BPS Wilson Loops and an exact result for the Bremsstrahlung function in ABJM model

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WILSON LOOPS

$$W[\Gamma] \sim \mathrm{Tr}\, P \exp\left(-i \int_\Gamma dx^\mu A_\mu\right)$$

Wilson loops, other than being fundamental physical objects, play a central role in testing the AdS/CFT correspondence. In fact, in SUSY gauge theories

- **BPS Wilson loops** can be constructed, which are invariant under a fraction of the original supersymmetry.

- **They are non-protected operators.** They correspond to fundamental strings in AdS. Results at strong and weak coupling.

- **Localization techniques** provide an interpolating function between strong/weak results, which can be used for non-trivial tests of the AdS/CFT

- Connections with Bremsstrahlung function and Entanglement Entropy

BPS WL in 3D theories

BPS WILSON LOOPS IN U(N)xU(M) ABJ(M)

N=6 SUSY quiver theory with U(N) x U(M) gauge symmetry:

Two gauge fields $A, \hat{A}$

(anti)bifundamental matter $C, \bar{C}$ $\quad \psi, \bar{\psi}$

Distinguishing feature: For dim. reasons the quadratic coupling to the scalars and linear couplings to fermions can be turned on

TWO FAMILIES OF WL: BOSONIC AND FERMIONIC WL

GENERALIZED FERMIONIC WILSON LOOPS

Drukker-Trancanelli 0912.3006

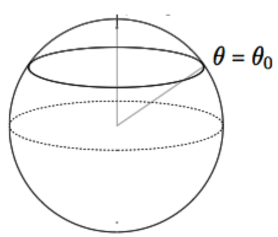
Cardinali, Griguolo, Martelloni, Seminara, 1209.4032

Holonomy of a superconnection of U(N|M)

$$\langle W_F[\Gamma]\rangle = \mathcal{N}\int D[A,\hat{A},C,\bar{C},\psi,\bar{\psi}]\, e^{-S}\, \mathrm{Str}\left[P\exp\left(-i\int_\Gamma d\tau\mathcal{L}(\tau)\right)\mathcal{T}\right]$$

$$\mathcal{L}(\tau) = \begin{pmatrix} \mathcal{A} & -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\eta_I\bar{\psi}^I \\ -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\psi_I\bar{\eta}^I & \hat{\mathcal{A}} \end{pmatrix}$$

$$\mathcal{A} = A_\mu\dot{x}^\mu - \frac{2\pi i}{k}\mathcal{M}_J^I C_I\bar{C}^J$$



$$x^\mu = (\sin\theta_0, \cos\theta_0\cos\tau, \cos\theta_0\sin\tau)$$

θ-latitude circle Γ

$$M_I{}^J = \hat{M}_I{}^J = l\left[\delta_I^J - 2is_I\bar{s}^J - 2il\cos 2\alpha\left(s_I\frac{\dot{x}\cdot\gamma}{|\dot{x}|}\bar{s}^J\right) - 2il\sin 2\alpha\,(s_I\gamma^\rho\bar{s}^J)\,\epsilon_{\rho\mu\nu}\frac{x^\mu\dot{x}^\nu}{|\dot{x}|}\right]$$

$$\eta_I^\beta(\tau) = \frac{i}{r_0}e^{i\nu\frac{\tau}{2}}\left[s_I\left(\cos\alpha\,\mathbb{I} - i\sin\alpha(x^\mu\gamma_\mu)\right)\left(\mathbb{I} + l\frac{\dot{x}\cdot\gamma}{|\dot{x}|}\right)\right]^\beta$$

$$\bar{\eta}_\beta^I(\tau) = ir_0e^{-i\nu\frac{\tau}{2}}\left[\left(\mathbb{I} + l\frac{\dot{x}\cdot\gamma}{|\dot{x}|}\right)\left(\cos\alpha\,\mathbb{I} + i\sin\alpha(x^\mu\gamma_\mu)\right)\bar{s}^I\right]_\beta$$

$$\nu = \sin 2\alpha\,\cos\theta_0$$

1/6 BPS

DUAL TO A FUNDAMENTAL STRING WITH WORLDSHEET ENDING ON WL CONTOUR AT THE BOUNDARY OF AdS AND DESCRIBING A CLOSED PATH IN CP3

(Aguilera-Damia, Correa, Silva, 1405.1396)

