Massive quarks in backreacted holographic QCD

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Outline

1. Motivation
2. Brief introduction to V-QCD models

   [MJ, Kiritsis arXiv:1112.1261]

3. V-QCD at finite quark mass

   [MJ, arXiv:1501.07272]

   - Bound state masses at finite quark mass
   - Four fermion operators

Motivation: Veneziano limit

QCD: $SU(N_c)$ gauge theory with N_f quark flavors (fundamental)

- Often useful: "quenched" or "probe" approximation, $N_f \ll N_c$, 't Hooft limit
- Here Veneziano limit: large N_f, N_c with $x = N_f/N_c$ fixed $\Rightarrow$ backreaction

Backreaction $\Rightarrow$ better modeling of (ordinary) QCD?

Important new features can be captured in the Veneziano limit:

- Phase diagram of QCD (at zero temperature, baryon density, and quark mass), varying $x = N_f/N_c$
- The QCD thermodynamics as a function of x
- Phase diagram as a function of baryon density

Motivation: finite quark mass

Why is detailed analysis at finite m_q necessary?

- Ordinary QCD has finite quark masses
- Important to make connection to lattice studies (which often have sizeable quark mass)
- Generally interesting scaling features
- May add constraints to the holographic model
  - E.g. at large quark mass many things can be computed from QCD

This talk: flavor independent mass

Holographic V-QCD: the fusion

The fusion:

1. IHQCD: model for glue by using dilaton gravity

   [Gursoy, Kiritsis, Nitti; Gubser, Nellore]

2. Adding flavor and chiral symmetry breaking via tachyon brane actions

   [Klebanov,Maldacena; Bigazzi,Casero,Cotrone,Iatrakis,Kiritsis,Paredes]

Consider 1. + 2. in the Veneziano limit with full backreaction
$\Rightarrow$ V-QCD models

[MJ, Kiritsis arXiv:1112.1261]

Defining V-QCD

Degrees of freedom:

- The tachyon $\tau \leftrightarrow \bar{q}q$, and the dilaton $\lambda \leftrightarrow \mathrm{Tr}F^2$
- $\lambda = e^{\phi}$ is identified as the 't Hooft coupling $g^2 N_c$

Terms relevant in the classical vacuum:

$$\mathcal{S}_{\mathrm{V-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$
$$- N_f N_c M^3 \int d^5x \, V_f(\lambda,\tau) \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau)}$$

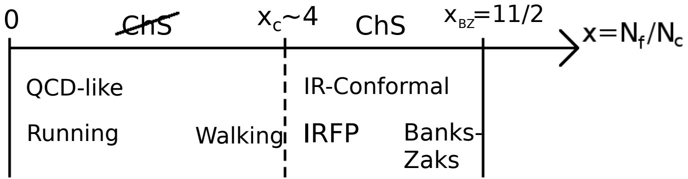
$$V_f(\lambda,\tau) = V_{f0}(\lambda) \exp(-a(\lambda)\tau^2) \; ; \qquad ds^2 = e^{2A(r)}(dr^2 + \eta_{\mu\nu}x^{\mu}x^{\nu})$$

Need to choose V_{f0}, a, and κ ... (V_g chosen as before)

The simplest and most reasonable choices do the job!

Phase diagram at $m_q = 0$

With reasonable potentials, at zero quark mass and temperature, constructing numerically all vacua, expected result:



- Conformal transition (BKT) at $x = x_c \simeq 4$
  [Kaplan,Son,Stephanov;Kutasov,Lin,Parnachev]
- Transition: violation of the BF bound
- Miransky scaling, $\langle \bar{q} q \rangle \sim \exp\left[-\frac{2K}{\sqrt{x_c - x}}\right]$, in walking regime
- For $x < x_c$, "good" IR singularity + tachyon
- For $x > x_c$, IR AdS_5, zero tachyon

Turning on finite m_q

Quark mass defined through the tachyon boundary conditions in the UV ($r \to 0$):

