Torsional Newton-Cartan geometry in Lifshitz holography and non-relativistic FTs

Gauge/Gravity Duality 2015

GGI, Firenze, April 17, 2015

Niels Obers, NBI

based on work with:

Jelle Hartong and Elias Kiritsis

1409.1519 [1] & 1409.1522 [2] & 1502.00228 [3] & to appear

and

Morten Holm Christensen, Jelle Hartong, Blaise Rollier

1311.4794 (PRD) & 1311.6471 (JHEP)

Introduction

- holography beyond original AdS-setup

- apply to study of strongly coupled CM systems
  non-relativistic scaling -> Schroedinger, Lifshitz, hyperscaling violating geometries

$$t \to \lambda^z t\,, \qquad \vec{x} \to \lambda \vec{x}\,.$$

- how general is the holographic paradigm ?
  (nature of quantum gravity, black hole physics)

- appearance of novel geometric structures on the boundary (this talk: TNC)

- exotic theories of gravity can be viewed as Schwinger source functionals
  of non-rel QFTs (``metric" couples to stress tensor)
  symmetries of FT -> symmetries of the coupled grav. theory and
  constrain form of source functionals

This talk: direct implementation of this in class of examples characterized
by Lifshitz scaling symmetry and extended Schroedinger sym.
\+ holographic realization within context of bulk Lifshitz spacetime

Lifshitz spacetimes

Aim: construct holographic techniques for (strongly coupled) systems with NR symmetries

Lifshitz holography $$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{1}{r^2}\left(dr^2 + d\vec{x}^2\right)$$

[Kachru,Liu,Mulligan]
[Taylor]

Taylor/Danielson,Thorlacius/Ross,Saremi/Ross/Baggio,de Boer,Holsheimer/Mann,McNees/Griffin,Horava,Melby-Thompson/Korovin,Skenderis,Taylor/Cheng,Hartnoll,Keeler/Baggio/Holsheimer/Christensen,Hartong,NO,Rollier/Chemissany, Papadimitriou/Hartong,Kiritsis,NO

Lifshitz

$$\overbrace{H, P_a, J_{ab}, D,}^{\text{Lifshitz}} \quad G_a, N, K (z=2)$$

Schroedinger

-> Lifshitz holography dual to field theories on torsional Newton-Cartan spacetime

[Hartong,Kiritsis,NO]

Overview of recent background

- bdry geometry for Lifshitz spacetimes is torsional Newton-Cartan (TNC) geometry
  (novel extension of NC)

- first observed for specific z=2 example (in 4D): [Christensen,Hartong,NO,Rollier]
  * Scherk-Schwarz dim. reduction (null on bdry) from 5D AlAdS solution

- generalized to large class of arbitrary z (in EPD model) [Hartong,Kiritsis,NO]1
  (sources: use vielbein formalism + appropriate lin. combo)

coupling of TNC to non-rel (bdry) FTs [Hartong,Kiritsis,NO]2,3
  -> vevs (stress tensor, mass current)
  & WIs in TNC covariant form

see also: [Jensen/Jensen,Karch]

- TNC geometry arises by gauging the Schroedinger algebra [Bergshoeff,Hartong,Rosseel]

- TNC natural geometry (warped geometry) that couples to WCFT [Hofmann,Rollier]

- recent activity using NC/TNC in CM (strongly-correlated electron system, FQH) [Son][Gromov,Abanov][Geracie,Son,Wu,Wu] [Brauner,Endlich,Monin,Penco][Geracie,Son] [Wu,Wu],[Geracie,Golkar,Roberts]

Plan & preview

1. Mini-intro to NC geometry

   time-like vielbein τ_μ, space-like vielbeins e_μ^a and a vector field M_μ,

2. holography for Lifshitz spacetimes and TNC geometry

3. scale invariant field theories on TNC backgrounds

   * the vector field can make a global U(1) into local sym.

4. flat NC spacetime

   * comes with function M (in $M_\mu = \partial_\mu M$)
   * local symmetries can generate non-trivial orbit of equivalent M

5. scale-invariant field theories on flat NC

   * novel mechanism: M can be eaten up by physical fields generating extra global symmetries (e.g. Galilean boost) beyond Lif (-> Sch.)