GENERALIZED BOSONIC WILSON LOOPS

Setting fermionic couplings to 0 and slightly modifying M

Bosonic Wilson loops $\langle W_B \rangle$, $\langle \hat{W}_B \rangle$

1/12 BPS WITH NO KNOWN DUAL STRING SOLUTION

CLASSICAL COHOMOLOGICAL EQUIVALENCE

$$W_F = \frac{N\, e^{-i\frac{\pi}{2}\nu}\, W_B + M\, e^{i\frac{\pi}{2}\nu}\, \hat{W}_B}{N e^{-i\frac{\pi}{2}\nu} - M e^{i\frac{\pi}{2}\nu}} \quad + QV$$

Special value $\nu = 1 \quad (\alpha = \pi/4\,,\ \theta_0 = 0)$

- Fermionic -> 1/2 BPS WL (Drukker-Trancanelli 0912.3006)

DUAL TO A FUNDAMENTAL STRING WITH WORLDSHEET ENDING ON WL CONTOUR AT THE BOUNDARY OF AdS AND LOCALIZED IN THE INTERNAL SPACE

- Bosonic -> 1/6 BPS WL (Drukker, Plefka, Young, 0809.2787; Chen, Wu, 0809.2863; Rey, Suyama, Yamaguchi, 0809.3786)

DUAL TO A CONFIGURATION OF STRINGS SMEARED ALONG A CP1 INSIDE CP3

$$w_{1/2} = \frac{N\, w_{1/6} + M\, \widehat{w}_{1/6}}{(N+M)} \quad + \text{QV}$$

VEV OF WILSON LOOPS

$$\langle W[\Gamma]\rangle = \mathcal{N}\int D[A,\hat{A},C,\bar{C},\psi,\bar{\psi}]\, e^{-S}\, \mathrm{Tr}\left[P\exp\left(-i\int_\Gamma d\tau \mathcal{A}(\tau)\right)\right]$$

$\lambda = N/k$ $\hat{\lambda} = M/k$

- **PERTURBATIVE EVALUATION** ($N/k,\ M/k \ll 1$)
- **HOLOGRAPHIC EVALUATION** ($N/k,\ M/k \gg 1$)
- **MATRIX MODEL EVALUATION** (LOCALIZATION) (N,M,k finite)

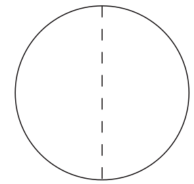
FOR GENERIC ν

- **No matrix model calculation available so far**
- **Holographic description only for the fermionic WL**
  (Aguilera-Damia, Correa, Silva, 1405.1396)
- **Perturbative evaluation at two loops**
  (M. Bianchi, L. Griguolo, M. Leoni, SP, D. Seminara, 1402.4128)

In dimensional regularization

$$\langle W_B(\nu)\rangle = 1 + \frac{\pi^2}{k^2}\left[\frac{1}{2}(1+\nu^2)MN - \frac{N^2-1}{6}\right].$$

$$\langle \hat{W}_B(\nu)\rangle = 1 + \frac{\pi^2}{k^2}\left[\frac{1}{2}(1+\nu^2)MN - \frac{M^2-1}{6}\right]$$



$$\langle W_F(\nu)\rangle = 1 + \frac{2\pi i\, MN\mathcal{R}}{k}\,\nu\,\cos\frac{\pi\nu}{2} - \frac{\pi^2\mathcal{R}}{6k^2}\Big[(N-M)(N^2+M^2+2(\nu^2-1)MN-1)\cos\frac{\pi\nu}{2} - i(N+M)(N^2+M^2-4MN-1)\sin\frac{\pi\nu}{2}\Big] + \mathcal{O}\left(k^{-3}\right)$$

For ABJM (M=N)

$$\langle W_F(\nu)\rangle_{ABJM} = 1 - \frac{N}{k}\pi\,\nu\,\mathrm{ctg}\frac{\pi\nu}{2} + \frac{N^2}{k^2}\frac{\pi^2}{6}\left(2+\frac{1}{N^2}\right).$$

FOR ν =1

Kapustin, Willett, Yaakov 0909.4559
Marino, Putrov 0912.3074, 1110.4066;
Drukker, Marino, Putrov 1007.3837;
Klemm, Marino, Schiereck, Soroush 1207.0611

Matrix model result available

Expanding the exact result at second order in N/k, M/k we obtain

$$\langle W_{1/6}[\Gamma]\rangle_{f=1} = e^{\frac{i\pi}{k}N}\left[1+\frac{\pi^2}{6\,k^2}\left(-N^2+6MN+1\right)\right]$$

$$\langle \hat{W}_{1/6}[\Gamma]\rangle_{f=1} = e^{-\frac{i\pi}{k}M}\left[1+\frac{\pi^2}{6\,k^2}\left(-M^2+6MN+1\right)\right]$$

