

nuclear shadowing in a holographic framework

based on

L. Agozzino, P. Castorina, PC: PRL 112 (2014) 141601
EPJ C 74 (2014) 2428

Pietro Colangelo
INFN – Bari – Italy

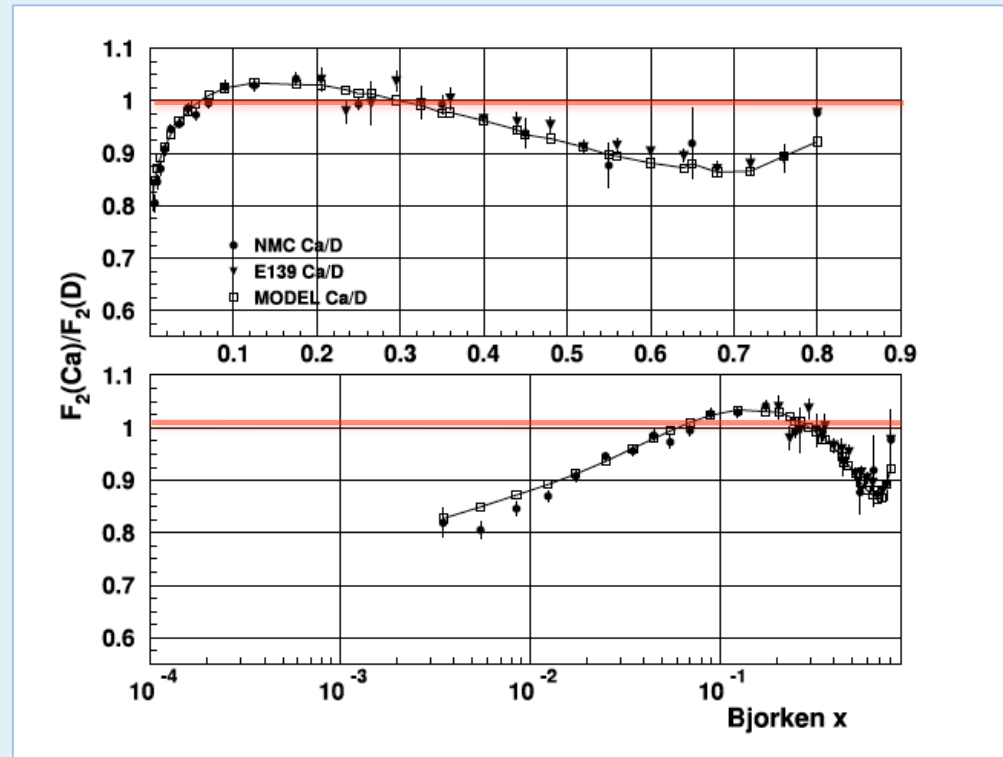
Gauge/Gravity Duality, GGI, 13 April 2015



a comparison of
the holographic approach to data

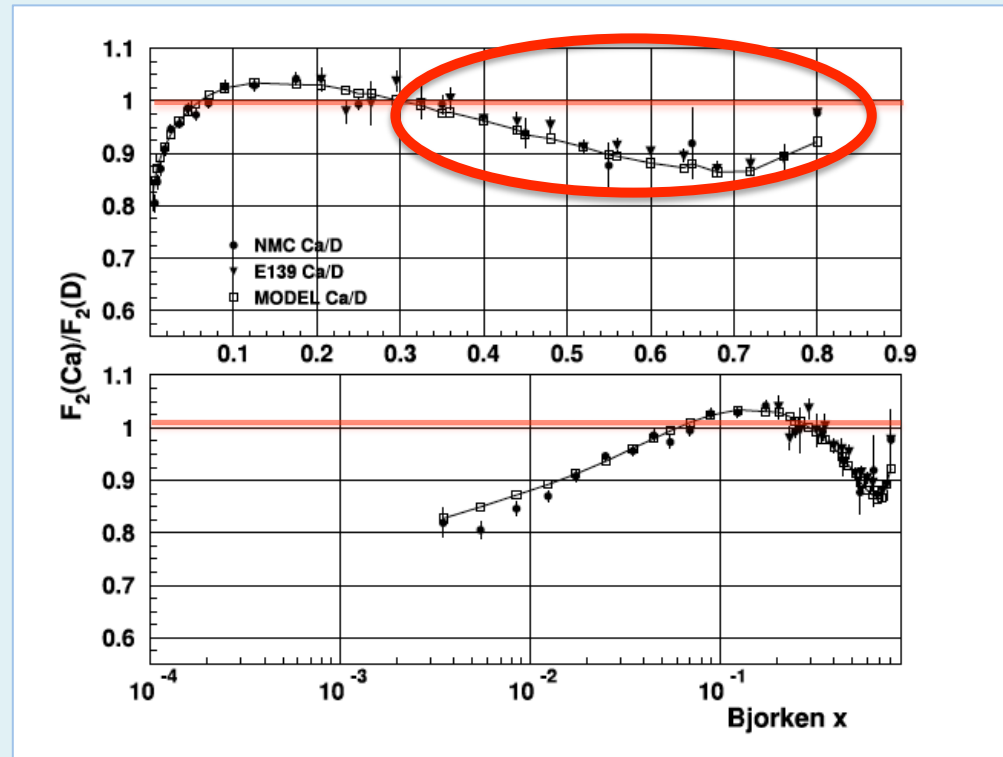
the analysis provides an insight into some
experimental results

nuclear versus nucleon structure functions



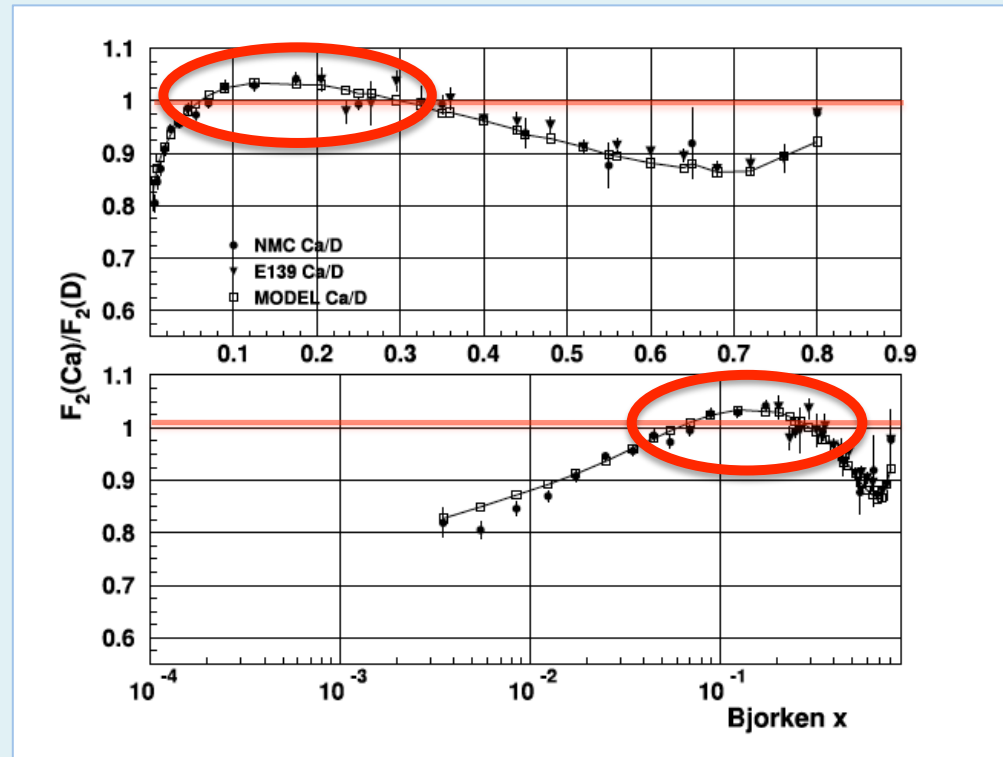
$$\frac{F_2(\text{Ca})}{F_2(\text{D})}$$