6 . Lifsthiz vacuum

   * exhibits source M transforming under local Sch (Lif realized by Killing)
   * scalar probes on Lif bgr that are Sch invariant by similar mechanism as in FT
   * conserved (or improved) current: global U(1)

Newton-Cartan makes Galilean local

Riemannian geometry: tangent space is Poincare invariant

NC geometry: tangent space is Bargmann (central ext. Gal.) invariant

Andringa,Bergshoeff,Panda,de Roo

here: only interested in geometrical framework; not in action/EOMs
boundary geometry in holographic setup is non-dynamical

→ dynamical (torsional) Newton-Cartan = HL gravity
[see talk Jelle Hartong (Tuesday)] Hartong,NO (to appear)

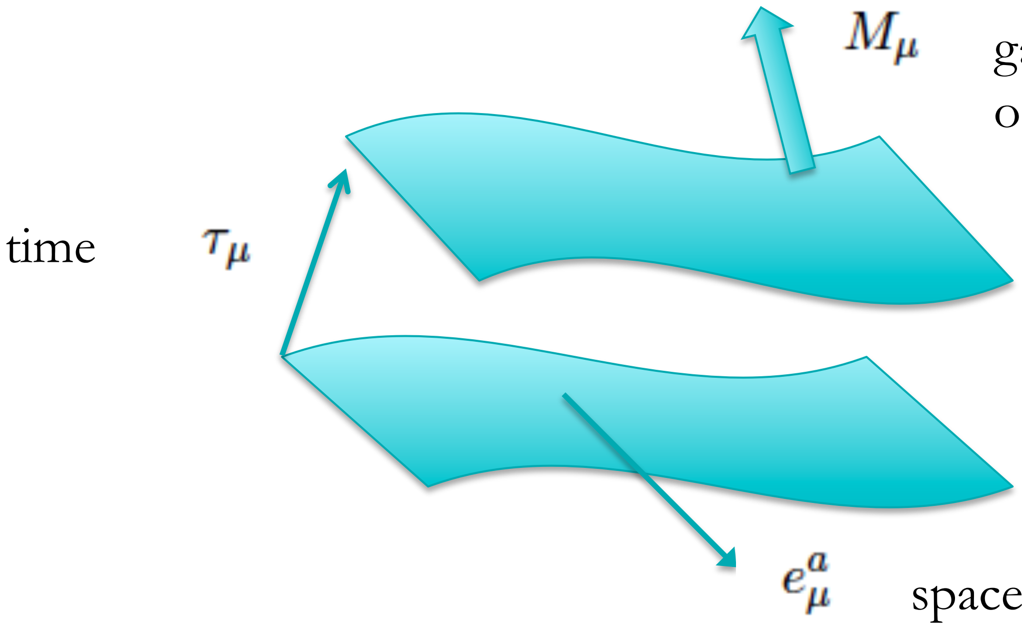
Note:

Galilean

$$\underbrace{\overbrace{H, P_a, J_{ab}, G_a}^{\text{Galilean}} \quad N}_{\text{Bargmann}}$$

Bargmann

Newton-Cartan geometry



gauge field of central extension of Galilean algebra (Bargmann)

- Newton-Cartan (NC): $\tau_\mu = \partial_\mu t$ notion of absolute time
- twistless torsional NC: $\tau_\mu = \mathrm{HSO}$ preferred foliation in equal time slices
- torsional NC (TNC): no conditions

EPD model and AlLif spacetimes

- bulk theory

$$S = \int d^4x\sqrt{-g}\left(R - \frac{1}{4}Z(\Phi)F^2 - \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}W(\Phi)B^2 - V(\Phi)\right)$$

- admits Lifshitz solutions with z>1

For AlLif BCs useful to write:

$$ds^2 = \frac{dr^2}{R(\Phi)r^2} - E^0E^0 + \delta_{ab}E^aE^b\,, \qquad B_M = A_M - \partial_M\Xi$$

[Ross],[Christensen,Hartong,NO,Rollier]
[Hartong,Kiritsis,NO]1

then AlLif BCs

$$E^0_\mu \simeq r^{-z}\tau_\mu\,, \qquad E^a_\mu \simeq r^{-1}e^a_\mu\,, \qquad \Phi \simeq r^\Delta\phi\,,$$

$$B_\mu - \alpha(\Phi)E^0_\mu \simeq -r^{z-2}M_\mu\,.$$

Stueckelberg decompostion: $M_\mu = \tilde{m}_\mu - \partial_\mu\chi\,.$

Transformation of sources

use local bulk symmetries:
local Lorentz, gauge transformations and diffs preserving metric gauge

these symmetries induce an action on sources: $\tau_\mu \quad e^a_\mu \quad M_\mu$

