Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Alexander von Humboldt
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Entanglement entropy associated to an out-of-equilibrium heat flow

Daniel Fernández

Max Planck Institute for Physics in Munich

Work in progress with:

Johanna Erdmenger
Eugenio Megías
Mario Flory
Ann-Kathrin Straub

Motivation

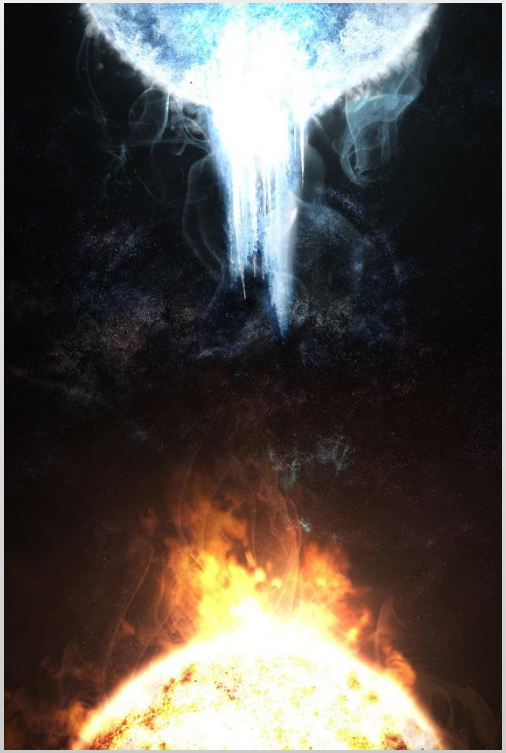
Far from equilibrium AdS/CFT

- Compute real time dynamics directly → Collective response.
- Transition to hydrodynamics at late times.
- In strongly correlated systems.

  - Organizing principles out of equilibrium?
  - Universality classes?

Examples

- Thermalization of quark-gluon plasma.
- Quenches in condensed matter systems.
- Fluctuations in the fractional Hall effect.



Simpler class of problems: Forced ~~equilibrium~~ steady state.

Concrete setup: Two heat reservoirs at different temperatures are put in contact at time t=0.

Initial form of energy density:

$$F(x) = \frac{c\pi}{12}\left[T_L^2\Theta(-x) + T_R^2\Theta(x)\right]$$

Experimental verification: Cold atom systems at the QCP.

Overview

Energy flow in strongly correlated systems

I. Steady state in CFT

II. Time evolution of bulk metric

Information flow

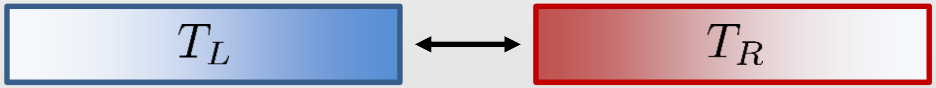
III. Analysis of entanglement entropy

A universal regime of thermal transport

D. Bernard, B. Doyon '12

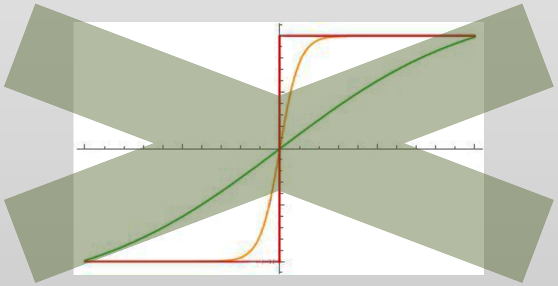
Consider:

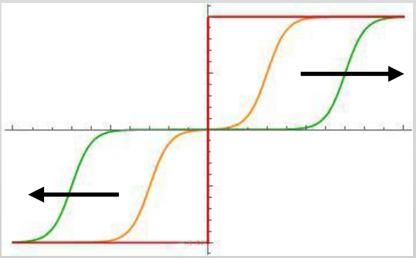
- Thermal quench in 1+1 dim. CFTs
- Two exact copies initially at equilibrium.
- Independently thermalized.



