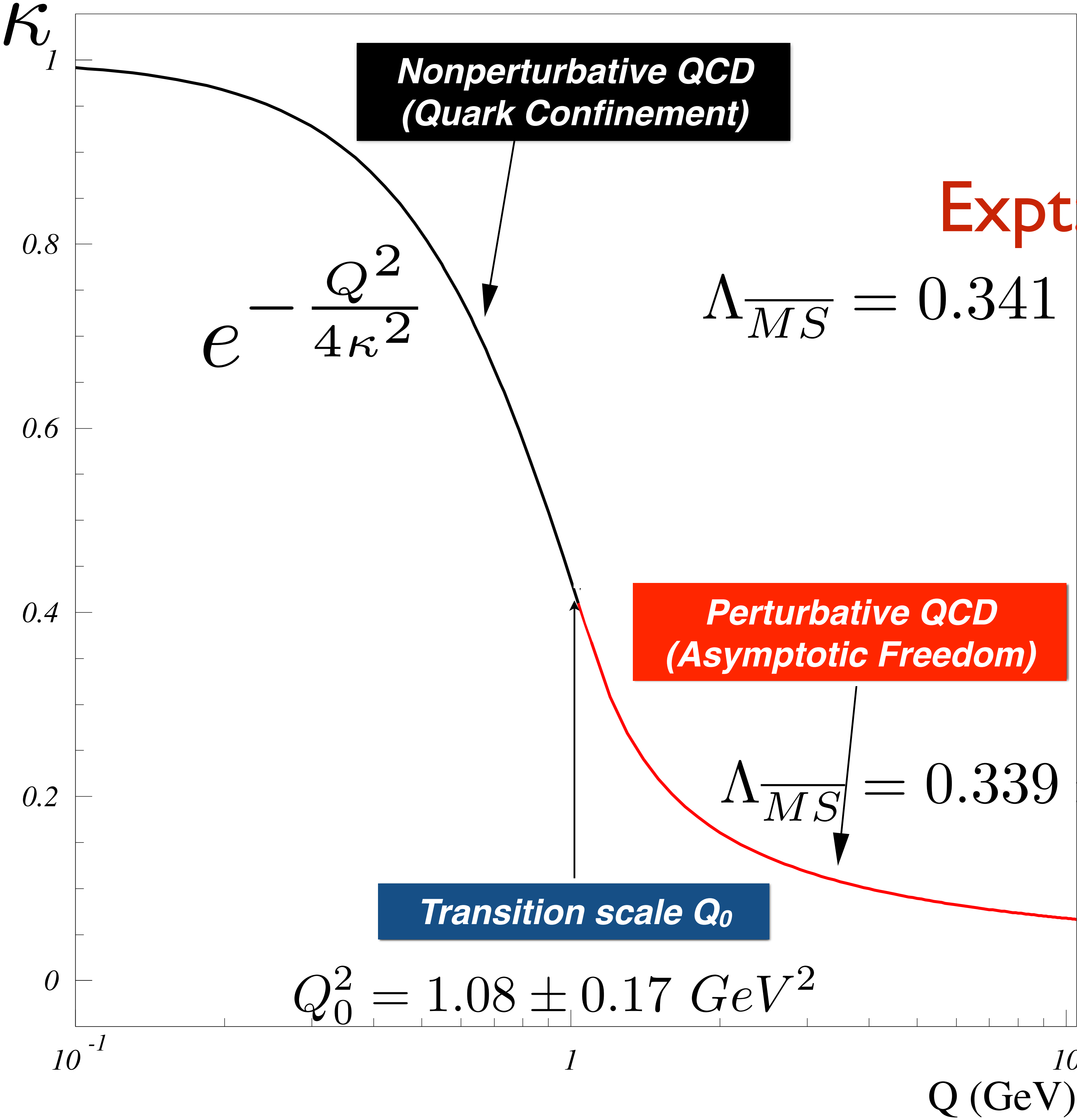
$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

