Transport in holographic systems with momentum dissipation

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Outline

- Outline of the talk:

  1). Introduction: transport properties of strange metals

  2). Momentum dissipation: fast and slow

  3). The simplest holographic example

  4). Conclusions

Introduction: the strange metals

- In experimental condensed matter physics, there are many materials with unusual and unexplained properties.

- The 'strange metal' phases of high-Tc superconductors are metallic states of matter with very odd transport properties (conductivities).

  e.g. the electrical resistivity $\rho_{DC} \propto T$

- Holography gives us easy theoretical access to qualitatively different kinds of states than those traditionally considered: strongly interacting states with no quasiparticles. How are energy and charge transported in these kinds of state?

Introduction: holography

- However, there are fundamental differences in the low energy physics of the strange metals, compared to the common strongly interacting field theory states with holographic duals.

- Most states studied holographically have translational symmetry: the total momentum of the dual field theory state is conserved.

- This symmetry has a big effect on the conductivities of the state: they are typically infinite, unlike in real strange metals.

- To learn anything about them from holography, we have to study examples where momentum is not conserved (e.g. where translational symmetry is broken).

Transport without quasiparticles

- The conductivity matrix $\begin{pmatrix} \vec{J} \\ \vec{Q} \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} \vec{E} \\ -\vec{\nabla} T / T \end{pmatrix}.$ controls the response of the system to small temperature gradients and electric fields.

- Its low energy properties are controlled by the long-lived excitations that arise from approximate conservation laws.

- In a strongly interacting system with no quasiparticles, we expect no long-lived excitations except those protected by approximate symmetries e.g. translational symmetry.

- There are two qualitatively different situations: when $\vec{P}$ is approximately conserved, and when it is totally unconserved.

Slow momentum dissipation

- When translational symmetry is broken weakly, momentum lives longer than everything else, and controls the low energy transport.

- In this case, the DC conductivities can be calculated perturbatively, and are all proportional to the momentum dissipation rate Γ :

$$\delta H = V \int dx e^{ik_L x} \mathcal{O}(x) \longrightarrow \Gamma = \frac{V^2 k_L^2}{\chi_{\vec{P}\vec{P}}} \lim_{\omega \to 0} \left. \frac{\mathrm{Im} G_{\mathcal{O}\mathcal{O}}(\omega, k_L)}{\omega} \right|_{V \to 0}$$

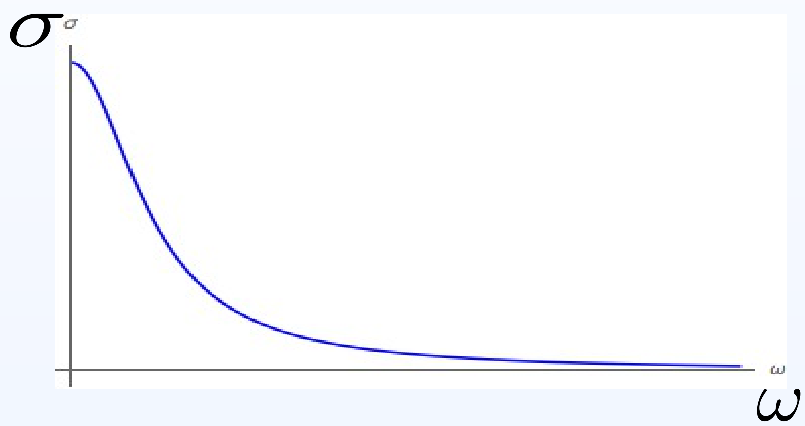
Hartnoll & Hofman (2012)

- At leading order, Γ depends only on properties of the translationally invariant state.

- Fundamentally, to understand the transport, one just has to understand the translationally invariant state.