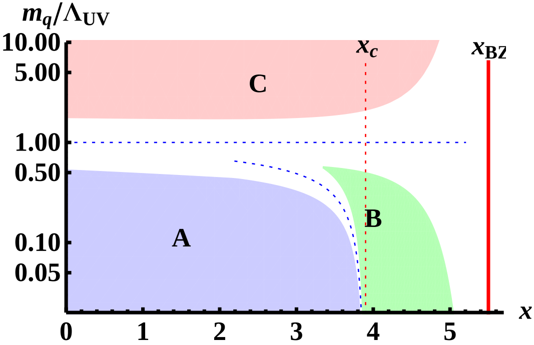
$$\tau(r) \simeq m_q(-\log r)^{-\gamma_0/\beta_0} r + \langle \bar{q}q \rangle (-\log r)^{\gamma_0/\beta_0} r^3$$

- Implies nonzero tachyon and chiral symmetry breaking
- Conformal transition becomes a crossover
- Discontinuous change of IR geometry in the conformal window

Analysis of the tachyon solution $\Rightarrow$ different regimes

- Physics "near" the BKT transition independent of the details of the model
- Results partially apply to BKT transitions in other models (e.g. D3-D7 and dynamic AdS/QCD)

[Kutasov,Lin,Parnachev; Alvares,Alho,Erdmenger,Evans,Kim,Scott,Tuominen]



Border between A and B $\sim \exp\left[-\frac{2K}{\sqrt{x_c - x}}\right] \sim \langle \bar{q}q \rangle$

A : m_q is a small perturbation

B : “Hyperscaling” regime: walking controlled by m_q

C : Regime of large quark mass

White regions: crossovers (no phase transitions at finite m_q)

Hyperscaling regime B
($\Delta_* =$ dimension of quark mass at IR fixed point)

$$m(\text{mesons}) \sim m_q^{1/\Delta_*} \qquad \langle \bar{q}q \rangle \sim m_q^{(4-\Delta_*)/\Delta_*}$$

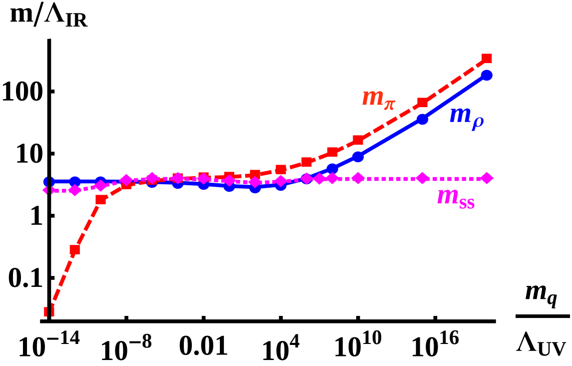
- Results agree with standard field theory analysis and dynamic AdS/QCD [Evans,Scott]
- Quite generic consequence of the tachyon flow in the conformal window

Large quark mass (regime C): only tachyon potentials $V_f(\tau) \propto e^{-C\tau^2}$ produce

- Meson mass gap $\mathcal{O}(m_q)$ and
- Suppressed splitting between the lowest meson states
- Asymptotically linear meson spectra at $m_q = 0$ also require $V_f(\tau) \propto e^{-C\tau^2}$ (!) [Arean, Iatrakis, MJ, Kiritsis]
- Glueball masses take their YM values, not enhanced with m_q

Example: masses for the walking case

$x_c - x \ll 1$, Masses in units of IR (glueball) scale

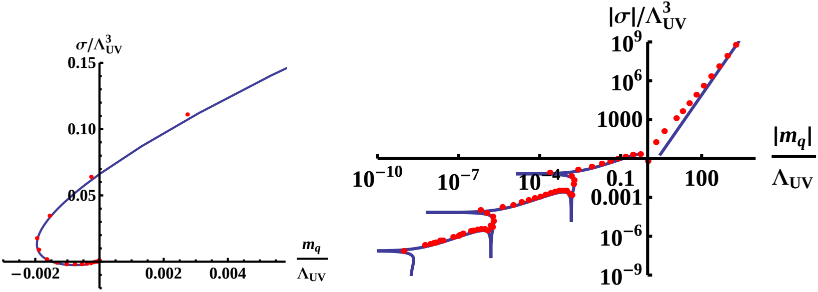


- All masses have the same behavior at intermediate m_q (regime B, hyperscaling)
- Meson masses enhanced wrt glueballs at large m_q