All-Scale QCD Coupling

Deur, de Teramond, sjb

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$

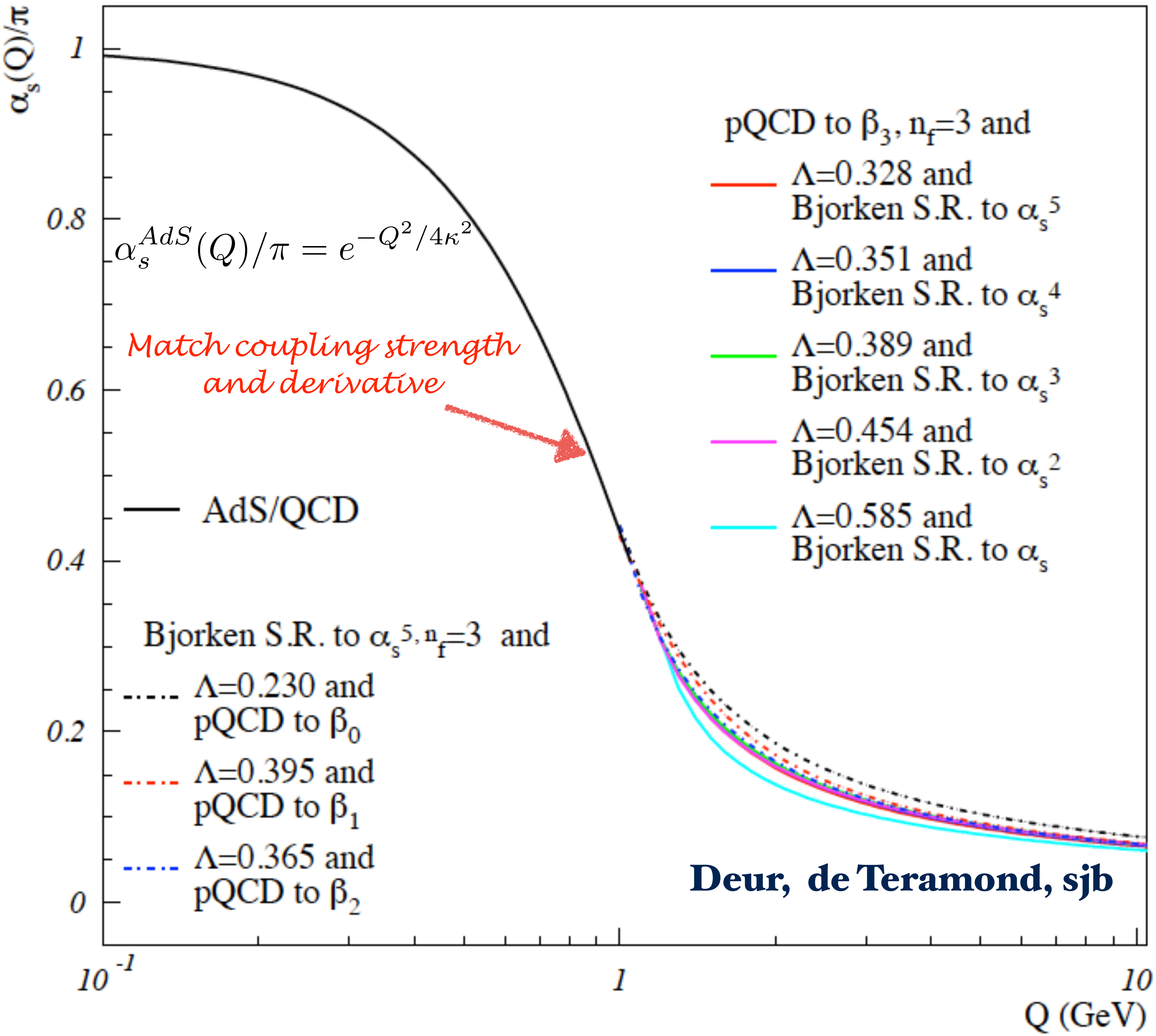


Expt:

$$\Lambda_{\overline{MS}} = 0.341 \pm 0.024 \; GeV$$

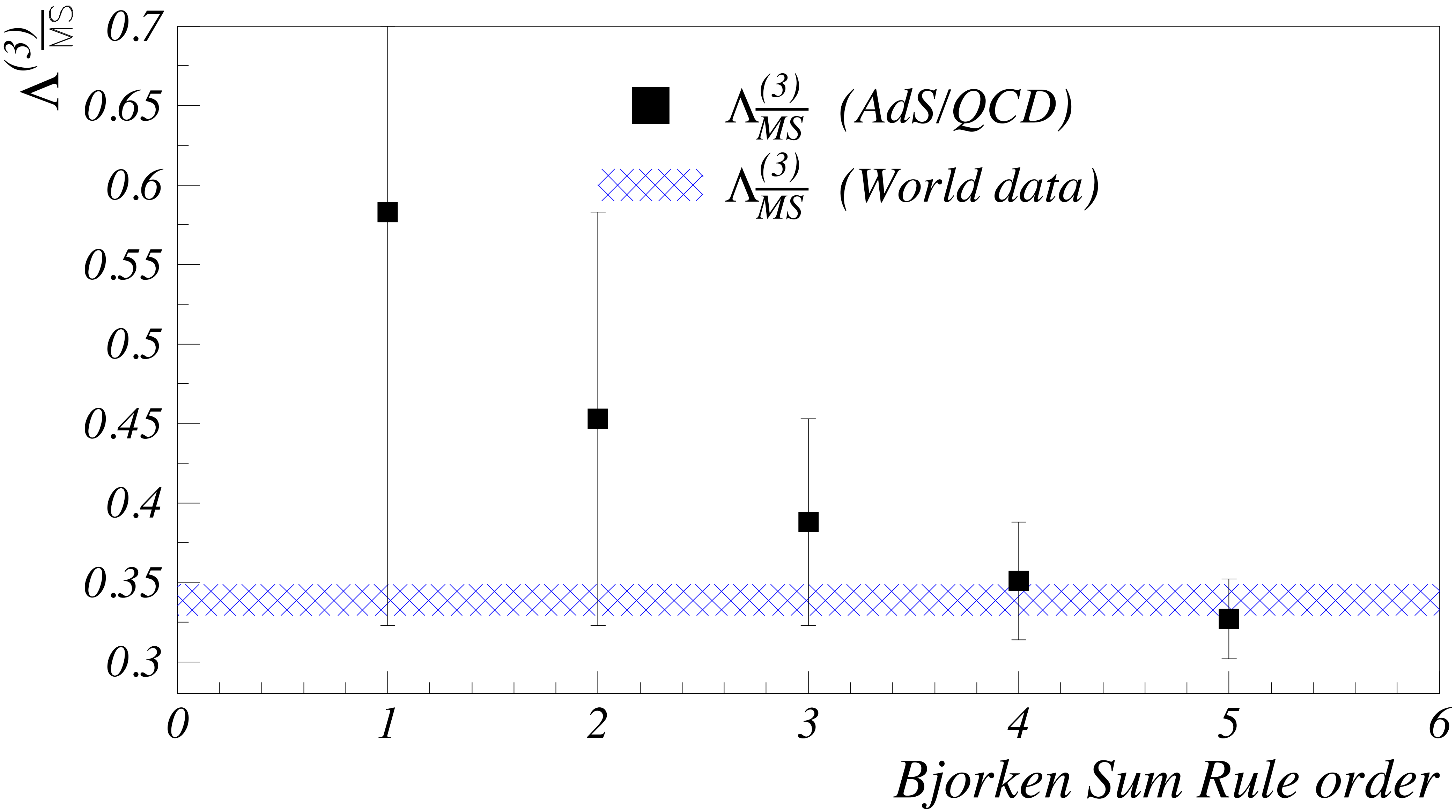
$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016 \; GeV$$

$$\lambda \equiv \kappa^2$$



$\alpha_s(Q)/\pi$
$\alpha_s^{AdS}(Q)/\pi = e^{-Q^2/4\kappa^2}$
Match coupling strength
and derivative
AdS/QCD
pQCD to β_3, n_f=3 and
Λ=0.328 and
Bjorken S.R. to α_s^5
Λ=0.351 and
Bjorken S.R. to α_s^4
Λ=0.389 and
Bjorken S.R. to α_s^3
Λ=0.454 and
Bjorken S.R. to α_s^2
Λ=0.585 and
Bjorken S.R. to α_s
Bjorken S.R. to α_s^5, n_f=3 and
Λ=0.230 and
pQCD to β_0
Λ=0.395 and
pQCD to β_1
Λ=0.365 and
pQCD to β_2
Deur, de Teramond, sjb
Q (GeV)

Deur, de Teramond, sjb



$$\Lambda_{\overline{MS}} = 0.5983\kappa = 0.5983\frac{m_\rho}{\sqrt{2}} = 0.4231 m_\rho = 0.328\ GeV$$

Connect $\Lambda_{\overline{MS}}$ to hadron masses!