Framing factors

$$\langle W_{1/2}[\Gamma]\rangle_{f=1} = e^{\frac{i\pi}{k}(N-M)}\left[1-\frac{\pi^2}{6k^2}\left(N^2+M^2-4NM-1\right)\right]$$

Matching with the perturbative result

$$\langle W_{1/6}[\Gamma]\rangle_{f=0} = e^{-\frac{i\pi}{k}N} \langle W_{1/6}[\Gamma]\rangle_{f=1} \qquad \langle \hat{W}_{1/6}[\Gamma]\rangle_{f=0} = e^{\frac{i\pi}{k}M} \langle \hat{W}_{1/6}[\Gamma]\rangle_{f=1}$$

$$\langle W_{1/2}[\Gamma]\rangle_{f=0} = e^{-\frac{i\pi}{k}(N-M)} \frac{N\langle W_{1/6}[\Gamma]\rangle_{f=1} + M\langle \hat{W}_{1/6}[\Gamma]\rangle_{f=1}}{N+M}$$

QUANTUM COHOMOLOGICAL EQUIVALENCE

$$W_F = \frac{N\, e^{-i\frac{\pi}{2}\nu}\, W_B + M\, e^{i\frac{\pi}{2}\nu}\, \hat{W}_B}{N e^{-i\frac{\pi}{2}\nu} - M e^{i\frac{\pi}{2}\nu}}$$

⇩

$$\langle W_F \rangle_\nu = \frac{N e^{-i\frac{\pi}{2}\nu}\, \langle W_B \rangle_\nu + M e^{i\frac{\pi}{2}\nu}\, \langle \hat{W}_B \rangle_\nu}{N e^{-i\frac{\pi}{2}\nu} - M e^{i\frac{\pi}{2}\nu}}$$

where

$$\langle W_B \rangle_\nu \equiv e^{i\pi\frac{N}{k}\nu} \langle W_B \rangle_0 \qquad \langle \hat{W}_B \rangle_\nu \equiv e^{-i\pi\frac{M}{k}\nu} \langle \hat{W}_B \rangle_0$$

$$\langle W_F \rangle_\nu \equiv e^{i\pi\frac{(N-M)}{k}\nu} \langle W_F \rangle_0$$

Framing ν ? It calls for a matrix model calculation

In 4D N=4 SYM we learn that WL depending on deformation parameters can be used to compute the Bremsstrahlung function

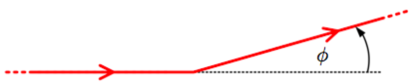
(Correa, Henn, Maldacena, Sever, 1202.4455;
Fiol-Garolera-Lewkowycz, 1202.5292)

We extend this result to the 3D case

BREMSSTRAHLUNG FUNCTION

"Deformed WL" in the planar limit

$$\Gamma_{cusp}(\phi,\theta)$$



$$\langle W_c \rangle \sim e^{-\Gamma_{cusp} \log L/\epsilon}$$

$$\Gamma_{cusp}(\phi,\theta) \sim B(\lambda)(\theta^2 - \phi^2)$$

$\theta, \phi << 1$

$$\langle W \rangle$$

$$B(\lambda) = -\frac{1}{4\pi^2}\partial_\theta^2 \log \langle W_o(\theta)\rangle|_{\theta=0}$$

$$B(\lambda)$$

Correa, Henn, Maldacena, Sever;
Fiol-Garolera-Lewkowycz

$$\Delta E = 2\pi B \int dt\, (\dot{v})^2$$

FOR 3D CASE (ABJM)

We look for generalization of $B(\lambda) = -\frac{1}{4\pi^2}\partial_\theta^2 \log \langle W_o(\theta)\rangle|_{\theta=0}$

Two different cusps

$\Gamma^{1/2}_{cusp} \sim B(\lambda)(\theta^2 - \phi^2)$ It vanishes for θ = ϕ (SUSY configuration)

$\Gamma^{1/6}_{cusp} \sim \theta^2 B^\theta(\lambda) - \phi^2 B^\phi(\lambda)$ It does not vanish for θ = ϕ (nonBPS)

$\nu = \sin 2\alpha\, \cos\theta_0$ can play the role of the parameter θ

In the ½ BPS case we make the following conjecture

$$B(\lambda) = \frac{1}{4\pi^2}\,\partial_\nu \log \langle W_F(\nu)\rangle_0|_{\nu=1}$$

Perturbative expansion of B(λ) contains only odd powers of λ

Applied to our perturbative result for ABJM $$B(\lambda) = \frac{\lambda}{8} + O(\lambda^3)$$