nuclear vs nucleon structure functions



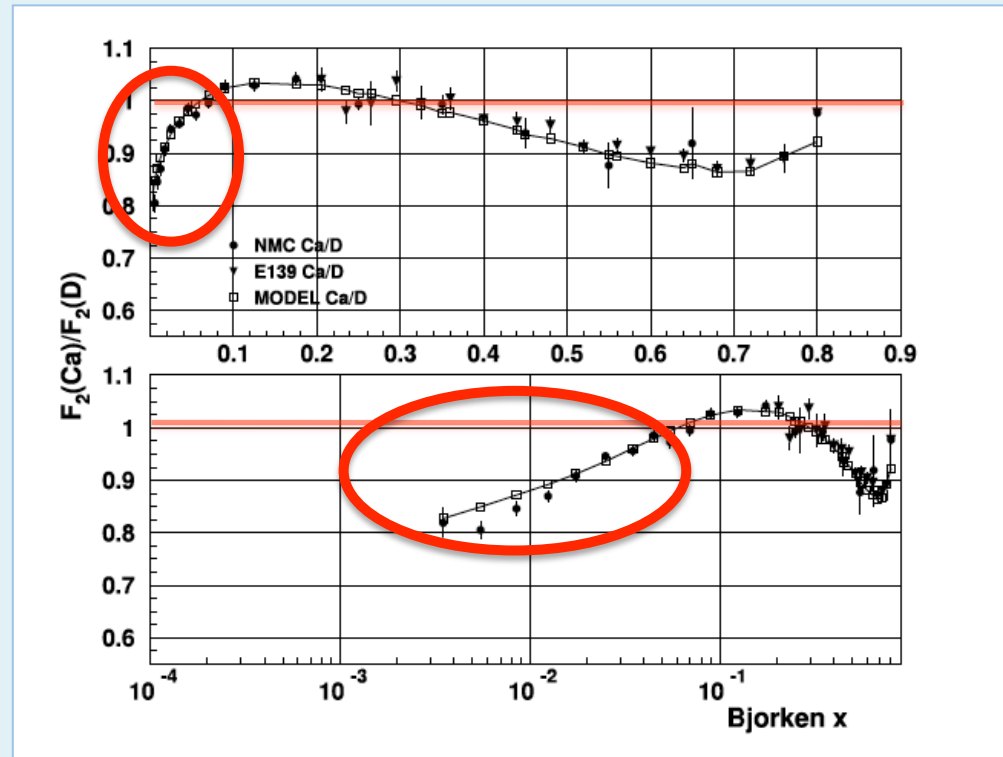
$$\frac{F_2(\text{Ca})}{F_2(\text{D})} < 1$$

nuclear vs nucleon structure functions



$$\frac{F_2(\text{Ca})}{F_2(\text{D})} > 1$$

nuclear vs nucleon structure functions



$$\frac{F_2(\text{Ca})}{F_2(\text{D})} < 1$$

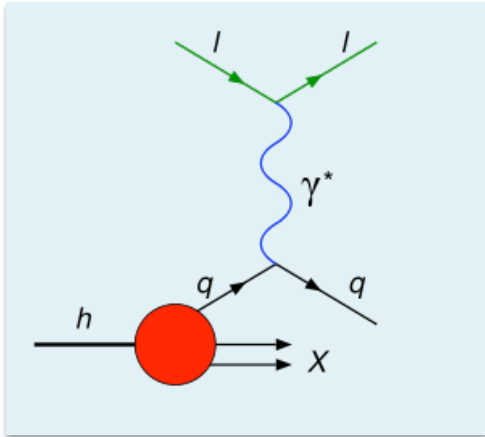
Kulagin and Petti, NPA 765 (2006) 126

small- x domain of interest here

SUMMARY

- DIS IN QCD AND IN A STRONGLY COUPLED THEORY
- SMALL X
- ANALYSIS OF NUCLEAR DATA
- CONCLUSIONS

PROLOGUE



$$\frac{d^2\sigma}{dE'd\Omega} = \frac{e^4}{16\pi^2 Q^4} \left(\frac{E'}{ME} \right) \ell_{\mu\nu} W^{\mu\nu}(p, q)$$

$$W_{\mu\nu} = F_1(x, q^2) \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \frac{2x}{q^2} F_2(x, q^2) \left(P_\mu + \frac{q_\mu}{2x} \right) \left(P_\nu + \frac{q_\nu}{2x} \right)$$

$$Q^2 = -q^2$$

$$0 \leq x = \frac{Q^2}{2p \cdot q} \leq 1 \quad \text{Bjorken } x$$

$F_{1,2}$ encode information on the structure of the target

PROLOGUE

DIS amplitudes from the imaginary part of the forward Compton scattering

$$T_{\mu\nu} = i \int d^4 y e^{iq \cdot y} \langle P\tilde{Q} | T [J_\mu(y) J_\nu(0)] | P\tilde{Q} \rangle$$

$$T_{\mu\nu} = \tilde{F}_1(x, \frac{q^2}{\Lambda^2}) \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \frac{2x}{q^2} \tilde{F}_2(x, q^2) \left(P_\mu + \frac{q_\mu}{2x} \right) \left(P_\nu + \frac{q_\nu}{2x} \right)$$

large Q^2 , $x \gg 1$: OPE for $T_{\mu\nu}$ in terms of operators $O_{n,j}$ (n =spin)
 $\Delta_{n,j} = \delta_{n,j} + \gamma_{n,j}$ total scaling dimension
 $\tau_{n,j} = \Delta_{n,j} - n$ twist

$$\tilde{F}_s(x, q^2) \approx i \sum_n \sum_j C_{n,j}^{(s)} A_{n,j} x^{-n} (2x)^{s-1} \left(\frac{\Lambda^2}{Q^2} \right)^{\frac{1}{2} \tau_{n,j} - 1} \quad (s=1,2)$$

large Q^2 : dominance of the operators with smallest twist

contour argument relates large x behaviour to the moments of the DIS structure functions

DIS and gauge/string duality

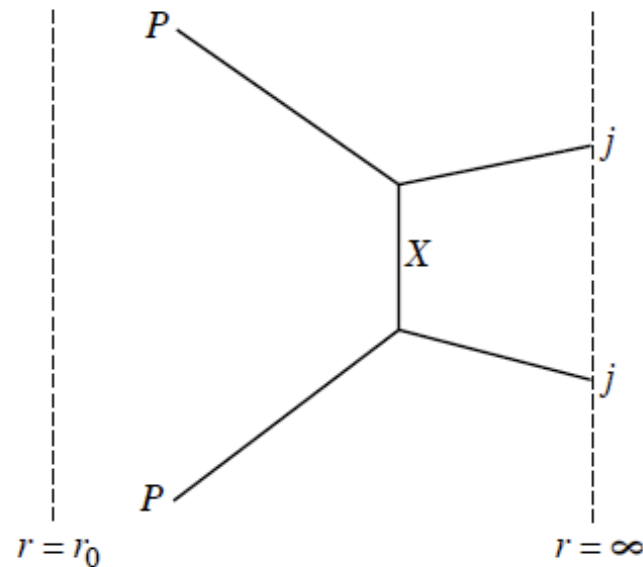
Polchinski Strassler JHEP 035 (2003) 012

Large 't Hooft coupling: gauge theories with dual description

conformal theories \rightarrow $\text{AdS}_5 \times W$ $ds^2 = \frac{r^2}{R^2} \eta_{\mu\nu} dy^\mu dy^\nu + \frac{R^2}{r^2} dr^2 + R^2 d\hat{s}_W^2$

confining theories \rightarrow geometry modified at $r \approx r_0 = \Lambda R^2$

$$\text{Im } T_{\mu\nu} = \pi \sum_{P_X, X} \langle P, Q | J_\nu(0) | P_X, X \rangle \langle P_X, X | \tilde{J}_\mu(q) | P, Q \rangle$$



DIS and gauge/string duality

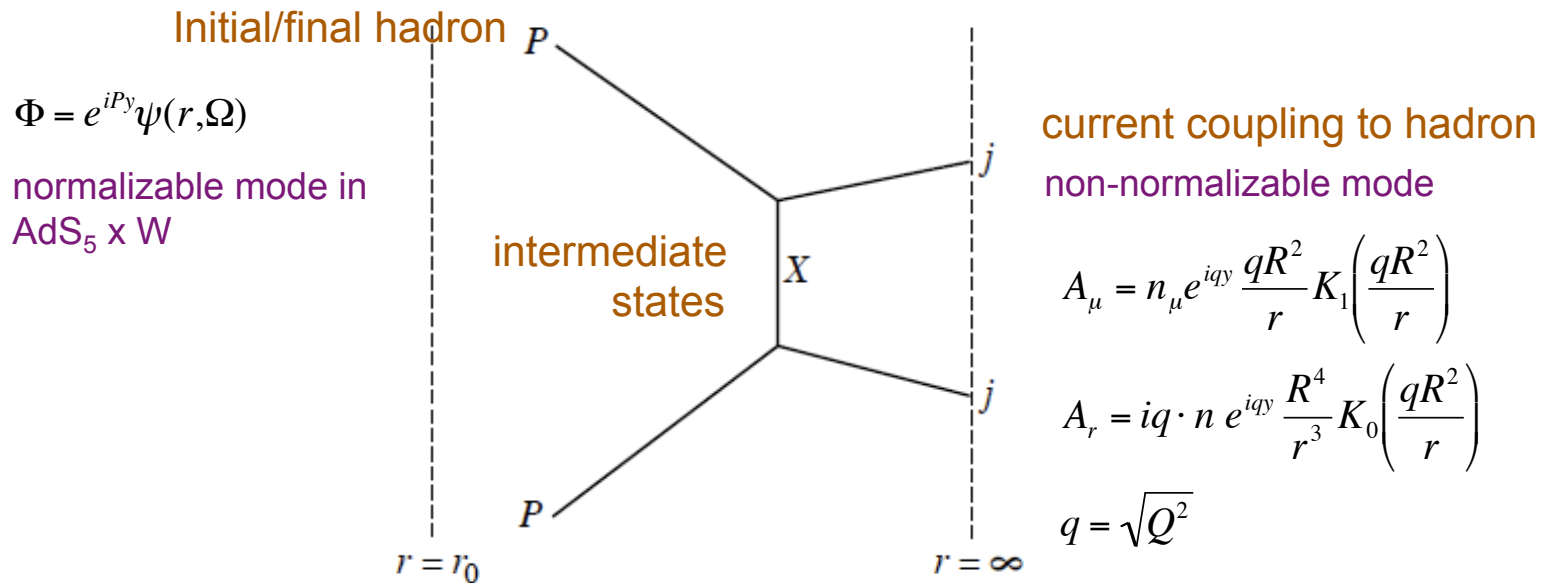
Polchinski Strassler JHEP 035 (2003) 012