Chiral condensate

The dependence of $\sigma \propto \langle \bar{q} q \rangle$ on the quark mass

- For $x < x_c$ spiral structure



- Dots: numerical data
- Continuous line: (semi-)analytic prediction

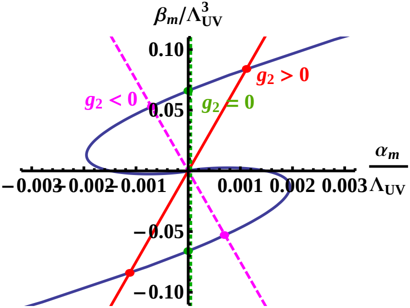
Allows to study the effect of double-trace deformations

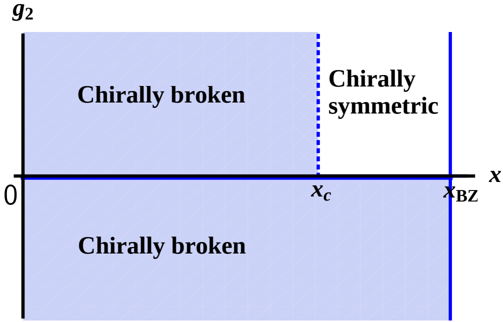
Witten's recipe: modified UV boundary conditions

$$W = -m_q \int d^4x \bar{q}q + \frac{g_2}{2}\int d^4x (\bar{q}q)^2$$

Denote $m_q \to \alpha_m$, $\sigma \to \beta_m$

$\alpha_m = g_2 \beta_m \qquad$ (for $m_q = 0$)





Conclusions

- Finite (flavor independent) quark mass can be easily implemented in V-QCD
- Dependence of mass spectra on m_q matches with QCD at qualitative level
- Most results close to the conformal transition independent of details
- Next step: fitting the potentials of the model quantitatively to QCD data

Extra slides

Matching to QCD

In the UV ($\lambda \to 0$):

- UV expansions of potentials matched with perturbative QCD beta functions $\Rightarrow$

$$\lambda(r) \simeq -\frac{1}{\beta_0 \log r}, \quad \tau(r) \simeq m_q(-\log r)^{-\gamma_0/\beta_0} r + \sigma(-\log r)^{\gamma_0/\beta_0} r^3$$

  the 5th coordinate $r \to 0$ and $\sigma \sim \langle \bar{q} q \rangle$

In the IR ($\lambda \to \infty$):

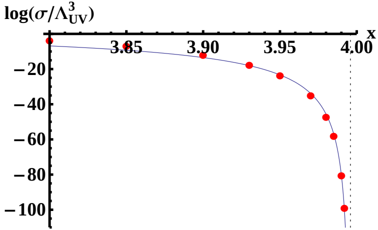
- $V_{f0}(\lambda)$, $a(\lambda)$, and $\kappa(\lambda)$ chosen to produce tachyon divergence: several possibilities ($\to$ Potentials I and II)
- Extra constraints from the asymptotics of the meson spectra and regularity at finite theta angle
  [Arean, Iatrakis, MJ, Kiritsis]
- Working potentials often string-inspired power-laws, multiplied by logarithmic corrections (i.e, first guesses usually work!)