$=$ action of Bargmann algebra plus local dilatations $=$ Schroedinger

$$\delta\tau_\mu = \mathcal{L}_\xi \tau_\mu + z\Lambda_D \tau_\mu \,,$$
$$\delta e^a_\mu = \mathcal{L}_\xi e^a_\mu + \lambda^a \tau_\mu + \lambda^a{}_b e^b_\mu + \Lambda_D e^a_\mu \,,$$
$$\delta M_\mu = \mathcal{L}_\xi M_\mu + e^a_\mu \lambda_a + (2-z)\Lambda_D M_\mu \,,$$

there is thus a Schroedinger Lie algebra valued connection given by

$$\mathcal{A}_\mu = H\tau_\mu + P_a e^a_\mu + G_a \omega_\mu{}^a + \frac{1}{2} J_{ab} \omega_\mu{}^{ab} + N m_\mu + D b_\mu$$

with appropriate curvature constrains that reproduces trafos of the sources

Torsional Newton-Cartan (TNC) geometry

the bdry geometry is novel extension of NC geometry

- inverse vielbeins (v^μ, e^μ_a)

$$v^\mu \tau_\mu = -1\,, \qquad v^\mu e^a_\mu = 0\,, \qquad e^\mu_a \tau_\mu = 0\,, \qquad e^\mu_a e^b_\mu = \delta^b_a$$

can build Galilean boost-invariants

$$h_{\mu\nu} = e^a_\mu e^b_\nu \delta_{ab}$$

$$\begin{aligned} \hat{v}^\mu &= v^\mu - h^{\mu\nu} M_\nu\,, \\ \bar{h}_{\mu\nu} &= h_{\mu\nu} - \tau_\mu M_\nu - \tau_\nu M_\mu\,, \\ \tilde{\Phi} &= -v^\mu M_\mu + \tfrac{1}{2} h^{\mu\nu} M_\mu M_\nu\,, \end{aligned}$$

affine connection of TNC

$$\Gamma^\rho_{\mu\nu} = -\hat{v}^\rho \partial_\mu \tau_\nu + \frac{1}{2} h^{\rho\sigma}\left(\partial_\mu \bar{h}_{\nu\sigma} + \partial_\nu \bar{h}_{\mu\sigma} - \partial_\sigma \bar{h}_{\mu\nu}\right)$$

with torsion $\Gamma^\rho_{[\mu\nu]} = -\frac{1}{2}\hat{v}^\rho(\partial_\mu \tau_\nu - \partial_\nu \tau_\mu)$

$$\nabla_\mu \tau_\nu = 0\,, \qquad \nabla_\mu h^{\nu\rho} = 0\,,$$

Coupling FTs to TNC

[Hartong,Kiritsis,NO]

- action functional $S = S[\hat{v}^\mu, h^{\mu\nu}, \tilde{\Phi}]\,.$

EM tensor: $T^\mu{}_\nu$

mass current T^μ

energy current (density +flux) → $-\tau_\nu T^\nu{}_\mu \delta\hat{v}^\mu$

momentum flux → $(\hat{e}^a_\nu \hat{v}^\mu T^\nu{}_\mu)$

$$\delta_{\rm bg} S = \int d^{d+1}x e \left[-\tau_\nu T^\nu{}_\mu \delta\hat{v}^\mu - (\hat{e}^a_\nu \hat{v}^\mu T^\nu{}_\mu)\, \hat{e}_{\sigma a}\tau_\rho \delta h^{\rho\sigma} + \frac{1}{2}(\hat{e}^b_\nu e^{\mu a} T^\nu{}_\mu)\, \hat{e}_{\rho b}\hat{e}_{\sigma a}\delta h^{\rho\sigma} + \tau_\mu T^\mu \delta\tilde{\Phi}\right],$$

spatial stress → $(\hat{e}^b_\nu e^{\mu a} T^\nu{}_\mu)$

mass density → $\tau_\mu T^\mu$

- from the various local symmetries:

$$e^{-1}\partial_\mu (e T^\mu) = \langle O_\chi \rangle\,,$$ particle number conservation (Stueckelberg U(1))

$$\hat{e}^a_\mu T^\mu - \tau_\nu e^{\mu a} T^\nu{}_\mu = 0\,.$$ mass current= momentum current (local boosts)

$$\hat{e}^{[a}_\nu e^{b]\mu} T^\nu{}_\mu = 0\,.$$ symmetric spatial stress (local rotations)