Slow momentum dissipation II

- In this limit, the AC conductivities have a simple "Drude-like" form



$$\sigma = \frac{\sigma_{DC}}{1 - i\omega/\Gamma}$$

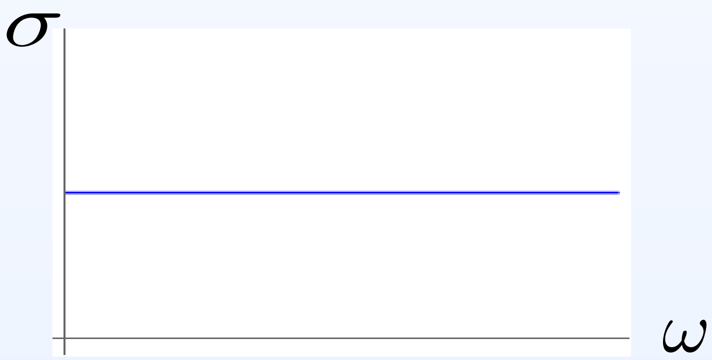
$$\sigma_{DC} \propto \Gamma^{-1}$$

- The conductivities are controlled by Γ and the thermodynamic properties of the state.

Fast momentum dissipation

- In the opposite limit, where translational symmetry is very strongly broken, momentum dissipates very quickly.

- There are no parametrically long-lived excitations in the theory, and the low energy conductivity has no peaks.



$$\sigma(\omega) = \sigma_{DC}$$

- There is no 'extrinsic' scale Γ in this low energy theory: the DC conductivities are intrinsic properties of the state.

Fast momentum dissipation II

- Unlike before, this is not a small perturbation around a translationally invariant state.
- To understand the state's transport properties, we need to understand the nature of the states that form when translational symmetry is strongly broken.
- e.g. a conjecture: the diffusion constants of a state like this are bounded from below:

$$DT \gtrsim v_F^2 \frac{\hbar}{k_B}$$

Hartnoll (2014)

- These two possibilities are at the ends of a spectrum: there is a smooth crossover between them as momentum dissipates faster.

Momentum dissipation in holography

- It is very difficult to theoretically access the transport properties of strongly interacting systems. This is where holography is very useful.
- Transport properties of systems like this can be determined from the perturbations of black hole solutions of the dual classical gravity.
- In reality, translational symmetry is broken in a complicated manner by periodic lattices and random distributions of impurities. This is difficult to do in holography.
- There are some simple toy models, which capture the essential physics. These break translational symmetry in a simple way such that they retain a homogeneous metric.

Holography: toy models

- One general class of these systems are solutions of the action

$$S = \int d^D x \sqrt{-g} \left[\mathcal{R} - \frac{Z(\phi)}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) - \frac{Y(\phi)}{2} \sum_{i=1}^{D-2} \partial_\mu \varphi_i \partial^\mu \varphi_i \right]$$

Bardoux, Caldarelli, Charmousis (2012), Andrade & Withers (2013), Gouteraux (2014), Donos & Gauntlett (2014),...

- This is the usual Einstein-Maxwell-Dilaton action, plus massless scalar fields that break translational symmetry $\varphi_i(r, t, x^i) = m x^i$

- This is (purportedly) dual to a strongly interacting field theory state with sources for the scalar operators dual to the fields φ_i .

- The finite DC conductivities of these states can be calculated exactly from the near-horizon black hole solution.

Blake & Tong (2013), Donos & Gauntlett (2014)

The simplest holographic example

- I am going to talk about the simplest example:

$$S = \int d^4x\sqrt{-g}\left(\mathcal{R} + 6 - \frac{1}{2}\sum_{i=1}^{2}\partial_\mu\varphi_i\partial^\mu\varphi_i\right)$$

RD & Gouteraux (2014)

- This action has the solution

$$ds^2 = -r^2 f(r)dt^2 + r^2\left(dx^2 + dy^2\right) + \frac{dr^2}{r^2 f(r)} \qquad \varphi_i = m x^i$$

$$f(r) = 1 - \frac{m^2}{2r^2} - \frac{r_0^3}{r^3}\left(1 - \frac{m^2}{2r_0^2}\right)$$

- This is a neutral state, so the interesting conductivity is the heat conductivity (energy conductivity): $\kappa_{DC} = \frac{4\pi sT}{m^2}$

- This very simple formula hides a variety of physical phenomena.