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DIS and gauge/string duality

Polchinski Strassler JHEP 035 (2003) 012

$$F_1 = 0 \quad F_2 = \pi A_0 \tilde{Q}^2 \left(\frac{\Lambda^2}{Q^2} \right)^{\Delta-1} x^{\Delta+1} (1-x)^{\Delta-2}$$

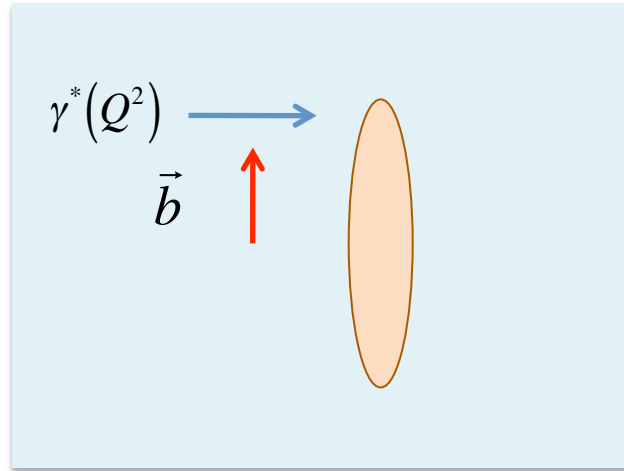
scalar hadrons

Δ dimension of the local operator
creating the initial/final state
from the vacuum

$$F_2 = 2F_1 = \pi A_0' \tilde{Q}^2 \left(\frac{\Lambda^2}{Q^2} \right)^{\tau-1} x^{\tau+1} (1-x)^{\tau-2}$$

spin 1/2 $\left(\tau = \Delta - \frac{1}{2} \right)$

emergence of 5D AdS



kinematical parameters:

2d Longitudinal $p_0 \pm p_3 \cong \exp\left[\pm \log\left(s/\Lambda_{QCD}\right)\right]$

2d Transverse $\vec{b} = \vec{x}_{perp} - \vec{x}'_{perp}$

1 Resolution $z = 1/Q$

Small-x
Regge region

Pomeron contribution

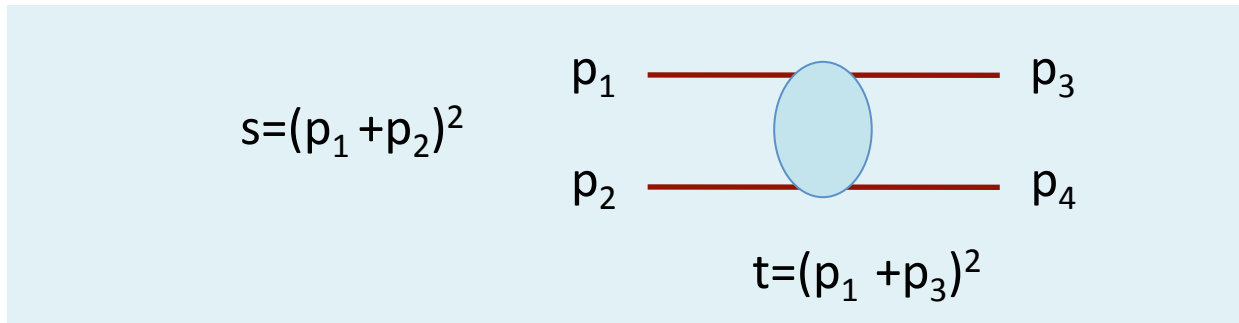
$$s \gg \Lambda_{QCD}^2 \quad |t| \approx \Lambda_{QCD}^2$$

$$x \approx \frac{Q^2}{s}$$

Regge diffusion impact parameter

$$A(s, t) \approx s^{\alpha(t)} = s^{\alpha' t + j_0} \rightarrow a(s, b_{perp}) \approx s^{j_0} \exp\left[-b_{perp}^2 / 4\alpha' \log(s)\right]$$

diffusion in transverse b (impact) space
“time”=rapidity

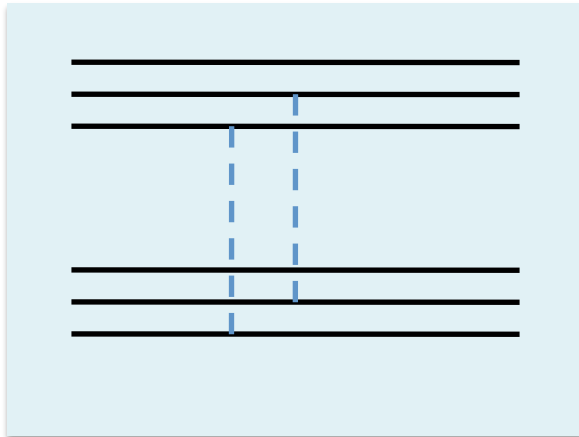


$$A(s, t = -q_{perp}^2) = \int \frac{d^2 b_{perp}}{4\pi^2} e^{-i q_{perp} \cdot b_{perp}} a(s, b_{perp})$$

bare Pomeron

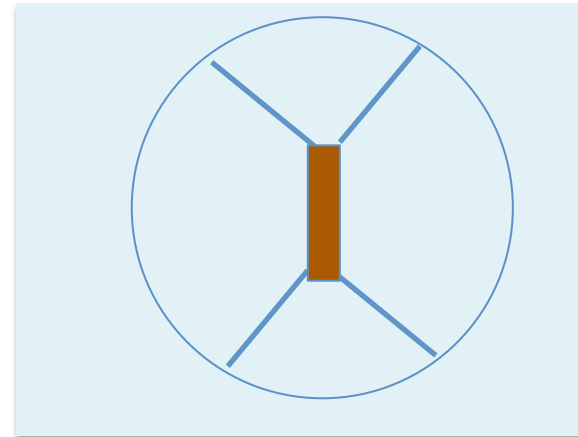
leading $1/N$ term exchange with cylinder topology

weak: two gluons



Low, PRD12 (1975) 163
Nussinov, PRL 34 (1975) 1286

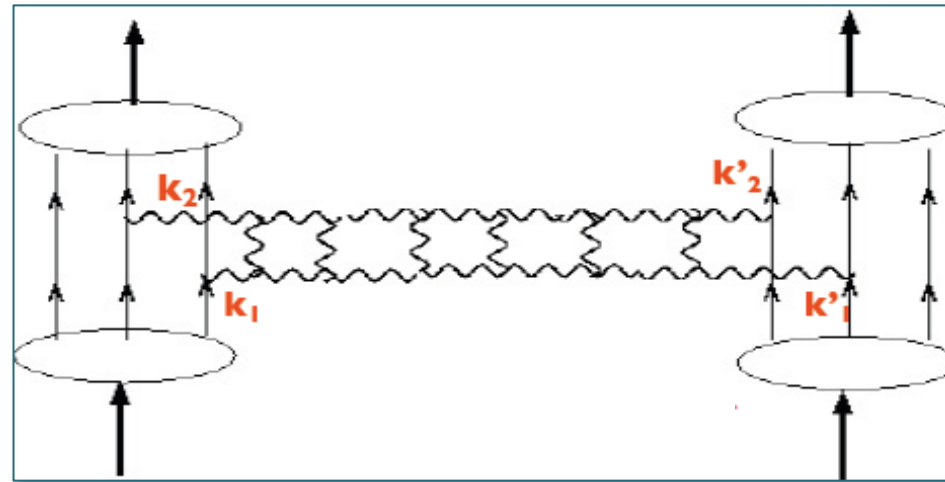
strong: AdS graviton



Witten
Adv.Theor.Math.Phys 2 (1998) 253

BFKL Pomeron

Balitsky Fadin Kuraev Lipatov



first order in λ and all orders in $(g^2 N_c \log s)^n$

“hard” Pomeron

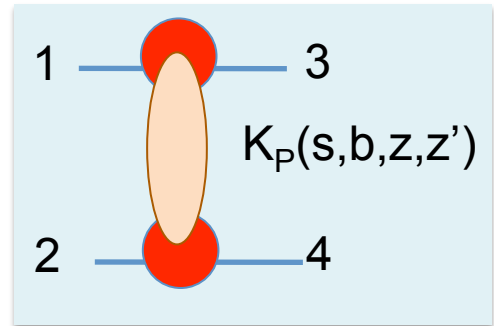
DIS at small-x and gauge/string duality

Brower Djuric Sarcevic Tan JHEP 11 (2010) 051

A,B → C,D
1,2 → 3,4

$$A(s,t) = P_{13} * K_P * P_{24}$$

scattering in 3-d transverse Euclidean AdS₃



$$A(s,t) = 2is \int d^2b e^{iq \cdot b} \int dz \int dz' P_{13}(z) P_{24}(z') \{1 - e^{i\chi(s,b,z,z')}\}$$

$$P_{ij}(z) = \sqrt{g} \left(\frac{z}{R} \right)^2 \phi_i^p(z) \phi_j^p(z) \quad \phi_i \text{ external normalizable wf}$$