Energy scales at zero quark mass

V-QCD reproduces the expected picture:

1. QCD regime: single energy scale Λ
2. Walking regime ($x_c - x \ll 1$): two scales related by Miransky/BKT scaling law

$$\frac{\Lambda_{\mathrm{UV}}}{\Lambda_{\mathrm{IR}}} \sim \exp\left(\frac{\kappa}{\sqrt{x_c - x}}\right)$$



3. Conformal window ($x_c \leq x < 11/2$): again one scale Λ, but slow RG flow

Gell-Mann-Oakes-Renner relation

Combination of two computations:

1. Pion mass at small m_q (analyzing the fluctuation equations)
2. Chiral condensate as $\frac{d}{dm_q} S_{\mathrm{on-shell}}$, when $m_q \to 0$

$$f_\pi^2 m_\pi^2 \simeq -m_q \langle \bar{q} q \rangle \left[1 + \mathcal{O}\left(m_q\right)\right]$$

- Expected linear corrections $\propto m_q$ (despite logarithmic running of m_q)
- Consistency check of UV structure

Vector correlators and S-parameter

1. Introduce bulk gauge fields dual to vector operators

$$A_\mu^{L/R} \leftrightarrow \bar{q}\gamma_\mu(1 \pm \gamma_5)q$$

2. Fluctuate full flavor action of V-QCD

$$S_f = -\frac{1}{2}M^3 N_c \mathbb{T}r \int d^4x\, dr \left(V_f(\lambda, T^\dagger T)\sqrt{-\det \mathbf{A}_L} + (L \to R) \right)$$

$$\mathbf{A}_{L/R\,MN} = g_{MN} + w(\lambda, T) F_{MN}^{(L/R)} + $$

$$+ \frac{\kappa(\lambda, T)}{2} \left[(D_M T)^\dagger (D_N T) + (D_N T)^\dagger (D_M T) \right]$$

Here T and $A^{(L/R)}$ matrices in flavor space

3. Compute vector-vector correlators using standard recipes

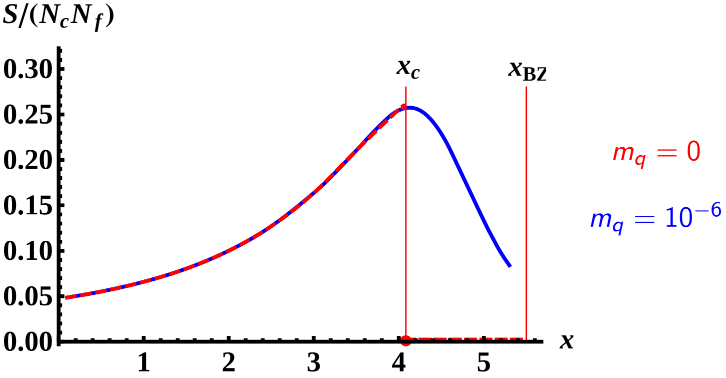
$$-i\langle J_\mu^{a\,(V)} J_\nu^{b\,(V)} \rangle \propto \delta^{ab} \left(q^2 \eta_{\mu\nu} - q_\mu q_\nu \right) \Pi_V(q^2)$$

$$-i\langle J_\mu^{a\,(A)} J_\nu^{b\,(A)} \rangle \propto \delta^{ab} \left[\left(q^2 \eta_{\mu\nu} - q_\mu q_\nu \right) \Pi_A(q^2) + q_\mu q_\nu \Pi_L(q^2) \right]$$

S-parameter

After adding gauge fields dual to vector operators in the DBI action

$$S = 4\pi \frac{\partial}{\partial q^2}\left[q^2\Pi_V(q^2) - q^2\Pi_A(q^2)\right]_{q^2=0}$$



- Discontinuity at $m_q = 0$ in the conformal window
- Qualitative agreement with field theory expectations

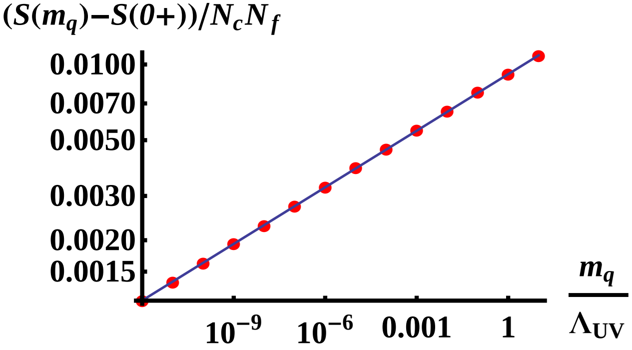
[Sannino]

Scaling of the S-parameter

As $m_q \to 0$ in the conformal window,

$$S(m_q) \simeq S(0+) + c\left(\frac{m_q}{\Lambda_{\rm UV}}\right)^{\frac{\Delta_{FF}-4}{\Delta_*}}$$

- Limiting value $S(0+) = \lim_{m_q\to 0+} S(m_q)$ is finite and positive (while $S(0) = 0$)
- Δ_{FF} is the dimension of $\mathrm{tr}F^2$ at the fixed point



How does the phase structure arise?

