Fermi gas for type $\hat{D}$ quiver theories and mirror symmetry

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hep-th 1505.xxxx B. A., N. Drukker, J. Felix.

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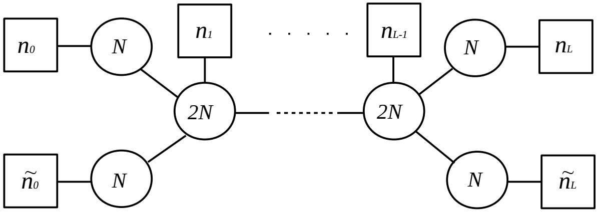
How far can we extend this approach ? Lower supersymmetry ? Yang-Mills IR fixed points ? Other gauge groups ? Interacting fermions ?

Outline

- $\hat{D}$ quivers and mirror dual theories,
- S^3 matrix model as a 1d free fermions partition function,
- Grand potential and holography.

$\hat{D}_{L+2}$ shaped quivers

Infrared fixed points of $d = 3$, $\mathcal{N} = 4$ quiver gauge theories,
$G = U(N)^2 \times U(2N)^{L-1} \times U(N)^2$, with (bi)fundamental hypermultiplets,



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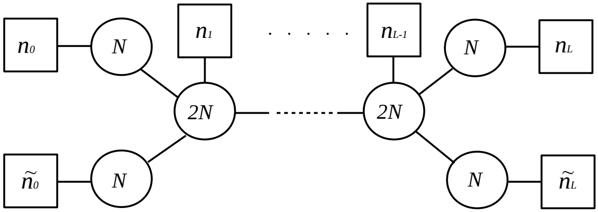

Type IIB brane realization with D3, D5, NS5-branes and orbifold 5-planes
Hanany, Zafaroni '99, Gaiotto, Witten '08

T-duality to type IIA and uplift to M-theory: $2N$ M2-branes on $\mathbb{C}^2/\mathbb{Z}_M \times \mathbb{C}^2/\mathbb{D}_L$.

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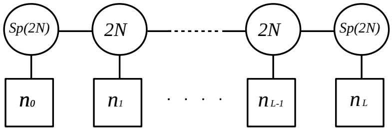

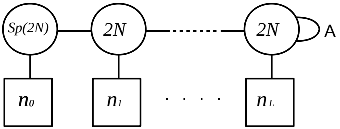

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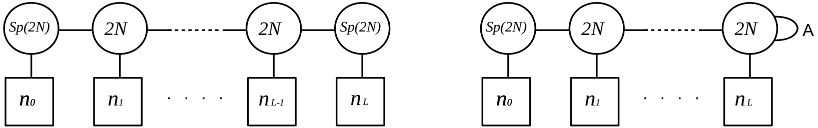

In the IIB brane realization, mirror symmetry = S duality.
NS5 and D5-branes are exchanged and orbifolds turn into $O5_0$ planes.
In M-theory: Exchange of the M-theory and T-duality circles inside $\mathbb{C}^2/\mathbb{Z}_M \times \mathbb{C}^2/\mathbb{D}_L$.

S^3 partition function

The S^3 partition function of $\mathcal{N} \geqslant 2$ theories can be computed exactly using the technique of supersymmetric localization. It reduces to an integral over constant scalars in the Cartan subalgebra (matrix model)
Kapustin, Willett, Yaakov '09

$$Z_{S^3} = \frac{1}{|\mathcal{W}|} \int_{\mathrm{Cartan}} d\lambda \;\; Z_{\mathsf{cl}}(\lambda) \cdot Z_{\mathsf{vector}}^{1-\mathrm{loop}}(\lambda) \cdot Z_{\mathsf{chiral}}^{1-\mathrm{loop}}(\lambda) \,.$$

The classical contribution depends on Chern-Simons and FI parameters. For instance the matrix model of the ABJM theory is given by

$$Z_{\mathrm{ABJM}} = \int \frac{d^N\lambda \, d^N\widetilde{\lambda}}{2^{2N} N!^2} \frac{\prod_{i<j} \sinh^2[\pi(\lambda_i - \lambda_j)] \sinh^2[\pi(\widetilde{\lambda}_i - \widetilde{\lambda}_j)]}{\prod_{i,j} \cosh^2[\pi(\lambda_i - \widetilde{\lambda}_j)]} \, e^{\pi i k \sum_i \lambda_i^2 - \widetilde{\lambda}_i^2}$$