$\chi(s,b,z,z')$ eikonal
related to the BPST Pomeron kernel

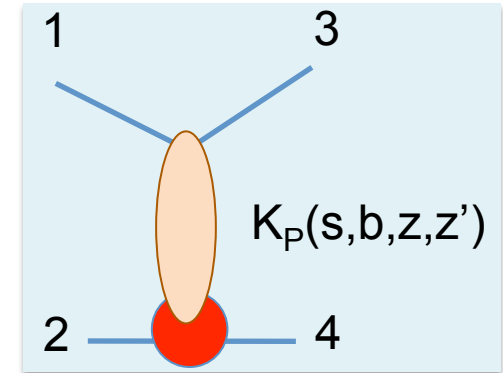
$$\chi(s,b,z,z') = \frac{g_0^2}{2s} \left(\frac{R^2}{zz'} \right)^2 K(s,b,z,z')$$

BPST= Brower Polchinski Strassler Tan JHEP 12 (2007) 005

DIS at small-x and gauge/string duality

Brower Djuric Sarcevic Tan JHEP 11 (2010) 051

$$\begin{array}{l} A, B \rightarrow C, D \\ 1, 2 \rightarrow 3, 4 \end{array} \quad \Longrightarrow \quad \begin{array}{l} \gamma^* p \rightarrow \gamma^* p \\ 1, 2 \rightarrow 3, 4 \end{array}$$



small-x DIS structure functions from the off-shell $\gamma^* p$ total cross section

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{em}} (\sigma_T + \sigma_L)$$

$$F_2(x, Q^2) = \frac{Q^2}{2\pi} \int d^2b \int dz \int dz' P_{13}(z, Q^2) P_{24}(z') \text{Re}\{1 - e^{i\chi(s, b, z, z')}\}$$

$$P_{13}(z) = \frac{1}{z} (Qz)^2 (K_0^2(Qz) + K_1^2(Qz))$$

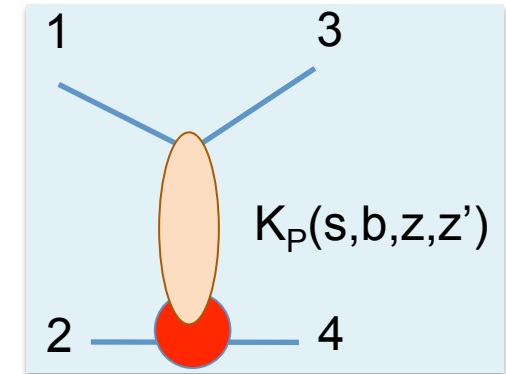
$$2xF_1(x, Q^2) = \frac{Q^2}{2\pi} \int d^2b \int dz \int dz' P_{13}(z, Q^2) P_{24}(z') \text{Re}\{1 - e^{i\chi(s, b, z, z')}\}$$

$$P_{13}(z) = \frac{1}{z} (Qz)^2 K_1^2(Qz)$$

DIS at small-x and gauge/string duality

Brower Djuric Sarcevic Tan JHEP 11 (2010) 051

$$\begin{array}{l} A, B \rightarrow C, D \\ 1, 2 \rightarrow 3, 4 \end{array} \quad \Rightarrow \quad \begin{array}{l} \gamma^* p \rightarrow \gamma^* p \\ 1, 2 \rightarrow 3, 4 \end{array}$$



small-x DIS structure functions from the off-shell $\gamma^* p$ total cross section

kernel K_P splitted in two contributions

$$F_2(x, Q^2) = F_{2,cr}(x, Q^2) + F_{2,ct}(x, Q^2)$$

DIS at small-x and gauge/string duality

Brower Djuric Sarcevic Tan JHEP 11 (2010) 051

conformal term

$$F_{2,cr}^p(x, Q^2) = \frac{g_0^2 \rho^{3/2}}{32\pi^{5/2}} \int dz \int dz' \frac{zz' Q^2}{\tau^{1/2}} P_{13}(z, Q^2) P_{24}(z') e^{(1-\rho)\tau} e^{\Phi(z, z', \tau)}$$

$$\Phi(z, z', \tau) = -\frac{(\log z - \log z')^2}{\rho\tau}$$

$$\tau = \log(\rho z z' s/2)$$

$$x \approx \frac{Q^2}{s}$$

g_0 ρ parameters

DIS at small-x and gauge/string duality

Brower Djuric Sarcevic Tan JHEP 11 (2010) 051

confinement term

$$F_{2,ct}^P(x, Q^2) = \frac{g_0^2 \rho^{3/2}}{32\pi^{5/2}} \int dz \int dz' \frac{zz' Q^2}{\tau^{1/2}} P_{13}(z, Q^2) P_{24}(z') e^{(1-\rho)\tau} e^{-\frac{\ln^2(zz'/z_0^2)}{\rho\tau}} G(z, z', \tau)$$

$$G(z, z', \tau) = 1 - 2\sqrt{\pi\rho\tau} e^{\eta^2} \operatorname{erfc}(\eta)$$

$$\eta = \frac{-\log(zz'/z_0^2) + \rho\tau}{\sqrt{\rho\tau}}$$

z_0 IR cutoff (hard wall)

conformal term

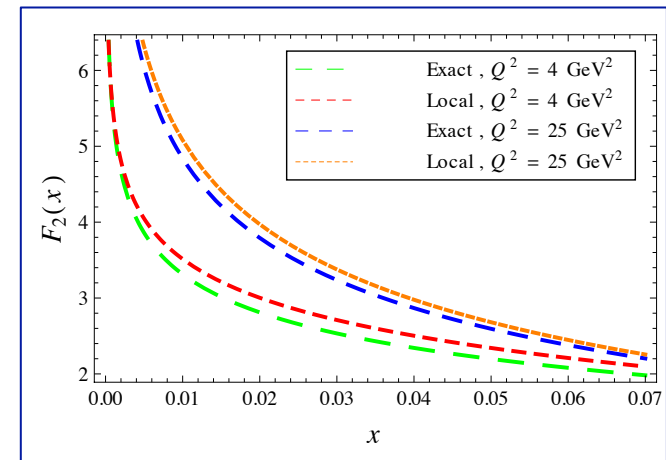
$$F_{2,cr}^P(x, Q^2) = \frac{g_0^2 \rho^{3/2}}{32\pi^{5/2}} \int dz \int dz' \frac{zz' Q^2}{\tau^{1/2}} P_{13}(z, Q^2) P_{24}(z') e^{(1-\rho)\tau} e^{\Phi(z, z', \tau)}$$

local approximation

$$P_{13}(z) = \frac{1}{z} (Qz)^2 (K_0^2(Qz) + K_1^2(Qz)) \rightarrow C \delta\left(z - \frac{1}{Q}\right)$$

$$P_{24}(z') \equiv \delta\left(z' - \frac{1}{Q'}\right)$$

$$F_{2,cr}^P(x, Q^2, Q') = \frac{g_0^2 \rho^{3/2}}{32\pi^{5/2}} \frac{Q}{Q'} \frac{1}{\tau^{1/2}} e^{(1-\rho)\tau} e^{-[\log^2(Q/Q')/\rho\tau]}$$