Turning on a tiny tachyon in the conformal window

$$\tau(r) \sim m_q r^{\Delta_*} + \sigma r^{4-\Delta_*}$$

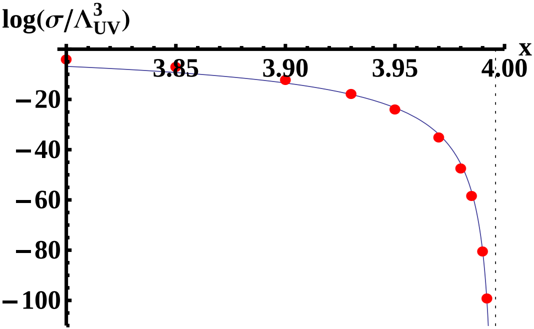
Breitenlohner-Freedman (BF) bound

$$\Delta_*(4-\Delta_*) = -m_\tau^2 \ell_*^2 \leq 4$$

Violation of BF bound $\Rightarrow$ instability

- $\Rightarrow$ bound saturated at the conformal phase transition ($x = x_c$)
- BF bound violation leads to a BKT transition quite in general

Consequences of the BKT transition



$$\langle \bar{q}q \rangle \sim \sigma \sim \exp\left(-\frac{\kappa}{\sqrt{x_c - x}}\right)$$

1. Miransky/BKT scaling as $x \to x_c$ from below
   - E.g., The chiral condensate $\langle \bar{q}q \rangle \propto \sigma$
2. Unstable Efimov vacua observed for $x < x_c$
3. Turning on the quark mass possible

The CP-odd term

Bulk axion a

- dual to $\mathrm{tr} F \wedge F$
- background value identified as θ/N_c, where θ is the theta angle of QCD

Tachyon Ansatz $T = \tau e^{i\xi}\,\mathbb{I}$

String motivated CP-odd term added in the action

$$S_a = -\frac{M^3 N_c^2}{2} \int d^5x \sqrt{-\det g}\, Z(\lambda) \times \left[da - x\left(2V_a(\lambda,\tau)\, A - \xi\, dV_a(\lambda,\tau)\right)\right]^2$$

[Casero, Kiritsis, Paredes]

Symmetry

$$A_\mu \to A_\mu + \partial_\mu \epsilon\ , \qquad \xi \to \xi - 2\epsilon\ , \qquad a \to a + 2x\, V_a \epsilon$$

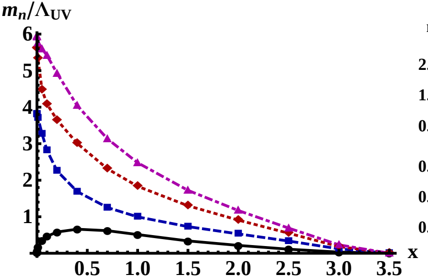
reflects the axial anomaly in QCD (with $\epsilon = \epsilon(x_\mu)$)

The mass of η'

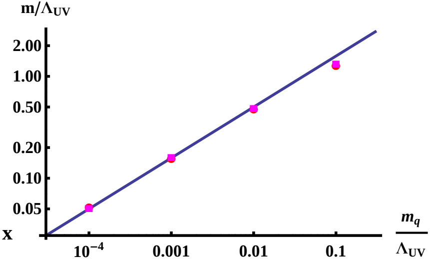
Perturbative analysis of the coupled flavor singlet (pseudoscalar meson+glueball) fluctuation equations $\Rightarrow$
The Witten-Veneziano relation: η' becomes light as $x \to 0$

