A menagerie of non-relativistic physics

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Major lesson from the 2000s

Deduce ~universal, non-perturbative features
in ~~SUSY~~ QFT via

1. Spacetime symmetries (AdS/CFT)
2. Anomalies (symmetry protected phases)

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→

hydrodynamics, low-energy T>0 transport

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in ~~SUSY~~ QFT via

1. Spacetime symmetries (AdS/CFT)
2. Anomalies (symmetry protected phases)

↓

existence of gapless edge states,
symmetry-protected response (e.g. QHE, TI)

Anomalies and QHE

Classic example:

- in 3d gapped phase of relativistic QFT, low-energy topological EFT

$$S_{eff} = \frac{k}{4\pi}\int A \wedge dA + c\int \mathrm{tr}\left(\Gamma \wedge d\Gamma + \frac{2}{3}\Gamma^3\right) + \ldots$$

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1. symmetry-protected bulk response
2. characterize anomalies of edge QFT

Anomalies and NR QHE

What one actually finds in (F)QHE is a 3d topological EFT for gapped phases of NR QFT:

$$S_{eff} = \frac{k}{4\pi}\int A \wedge dA + \tilde{\eta}\int A \wedge d\omega + c\int \omega \wedge d\omega + \ldots$$

Basic questions:

1. What precisely is ω ?
2. Do $(\tilde{\eta}, c)$ correspond to genuine anomalies?
3. What about QHE in Galilean-invariant theories?

Thermal transport

Lesson from applied AdS/CFT, fluid/gravity, hydro.. :

Spacetime symmetries imply non-perturbative interrelations between charge, thermal transport

Manifested in hydrodynamic effective description

What about in NR systems? Like anomalous Hall effect?

(2+1d systems with ~~parity~~,
theory of transport developed in 1970s)

The plan

i. Motivation

ii. Geometry for NR QFT

iii. Transport

iv. Warped CFT

Galilean QFT

Galilean theories possess a boost symmetry and U(1) charge: $[P_i, K_j] = -i\delta_{ij} M$

simplest example:

$$S_{free} = \int d^d x \left\{ \frac{i}{2} \Psi^\dagger \overleftrightarrow{\partial}_0 \Psi - \frac{\delta^{ij}}{2m} \partial_i \Psi^\dagger \partial_j \Psi \right\}$$

Geometry and symmetries

After some work, can deduce that Galilean theories naturally couple to [Son][3], [Geracie, Son, Wu[2]], [KJ], [others]

$$(n_\mu, h_{\nu\rho}, A_\sigma) \longleftrightarrow (v^\mu, h^{\nu\rho}, A_\sigma)$$

$h_{\mu\nu}$ gives spatial metric $\qquad \gamma_{\mu\nu} = n_\mu n_\nu + h_{\mu\nu}$

$$S_{free} = \int d^d x \sqrt{\gamma} \left\{ \frac{i v^\mu}{2} \Psi^\dagger \overleftrightarrow{D}_\mu \Psi - \frac{h^{\mu\nu}}{2m} D_\mu \Psi^\dagger D_\nu \Psi \right\}$$

Geometry and symmetries

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One ought to impose invariance under