The simplest holographic case: zero m

- The simplest case is when there is translational symmetry: m=0.
- At low energies and long distances, the strongly interacting field theory obeys the laws of conformal, relativistic hydrodynamics. These laws tell us how energy/heat is transported.

Policastro, Son, Starinets, Herzog, …..

- The equations of motion are simply the conservation of energy and momentum.
- As momentum is conserved, the DC heat conductivity is infinite

$$\kappa(\omega) = s\left(\delta(\omega) + \frac{i}{\omega}\right)$$

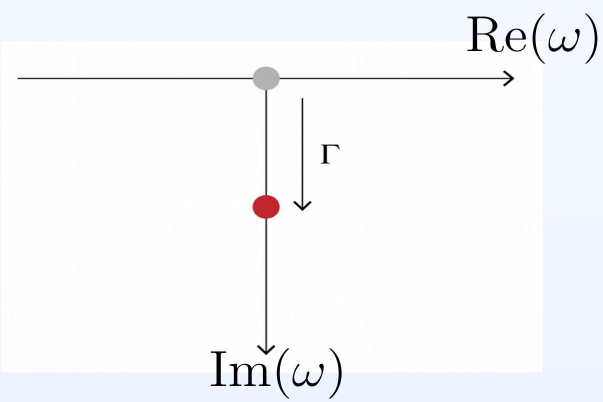
- The low energy excitations which carry heat are sound waves.

The simplest holographic case: small m

- At leading order in m, the effect of the translational symmetry breaking is captured by changing the momentum conservation equation to $$\partial_t P_i + \partial_j \Pi_{ij} = -\Gamma P_i$$

- This modified hydrodynamics has a Drude-like heat conductivity, as expected, with $\kappa_{DC} = s/\Gamma$.

- The pole at $\omega = 0$ has moved to $\omega = -i\Gamma$



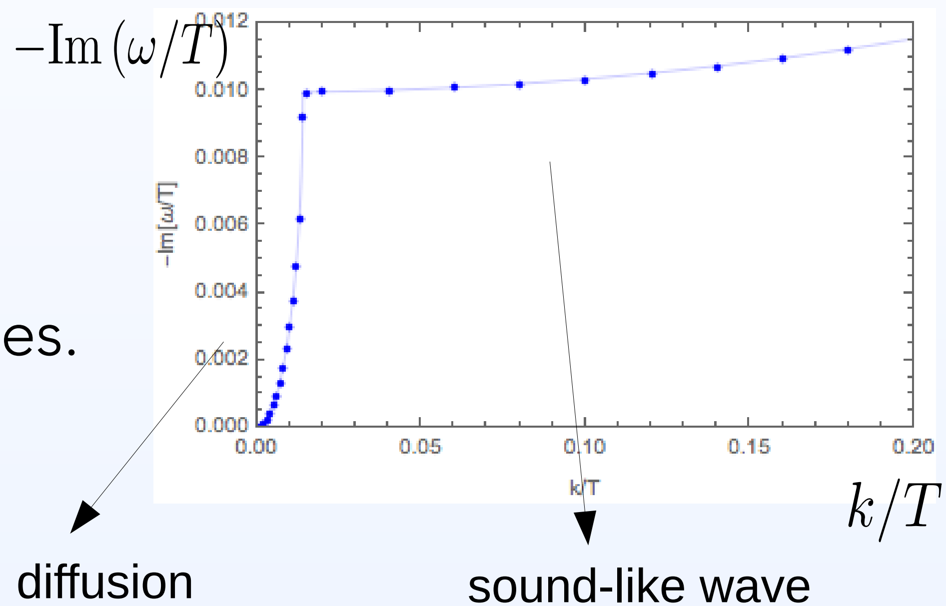
- From the gravitational theory, the momentum dissipation rate is

$$\Gamma = \frac{m^2}{4\pi T} + O(m^4)$$

RD (2013)

The simplest holographic case: small m

- We can also look at the spatially resolved heat conductivity:
- Over short distances, heat is carried by sound-like waves.
- Over long distances, heat diffuses.