F_2^P vs $F_2^P|_{local}$

confinement term

$$F_{2,ct}^P(x, Q^2) = \frac{g_0^2 \rho^{3/2}}{32\pi^{5/2}} \int dz \int dz' \frac{zz' Q^2}{\tau^{1/2}} P_{13}(z, Q^2) P_{24}(z') e^{(1-\rho)\tau} e^{-\frac{\ln^2(zz'/z_0^2)}{\rho\tau}} G(z, z', \tau)$$

local approximation

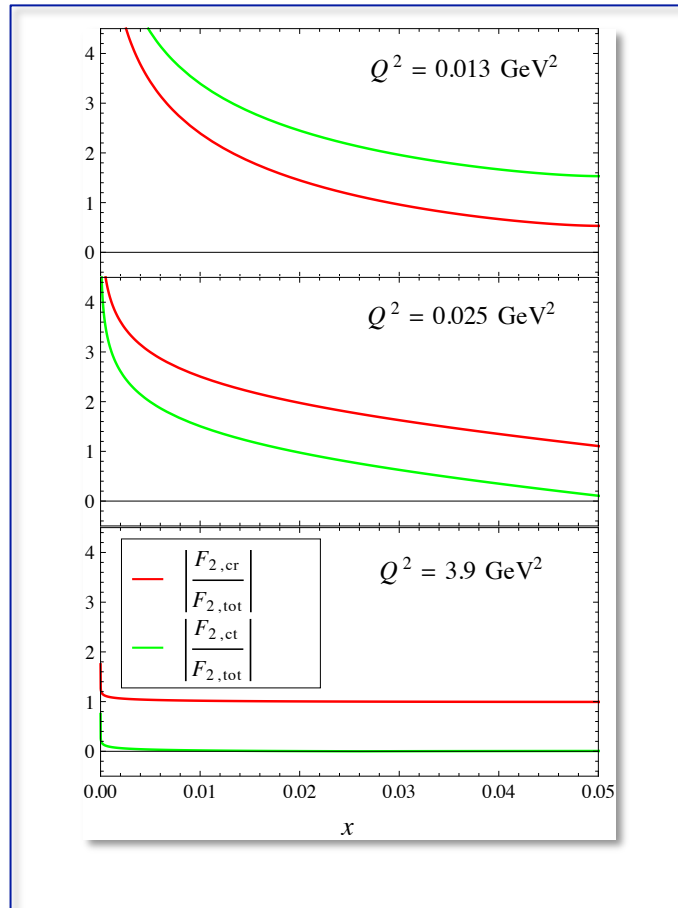
$$P_{13}(z) = \frac{1}{z} (Qz)^2 (K_0^2(Qz) + K_1^2(Qz)) \rightarrow C \delta\left(z - \frac{1}{Q}\right)$$

$$P_{24}(z') \cong \delta\left(z' - \frac{1}{Q'}\right)$$

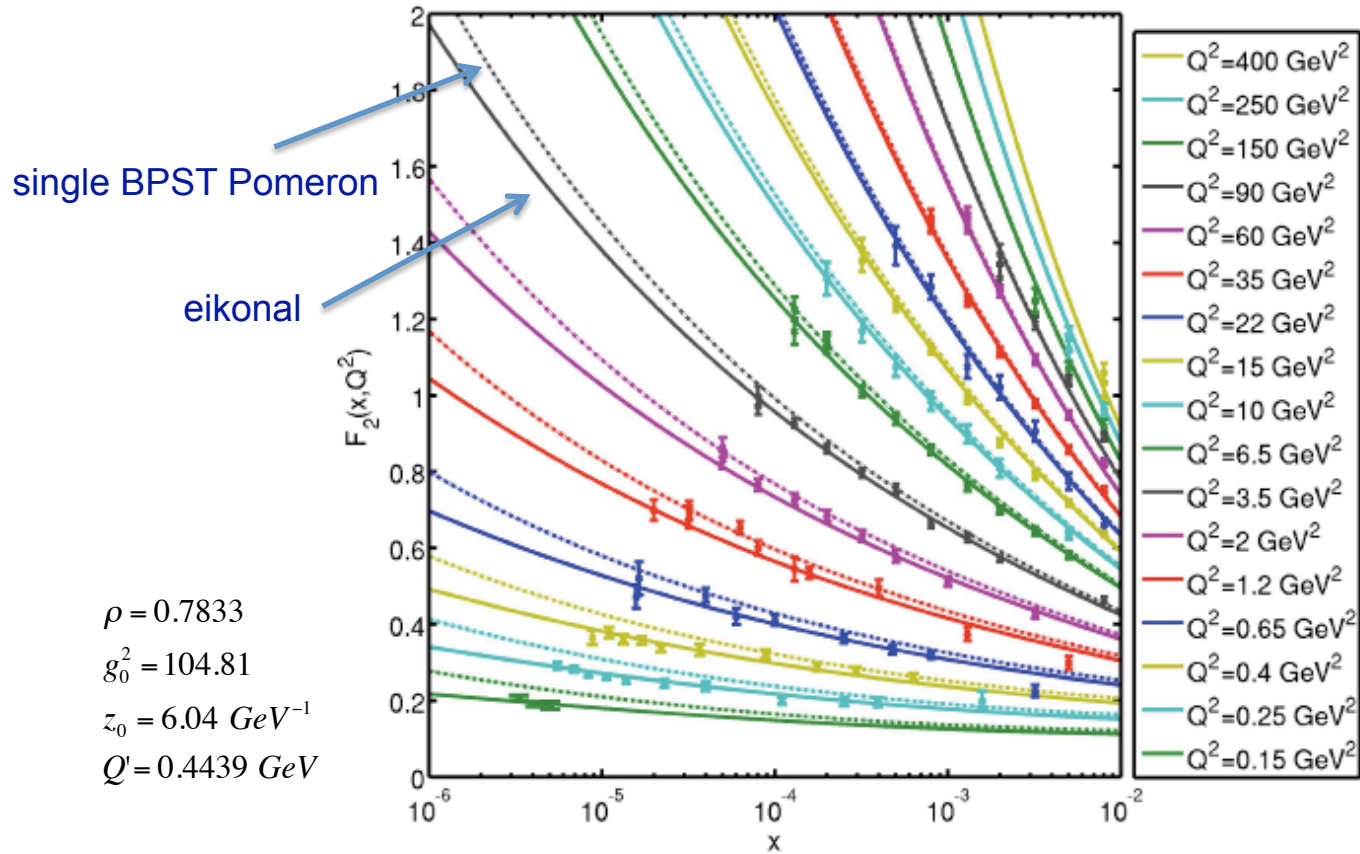
$$Q_0 = 1/z_0$$

$$F_{2,ct}^P(x, Q^2, Q', Q_0^2) = \frac{g_0^2 \rho^{3/2}}{32\pi^{5/2}} \frac{(Q/Q')}{\tau^{1/2}} e^{(1-\rho)\tau} e^{-\frac{\ln^2(Q_0^2/(QQ'))}{\rho\tau}} G\left(\frac{1}{Q}, \frac{1}{Q'}, \tau\right)$$

$$F_2^p = F_{2cr}^p + F_{2ct}^p$$



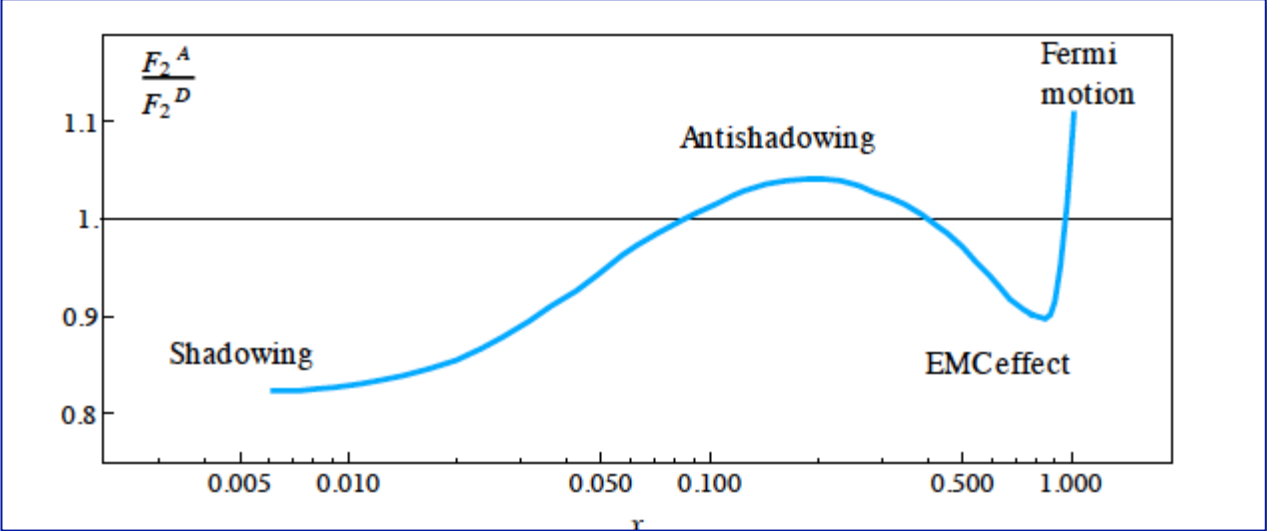
H1-ZEUS small-x measurements



Brower et al. 2010

good description of the data in the small-x region

inclusion of eikonal not necessary – the saturation regime is not reached



isospin violation can be included in the formula for F_2

$$F_2^n = F_{2cr}^p(x, Q^2, Q_n') + F_{2ct}^p(x, Q^2, Q_n', Q_{0n}^2)$$

similar proton and neutron confinement scale $Q_{0n} \cong Q_0$

experimental information from DIS on deuterium

nuclear structure function F_2 in the holographic framework

conformal term

$$F_{2cr}^p(x, Q^2, Q') = \frac{g_0^2 \rho^{3/2}}{32\pi^{5/2}} \frac{Q}{Q'} \frac{1}{\tau^{1/2}} e^{(1-\rho)\tau} e^{-[\log^2(Q/Q')/\rho\tau]}$$