Diffeomorphism and scale Ward identities

- diffeos -> on-shell WI

$$0 = e^{-1}\partial_\nu \left(eT^\nu{}_\mu\right) + T^\rho{}_\nu \left(\hat{v}^\nu \partial_\mu \tau_\rho - e^\nu_a \partial_\mu \hat{e}^a_\rho\right) + \tau_\nu T^\nu \partial_\mu \tilde{\Phi} \,.$$

* conserved currents $\partial_\nu \left(eK^\mu T^\nu{}_\mu\right) = 0 \,.$

for K a TNC Killing vector:

$$\mathcal{L}_\xi \hat{v}^\mu = 0 \,, \qquad \mathcal{L}_\xi h^{\mu\nu} = 0 \,, \qquad \mathcal{L}_\xi \tilde{\Phi} = 0 \,,$$

- if theory has scale invariance:
  can use TNC analogue of dilatation connection

$$-z\tau_\nu \hat{v}^\mu T^\nu{}_\mu + \hat{e}^a_\nu e^{\mu a} T^\nu{}_\mu + 2(z-1)\tau_\mu T^\mu \tilde{\Phi} = 0 \,.$$

z-deformed trace WI

Schroedinger model & Lifshitz model

- simplest toy model for coupling non-rel. scale-inv theory to TNC (z=2)

$$S = \int d^{d+1}x e \left(-i\phi^\star \hat{v}^\mu \partial_\mu \phi + i\phi \hat{v}^\mu \partial_\mu \phi^\star - h^{\mu\nu}\partial_\mu \phi \partial_\nu \phi^\star - 2\tilde{\Phi}\phi\phi^\star - V_0(\phi\phi^\star)^{\frac{d+2}{d}} \right)$$

-> gives Schr. equation

* can also consider deformations preserving local scale inv

- other possibility: do not couple to $\tilde{\Phi}$ -> e.g. z=2 Lifshitz model

$$S = \int d^{d+1}x e \left[\frac{1}{2} (\hat{v}^\mu \partial_\mu \phi)^2 - \frac{\lambda}{2} (h^{\mu\nu} \nabla_\mu \partial_\nu \phi)^2 \right]$$

- more generally: $S = S[\hat{v}^\mu, h^{\mu\nu}]$. --> $S = S[\hat{v}^\mu, g^{\mu\nu}]$.

$$g^{\mu\nu} = -\hat{v}^\mu \hat{v}^\nu + h^{\mu\nu}$$

situation considered in [Hoyos,Kim,Oz]

* special case: $S = S[g^{\mu\nu}]$.

Flat NC spacetime

to study FTs on flat NC: first need to define notion of flat NC
- use global inertial coordinates (t, x^i)

$$\tau_\mu = \delta_\mu^t \,, \qquad e_\mu^a = \delta_\mu^i \delta_i^a \,. \longrightarrow \begin{array}{l} h^{tt} = h^{ti} = 0 \,, \quad h^{ij} = \delta^{ij} \,, \\ v^\mu = -\delta_t^\mu \,, \\ h_{tt} = h_{ti} = 0 \,, \quad h_{ij} = \delta_{ij} \,. \end{array}$$

$$\Gamma^\rho_{\mu\nu} = 0 \to M_\mu = \partial_\mu M \,.$$

choice of vector field is motivated by looking at geodesics

- flat space should include M=const.
  * will see that we can allow for more general choices:
    equiv. to M=const by local syms of the theory
      -> defines the notion of orbit of M

residual coordinate trafos of flat NC

- trafos of the TNC geometry that leave flat NC invariant (up to local rescaling) ?