The matrix model of $\mathcal{N} \geqslant 3$ Chern-Simons circular quiver theories with $U(N)$ nodes ($\hat{A}$ shaped quivers) can be written as the partition function of a gas of non-interacting fermions on a line Marino, Putrov '10

$$Z_{\hat{A}}(N) = \frac{1}{N!} \sum_{\sigma \in S_N} (-1)^\sigma \int d^N\lambda \prod_{i=1}^{N} \langle \lambda_i | \, \hat{\rho} \, | \lambda_{\sigma(i)} \rangle \, .$$

$\hat{\rho} = e^{-\hat{H}}$ is the density operator of the theory. $\{\lambda_i\}$ are the positions of the fermions.

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For the ABJM theory,

$$\hat{\rho}_{\mathrm{ABJM}} = \frac{1}{4\cosh(\pi \hat{p})\cosh(\pi \hat{q})}$$

with $\hat{q}, \hat{p}$ the position and momentum operators satisfying $[\hat{q}, \hat{p}] = \frac{i\,k}{2\pi}$ $(\hbar = \frac{k}{2\pi})$.

For $\mathcal{N} = 4$ $\hat{D}$ quivers and their mirror-dual theories, we find that Z_{S^3} can be recast as the partition function of **non-interacting fermions on a half-line** with Neumann (or Dirichlet) boundary condition at the origin.

$$Z_{\hat{D}}(N) = \frac{1}{N!} \sum_{\sigma \in S^N} (-1)^\sigma \int \prod_{i=1}^{N} d\lambda_i \prod_{i=1}^{N} \langle \lambda_i | \, \hat{\rho} \left(\frac{1 + \hat{R}}{2} \right) | \lambda_{\sigma(i)} \rangle \, ,$$

with $\hat{R} \, |\lambda\rangle = |-\lambda\rangle$. Mezei, Pufu '13

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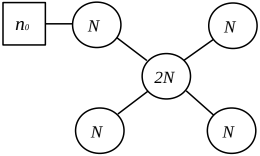
For instance,

$$\hat{\rho} = \frac{1}{2}\left(\frac{1}{\mathrm{ch}^{n_0}\hat{q}}\frac{\mathrm{sh}\hat{p}}{\mathrm{ch}\hat{p}} + \frac{\mathrm{sh}\hat{p}}{\mathrm{ch}\hat{p}}\frac{1}{\mathrm{ch}^{n_0}\hat{q}}\right)\frac{\mathrm{sh}\hat{p}}{\mathrm{ch}^5\hat{p}}\,,$$



with $[\hat{q},\hat{p}] = \frac{i}{2\pi}$ and $\mathrm{sh}(x) = 2\sinh(\pi x)$, $\mathrm{ch}(x) = 2\cosh(\pi x)$.

Mirror symmetry

Mirror symmetry is realized by a simple canonical transformation

$$\hat{p} \rightarrow \hat{q} \,, \quad \hat{q} \rightarrow -\hat{p} \,,$$

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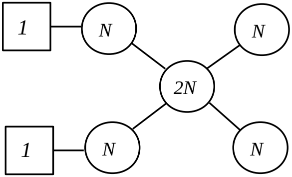

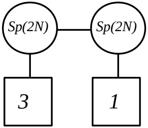

$$\rho = \frac{\mathrm{sh}\hat{p}}{\mathrm{ch}^5\hat{p}} \frac{1}{\mathrm{ch}\hat{q}} \frac{\mathrm{sh}\hat{p}}{\mathrm{ch}\hat{p}} \frac{1}{\mathrm{ch}\hat{q}} \,, \qquad \tilde{\rho} = \frac{\mathrm{sh}\hat{q}}{\mathrm{ch}^5\hat{q}} \frac{1}{\mathrm{ch}\hat{p}} \frac{\mathrm{sh}\hat{q}}{\mathrm{ch}\hat{q}} \frac{1}{\mathrm{ch}\hat{p}} \,.$$

This holds in the presence of mass and FI deformation parameters, which get exchanged under mirror symmetry.