Conservation eqs. & Tracelessness:

$$\boxed{\begin{aligned} \partial_x \langle T^{xx} \rangle &= -\partial_t \langle T^{tx} \rangle = 0 \\ \langle T^{xx} \rangle &= \langle T^{tt} \rangle \end{aligned}}$$





$$\left.\begin{aligned} \langle T^{tx} \rangle &= F(x-t) - F(x+t) \\ \langle T^{tt} \rangle &= F(x-t) + F(x+t) \end{aligned}\right\}$$

"shockwaves" emanating from interface,

converge to non-equilibrium **Steady State.**

Long Time Limit: $\langle T^{tx} \rangle = cg(T_L - T_R)$ $\longleftrightarrow$ $\langle J_E \rangle \neq 0$

with thermal conductance: $g = \dfrac{\pi^2}{6}(T_L + T_R)(k_B^2/h)$

Energy flow:
Lorentz-boosted thermal distribution.

Universality in d >1

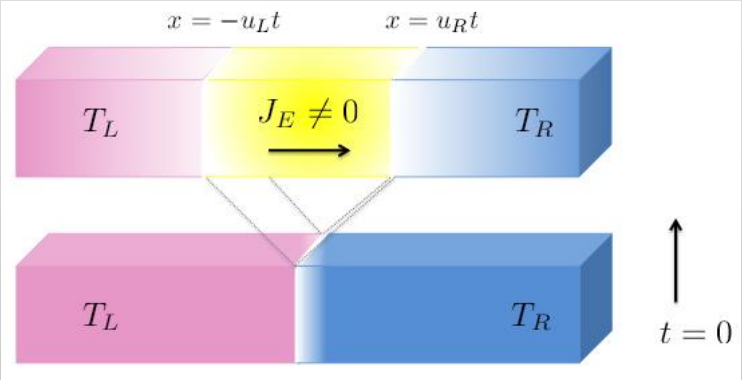
M.J. Bhaseen, B. Doyon, A. Lucas, K. Schalm '13

Motivation from holography

- In 1+1, holographic dual has a unique solution: A boosted BTZ black hole.
- In d+1, assume ctant. homogeneous heat flow as well.

also unique non-singular solution:
A boosted black brane.

$$\langle T^{\mu\nu}\rangle = a_d\, T^{d+1}\left(\eta^{\mu\nu} + (d+1)u^\mu u^\nu\right)$$



$\langle \vec{J}_E \rangle \neq 0$ even if systems asymptotically far apart.

Generalization to any d:

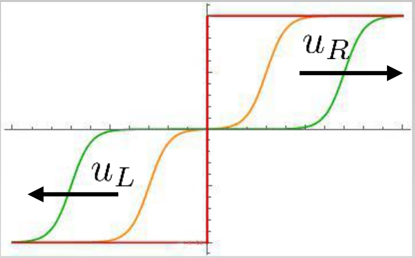
- Effective dimension reduction to 1+1.
- *Dissipation*: Energy can be exchanged among the various constituents.
- Linear response regime:

$$|T_L - T_R| << T_L + T_R$$

→ Hydro eqs. explicitly solvable.

❖ Heat transport dominated by diffusion instead of flow?

$$\begin{cases} \text{width} \propto \sqrt{t} \\ \text{distance} \propto t \end{cases}$$



Hydrodynamics

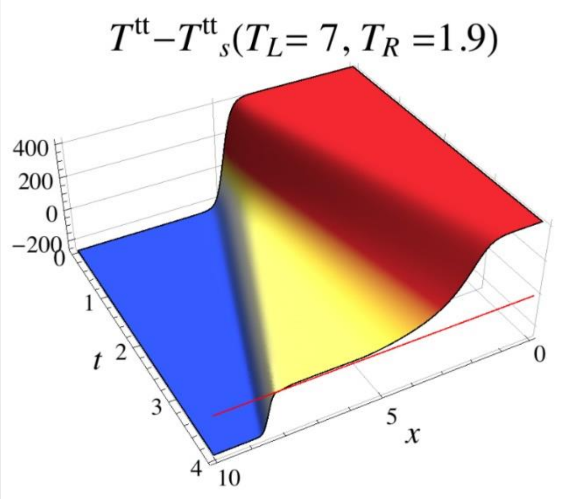
M.J. Bhaseen, B. Doyon, A. Lucas, K. Schalm '13

- 1+1 case prediction:

Lorentz-boosted energy density with $u^\mu = (\cosh\theta, \sinh\theta, \vec{0})$

and temperature given by

$$\left.\begin{aligned} T_L &= T\,e^{\theta} \\ T_R &= T\,e^{-\theta} \end{aligned}\right\} \quad \langle T^{tx}\rangle = \frac{c\pi}{12}(T_L^2 - T_R^2)$$



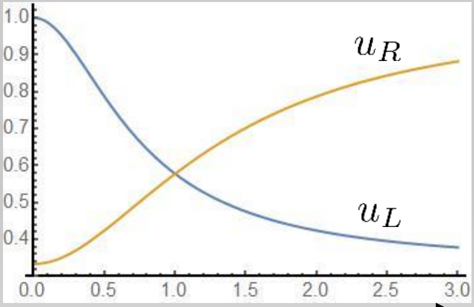
Numerical result from hydro

- Generic d prediction:

Final steady state characterized by

$$\langle T^{tx}\rangle = a_d \frac{T_L^{d+1} - T_R^{d+1}}{u_L + u_R} \quad \text{where} \quad a_d \sim \frac{L^d}{G_N}$$

and



T_L/T_R

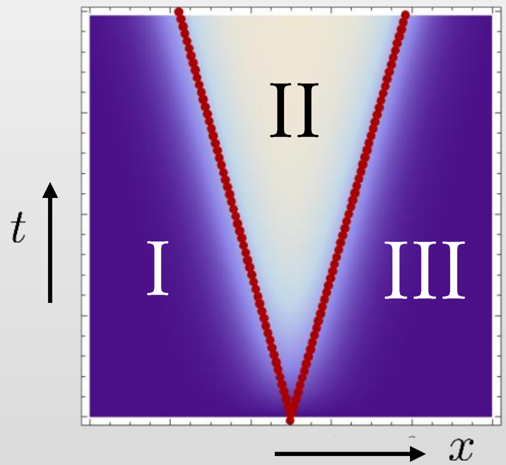
- Boosted thermal state:

$$\begin{cases} T = \sqrt{T_L T_R} & \text{Temperature} \\ \chi = \left(\dfrac{T_L}{T_R}\right)^{\frac{d+1}{2}} & \sim \text{Boost velocity} \end{cases}$$

- Shockwave velocities: $u_L u_R = \dfrac{1}{d} = c_s^2$

Step Ansatz Hydrodynamics

H.-C. Chang, A. Karch, A. Yarom '13



Ingredients:

- Configuration with 2 steps
- Conservation of energy & momentum

Dynamics encapsulated by constitutive rels. of hydro

Go to 2nd order in gradient expansion:

$$T^{\mu\nu} = P(d\, u^\mu u^\nu + \eta^{\mu\nu}) + \pi^{\mu\nu} + \mathcal{O}(\partial^2)$$

- Specialize to conformal case:

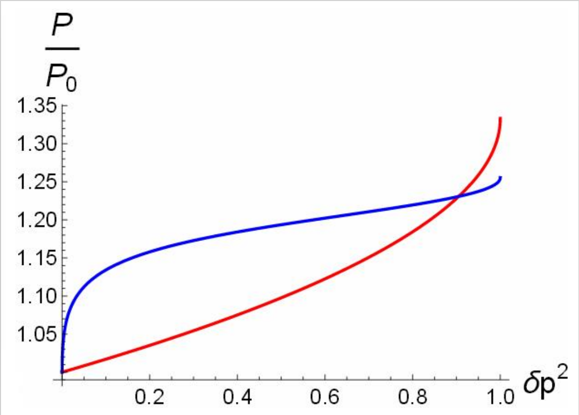
$$\epsilon(P) = (d-1)P$$

$\delta p \propto |P_L - P_R|$ determines shock velocities v_L, v_R

Two posible sets of Steady State configurations:
Thermodynamic branch, second branch

Hydrodynamical evolution of 3 regions

Match solutions → asymptotics of the central region?



- Numerical hydro solutions

Looking for sols. of the form $u^\mu = (\cosh[\theta(t,x)], \sinh[\theta(t,x)], \vec{0})$

Unknown functions: $\{T(t,x), \theta(t,x), \pi_{xx}(t,x)\}$

→ Late time prediction is matched

Gravitational Dual

I. Amado, A. Yarom '15
(see Yarom's talk tomorrow)

Energy transport: Lorentz-boosted thermal distribution

→ Gravity dual: Boosted black brane

$$ds^2 = \frac{L^2}{z^2}\left[\frac{dz^2}{f(z)} - f(z)\left(\cosh\theta\, dt - \sinh\theta\, dx\right)^2 + \left(\cosh\theta\, dx - \sinh\theta\, dt\right)^2 + dx_\perp^2\right]$$

Check via holography

Numerical GR

- Full out-of-equilibrium evolution of Einstein's equations.