  (analogue of Poincare (conformal) for Minkowski)

* finite versions:

$$
\begin{array}{lll}
M'(x) = M(x) + C & & \\
t' = t + a & & M'(x') = M(x) \\
x'^i = x^i + a^i & & M'(x') = M(x) \\
x'^i = R^i{}_j x^j & & M'(x') = M(x) \\
t' = \lambda^z t & x'^i = \lambda x^i & M'(x') = \lambda^{2-z} M(x) \\
x'^i = x^i + v^i t & t' = t & M'(x') = M(x) - \frac{1}{2} v^i v^i t + v^i x^i
\end{array}
$$

plus special conformal transformation for z=2

$$
t' = \frac{t}{1-ct}\,, \qquad x'^i = \frac{x^i}{1-ct}\,, \qquad M'(x') = M(x) + \frac{c}{2}\frac{x^i x^i}{1-ct}\,.
$$

Scale invariant FTs on flat NC

- role of M is non-trivial: consider the toy FT models

  - (deformed) Schroedinger model:

$$S=\int d^{d+1}x\left(-\varphi^2\left[\partial_t\left(\theta+M\right)+\frac{1}{2}\partial_i\left(\theta+M\right)\partial^i\left(\theta+M\right)+a\partial_i\partial^i\left(\theta+M\right)\right]-\frac{1}{2}\partial_i\varphi\partial^i\varphi-V_0\varphi^{\frac{2(d+2)}{d}}\left(1+b\theta^2\right)\right),$$

  - Lifshitz model:

$$S=\int d^{d+1}x\left[\frac{1}{2}\left(\partial_t\phi+\partial^i M\partial_i\phi\right)^2-\frac{\lambda}{2}\left(\partial_i\partial^i\phi\right)^2\right].$$

can we remove M by local transformations (field redefinitions) ?
and get M=const. : depends on the model in question

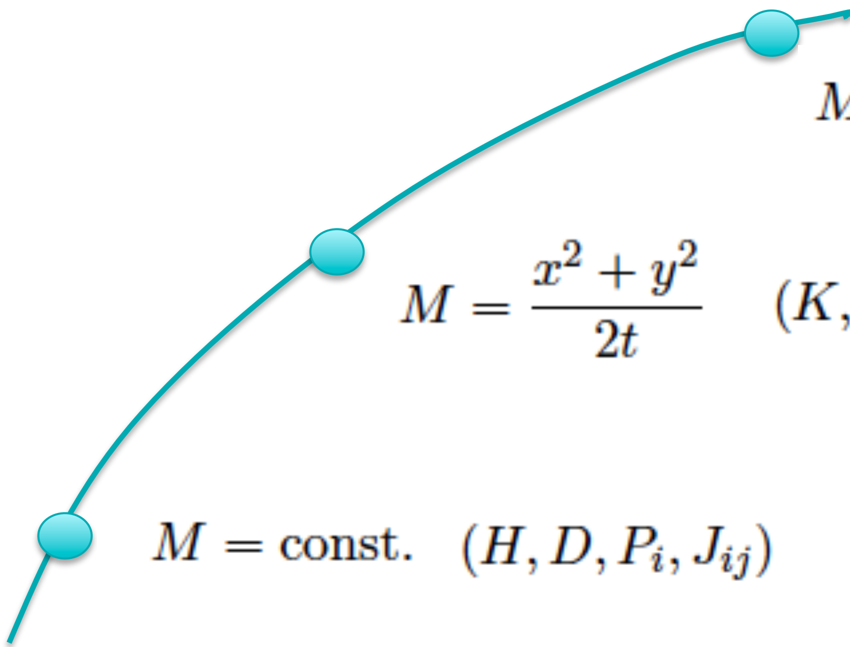
b=0: $\tilde{\theta}=\theta+M$ ⇒ Sch-invariant for a=0
Lif + Galilean boost for a not zero

(consequence of local U(1) symmetry)

b not zero
& Lifsthiz model ⇒ only Lif invariance

Orbits of M

- the M functions related to M=const by residual trafos define orbit
    * maximal orbit underlies Sch symmetry (as in undeformed Sch model)



$$M = -\frac{1}{2}V^iV^it + V^ix^i$$

$$M = \frac{x^2+y^2}{2t} \quad (K, D, G_i, J_{ij})$$

$$M = \text{const.} \quad (H, D, P_i, J_{ij})$$

- for each choice of M: CKVs form a Lifshitz subalgebra
    - residual trafos of flat NC with $\delta M = 0$

-> will be useful next when we look at Lif vacuum (in holography)

Symmetries of the Lifshitz vacuum (back to holography)

[Kiritsis,Hartong,NO]3

- what is bulk realization of residual syms of flat NC ?