Experiment: $M_\rho = 0.7753 \pm 0.0003\ GeV$

$$m_\rho = \sqrt{2}\kappa$$

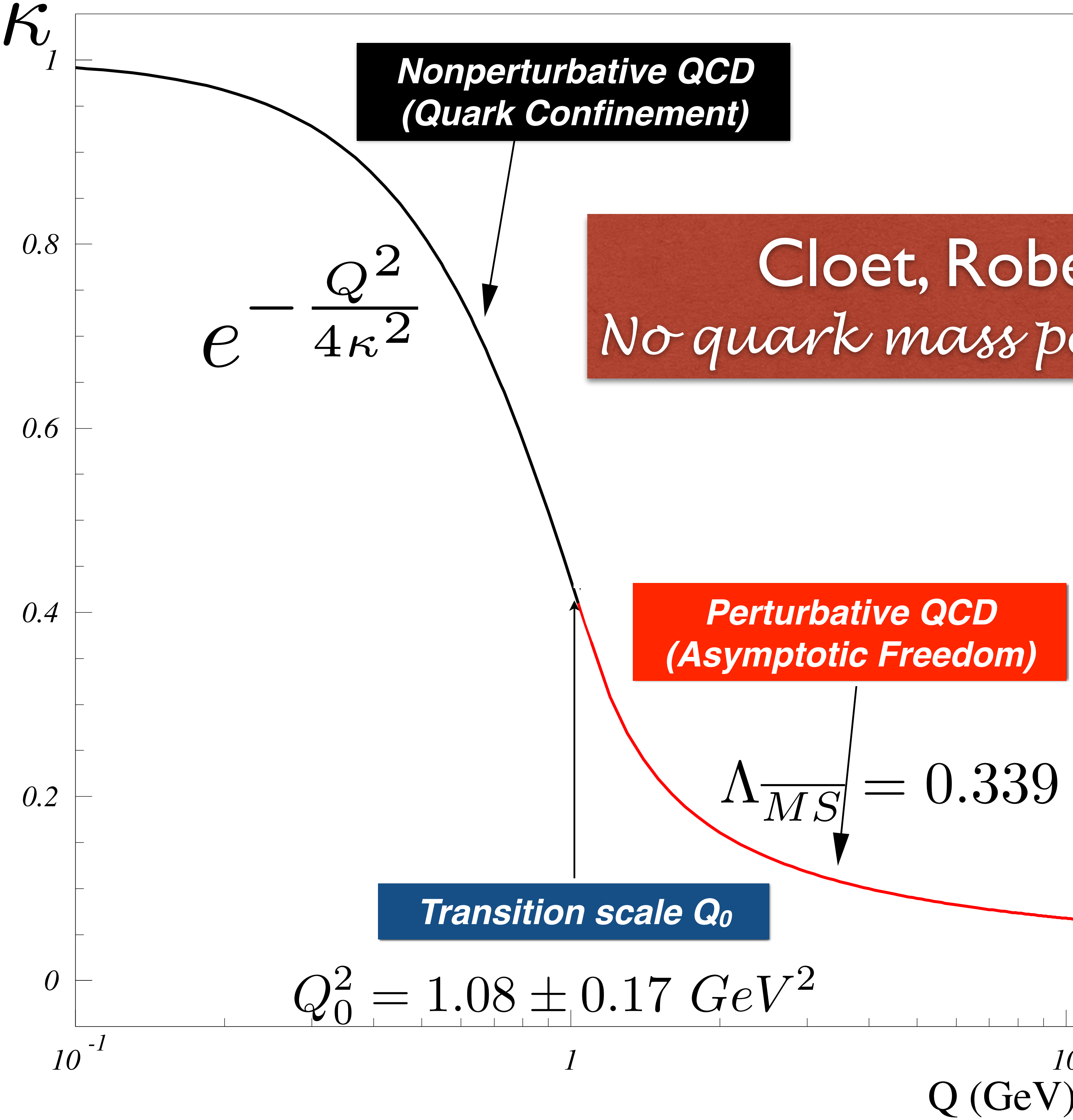
$$m_p = 2\kappa$$

All-Scale QCD Coupling

Deur, de Teramond, sjb

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$

$$e^{-\frac{Q^2}{4\kappa^2}}$$



Cloet, Roberts:
No quark mass pole in DSE

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016\ GeV$$

$$Q_0^2 = 1.08 \pm 0.17\ GeV^2$$

$$\lambda \equiv \kappa^2$$

de Tèramond, Dosch, sjb

Interpretation of Mass Scale κ

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of κ
- Value of κ itself not determined -- place holder
- Need external constraint such as f_π

Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

↕

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Connection to the Linear Instant-Form Potential

- Compare invariant mass in the instant-form in the hadron center-of-mass system $\mathbf{P} = 0$,

$$M^2_{q\bar{q}} = 4\, m_q^2 + 4\mathbf{p}^2$$

with the invariant mass in the front-form in the constituent rest frame, $\mathbf{k}_q + \mathbf{k}_{\bar{q}} = 0$

$$M^2_{q\bar{q}} = \frac{\mathbf{k}^2_\perp + m_q^2}{x(1-x)}$$

obtain

$$U = V^2 + 2\sqrt{\mathbf{p}^2 + m_q^2}\, V + 2\, V\sqrt{\mathbf{p}^2 + m_q^2}$$

where $\mathbf{p}^2_\perp = \frac{\mathbf{k}^2_\perp}{4x(1-x)}, \quad p_3 = \frac{m_q(x-1/2)}{\sqrt{x(1-x)}}$, and V is the effective potential in the instant-form

