

# Holographic Goldstino



Matteo Bertolini



based on **1412.6499**

w/ Argurio, Musso, Porri & Redigolo

[plus work in progress w/ Musso, Papadimitriou & Raj]

see also: 1205.4709, 1208.3615, 1304.1481, 1310.6897

w/ Argurio, Di Pietro, Porri & Redigolo

# Motivations

- Spontaneously broken global symmetries leave universal fingerprints in the IR behavior of QFTs. In particular, Goldstone theorem ensures that massless modes should appear in IR spectrum.
- In supersymmetric QFTs with spontaneously broken supersymmetry, a massless **Goldstino** is expected.
- Goldstino appears as massless pole in **supercurrent** 2-point correlator. Pole residue is related to vacuum energy by specific supersymmetry **Ward identities**.

# Motivations

- If QFT dynamics **strongly coupled** (e.g. DSB) things get hard → *AdS/CFT* provides a powerful tool, allowing a dual weakly coupled supergravity description.
- Need a complete control of the holographic map in the supercurrent sector, i.e. the **gravitino**. Not much so far for non-AdS bg (necessary condition for broken SUSY).
- *Primary goal*: fill above gap + provide a holographic derivation of SUSY **Ward identities** and show, in turn, the appearance of **Goldstino** in theories which are inherently strongly coupled.

# Supersymmetric RG-flows

- Let us consider 4d SQFTs described by RG-flows triggered by some **relevant** operator perturbing a strongly coupled UV fixed point. Most generally one can write

$$\mathcal{L} = \mathcal{L}_{\text{SCFT}} + \lambda \int d^2\theta \mathcal{O} = \mathcal{L}_{\text{SCFT}} + \lambda \mathcal{O}_{\text{F}} \quad [\text{GREEN ET AL. '10} + \text{OTHERS}]$$

where  $\mathcal{O} = \mathcal{O}_{\text{S}} + \sqrt{2}\theta \mathcal{O}_{\psi} + \theta^2 \mathcal{O}_{\text{F}}$ .

- The theory can flow to a SUSY or **SUSY breaking** vacuum, depending whether the operator  $\mathcal{O}_{\text{F}}$  acquires a VEV.  
→ 1 & 2-point functions of supercurrent multiplet can tell!
- **Note:** we do not consider QFTs without an interacting UV-fixed point, e.g. cascading gauge theories as KS/KT or quiver embeddings of ISS-like models. [WORK IN PROGRESS...]

# Supercurrent multiplet

- The **supercurrent** multiplet, which contains the EM tensor and the SUSY current, can be described as (**FZ** multiplet)

$$(\mathcal{J}_\mu, \mathbf{X}) \quad \begin{array}{l} \swarrow \text{Chiral superfield} \\ \nwarrow \text{Real superfield} \end{array} \quad \sigma_{\alpha\dot{\alpha}}^\mu \bar{\mathbf{D}}^{\dot{\alpha}} \mathcal{J}_\mu = -\frac{1}{2} \mathbf{D}_\alpha \mathbf{X}$$

$$\mathcal{J}_\mu = \mathbf{j}_\mu + \theta \mathbf{S}_\mu + \bar{\theta} \bar{\mathbf{S}}_\mu + \theta \sigma^\nu \bar{\theta} \mathbf{2T}_{\mu\nu} + \dots$$

$$\mathbf{X} = \mathbf{x} + \frac{2}{3} \theta \mathbf{S} + \theta^2 \left( \frac{2}{3} \mathbf{T} + \mathbf{i} \partial^\mu \mathbf{j}_\mu \right) + \dots$$

- For a SCFT  $\mathbf{X} = \mathbf{0}$ . In our case, choosing without loss of generality  $\Delta_{\mathcal{O}} = \mathbf{2}$ , we have  $\mathbf{X} = \mathbf{4/3} \lambda \mathcal{O}$ .