Lif metric for any M in flat NC-orbit

$$ds^2 = \left(\frac{dr}{r} - \frac{1}{d}\partial_i\partial^i M dt\right)^2 - \frac{dt^2}{r^{2z}} + \frac{1}{r^2}\left(dx^i - \partial^i M dt\right)^2 .$$

- sources for Lif vacuum transform under Sch group (via bulk PBH trafos)
    - trafos for given M in M=const orbit: Lifshitz
    - delta M trafos lie in Sch algebra

- in suitable bulk coords this is dual to:

flat NC with CKVs spanning Lif and Sch realized locally on $M_\mu = \partial_\mu M$.

- possible to have conserved particle number associated to local shifts in M (generated by Galilean and special conformal)

Schroedinger invariant probe actions

have seen: FTs on flat NC realize Sch with mechanism in which
M is ``eaten'' up
(generators outside Lif are realized as projective transformations)

-> projective realizations of spacetime syms cannot be predicted
by looking at Killing vectors

- can construct z=2 probe actions on Lifshitz bulk geometries that are invariant under Sch (in same manner as in FT setting)

use covariant characterization of Lif

$$ds^2 = (-B_M B_N + \gamma_{MN})\, dx^M dx^N$$

$B^2 = -1$ $\quad \gamma_{MN}$ is orthogonal to B^M

$$S = \int d^4x \sqrt{-g}\, (\gamma^{MN}\partial_M\phi^\star\partial_N\phi + iq\phi^\star B^M\partial_M\phi - iq\phi B^M\partial_M\phi^\star - (m^2 - q^2)\phi^\star\phi)$$

eat up M: $\phi = \exp[-iqM - \frac{i}{4}qr^2\partial^2 M]\tilde{\phi}$ + use all props of M

$$r^2\left(\partial_i\partial^i\tilde{\phi} + 2iq\partial_t\tilde{\phi}\right) + r^2\partial_r^2\tilde{\phi} - 3r\partial_r\tilde{\phi} - (m^2 - q^2)\tilde{\phi} = 0$$

Summary

- holography for Lifshitz spacetimes: Schroedinger symmetry acting on sources (TNC geometry)
  -> strongly suggest that bdry theory can have Schroedinger invariance

[Hartong,Kiritsis,NO]1,2,3

Main overall points of this talk:

- appearance of global symmetries in non-relativistic field theories exhibits a new mechanism:
  * interplay between conserved currents and space-time isometries is different compared to relativistic case

- supported by considering the Lifshitz vacuum:
  = holographic dual of flat NC spacetime

-> Lifshitz holography dual to field theories on TNC spacetime

Next steps

- new perspective on existing results: comparison to linearized perturbations

  relation between $\tilde{\Phi}$ and ψ.

- EMD model (emergence of TNC, and role of U(1) ?), adding charge ?

- adding other exponents: (logarithmic running of scalar) alpha/zeta-deformation

$$A_a = r^{-z-\zeta}\alpha_{(0)}\tau_{(0)a}$$

[Kiritsis,Goutereaux][Gath,Hartong,Monteiro,NO]
[Khveshchenko][Karch][Hartnoll,Karch]

- applications to non-rel. hydrodynamics:

  Lifsthiz: [Hoyos,Kim,Oz]
  Galilean: [Jensen]

  fluid/gravity: black branes with zero/non-zero particle no. ? Galilean perfect fluids

- TNC right ingredients to start constructing effective TNC theories and their coupling to matter (e.g. QH-effect) [Son] et al

- Schroedinger holography [Andrade,Keeler,Peach,Ross,][Armas,Blau,Hartong(in progress)]

- 3D bulk (Virasoro-Schroedinger) & connection to Warped CFTs [Hofman,Rollier]

- HL gravity (see talk Jelle Hartong) [Hartong,NO(to appear)]

The end

Concluding workshop of HOLOGRAV

Current Themes in Holography

April 23-27, 2016

Niels Bohr Institute, Copenhagen

local organizers:
Troels Harmark, Cindy Keeler, Charlotte Kristjansen, Marius de Leeuw, NO