- For small quark masses a linear instant-form potential V implies a harmonic front-form potential U and thus linear Regge trajectories

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

AdS/QCD and Light-Front Holography

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

- **Zero mass pion for m_q =0 (n=J=L=0)**
- **Regge trajectories: equal slope in n and L**
- **Form Factors at high Q^2: Dimensional counting** $[Q^2]^{n-1} F(Q^2) \to \text{const}$
- **Space-like and Time-like Meson and Baryon Form Factors**
- **Running Coupling for NPQCD** $\alpha_s(Q^2) \propto e^{-\frac{Q^2}{4\kappa^2}}$
- **Meson Distribution Amplitude** $\phi_\pi(x) \propto f_\pi \sqrt{x(1-x)}$

Features of AdS/QCD

de Tèramond, Dosch, Deur, sjb

- **Color confining potential $\kappa^4\zeta^2$ and universal mass scale from dilaton**

$$e^{\phi(z)} = e^{\kappa^2 z^2} \qquad \alpha_s(Q^2) \propto \exp -Q^2/4\kappa^2$$

- **Dimensional transmutation** $\Lambda_{\overline{MS}} \leftrightarrow \kappa \leftrightarrow m_H$

- **Chiral Action remains conformally invariant despite mass scale** *DAFF*

- **Light-Front Holography: Duality of AdS and frame-independent LF QCD**

- **Reproduces observed Regge spectroscopy — same slope in n, L, and J for mesons and baryons**