$$Q'_A = \lambda_A Q' \Rightarrow Q^2 \rightarrow Q^2 / \lambda_A^2$$

$$F_2^A(x, Q^2) = F_{2cr}^N\left(x, \frac{Q^2}{\lambda_A^2}, Q'\right)$$

peak of the wave function
of the bound nucleon

confinement term

$$F_{2ct}^p(x, Q^2, Q') = \frac{g_0^2 \rho^{3/2}}{32\pi^{5/2}} \frac{Q}{Q'} \frac{1}{\tau^{1/2}} e^{(1-\rho)\tau} e^{-[\log^2(Q_0^2/QQ')/\rho\tau]} G\left(\frac{1}{Q}, \frac{1}{Q'}, \tau\right)$$

$$Q'_A = \lambda_A Q' \Rightarrow Q_0^2 \rightarrow Q_0^2 / \lambda_A^2$$

Q^2 rescaling rule:

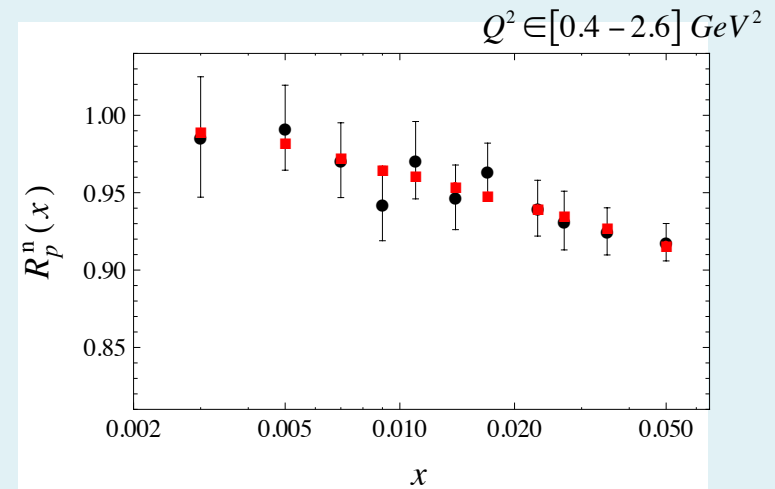
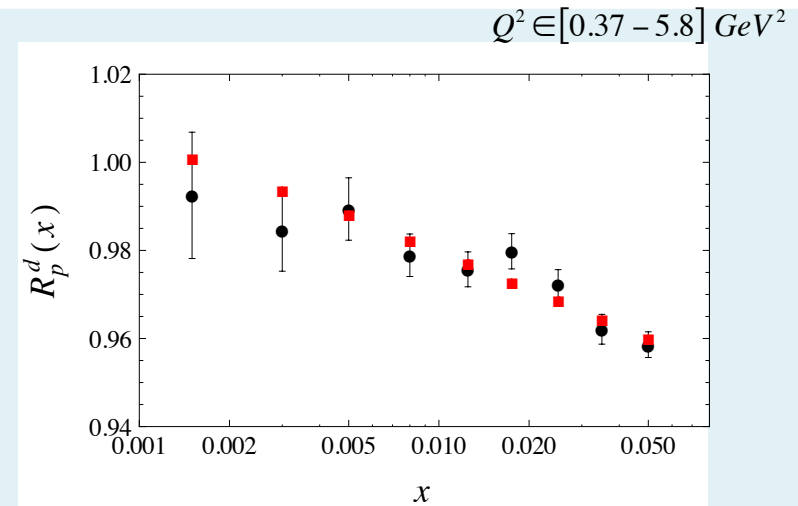
$$F_2^A(x, Q^2) = F_{2cr}^N\left(x, \frac{Q^2}{\lambda_A^2}, Q'\right) + F_{2ct}^N\left(x, \frac{Q^2}{\lambda_A^2}, Q', \frac{Q_0^2}{\lambda_A^2}\right)$$

deuterium structure function

$$F_2^D = \frac{1}{2} [F_2^{pD} + F_2^{nD}]$$

$$F_2^{pD} = F_2^p \left(x, \frac{Q^2}{\lambda_D^2}, Q', \frac{Q_0^2}{\lambda_D^2} \right)$$

$$F_2^{nD} = F_2^n \left(x, \frac{Q^2}{\lambda_D^2}, Q_n', \frac{Q_0^2}{\lambda_D^2} \right)$$