# Supersymmetry Ward identities

- Regardless the vacuum, the SUSY algebra implies that the following **Ward identities** should hold

$$\langle \partial^\mu \mathbf{S}_{\mu\alpha}(\mathbf{x}) \bar{\mathbf{S}}_{\nu\dot{\beta}}(\mathbf{0}) \rangle = -2 \delta^4(\mathbf{x}) \langle \delta_\alpha \bar{\mathbf{S}}_{\nu\dot{\beta}} \rangle = -2 \sigma_{\alpha\dot{\beta}}^\mu \langle \mathbf{T}_{\mu\nu} \rangle \delta^4(\mathbf{x})$$

$$\langle \partial^\mu \mathbf{S}_{\mu\alpha}(\mathbf{x}) \mathcal{O}_{\psi\beta}(\mathbf{0}) \rangle = -2 \delta^4(\mathbf{x}) \langle \delta_\alpha \mathcal{O}_{\psi\beta} \rangle = \sqrt{2} \langle \mathcal{O}_{\mathbf{F}} \rangle \delta^4(\mathbf{x})$$

- The Ward identities imply

$$\langle \mathbf{S}_{\mu\alpha}(\mathbf{x}) \bar{\mathbf{S}}_{\nu\dot{\beta}}(\mathbf{0}) \rangle = \dots - \frac{i}{4\pi^2} \langle \mathbf{T} \rangle (\sigma_\mu \bar{\sigma}^\rho \sigma_\nu)_{\alpha\dot{\beta}} \frac{\mathbf{x}_\rho}{\mathbf{x}^4}$$

$$\langle \mathbf{S}_{\mu\alpha}(\mathbf{x}) \mathcal{O}_{\psi\beta}(\mathbf{0}) \rangle = \dots - \frac{i}{2\pi^2} \sqrt{2} \langle \mathcal{O}_{\mathbf{F}} \rangle \epsilon_{\alpha\beta} \frac{\mathbf{x}_\mu}{\mathbf{x}^4}$$

[ARGURIO ET AL. '13]

Upon Fourier transform it displays the massless pole associated to **Goldstino**, which is the lowest energy excitation in  $\mathbf{S}_\mu$ : in IR  $\mathbf{S}_\mu = \sigma_\mu \bar{\mathbf{G}}$ .

# Field/operator map

- The SQFT

$$\mathcal{L} = \mathcal{L}_{\text{SCFT}} + \lambda \int d^2\theta \mathcal{O}$$

can be described holographically by 5d **N=2 SUGRA** coupled to **one hypermultiplet** (made of one hyperino  $\zeta$  + two complex scalars  $\rho$  and  $\phi$ )

- Since  $\Delta(\mathcal{O}) = 2$  dual bg is AAdS and  $\mathbf{m}_{(\rho,\phi)}^2 = -4, -3$ .

$$ds^2 = \frac{1}{z^2} (\mathbf{F}(z) dx^2 + dz^2) \quad , \quad \mathbf{F}(0) = 1$$

$$\rho \sim z^2 (\mathbf{c} \log z + \mathbf{d}) + \mathbf{O}(z^4) \quad , \quad \phi \sim z(\mathbf{a} + \mathbf{b}z^2) + \mathbf{O}(z^5)$$

$$\rho \longleftrightarrow \mathcal{O}_{\mathbf{S}}$$

$$\phi \longleftrightarrow \mathcal{O}_{\mathbf{F}}$$

$$\mathbf{c} \longleftrightarrow \text{source } \mathcal{O}_{\mathbf{S}} \quad (\text{SUSY})$$

$$\mathbf{a} \longleftrightarrow \text{source } \mathcal{O}_{\mathbf{F}} \quad (\text{SUSY})$$

$$\mathbf{d} \longleftrightarrow \text{VEV } \mathcal{O}_{\mathbf{S}} \quad (\text{SUSY})$$

$$\mathbf{b} \longleftrightarrow \text{VEV } \mathcal{O}_{\mathbf{F}} \quad (\text{SUSY})$$

# Supergravity model

- The model we consider is **N=2 SUGRA** coupled to one **hypermultiplet** with  $\sigma$ -model metric  $\frac{\text{SU}(2,1)}{\text{U}(1) \times \text{SU}(2)}$  of which graviphoton gauges a **U(1)** isometry. [CERESOLE ET AL. '01]

- One can choose a **gauging** for which the two complex hyperscalars  $(\rho, \phi)$  have  $\mathbf{m}^2 = -4, -3$ . We further truncate the action to  $\rho = 0$  and take  $\phi$  real. Bosonic action becomes

$$\mathbf{S}_{5D}^B = \int d^5 \mathbf{x} \sqrt{-\mathbf{G}} \left\{ \frac{1}{2} \mathbf{R} - \partial_{\mathbf{M}} \phi \partial^{\mathbf{M}} \phi - \mathbf{U}(\phi) \right\}$$

$$\mathbf{U}(\phi) = \frac{1}{12} (10 - \cosh(2\phi))^2 - \frac{51}{4}$$

- We look for AAdS solutions preserving 4d Poincare' invariance, hence we require  $\phi = \phi(\mathbf{z})$ .