- **Massless pion for massless quark**

- **Supersymmetric meson-baryon dynamics and spectroscopy: $L_M = L_B + 1$**

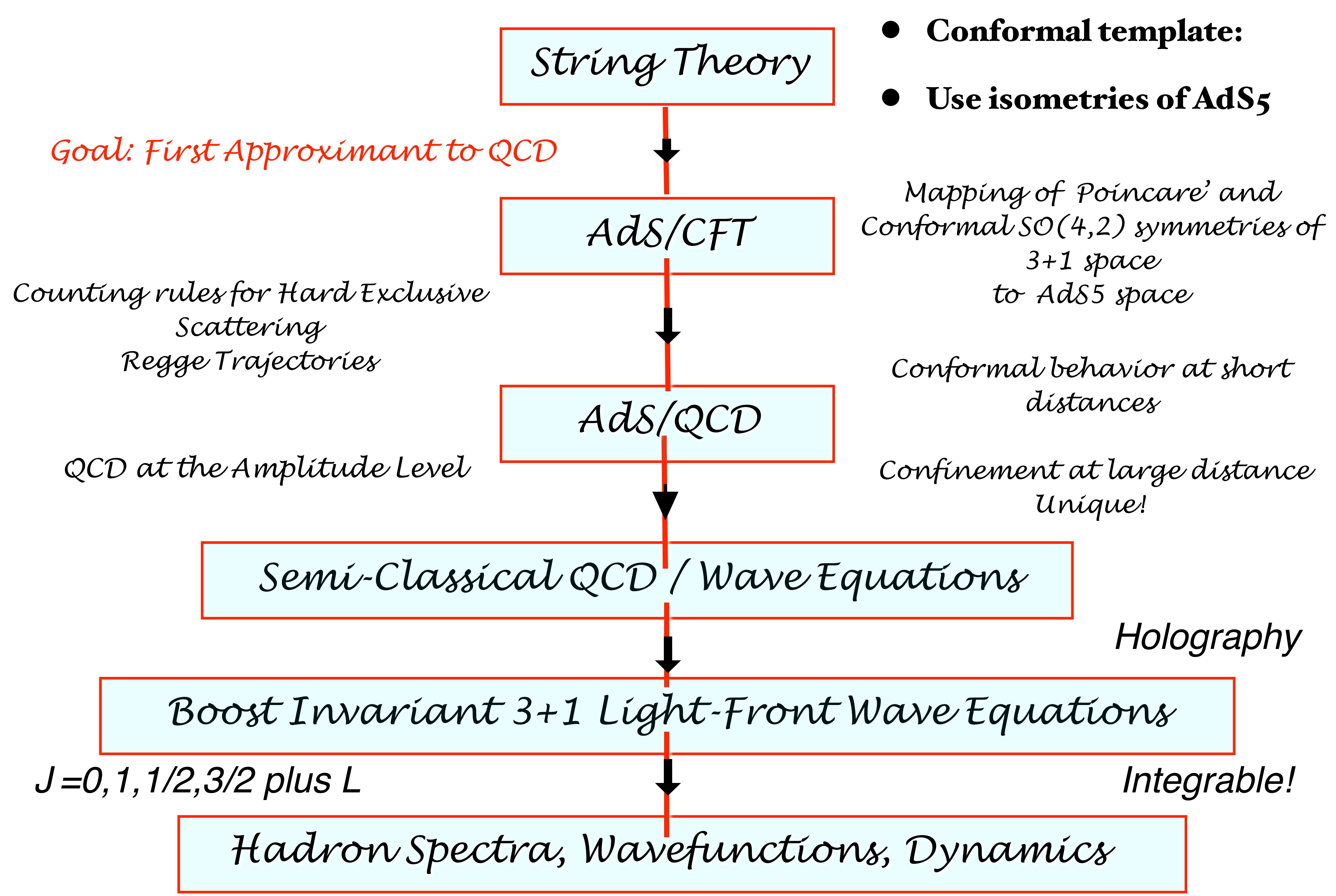
Superconformal Algebra
Fubini and Rabinovici

- **Dynamics: LFWFs, Form Factors, GPDs**

An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- As Simple as Schrödinger Theory in Atomic Physics
- LF radial variable ζ conjugate to invariant mass squared
- Relativistic, Frame-Independent, Color-Confining
- Unique confining potential!
- QCD Coupling at all scales: Essential for Gauge Link phenomena
- Hadron Spectroscopy and Dynamics from one parameter
- Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates: Zero cosmological constant!
- Systematically improvable with DLCQ-BLFQ Methods