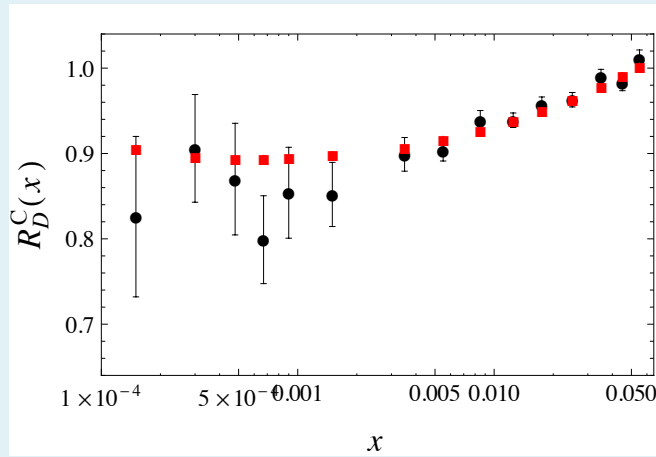


- our model
- New Muon Collaboration (CERN)
NPB 487 (1997) 3
NPB 371 (1992) 3

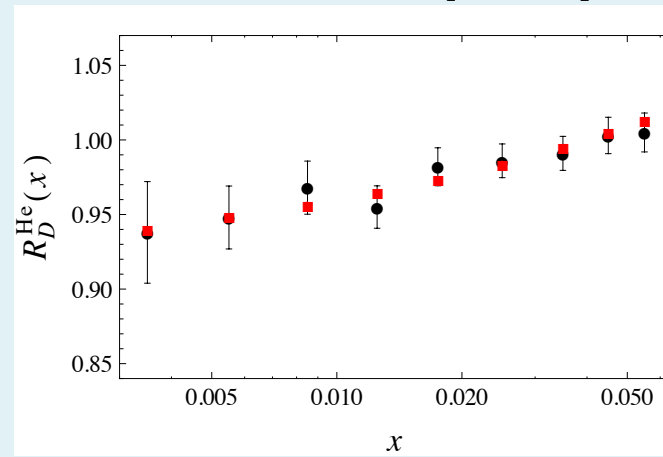
nuclear structure function F_2

nucleus	data set
He	9
Li	9
Be	6
C	15
Al	6
Ca	9
Fe	6
Pb	6

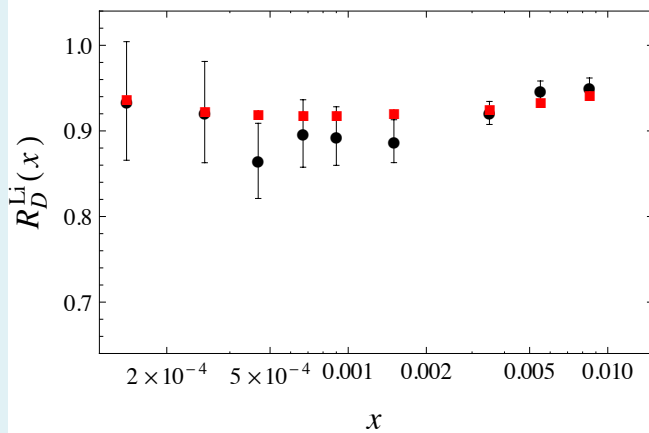
$Q^2 \in [0.03 - 6.4] \text{ GeV}^2$



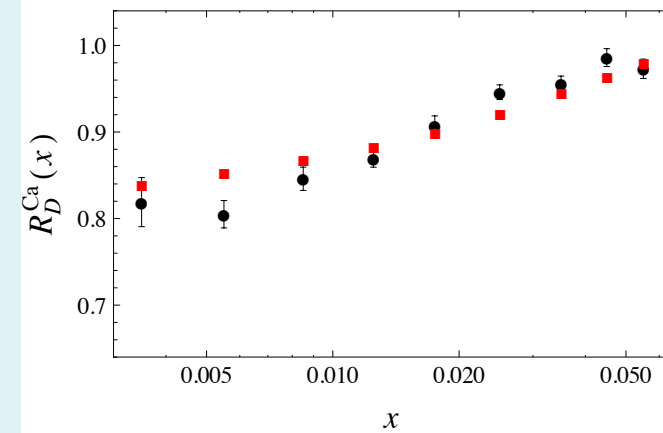
$Q^2 \in [0.77 - 6.3] \text{ GeV}^2$



$$R_D^A = \frac{F_2^A}{F_2^D}$$



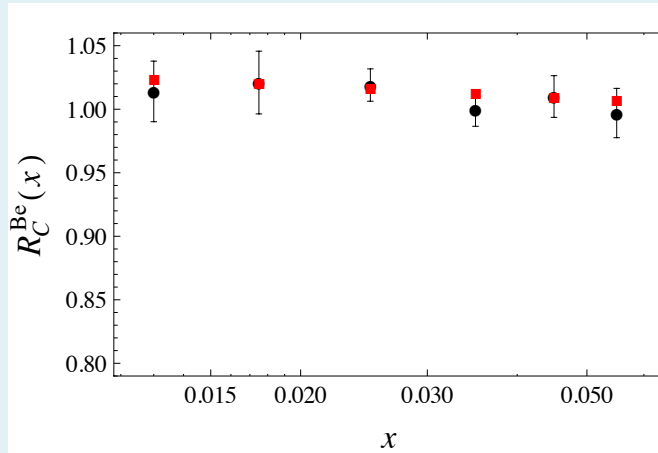
$Q^2 \in [0.03 - 1.4] \text{ GeV}^2$



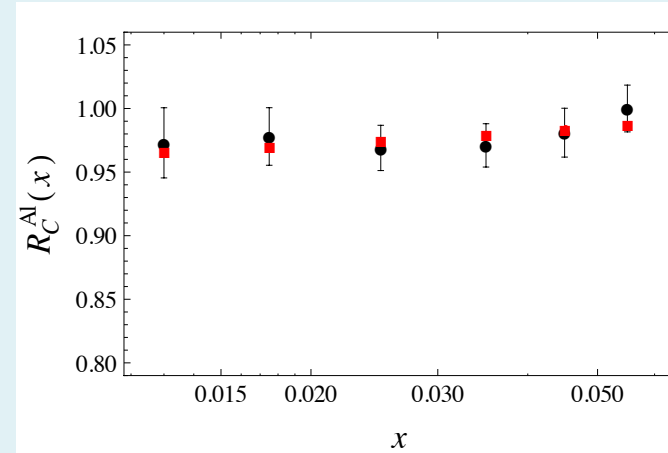
$Q^2 \in [0.6 - 6.8] \text{ GeV}^2$

- our model
- New Muon Collaboration
PLB 441 (1995) 3, NPB 441(1995) 12
NPB 481 (1996) 3

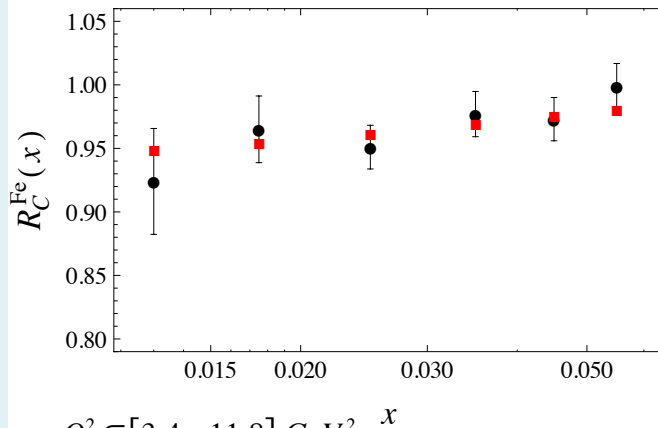
$Q^2 \in [3.4 - 11.4] \text{ GeV}^2$



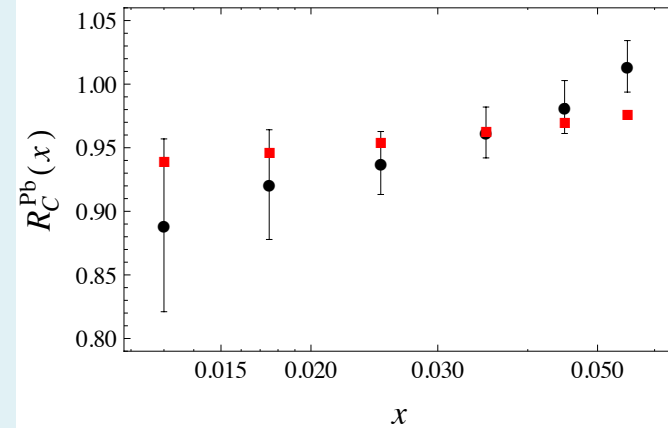
$Q^2 \in [3.4 - 11.6] \text{ GeV}^2$



$$R_C^A = \frac{F_2^A}{F_2^C}$$



$Q^2 \in [3.4 - 11.8] \text{ GeV}^2$



$Q^2 \in [3.4 - 11.6] \text{ GeV}^2$

- our model
- New Muon Collaboration
PLB 441 (1995) 3, NPB 441(1995) 12
NPB 481 (1996) 3

nucleus	n. points	$\chi_{d.o.f}^2$	n. points	$\chi_{d.o.f}^2$	range of $\langle Q^2 \rangle$
He	9	1.09	9	0.24	[0.77 – 6.3]
Li	9	0.93	9	0.79	[0.03 – 1.4]
Be	6	0.21	6	0.30	[3.4 – 11.4]
C	9	1.61	15	0.89	[0.03 – 6.4]
Al	6	0.23	6	0.21	[3.4 – 11.6]
Ca	9	8.0	9	3.87	[0.6 – 6.8]
Fe	6	0.41	6	0.42	[3.4 – 11.8]
Pb	6	1.11	6	0.93	[3.4 – 11.6]

Table 1 Experimental data sets [21] and $\chi_{d.o.f}^2$ of the fit of the structure function F_2^A for each nucleus. The third column reports the $\chi_{d.o.f}^2$ of fits without isospin breaking, the fourth and fifth columns correspond to fits with the isospin breaking effect included. In the last column, the experimental average Q^2 ranges (in GeV^2) for the various cases are indicated, from the first to the last bin of the Bjorken x .

Isospin effect included

$$F_2^A(x, Q^2) = \frac{Z}{A} F_{2ct}^p \left(x, \frac{Q^2}{\lambda_A^2}, Q', \frac{Q_0^2}{\lambda_A^2} \right) + \left(1 - \frac{Z}{A} \right) F_{2ct}^n \left(x, \frac{Q^2}{\lambda_A^2}, Q_n', \frac{Q_0^2}{\lambda_A^2} \right)$$

conventional approach:
QCD dipole model

γ^* produces a qq pair which interacts with the target

$$\frac{\sigma^{\gamma^* A}(\tau^A)}{\pi R_A^2} = \frac{\sigma^{\gamma^* N}(\tau^N)}{\pi R_N^2}$$

τ_A saturation scale

$$\lambda_A = \left(\frac{AR_N^2}{R_A^2} \right)^{1/2\delta}$$

$$\pi R_N^2 = 1.55 \text{ fm}^2$$

$$\delta = 0.79$$

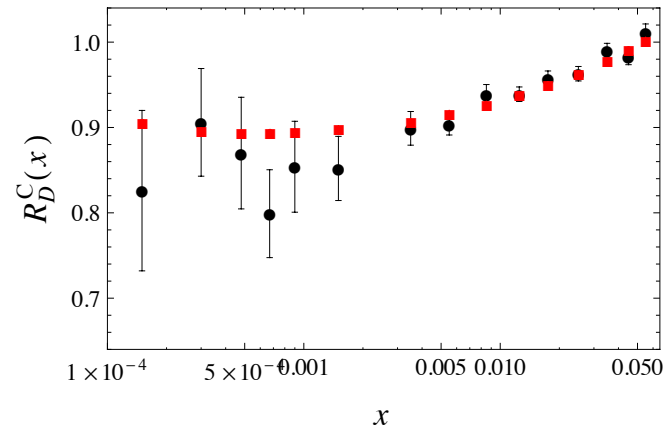
$$R_A = (1.12A^{1/3} - 0.86A^{-1/3}) \text{ fm}$$

nucleus	λ_A (holography)	$\lambda_{A,dip}$ [24]
Li	1.843	1.130
Be	1.764	1.140
C	1.775	1.160
Al	1.972	1.264
Ca	2.006	1.338
Fe	2.090	1.413
Pb	2.286	1.780

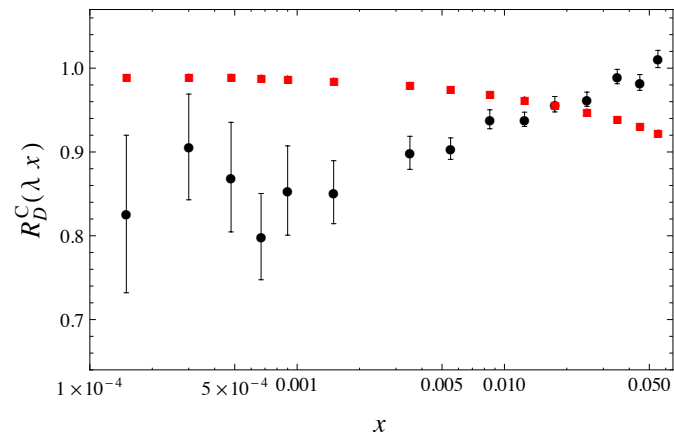
Table 2 Rescaling parameter λ_A obtained using the holographic expression for F_2^A and taking into account the isospin breaking. The values in the last column are obtained within the QCD dipole model [24].