# Supergravity model

- **SUSY** solutions

$$\phi(\mathbf{z}) = \frac{1}{2} \log \left( \frac{1 + \mathbf{a}\mathbf{z}}{1 - \mathbf{a}\mathbf{z}} \right), \quad \mathbf{F}(\mathbf{z}) = (1 - \mathbf{a}^2 \mathbf{z}^2)^{1/3}$$

- **SUSY breaking** solutions

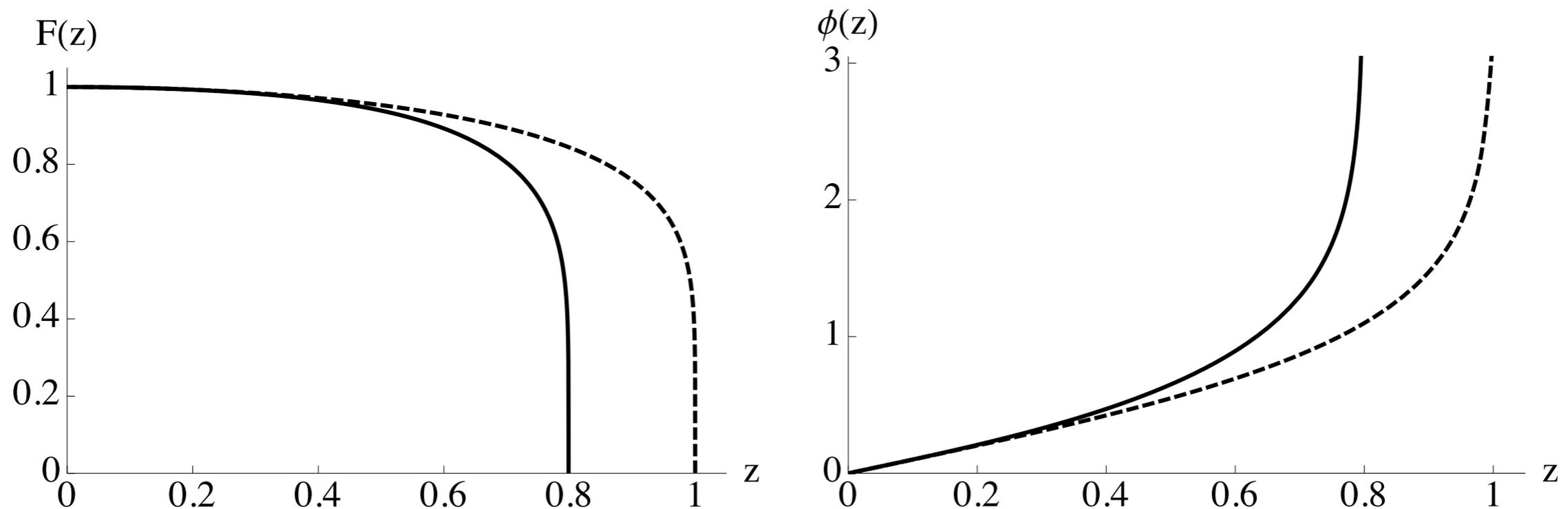
$$\phi(\mathbf{z}) = \mathbf{a}\mathbf{z} + \mathbf{b}\mathbf{z}^3 + \mathbf{O}(\mathbf{z}^5)$$

$$\mathbf{F}(\mathbf{z}) = 1 - \frac{\mathbf{a}^2}{3} \mathbf{z}^2 + \frac{\mathbf{a}^4 - 9\mathbf{a}\mathbf{b}}{18} \mathbf{z}^4 + \mathbf{O}(\mathbf{z}^6)$$

- **Note:**  $\mathbf{b} = \mathbf{a}^3/3$  is the SUSY solution  $\rightarrow \beta = \mathbf{a}^3/3 - \mathbf{b}$  is SUSY breaking parameter (related to VEV of  $\mathcal{O}_{\mathbf{F}}$ ).
- **Comment:** all solutions are singular. But remarkably, this won't affect final results!

# Supergravity model

- The BPS and ~~SUSY~~ **solutions** for the warp factor  $\mathbf{F}$  and the scalar  $\phi$  look



where **dashed** line is the BPS solutions and **solid** line the ~~SUSY~~ ones (parameters are  $\mathbf{a} = \mathbf{1}$  and  $\beta = \mathbf{0}, -\mathbf{2}/\mathbf{3}$ ).