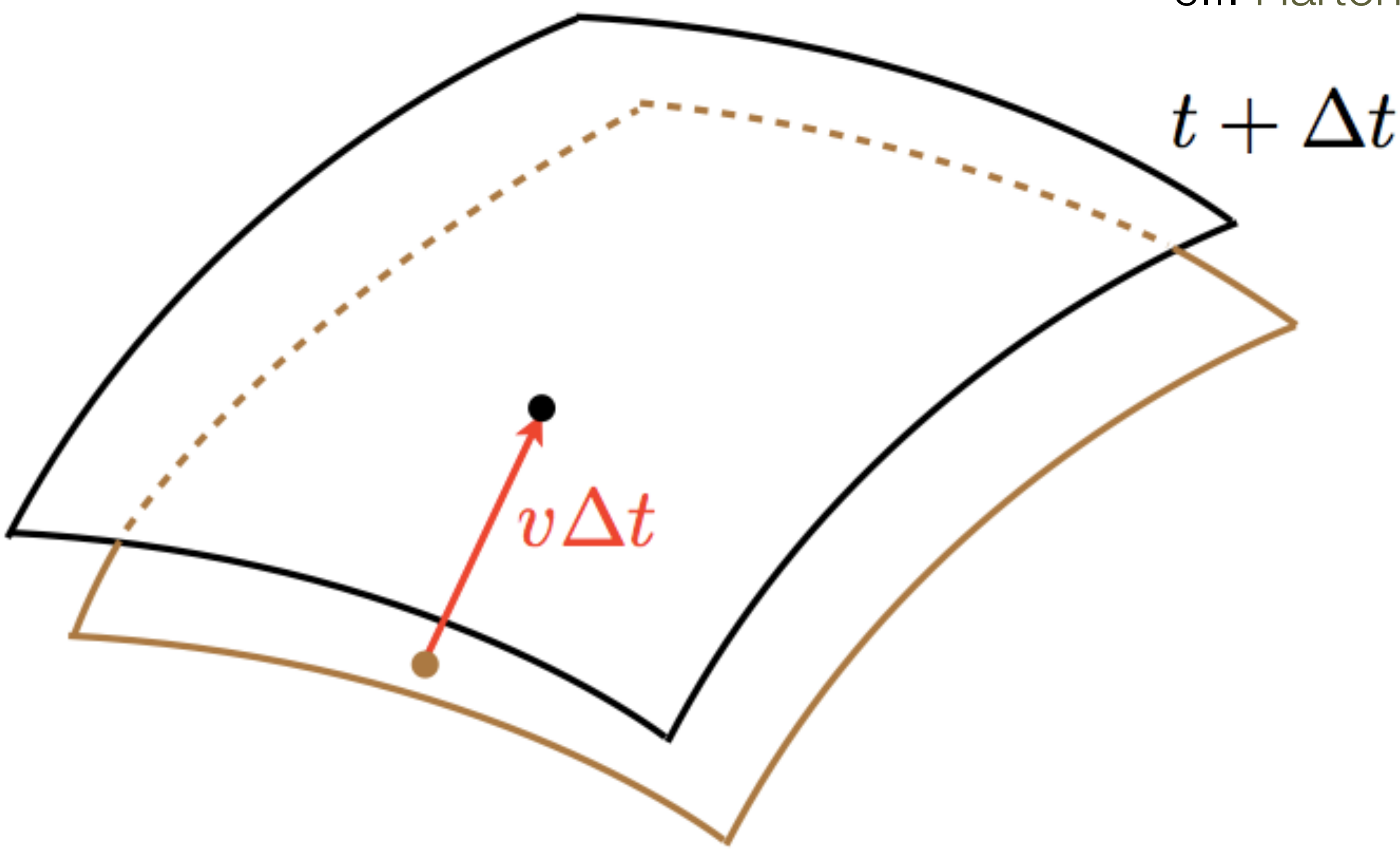
1. U(1) gauge transformations
2. Reparameterizations
3. "Milne boosts," which act as $\begin{cases} v^\mu \to v^\mu + \psi^\mu \\ A_\mu \to A_\mu + \psi_\mu - \frac{1}{2} n_\mu \psi^2 \end{cases}$

$(v^\mu \psi_\mu = 0)$

Newton-Cartan geometry

We've unwittingly generated (one version of) Newton-Cartan (NC) geometry

c.f. Hartong's, Obers' talks



Cross-checks

1. For NR theories obtained either from large c limits [KJ, Karch] or from DLCQ [Duval, et al] [Christensen, et al], NR QFT couples to NC geometry in a Milne-invariant way [KJ]

$$G = 2n_\mu dx^\mu (dx^- + A) + h_{\mu\nu} dx^\mu dx^\nu$$

2. So this background geometry appears on boundary of asymptotically Schrodinger spacetimes [Christensen, et al]
3. (Others)