$$m_{\eta'}^2 \simeq m_\pi^2 + x \frac{N_f N_c \chi}{f_\pi^2}$$

$m_q = 0$



$x = 0.0001$



Finite T and μ – definitions

Add gauge field

$$\mathcal{S}_{\mathrm{V-QCD}} = N_c^2 M^3 \int d^5x\sqrt{g}\left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)\right]$$
$$- N_f N_c M^3 \int d^5x V_f(\lambda,\tau)$$
$$\times\sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau + w(\lambda)F_{ab})}$$

$$F_{r0} = \partial_r\Phi \qquad \Phi = \mu - nr^2 + \cdots$$

A more general metric (A and f solved from EoMs)

$$ds^2 = e^{2A(r)}\left(\frac{dr^2}{f(r)} - f(r)dt^2 + d\mathbf{x}^2\right)$$

Nontrivial blackening factor f: black hole solutions possible

Various solutions

Two classes of IR geometries:

1. Black hole solutions $\rightarrow$ temperature and entropy through BH thermodynamics
   - $f'(r_h) = -4\pi T$; $\quad s = 4\pi M^3 N_c^2 e^{3A(r_h)}$
2. Thermal gas solutions ($f \equiv 1$)
   - Any T and μ, zero s

Two types of tachyon behavior ($\tau \leftrightarrow \bar{q}q$, quark mass and condensate from UV boundary behavior):

1. Vanishing tachyon – chirally symmetric
2. Nontrivial tachyon – chirally broken

$\Rightarrow$ four possible types of background solutions

Computation of pressure

Three phases turn out to be relevant (at small x)

- Tachyonic Thermal gas (chirally broken)
- Tachyonic BH (chirally broken)
- Tachyonless BH (chirally symmetric)

Nontrivial numerical analysis:

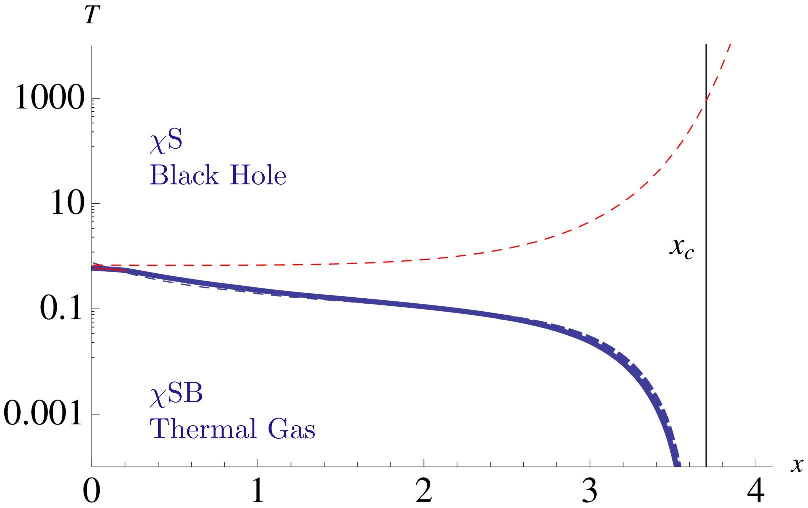
1. T, μ not input parameters, they need to be calculated first
2. Integrate numerically for each phase

$$dp = s\,dT + n\,d\mu$$

3. Phase with highest p dominates

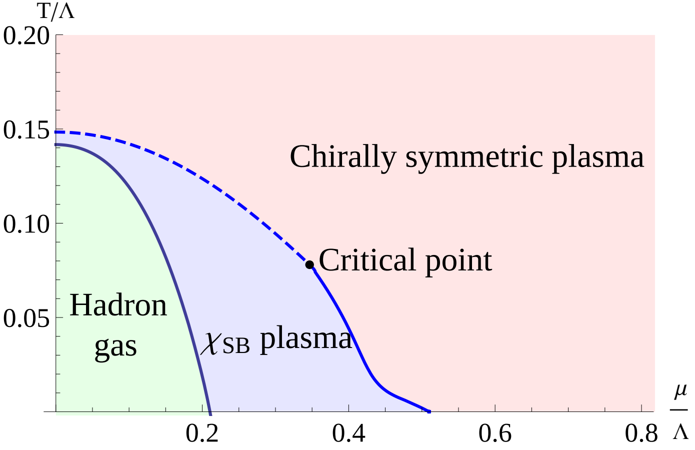
Phase diagram: example at zero μ

Phases on the (x, T)-plane – as expected from QCD



Phase diagram at finite μ (example at fixed x)