# AdS/CFT & Hol. Renormalization

- QFT correlators are computed in AdS/CFT as

$$\langle \mathcal{O}(\mathbf{x}_1) \dots \mathcal{O}(\mathbf{x}_n) \rangle = \frac{\delta^n \mathbf{S}_{\text{on-shell}}}{\delta \phi_0(\mathbf{x}_1) \dots \delta \phi_0(\mathbf{x}_n)} \Big|_{\phi_0=0}$$

- $\mathbf{S}_{\text{on-shell}}$  diverges at  $\mathbf{z} = 0$  (because of the infinite volume of (A)AdS space) and should then be **holographically renormalized**. [BIANCHI-FREEDMAN-SKENDERIS '01A]

-  Regularize the action at  $\mathbf{z} = \epsilon$ , add a 4d covariant counter-term action  $\mathbf{S}_{\text{c.t.}}$  and define a renormalized action

$$\mathbf{S}_{\text{ren}} = \lim_{\epsilon \rightarrow 0} (\mathbf{S}_{\text{on-shell}}[\epsilon] + \mathbf{S}_{\text{c.t.}}[\epsilon])$$

- **Note:** in general  $\mathbf{S}_{\text{c.t.}}$  provides extra finite terms, which corresponds to arbitrariness in renormalization scheme.

# Hol Ren: Bosonic sector

- Have to evaluate 1-pt functions only  $\rightarrow$  focus on on-shell action at **linear order** in the fluctuations of metric and scalar

$$\phi = \phi(\mathbf{z}) + \varphi(\mathbf{z}, \mathbf{x})$$

$$ds^2 = \frac{1}{z^2} [\mathbf{dz}^2 + \mathbf{F}(\mathbf{z}) (\eta_{\mu\nu} + \mathbf{h}_{\mu\nu}(\mathbf{z}, \mathbf{x})) \mathbf{dx}^\mu \mathbf{dx}^\nu]$$

where we fixed the gauge  $\mathbf{h}_{zz} = \mathbf{h}_{\mu z} = \mathbf{0}$  and we decompose the metric as

$$\mathbf{h}_{\mu\nu} = \mathbf{h}_{\mu\nu}^{\mathbf{tt}} + \eta_{\mu\nu} \mathbf{h} + \partial_{(\mu} \mathbf{H}_{\nu)}$$

- Evaluating the on-shell SUGRA action

$$\mathbf{S} = \mathbf{S}_{5D}^{\mathbf{B}} + \mathbf{S}_{\text{GH}} \quad , \quad \mathbf{S}_{\text{GH}} = \int d^4 \mathbf{x} \sqrt{-\mathbf{g}} \mathcal{K}$$

one gets a **boundary term**, only.

# Hol Ren: Bosonic sector

- The boundary action (to linear order) reads

$$S_{\text{bdy}}^{\text{B}} = \int d^4x \sqrt{-g} \left[ \left( 3 - \frac{3zF'}{2F} \right) (1 + 2h + \dots) + 2z\phi'\varphi \right]$$

- $S_{\text{bdy}}^{\text{B}}$  diverges at  $z = 0$ . The (SUSY preserving scheme) c.t. action is

[BIANCHI-FREEDMAN-SKENDERIS '01 B + OTHERS]

$$S_{\text{c.t.}}^{\text{B}} = - \int_{z=\epsilon} d^4x \sqrt{-g} 3 \mathbf{W}(\phi + \varphi)$$

with  $\mathbf{W}$  the SUGRA superpotential evaluated at the bdy.

- The end result is

$$S_{\text{ren}}^{\text{B}} = - \int d^4x [\mathbf{a}\beta (1 + 2h_0 + \dots) + 4\beta\varphi_0]$$

Consistently, the renormalized action vanishes for  $\beta = 0$ .