Conformal template:
Use isometries of AdS5
String Theory
Goal: First Approximant to QCD
AdS/CFT
Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space
Counting rules for Hard Exclusive Scattering
Regge Trajectories
AdS/QCD
Conformal behavior at short distances
QCD at the Amplitude Level
Confinement at large distance
Unique!
Semi-Classical QCD / Wave Equations
Holography
Boost Invariant 3+1 Light-Front Wave Equations
J =0,1,1/2,3/2 plus L
Integrable!
Hadron Spectra, Wavefunctions, Dynamics

de Tèramond, Dosch, Lorce, sjb

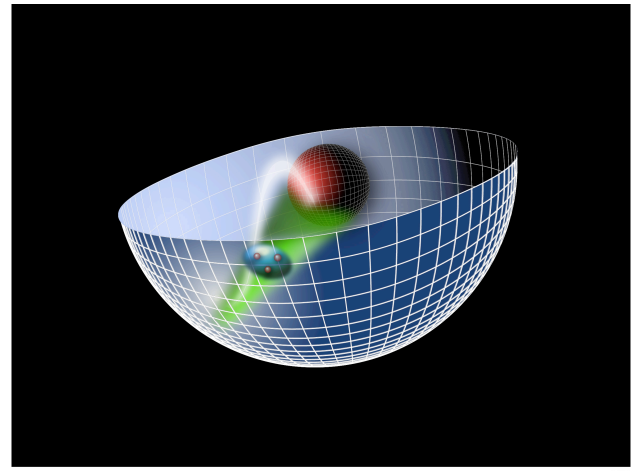
Future Directions for AdS/QCD

- **Hadronization at the Amplitude Level**
- **Diffractive dissociation of pion and proton to jets**
- **Identify the factorization Scale for ERBL, DGLAP evolution: Q_o**
- **Compute Tetraquark Spectroscopy Sequentially**
- **Update SU(6) spin-flavor symmetry**
- **Heavy Quark States: Supersymetry, not conformal**
- **Compute higher Fock states; e.g. Intrinsic Heavy Quarks**
- **Nuclear States — Hidden Color**
- **Basis LF Quantization**

