Scale vs. Conformal Invariance in Holography with Higher Derivative Corrections

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Firenze, 15 aprile 2015

References

Based on work in progress with Kostas Skenderis

Related work includes

[Polchinski (1988); Osborn (1991); Dorigoni, Rychkov (2009); de Boer, Kulaxizi, Parnachev (2009); Luty, Polchinski, Rattazzi (2012); Dymarsky, Komargodski, Schwimmer, Theisen (2013); Nakayama (2011, 2013); Bzowski, Skenderis (2014); Dymarsky, Farnsworth, Komargodski, Luty, Prilepina (2014)...]

Motivation

- There are real-world critical phenomena about which we do not know if the corresponding fixed point is conformally or "only" scale invariant
- Do CFTs exhaust all possible second order phase transitions?

Outline

- Field Theory Formulation
- Holographic Approach

Dilatation and Virial currents

In a CFT the conserved current associated to a conformal Killing vector ξ^μ is given by $j^\mu = T^\mu_\nu \xi^\nu$. Dilatation current

$$j^\mu = T^\mu_\nu x^\nu.$$

In a scale invariant theory the stress-energy tensor is not traceless. The dilatation current is

$$j^\mu = T^\mu_\nu x^\nu + V^\mu,$$

with

$$0 = \partial_\mu j^\mu = T^\mu_\mu + \partial_\mu V^\mu.$$

The virial current V^μ is not conserved.

Scale vs. Conformal

Criterium

If

$$V^\mu = \partial^\mu L,$$

where L is a local operator, then the stress-energy tensor can be improved and made traceless. Thus in this case the theory is conformal.

Trace anomaly as a signature

Scaling anomaly

In 4d scale invariant theory allows more general anomaly when coupled to the background metric:

$$T^{\mu}_{\mu} = aE_4 - c\,Weyl^2 + eR^2.$$

Presence of the R^2 term in the trace anomaly is a clear signal of a scale but not conformally invariant field theory.

Current state

Under the assumptions of locality and unitarity:

- In 2D scale invariance implies conformal invariance ([Polchinski (1988)).
- In 4D there are strong arguments that scale invariance implies conformal invariance[Dymarsky, Komargodski, Schwimmer, Theisen (2013)]
- In higher even dimensions or in odd dimensions - answer unknown

Holography

The natural questions are:

- Can we use holography to prove "scale $=>$ conformal"?
- Can we construct examples of scale but not conformally invariant theories?
- What is the holographic dual of the virial current?

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Einstein-Hilbert Gravity

- R^2 anomaly in Einstein-Hilbert gravity?
- NO! Analysis by [Henningson, Skenderis (1998)] demonstrated that no R^2 anomaly may appear if the gravitational sector is described by Einstein-Hilbert gravity.

Higher derivative corrections

Generic gravitational theories with the higher derivative corrections are believed to be dual to non-unitatry field theories. There are known examples of non-unitary scale invariant theories [Riva, Cardy (2005); El-Showk, Nakayama, Rychkov (2011)]. Can one construct a holographic example of non-unitary scale invariant theory using R^2 type corrections?

The Model

$$S = \frac{1}{16\pi G_{d+1}} \int d^{d+1}x \sqrt{-G} \Big[R + \frac{d(d-1)}{L^2} + L^2 (\lambda_1 R_{abcd} R^{abcd} + \lambda_2 R_{ab} R^{ab} + \lambda_3 R^2) \Big].$$

Our goal here is to perform the Fefferman-Graham type analysis of this theory

Variational problem

As clarified in [Skenderis, Taylor, van Rees (2009)] and emphasized in [Smolic, Taylor (2013)] the higher derivative terms in the action generically lead to the new degrees of freedom. These have to be taken care of when setting up the variational problem. Imposing $\delta g = 0$ at the boundary of AdS is not enough to set the variational problem. Other sources have to be fixed as well.

Presence of an AdS vacuum

Looking for a solution of the form

$$ds^2 = dr^2 + e^{2r/l}\eta_{ij},$$

we find a fourth order characteristic equation for the radius of AdS l. The solution is

$$\frac{4}{l^2} = \frac{1 \pm \sqrt{1 - 64\lambda}}{8\lambda L^2}$$

with

$$\lambda = \frac{d-3}{8(d-1)}\Big(\lambda_1 + \frac{d}{2}\lambda_2 + \frac{d(d+1)}{2}\lambda_3\Big).$$

The smaller root is continuously connected to the pure AdS solution of Einstein's gravity. A priori the other mode may lead to nonunitarity and possibly to a scale (but not conformal) anomaly.

Fefferman-Graham Expansion

We parametrize the metric as

$$ds^2 = dr^2 + e^{2r/l} g_{ij}(x,r) dx^i dx^j,$$
$$g_{ij}(x,r) = g_{(0)ij}(x,r) + e^{-2r/l} g_{(2)ij}(x,r) + e^{-4r/l} g_{(4)ij}(x,r) + \dots$$

Some of the results

At the next to leading order we find

$$g(\lambda_i)(l^2 R + 2(d-1)\mathrm{tr}(g_{(2)})) = 0,$$

If

$$g(\lambda_i) = 0 \implies \text{Degeneracy.}$$

In non-degenerate cases we get

$$g_{(2)ij} = \frac{1}{d-2}\Big(\frac{R_{(0)}}{2(d-1)} g_{(0)ij} - R_{(0)ij},\Big)$$

which is the same as in the Einstein-Hilbert gravity!

Some of the results

For the $\mathrm{tr}(g_{(4)})$ we obtain the equation (in the non-degenerate case)

$$f(\lambda_i)\Big[\mathrm{tr}(g_{(2)}^2) - 4\mathrm{tr}(g_{(4)})\Big] - \frac{l^2 L^2}{d-1}\lambda_1 \, Weyl_{(0)}^2 = 0$$

This corresponds to a shift in the c coefficient of the (conformal) trace anomaly:

$$< T_\mu{}^\mu >= aE_4 - c\,Weyl^2, \qquad a \neq c.$$

Interestingly only the $Riem^2$ term in the action contributes to this shift [see also Nojiri, Odintsov (1999); Blau, Gava, Narain (1999); Schwimmer, Theisen (2003)].

Degenerate cases / Chern-Simons gravity

Chern-Simons gravity in 5D is a special Lovelock type gravity ($-4\lambda_1 = -4\lambda_3 = \lambda_2 = \lambda_\star$) such that the two AdS vacua degenerate ($\lambda = 1/64$).

The coefficients

$$g_{(2)ij}, \qquad \mathrm{tr}(g_{(4)}), \qquad \nabla^j g_{(4)ij}$$

are not determined by the near-boundary analysis!

Degenerate cases / Chern-Simons gravity

What is the holographic interpretation of this phenomenon?

- There are solutions in CS gravity involving arbitrary undetermined functions [J.T. Wheeler (1986); Charmousis, J.-F. Dufaux (2002); ...]
- There are no standard perturbative expansion around general solutions
- From the Hamiltonian point of view CS gravity is a 'degenerate' and 'irregular' system.

Conclusions and Open Questions

- The asymptotic structure of solutions in gravity with higher curvature corrections is very rich
- In the non-degenerate cases only conformal field theories are realized.

- Degenerate cases represent a challenge on its own
- Variational problem in the presence of higher curvature corrections?
- What is the holographic dual of a virial current?