(λ = N/k)

It agrees with Γ computed in Griguolo, Marmiroli, Martelloni, Seminara, 1208.5766

CHECK AT STRONG COUPLING

Our conjecture checked at strong coupling by Aguilera-Damia, Correa, Silva (1405.1396)

They have found the string solution dual to the fermionic WL

$$\langle W_F \rangle = e^{-S^{on-shell}} + \mathcal{O}(1) = e^{\pi\sqrt{2\lambda}\nu} + \mathcal{O}(1)$$

Applying our prescription one obtains

$$B_{1/2} = \frac{\sqrt{2\lambda}}{4\pi} + \mathcal{O}(1)$$

in agreement with Γ computed at strong coupling (Forini, Giangreco-Puletti, Ohlsson-Sax, 1204.3302)

In the 1/6 BPS case the same expression should work for B_θ

$$B^\theta(\lambda) = \frac{1}{4\pi^2}\,\partial_\nu \log \langle W_B(\nu)\rangle_0|_{\nu=1}$$

Applied to our perturbative result for ABJM $\quad B^\theta(\lambda) = \frac{\lambda^2}{4} + O(\lambda^3)$

- It agrees with the perturbative result found in Griguolo, Marmiroli, Martelloni, Seminara, 1208.5766

- It agrees with the result from the Maldacena-Lewkowycz prescription

$$B = \frac{1}{4\pi^2}\partial_m \log |W_m|\Big|_{m=1}$$

- Aguilera-Damia, Correa, Silva (1405.1396) have proved this formula **at all orders,** on general grounds

CONCLUSIONS AND PERSPECTIVES

- For the fermionic 1/6 BPS WL and bosonic 1/12 BPS WL we have found an analytic expression for the perturbative result up to two loops and discussed the identification of a generalized (non-integer) framing factor.
- We have related these WL to the Bremsstrahlung function and proposed an exact expression for B(λ).

SOME OPEN QUESTIONS …..

- Proof of the conjecture for the fermionic case
- Check at three loops for

$$B_{1/2}(\lambda) = \frac{\lambda}{8} - \frac{\pi^2}{48}\lambda^3 + \mathcal{O}\left(\lambda^5\right)$$ **……Work in progress**

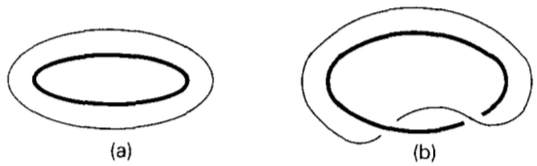
- Matrix model for *generic* WL (framing ν ?)
- String dual of the bosonic WL

CHANGE OF FRAMING

Appearance of short distance divergences requires REGULARIZATION

- DIMENSIONAL REGULARIZATION D = 3 -2ε $\langle A_\mu(x_1) A_\nu(x_2) \rangle$

- POINT-SPLITTING REGULARIZATION



$$\langle WL(C) \rangle = e^{-i\frac{2\pi}{k}\phi_f(C)}\,\rho(C)$$

Witten, Comm. Math. Phys. 121 (1989) 351
Guadagnini-Martellini-Mintchev NPB330 (1990) 575

ABJ(M) THEORIES

Aharony, Bergman, Jafferis, Maldacena 0806.1218
Aharony, Bergman, Jafferis 0807.4924

N=6 SUSY quiver theory with U(N) x U(M) gauge symmetry

$$S_{CS} = -i\frac{k}{4\pi}\int d^3x\, \varepsilon^{\mu\nu\rho}\Big[\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho + \frac{2}{3}iA_\mu A_\nu A_\rho\right) - \mathrm{Tr}\left(\hat{A}_\mu\partial_\nu \hat{A}_\rho + \frac{2}{3}i\hat{A}_\mu \hat{A}_\nu \hat{A}_\rho\right)\Big]$$

$$S_{matter} = \int d^3x\, \mathrm{Tr}\Big[D_\mu C_I D^\mu \bar{C}^I + i\bar{\psi}^I\gamma^\mu D_\mu\psi_I\Big] + S_{int}$$

$$S_{gf} = \frac{k}{4\pi}\int d^3x\, \mathrm{Tr}\Big[\frac{1}{\xi}(\partial_\mu A^\mu)^2 + \partial_\mu\bar{c}D^\mu c - \frac{1}{\xi}(\partial_\mu\hat{A}^\mu)^2 - \partial_\mu\bar{\hat{c}}D^\mu\hat{c}\Big]$$

Dual to type IIA string theory on AdS4 x CP3