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

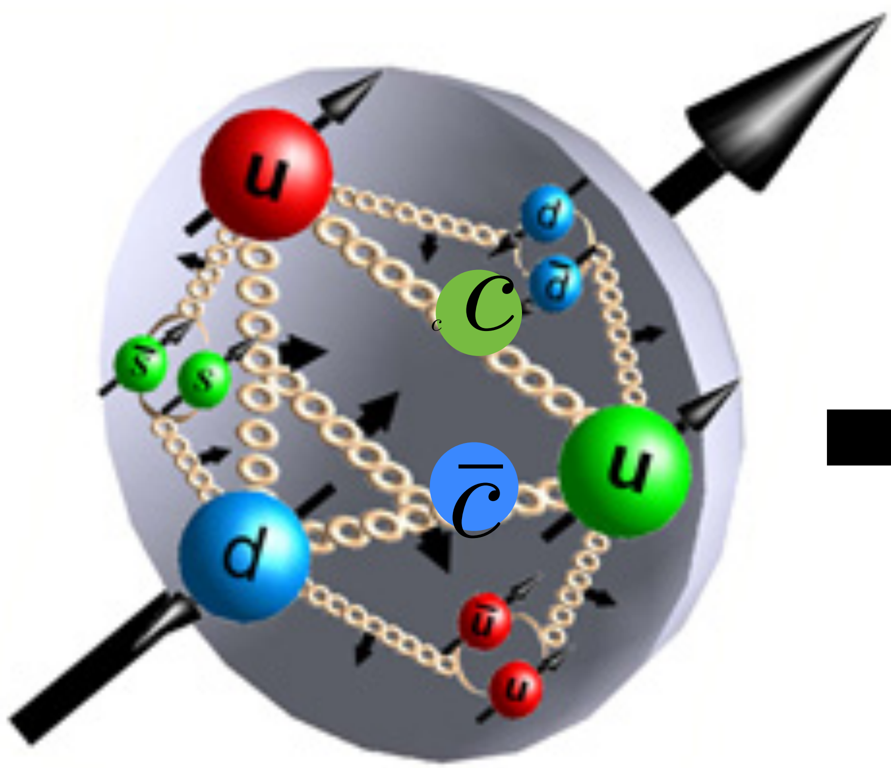
$$\kappa \simeq 0.6 \ GeV$$

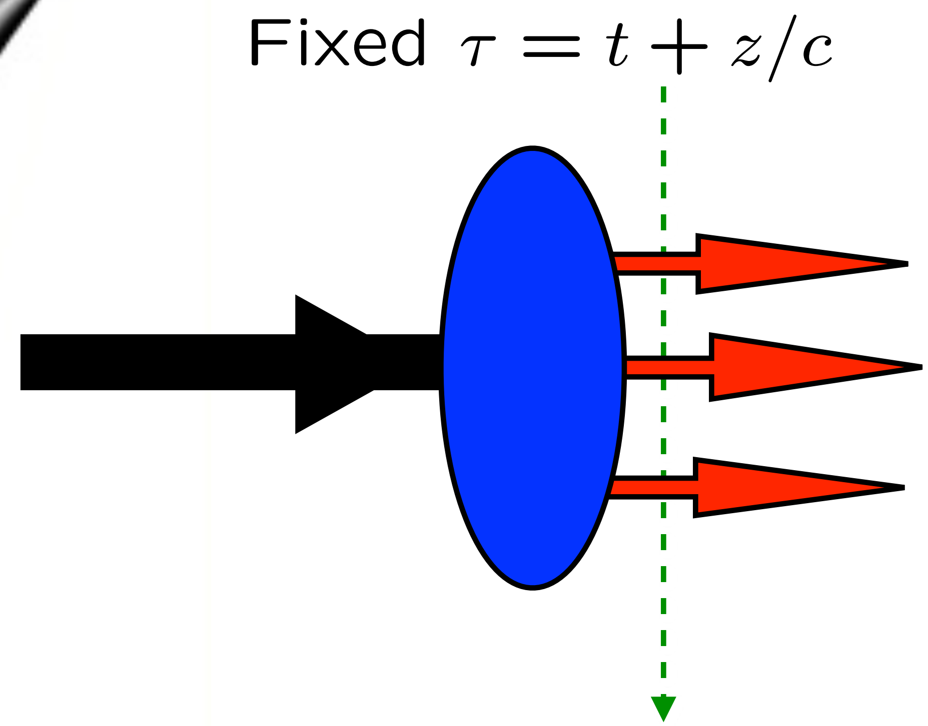
$$1/\kappa \simeq 1/3 \ fm$$

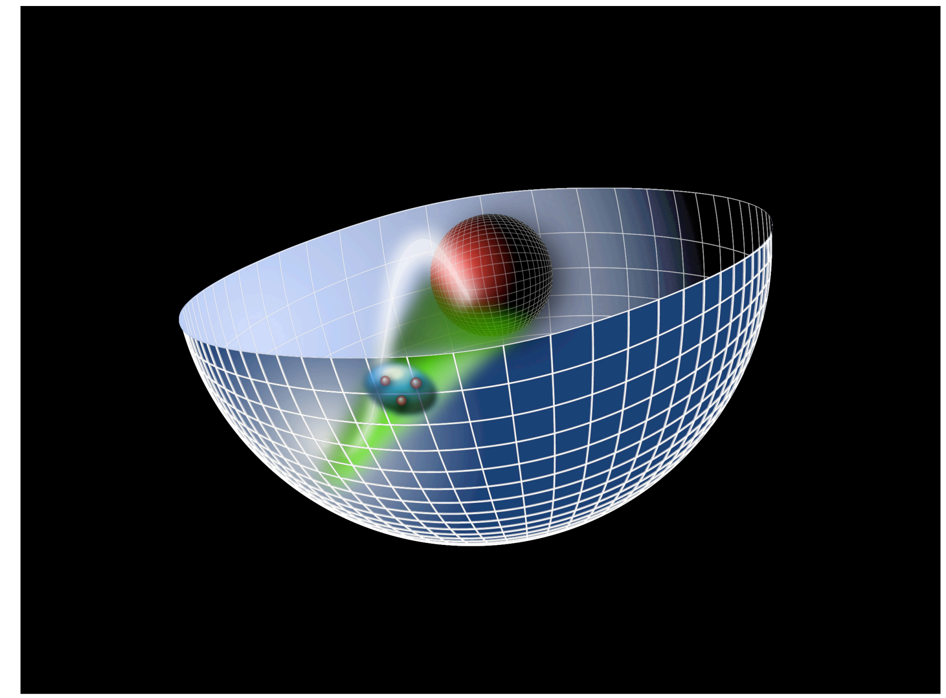
- **de Alfaro, Fubini, Furlan:**
  - **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Light-Front Holography and New Advances in Nonperturbative QCD







Stan Brodsky

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