- Ansatz for Characteristic Evolution (Eddington-Finkelstein coord.):

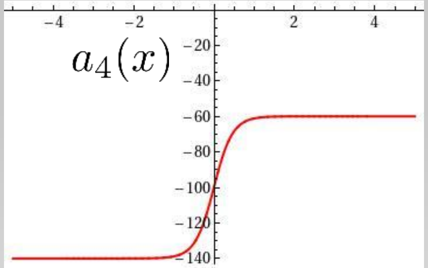
$$ds^2 = 2dt\left(dr - A\, dt - F\, dx\right) + \Sigma^2\left(e^{B}\, dx_\perp^2 + e^{-B}\, dx^2\right)$$

- Use gauge reparametrization to manage time evolution of the Apparent Horizon:

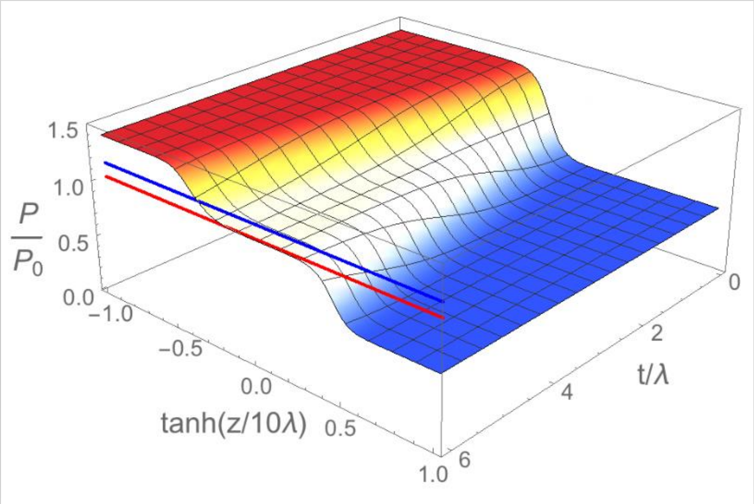
$$r \to r + \xi(t,x)$$

- Initial data:

$$B = 0,$$
$$f_4 = 0,$$
and



evolution

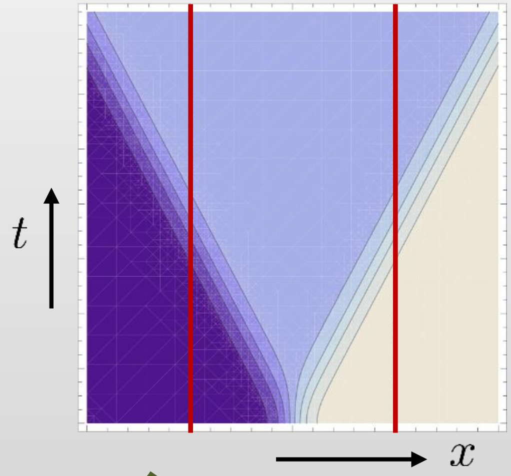


Information Flow

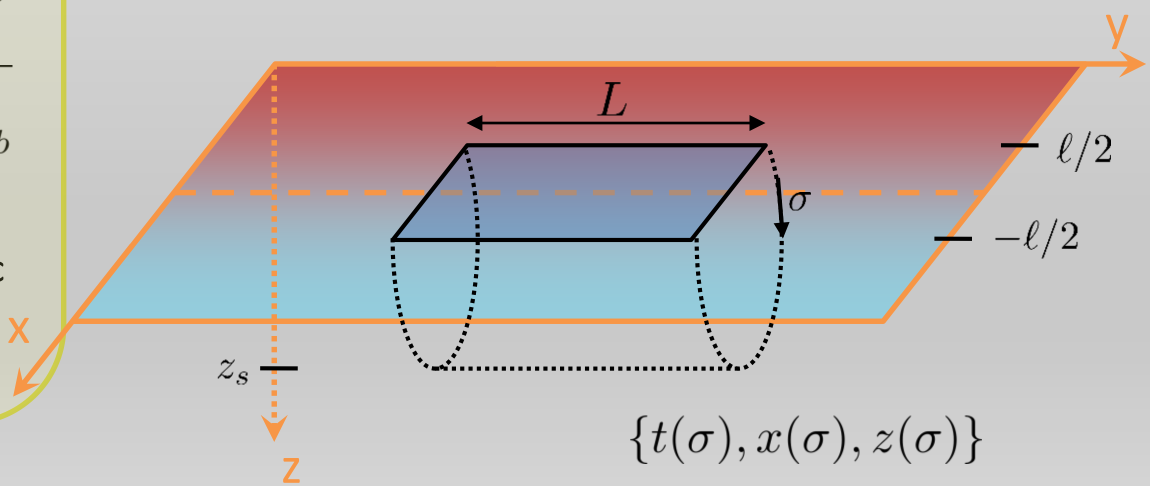
How does information get exchanged between the systems which are isolated at t=0?

- Entanglement entropy
  → Holographic measure: $$S_E = \frac{\mathrm{Area}(\gamma_A)}{4G_N^{d+2}}$$

- Regularization: Substract result at T=0: $$S_{EE}^{div} \sim \frac{1}{z_{uv}}$$

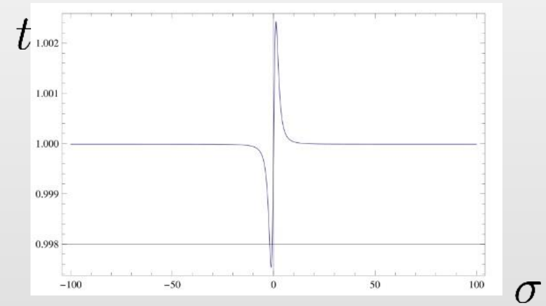


- Spatial induced metric on surface:
$$ds^2 = h_{\sigma\sigma} d\sigma^2 + g_{yy} dy^2$$
- Action:
$$\mathcal{I} = L \int ds \sqrt{\frac{\partial x^a}{\partial \sigma}\frac{\partial x^b}{\partial \sigma}\tilde{g}_{ab}}$$
- Geodesic eqs. for effective metric
$$\tilde{g}_{ab} = g_{yy} g_{ab}$$