First attempt: $x = N_f/N_c = 1$, Veneziano limit, zero quark mass



- $\mathrm{AdS}_2 \times \mathbb{R}^3$ IR geometry as $T \to 0$
- Finite entropy at zero temperature $\Rightarrow$ instability?

Fluctuation analysis

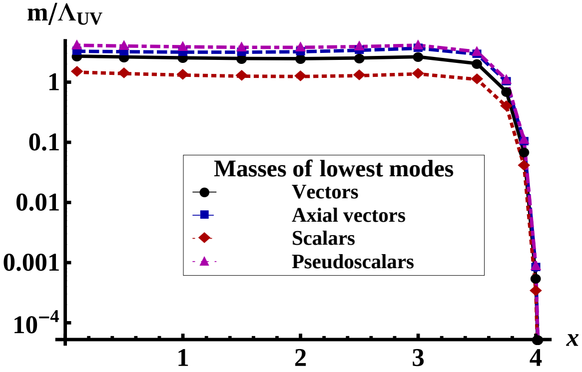
1. Meson spectra (at zero temperature and quark mass)
   - Implement (left and right handed) gauge fields in $\mathcal{S}_{\mathrm{V-QCD}}$
   - Four towers: scalars, pseudoscalars, vectors, and axial vectors
   - Flavor singlet and nonsinglet ($SU(N_f)$) states

In the region relevant for "walking" technicolor ($x \to x_c$ from below):

- Possibly a light "dilaton" (flavor singlet scalar): Goldstone mode due to almost unbroken conformal symmetry.
  Could the dilaton be the 125 GeV Higgs?

Meson masses

Flavor nonsinglet masses (Example: PotI)



- Miransky scaling:

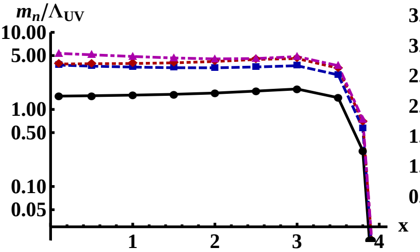
$$m_n \sim \exp\left(-\frac{\kappa}{\sqrt{x_c - x}}\right)$$

- Radial trajectories $m_n^2 \sim n$ or $m_n^2 \sim n^2$ depending on potentials

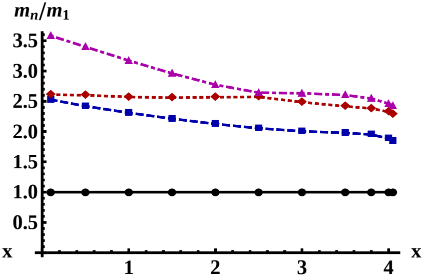
Scalar singlet masses

Scalar singlet (0^{++}) spectrum (PotI):

In log scale



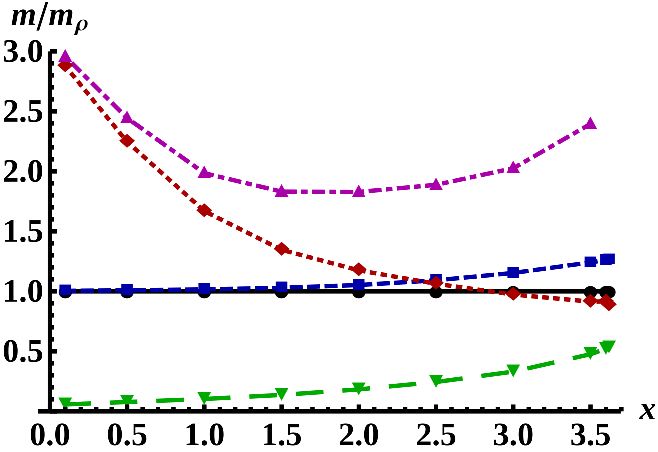
Normalized to the lowest state



- No light dilaton state as $x \to x_c$?