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Hydrodynamics - I

- Hydrodynamics is near-universal effective theory at T>0
- Describes evolution of $\langle T^{\mu\nu} \rangle$, $\langle J^{\mu} \rangle$ near equilibrium
- ~ Current algebra
- Intimately tied to spacetime symmetries
- AdS/CFT-inspired mini-revolution (many new transport coefficients which do not appear in the textbook treatment)

Hydrodynamics - II

Hydro in one slide:

1. Promote thermodynamic quantities (T, μ, u^μ) to classical dofs (hydro variables)
2. Constitutive relations for $(T^{\mu\nu}, J^\mu)$, e.g.

$$J^\mu = \rho u^\mu + \mathcal{O}(\partial)$$

3. Impose Ward identities as eoms $\begin{cases} D_\nu T^{\mu\nu} = F^\mu{}_\nu J^\nu \,, \\ D_\mu J^\mu = 0 \,, \end{cases}$

4. Demand a local Second law:

$$\exists\, S^\mu \;\; s.t. \;\; D_\mu S^\mu \geq 0$$

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4. Demand a local Second law:

$$\exists\, S^\mu \;\; s.t. \;\; D_\mu S^\mu \geq 0 \longrightarrow$$

~understood from first principles!

Hydrodynamics - III

What do we have for NR hydro? [KJ]

1. Hydro variables: (T, μ, u^μ) $\quad (u^\mu n_\mu = 1)$
2. Operators conjugate to background spacetime:

$$\underbrace{\langle J^\mu \rangle \,,\ \langle \mathcal{P}_\mu \rangle = h_{\mu\nu} \langle J^\nu \rangle \,,\ \langle T_{\mu\nu} \rangle}_{\langle \mathcal{T}^{\mu\nu} \rangle} \,,\ \langle \mathcal{E}^\mu \rangle$$

$$\langle \tilde{\mathcal{E}}^\mu \rangle = \langle \mathcal{E}^\mu \rangle - \left(u_\nu - \frac{1}{2} n_\nu u^2 \right) \langle \mathcal{T}^{\mu\nu} \rangle$$

Hydrodynamics - III

What do we have for NR hydro? [KJ]

3. Ward identities:

[Geracie, Son, Wu[2]], [KJ]

$$\begin{cases} \left(\tilde{D}_\mu - 2E^n_\mu\right)\tilde{\mathcal{E}}^\mu = \tilde{D}_\mu u_\nu \mathcal{T}^{\mu\nu} \\ \left(\tilde{D}_\nu - E^n_\nu\right)\mathcal{T}^{\mu\nu} = (F^n)^\mu{}_\nu \tilde{\mathcal{E}}^\nu \end{cases}$$

4. Impose local Second law

P-violating hydrodynamics in 2+1d

Consider fluid mechanics of T>0 system with ~~parity~~

Relevant for systems which display anomalous Hall effect

A careful entropy analysis shows that there are **6** new first-order, dissipationless response coefficients (4 independent)

$$\tilde{\eta}, \tilde{\kappa} \qquad \mathcal{M}, \mathcal{M}_n$$

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Hall viscosity
anomalous thermal Hall conductivity

P-violating hydrodynamics in 2+1d

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magnetic susceptibilities

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Warped CFTs and holography

- Many near-extremal BHs have BTZ hear-horizon geometry
- However, most (like Kerr) have a “warped BTZ” near-horizon

$$g = R^2 \left(-\cosh^2 r d\tau^2 + dr^2 + c^2 (d\phi + \sinh \sigma d\tau)^2\right)$$

Is there a dual CFT? What is it?