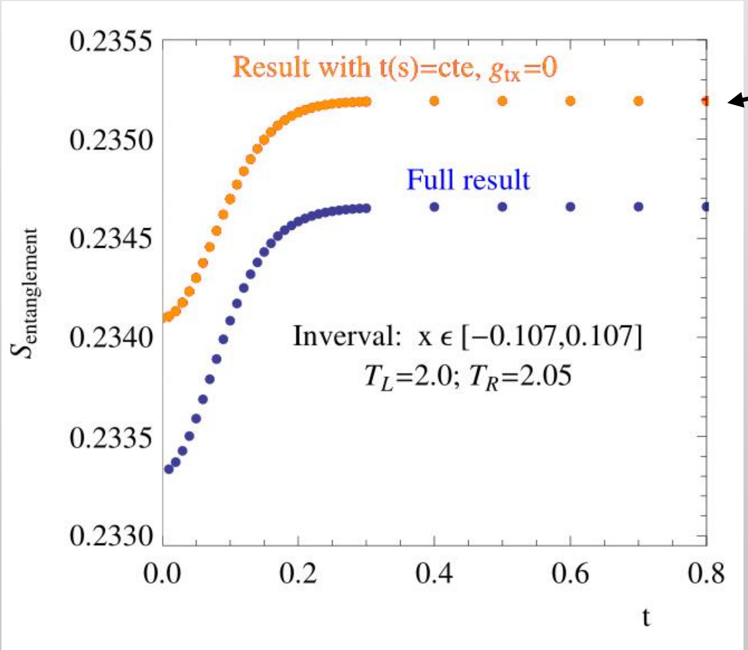


Entanglement Entropies

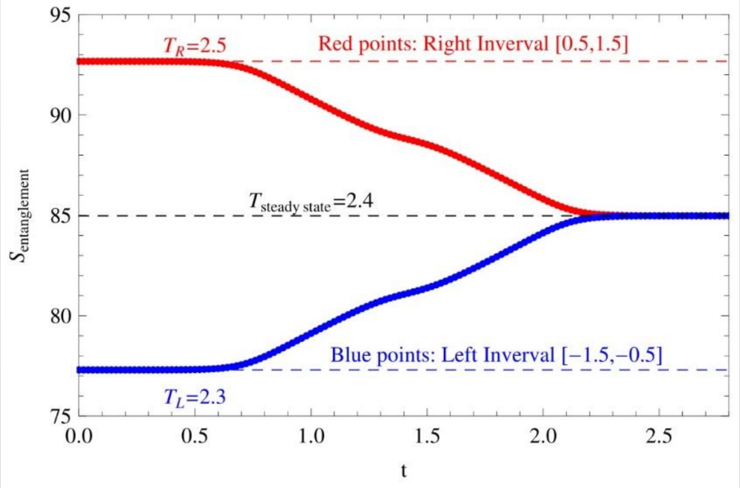
- Time dependent geodesics: $t(\sigma) \neq \mathrm{ctant.}$
  Notice deviation around the center due to $g_{tx} \neq 0$:

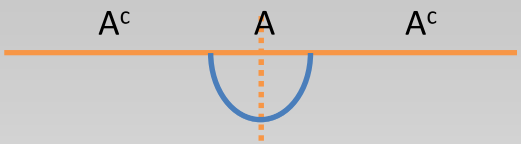


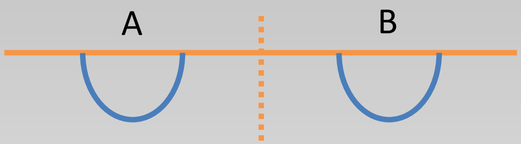
- Results for small temperature difference:



Upper bound







Mutual Information

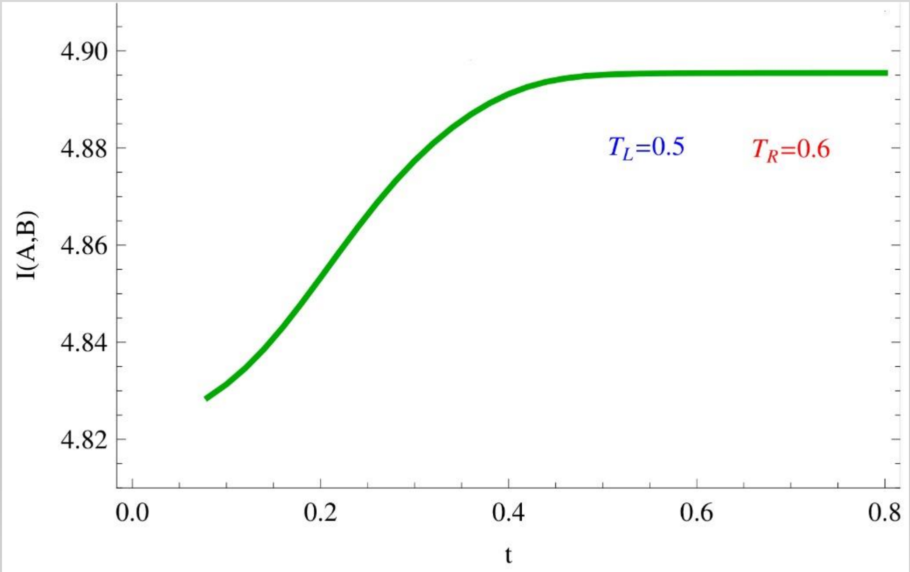
What do we learn about A by looking at B?

→ Mutual Information, definition: $I(A,B) = S(A) + S(B) - S(A+B)$

Holographic procedure:

Basic features:

- Shockwaves transport information.
- When does system 1 find out about the existence of system 2?
- M.I. grows as shocks pass through the region.
- Generically, left and right sides are asymmetric, since $u_L \neq u_R$



Outlook

- Complete analysis for full range: $0 < T_L/T_R < 1$
  a. *Qualitatively different for greater ΔT ?*
  b. *Any role for the 2^{nd} branch?*

- Higher dimensions
  a. *Generalize: 2+1 → 3+1, what changes?*

- Find an interpretation
  a. *Picture - The shockwaves transport information?*
  b. *Asymmetry – Intervals are not reached simultaneously*

Thank you!