# Hol Ren: Fermionic sector

- As for the fermions, we need to work at quadratic order

$$\begin{aligned} \mathbf{S}_{5D}^F = \int d^5 \mathbf{x} \sqrt{-\mathbf{G}} \left\{ - \bar{\Psi}_M \Gamma^{MNP} \mathbf{D}_N \Psi_P - 2 \bar{\zeta} \Gamma^M \mathbf{D}_M \zeta \right. \\ \left. + i \partial_N \phi \left( \bar{\zeta} \Gamma^M \Gamma^N \Psi_M - \bar{\Psi}_M \Gamma^N \Gamma^M \zeta \right) - 2 \mathcal{M}(\phi) \bar{\zeta} \zeta \right. \\ \left. + 2 \mathcal{N}(\phi) \left( \bar{\Psi}_M \Gamma^M \zeta + \bar{\zeta} \Gamma^M \Psi_M \right) + \mathbf{m}(\phi) \bar{\Psi}_M \Gamma^{MN} \Psi_N \right\} \end{aligned}$$

- In AAdS space has to be supplemented by a bdy term, to make it stationary on the EOM. We get

$$\mathbf{S}_{\text{bdy}}^F = \int d^4 \mathbf{x} \sqrt{-\mathbf{g}} \left\{ -\frac{1}{2} \bar{\Psi}_m \Gamma^{mn} \Psi_n - \bar{\zeta} \zeta \right\}$$

- We work in axial gauge  $\Psi_z = 0$  and split the gravitino as

$$\Psi_m = \psi_m^{\text{tt}} + \partial_m \vartheta + \Gamma_m \chi$$

→ EOM for  $\psi_m^{\text{tt}}$  **decouple**, those for  $\vartheta$  and  $\chi$  **coupled** to  $\zeta$ .

# Hol Ren: Fermionic sector

- The bulk action vanishes on-shell, only bdy term matters ... and diverges. The action has  $\epsilon^{-2}$  and  $\log \epsilon$  divergent terms and should be **holographically renormalized**. The **counter-term** action is

$$\begin{aligned}
 \mathbf{S}_{\text{c.t.}}^{\text{F}} = \int_{\mathbf{z}=\epsilon} d^4 \mathbf{x} \sqrt{-\mathbf{g}} \left\{ \frac{1}{2} \bar{\Psi}_m \Gamma^{\text{mrn}} \partial_r \Psi_n + \log \epsilon \left[ - 2 \bar{\zeta} \not{\partial} \zeta \right. \right. \\
 - \frac{1}{4} \bar{\Psi}_m \Gamma^{\text{mrn}} \square \partial_r \Psi_n + \frac{1}{3} \phi^2 \bar{\Psi}_m \Gamma^{\text{mnr}} \partial_r \Psi_n \\
 - \frac{1}{6} (\partial_n \bar{\Psi}_m \Gamma^{\text{mn}}) \not{\partial} (\Gamma^{\text{rs}} \partial_r \Psi_s) \\
 \left. \left. - \frac{2}{3} \mathbf{i} \phi [\bar{\zeta} (\Gamma^{\text{rs}} \partial_r \Psi_s) - (\partial_n \bar{\Psi}_m \Gamma^{\text{mn}}) \zeta] \right] \right\}
 \end{aligned}$$

where  $m, n, \dots$  4d curved indices.

# Hol Ren: Fermionic sector

- The end result for the renormalized action is

$$\mathbf{S}_{\text{ren}}^{\text{F}} = \int d^4\mathbf{x} \left\{ i \frac{\beta \mathbf{a}}{2} \bar{\vartheta}_0 \not{\partial} \vartheta_0 + 2\beta \vartheta_0 (\mathbf{i}\zeta_0 + \mathbf{a}\chi_0) - 2\beta \bar{\vartheta}_0 (\mathbf{i}\bar{\zeta}_0 - \mathbf{a}\bar{\chi}_0) + \text{non-local} + \text{scheme-dep} \right\}$$

where  $\zeta_0$ ,  $\vartheta_0$  and  $\chi_0$  are bdy leading modes of hyperino and gravitino, resp.

- Note:** **local** dependence of *all* subleading modes on leading ones can be uniquely fixed *just* by near boundary analysis + on-shell SUSY invariance of EOM! In particular we find

$$\tilde{\zeta}_1 = -\mathbf{i}\beta\vartheta_0 + \not{\partial}\mathbf{f}(\square)(\bar{\zeta}_0 + \mathbf{ia}\bar{\chi}_0) + \bar{\mathbf{f}}(\square)\mathbf{a}(\zeta_0 - \mathbf{ia}\chi_0)^{\text{T}}$$