Meson mass ratios

Mass ratios (PotII): Lowest states normalized to ρ



All ratios tend to constants as $x \to x_c$: indeed no dilaton

S-parameter

$$S \sim \frac{d}{dq^2} q^2 \left[\Pi_V(q^2) - \Pi_A(q^2)\right]_{q^2=0}$$

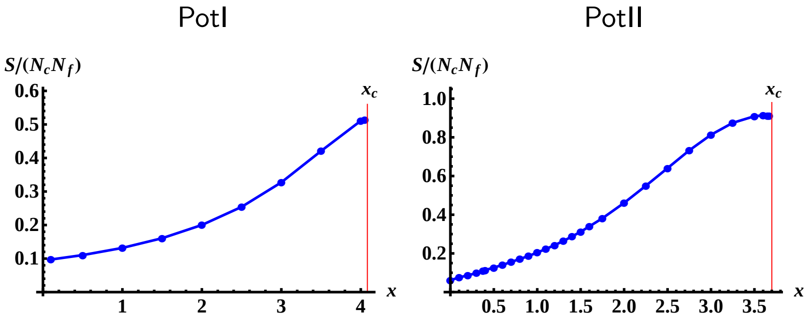
where (at zero quark mass)

$$\Pi_{V/A}(q^2)\left(q^2 g^{\mu\nu} - q^\mu q^\nu\right)\delta^{ab} \propto \langle J^{\mu\, a}_{V/A} J^{\nu\, b}_{V/A}\rangle$$

in terms of the vector-vector and axial-axial correlators

- The S-parameter might be reduced in the walking regime

Results:

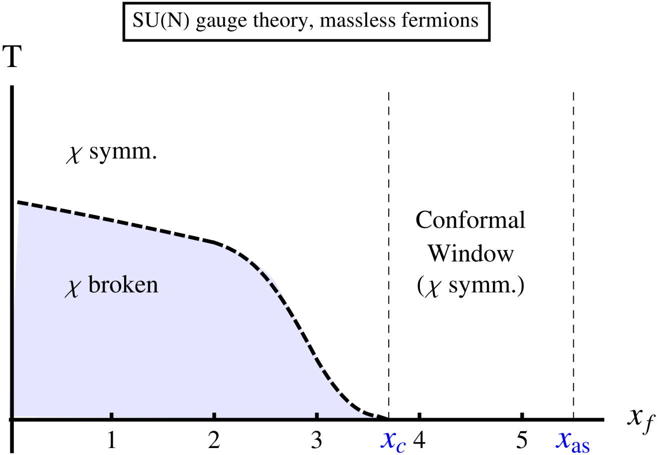


The S-parameter increases with x: expected suppression absent

Jumps discontinuously to zero at $x = x_c$

QCD at finite T (and x)

Expected phase structure at finite temperature (and x)



Potentials I

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda,\tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473-2726x+92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x) \\
\kappa(\lambda) &= \frac{1}{\left(1+\frac{115-16x}{288\pi^2}\lambda\right)^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges exponentially:

$$
\tau(r) \sim \tau_0 \exp\left[\frac{81\ 3^{5/6}(115-16x)^{4/3}(11-x)}{812944\ 2^{1/6}}\frac{r}{R}\right]
$$

Potentials II

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda,\tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473-2726x+92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x)\frac{1+\frac{115-16x}{216\pi^2}\lambda+\lambda^2/(8\pi^2)^2}{(1+\lambda/(8\pi^2))^{4/3}} \\
\kappa(\lambda) &= \frac{1}{(1+\lambda/(8\pi^2))^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges as

$$\tau(r) \sim \frac{27\ 2^{3/4}3^{1/4}}{\sqrt{4619}}\sqrt{\frac{r-r_1}{R}}$$