Warped CFTs and holography

$$g = R^2 \left(-\cosh^2 r d\tau^2 + dr^2 + c^2 (d\phi + \sinh\sigma d\tau)^2 \right)$$

1. One set of bc (e.g. [Compere, Detournay]) implies

   Virasoro x U(1) Kac-Moody symmetries (`WCFT")

2. Another [Guica] implies dual is ordinary CFT

WCFTs from field theory - I

Suppose you have a 2d theory invariant under:

$$x^{\pm} \to x^{\pm} + c^{\pm}\,, \qquad x^{-} \to \lambda x^{-}$$ [Hofman, Strominger]

Unitarity, locality enhance these to affine algebras

Two minimal cases:

1. Lorentz-invariance : get ordinary 2d CFT
2. R-moving translation enhances to Virasoro, L-moving translation to Kac-Moody

→ WCFT!

WCFTs from field theory - II

$$x^- = x^-(X^-)\,,$$

$$x^+ = X^+ + f(X^-)$$

Virasoro:

$$T(x)T(0) \sim \frac{c}{2(x^-)^4}$$

Kac-Moody:

$$P(x)P(0) \sim \frac{k}{(x^-)^2}$$

(R-moving global symmetries also get enhanced to KM)

WCFTs from field theory - III

Can use symmetries to deduce modular properties of torus partition function, ``Cardy formula”

[Detournay, Hartman, Hofman]

Correctly reproduces entropy of WAdS3 BHs

Q: Should we really care about WCFT?

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One route:

- deduce anomalies of WCFT
- can these be matched to anything?
  (putative holographic duals, edge theory of QHE, &c)

WCFTs are Galilean CFTs

[Hofman, Rollier]

Show that WCFTs naturally couple to a version of NC geometry which they dub ``warped geometry”

Easy to then show that WCFTs are (massless) Galilean CFTs with $z = \infty$ [KJ, WIP]

$$h_{\mu\nu} = h_\mu h_\nu \ (d = 2)$$

Free scalar WCFT

With this in hand we can easily write free scalar WCFTs:

$$S = \frac{1}{2}\int d^2x\sqrt{\gamma}\left\{(w^\mu\partial_\mu\varphi)^2 + \left(\frac{N^2}{2} - w^\mu\partial_\mu N\right)\varphi^2 + m^2\varphi^2\right\}$$

$$(N = \varepsilon^{\mu\nu}\partial_\mu n_\nu)$$

"conformal mass"

$$\varphi \to e^{-\Omega/2}\varphi$$

Kac-Moody: k ~ m

Anomalies and central charges

WZ consistency => four types of anomalies:

1. Pure boost: $\delta_\psi W \propto k \int d^2x \sqrt{\gamma}\psi$
2. Mixed boost/Weyl: $\delta_\chi W \propto c \int d^2x \sqrt{\gamma} \left\{ -\dot{\Omega} N + \frac{\psi}{2} N^2 \right\}$
3. Gravitational: $\delta_\xi W = \tilde{c} \int \partial_\mu \xi^\nu d\Gamma^\mu{}_\nu + \dots$
4. Flavor: $\delta_\Lambda W \propto \tilde{k} \int \Lambda \cdot dA$

Anomalies and central charges

WZ consistency => four types of anomalies:

1. Pure boost: $$\delta_\psi W \propto k \int d^2x \sqrt{\gamma}\psi$$

I don't see how this can be matched

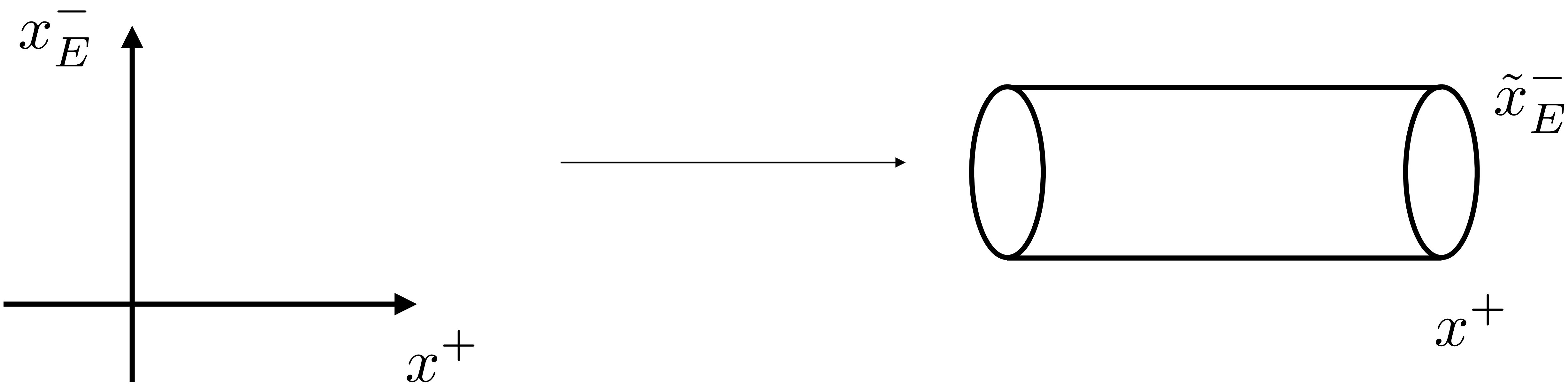
to holography, or anything else for that matter

Challenge: Match (?) boost anomaly to WAdS3

Thermodynamics

Just like in 2d CFT, there is a Weyl/boost

which takes the Euclidean plane to a thermal cylinder



Get thermodynamics from ``Schwarzian”

(can also get entire hydrostatic partition function)

Conclusions

1. Galilean-invariant theories couple to NC geometry
2. Crucial ingredient: Milne boosts
3. NR hydro modernized; ~ first-principles approach to AHE
4. Warped CFTs are Galilean CFTs with $z = \infty$,

   puzzles with anomaly matching to WAdS3
5. [WIP] Anomaly inflow and QHE

Thank you!

Extra slides

Another proposal

[Geracie, Prabhu, Roberts] Give alternative: “Bargmann structure”

Most natural in terms of frame, spin connection

$$g = \begin{pmatrix} 1 & 0 \\ K^i & R^i{}_j \end{pmatrix} \qquad \omega = \begin{pmatrix} 0 & 0 \\ \omega^i{}_0 & \omega^i{}_j \end{pmatrix}$$

$$\begin{cases} f_\mu^A = (n_\mu, e_\mu^i) \\ F_A^\mu = (v^\mu, E_i^\mu) \end{cases}$$