Grand potential

Quantum/statistical mechanics techniques are suitable for computing the grand potential $J(\mu)$, instead of $Z(N)$.

$$\Xi(\mu) = 1 + \sum_{N=1}^{\infty} Z(N) e^{\mu N} \equiv e^{J(\mu)}$$

$$J(\mu) = -\sum_{l=1}^{\infty} \frac{1}{2} \frac{(-1)^l Z_l e^{\mu l}}{l}, \quad Z_l = \mathrm{Tr}\, \hat{\rho}^l .$$

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The large μ behaviour of J is related to the large N behaviour of $Z(N)$. We are interested in the perturbative part of J at large μ,

$$J(\mu) = J_{\mathrm{pert}}(\mu) + \mathcal{O}(e^{-\alpha\mu}), \quad \alpha > 0 .$$

The strategy to compute $Z_l = \mathrm{Tr}\,\hat{\rho}^l$ is to go to *phase space* using the Wigner transformation,

$$\hat{A} \;\rightarrow\; A_W(p,q) \;=\; \int dq' \left\langle q - \frac{q'}{2} \right| \hat{A} \left| q + \frac{q'}{2} \right\rangle e^{2\pi i p q'} \,,$$

$$(\hat{A}\hat{B})_W \;=\; A_W \star B_W \,, \qquad \star \;=\; \exp\left[\frac{i}{4\pi} \left(\overleftarrow{\partial}_q \overrightarrow{\partial}_p - \overrightarrow{\partial}_q \overleftarrow{\partial}_p \right) \right] ,$$

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This leads to

$$Z_l = \int dpdq (\hat{\rho}^l)_W \;=\; \int dpdq\, \overbrace{\rho_W \star \rho_W \cdots \star \rho_W}^{l} \,.$$

Z_l can be computed in a derivative expansion of the $\star$ products.

We find at order four in derivatives the generic form

$$J_{\mathrm{pert}}(\mu) = \frac{C}{3}\mu^3 + B\mu + A\,,$$

where A, B, C are constants given in terms of the quiver data.

It is expected that the contributions to C and B in the derivative expansion end at order four, so that we obtain the **exact coefficients** C **and** B. The perturbative part of $Z(N)$ can be extracted from $J_{\mathrm{pert}}(\mu)$,

$$Z(N) = C^{-\frac{1}{3}} e^{A}\,\mathrm{Ai}\left[C^{-\frac{1}{3}}(N-B)\right] + \mathcal{O}\left(e^{-\sqrt{N}}\right)\,,$$

with Ai denoting the Airy function. At large N,

$$-\log Z(N) \; = \; \frac{2}{3\sqrt{C}}N^{\frac{3}{2}} - \frac{B}{\sqrt{C}}N^{\frac{1}{2}} + \cdots\,.$$

Holography

- The holographic map for $\hat{D}_{L+2}$ quivers with a full 10d or 11d backgrounds is not known in general. The M-theory geometry is known: $AdS_4 \times S^7/(\mathbb{Z}_M \times \mathbb{D}_L)$. Dey '12

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- M-theory backgrounds $AdS_4 \times SE_7$ (Sasaki-Einstein manifold) admit consistent truncations to 4d gauged (conformal) supergravity on AdS_4.
  The exact partition function on AdS_4 Drukker, Dabholkar, Gomes '14 reproduces the Airy function for ABJM. This is the full non-perturbative supergravity result (finite N). It does not capture the worlsheet and membrane instanton corrections.

Holography

- General result for the C coefficient Hertog et al '11,

$$C = \frac{6}{\pi^6}\mathrm{Vol}(SE_7) \; = \; \frac{1}{4\pi\, M\, L}\,.$$

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- In principle worldsheet and membrane instanton corrections can be computed from the matrix models. Marino, Moriyama, ... '11, '14

Main ideas

- IR fixed points of $\mathcal{N} = 4$ YM type $\hat{D}$ quivers
  $\rightarrow$ non-interacting fermions on a half-line.
- Mirror symmetry is simply realized on the QM density operator.
- Powerful approach to test AdS/CFT.
- It suggests that a fermion formalism exists for all SCFTs realized on M2-branes in some geometry.