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Q² rescaling versus x rescaling



$$x \rightarrow \lambda_A x$$

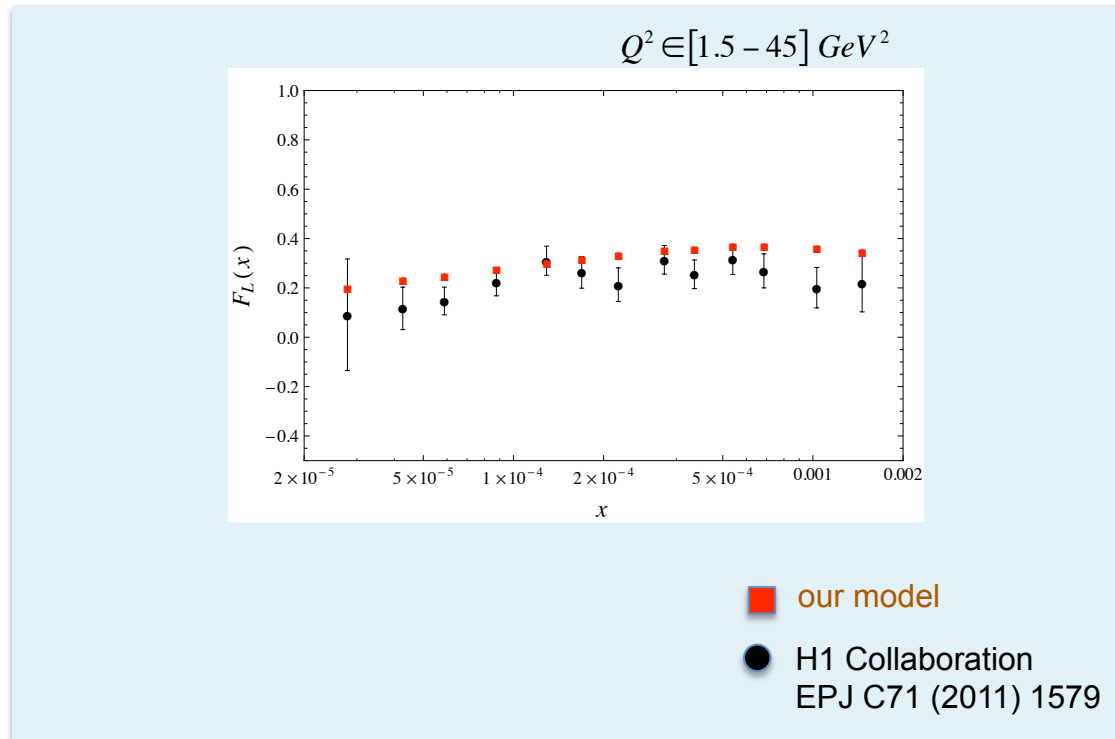


● data
■ our model

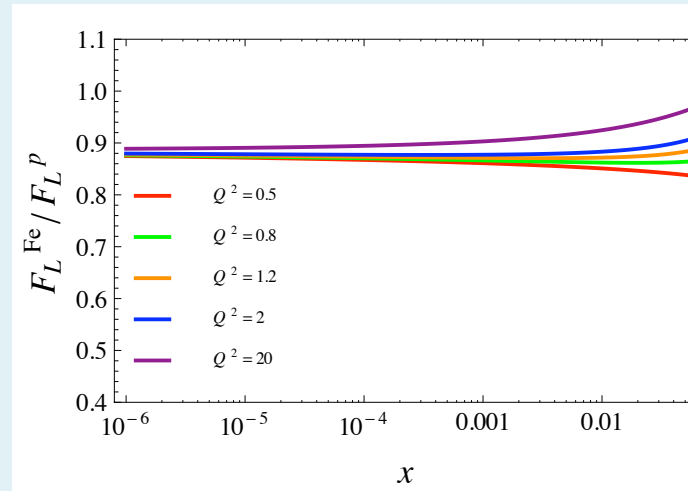
longitudinal structure function

$$F_L = F_2 - 2xF_1$$

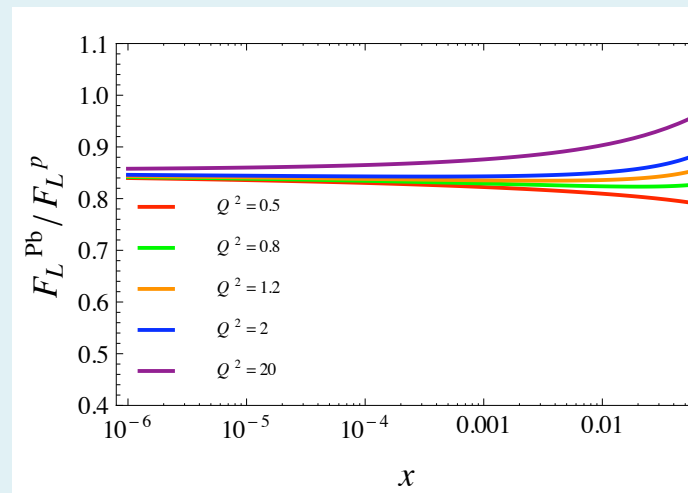
proton



nuclear modification of the longitudinal structure functions



$$\frac{F_L^A}{F_L^p}$$



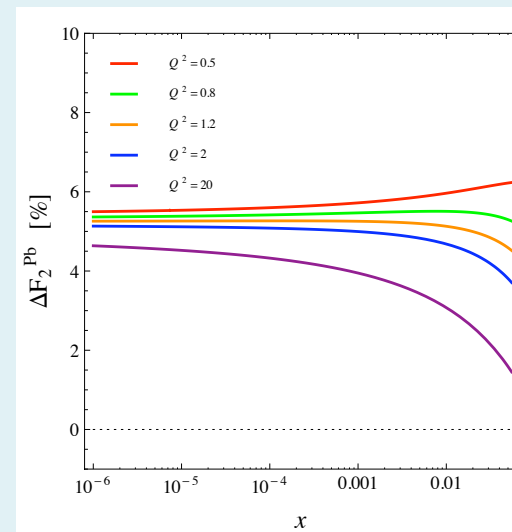
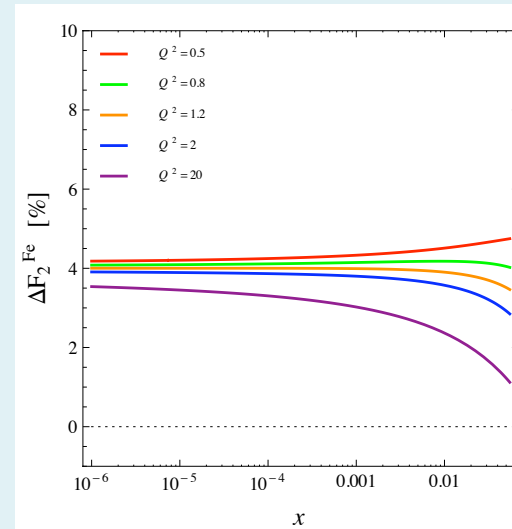
structure functions from cross section measurements

reduced cross section

$$\sigma_r = F_2 \left[1 - f(y) \frac{F_L}{F_2} \right]$$

maximum uncertainty on F_2

$$\Delta F_2 = 1 - \frac{F_2^A}{F_2^A + f(y)(F_L^N - F_L^A)}$$



by Q^2 rescaling at low- x , the expression of F_2 obtained in the gauge/gravity formulation reproduces the set of nuclear data

rescaling can be understood in terms of a modification of the z_{IR} in nuclei and in the change of the peak of the wave function of the bound nucleon

geometrical interpretation of nuclear shadowing

Questions:

anti-shadowing by momentum rescaling?

holographic description of the EMC effect?

Anti-shadowing usually analyzed by energy-momentum
sum rules

Nature of the EMC effect debated

Conclusions

A unified description of nuclear structure functions in the full x -range is missing

The gauge/gravity duality formulation seems to capture the relevant dynamics to describe nuclear DIS phenomena at small- x

Possibilities for the description of anti-shadowing and of the EMC effect need to be investigated

Interest for neutrino DIS on nuclear targets \rightarrow neutrino cross section at high energy and small- x : a definition of the charged current in the dual approach is needed

SPARES