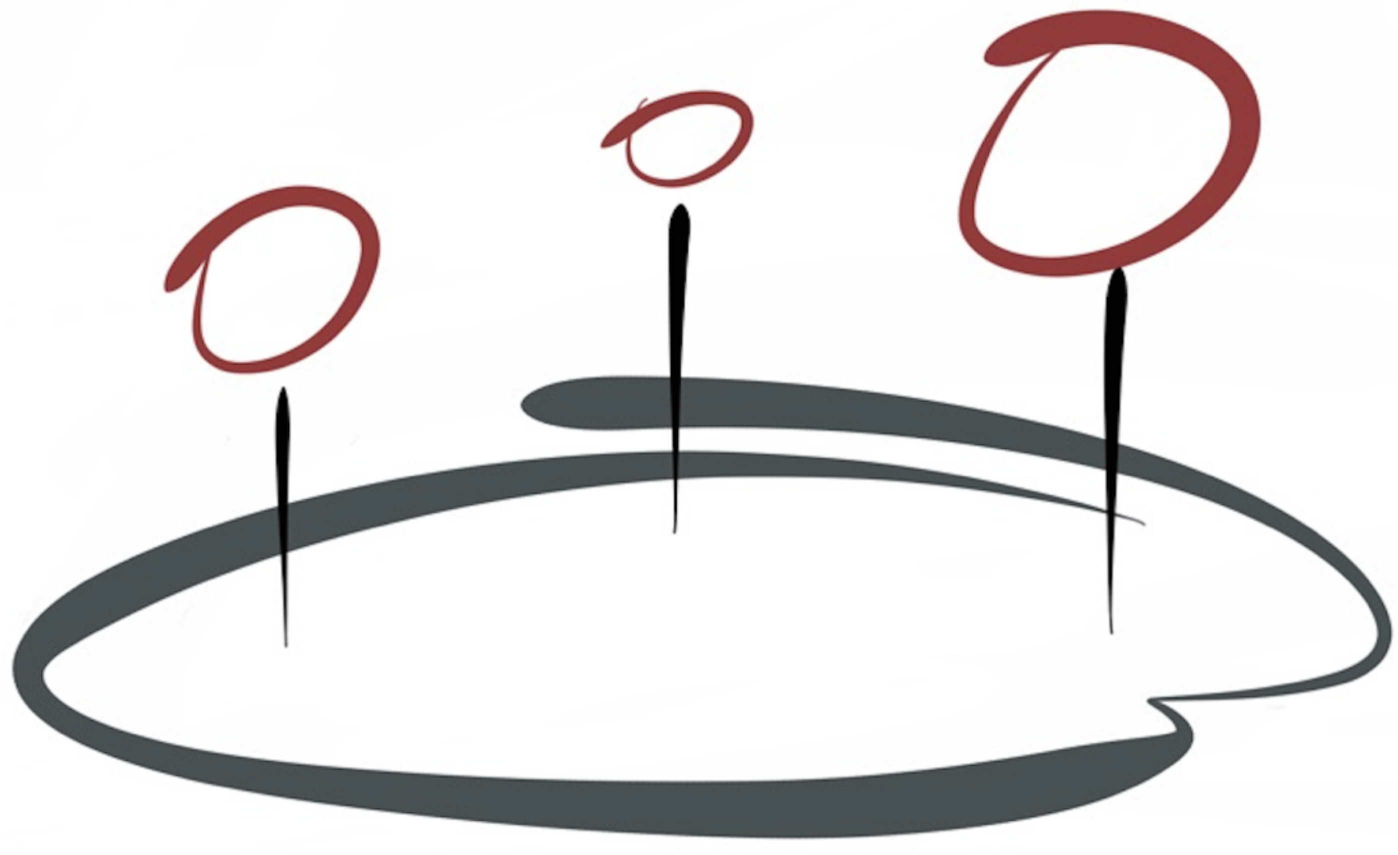
reduces to above upon restricting

$$\omega^i{}_0 \wedge e_i \sim F$$

Hydrostatics

Just as in the relativistic case [KJ, et al] [Banerjee, et al]

Intimate relation between local Second law and hydrostatic equilibrium



Finite correlation length:
effective Euclidean theory
gapped, leads to simple

$$W = \int d^d x \sqrt{\gamma}\, P\,(T, \mu) + \mathcal{O}(\partial)$$

WCFTs are Galilean CFTs

Easy to then show that WCFTs are (massless) Galilean CFTs with $z = \infty$

How do we see this?

Consider a theory of m=0 Galilean fields, coupled to $(n_\mu, h_{\mu\nu})$

$$h_{\mu\nu} = h_\mu h_\nu \qquad h^{\mu\nu} = w^\mu w^\nu \longrightarrow \text{Under boosts,} \quad \begin{cases} n_\mu \to n_\mu \\ h_\mu \to h_\mu - \psi n_\mu \end{cases}$$

$$z = \infty \longrightarrow \text{Under Weyl,} \quad \begin{cases} n_\mu \to e^\Omega n_\mu \\ h_\mu \to h_\mu \end{cases}$$

WCFTs are Galilean CFTs

Global symmetries in flat space fix $n = dx^- \,, \; h = dx^+$

These are exactly

$$x^- = x^-(X^-) \,,$$

$$x^+ = X^+ + f(X^-)$$