# Holographic Ward identities

- The **field/operator map** can be read from couplings between FZ and  $\mathcal{O}$  multiplets with bulk fields at the boundary. This is

$$\int d^4x \left[ \frac{1}{2} h_0^{\mu\nu} \mathbf{T}_{\mu\nu} + \frac{1}{2} (i\Psi_0^\mu \mathbf{S}_\mu + \text{c.c.}) + \right. \\ \left. + 2(\varphi_0 \mathcal{O}_F + \text{c.c.}) - \sqrt{2}(i\zeta_0 \mathcal{O}_\psi + \text{c.c.}) + \dots \right]$$

which gives

$$h_0^{\mu\nu} \longleftrightarrow \frac{1}{2} \mathbf{T}_{\mu\nu} \quad , \quad \Psi_0^{\mu\alpha} \longleftrightarrow \frac{i}{2} \mathbf{S}_{\mu\alpha} \\ \varphi_0 \longleftrightarrow 2 \mathcal{O}_F \quad , \quad \zeta_0^\alpha \longleftrightarrow -i\sqrt{2} \mathcal{O}_{\psi\alpha}$$

- As far as gravitational sector, the relevant maps hence is

$$h_0 \longleftrightarrow \frac{1}{2} \mathbf{T} \quad , \quad \vartheta_0^\alpha \longleftrightarrow -\frac{i}{2} \partial^\mu \mathbf{S}_{\mu\alpha} \quad , \quad \bar{\chi}_{0\dot{\alpha}} \longleftrightarrow \frac{1}{2} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \mathbf{S}_{\mu\alpha}$$

# Holographic Ward identities

- Using AdS/CFT prescription, from  $\mathbf{S}_{\text{ren}}^{\text{B}}$  we get

$$\langle \mathbf{T} \rangle = 2 \frac{\delta \mathbf{S}_{\text{ren}}^{\text{B}}}{\delta \mathbf{h}_0} = -4\beta \mathbf{a}, \quad \langle \mathcal{O}_{\text{F}} \rangle = \frac{1}{2} \frac{\delta \mathbf{S}_{\text{ren}}^{\text{B}}}{\delta \varphi_0} = -2\beta$$

and from  $\mathbf{S}_{\text{ren}}^{\text{F}}$

$$\langle \partial^\mu \mathbf{S}_{\mu\alpha} (\sigma^\nu \bar{\mathbf{S}}_\nu)_\beta \rangle = -4 \frac{\delta^2 \mathbf{S}_{\text{ren}}^{\text{F}}}{\delta \vartheta_0^\alpha \delta \chi_0^\beta} = -8\beta \mathbf{a} \varepsilon_{\alpha\beta}$$

$$\langle \partial^\mu \mathbf{S}_{\mu\alpha} \mathcal{O}_{\psi\beta} \rangle = \sqrt{2\mathbf{i}} \frac{\delta^2 \mathbf{S}_{\text{ren}}^{\text{F}}}{\delta \vartheta_0^\alpha \delta \zeta_0^\beta} = -2\sqrt{2}\beta \varepsilon_{\alpha\beta}$$

which **exactly** reproduce the QFT Ward identities!

- Note:** dependence of  $\mathbf{S}_{\text{ren}}^{\text{F}}$  on combination  $\mathbf{i}\zeta_0 + \mathbf{a}\chi_0$  provides holographic derivation of QFT identity  $\sigma^\mu \bar{\mathbf{S}}_\mu = 2\sqrt{2}\lambda \mathcal{O}_\psi$ . Corresponding bosonic operator identity from  $\mathbf{S}_{\text{ren}}^{\text{B}}$ , too.

# Summary

- We provided a holographic description of a general class of SQFT where SUSY is broken at strong coupling.
- Performing holographic renormalization in the **gravitino** sector, we have recovered a set of Ward identities, which encodes the presence of the **Goldstino** mode. Non-trivial check of AdS / CFT!
- Focus was on a prototype model, but results have wider applicability → **Notice**: relevant contact terms do not depend on deep interior, or on nature of IR singularity. This is consistent with QFT expectations!

# Outlook

- **Short-term** goal:
  - Repeat the analysis for non-AAAdS backgrounds, where QFT ~~SUSY~~ vacua arise due to strong coupling dynamics (like in **cascading theories**, cfr. KS, quiver completions of ISS-like models, ...).

[WORK IN PROGRESS W/ MUSSO-PAPADIMITRIOU&RAJ]

- **Longer-term** goals:
  - Full control on fermionic sector, in particular gravitino's, could allow for holographic derivation of Goldstino **effective action**.

[CFR. KOMARGODSKI-SEIBERG '09, HOYOS ET AL. '13]

- Thorough study of **stability properties** of e.g. proposed SUSY breaking (string theory) vacua.

[CFR. ARGURIO-MUSSO-REDIGOLO '14]