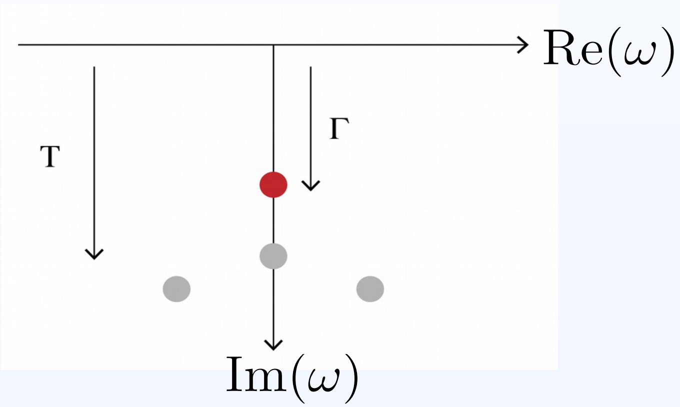


- The low energy, long distance transport properties of the state in this limit can be understood in terms of a simple, non-holographic effective theory.

The simplest holographic case: large m

- When m~T, there is a qualitative change in the conductivity, even though the DC value does not change.

  The momentum dissipation rate is comparable to the intrinsic dissipation rate of the system.



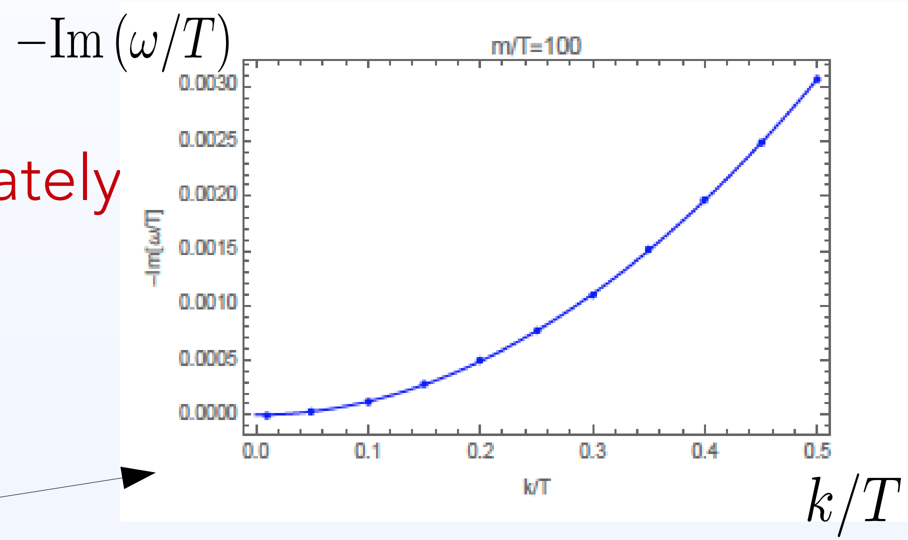
- At very large m/T, momentum is not approximately conserved and we remove it from our effective theory. Only energy is conserved:

$$\partial_t \epsilon + \partial_i P_i = 0 \qquad\qquad P_i = -\kappa \partial_i T + \dots$$

- This is a simple example of a strongly interacting state, with no quasiparticles, in which momentum dissipates quickly.

Large m: diffusion constant

- For m>>T, there is agreement with this effective theory.
- The low frequency heat conductivity is approximately constant.



- Heat diffuses on all distance scales.
- We can test the conjectured bound on the diffusion constant:

$$DT = \frac{T^2}{m^2}\sqrt{4\pi^2 + \frac{3m^2}{2T^2}} \to \sqrt{\frac{3}{2}}\frac{T}{m} \to 0$$

- DT can be made arbitrarily small by increasing m/T.

Is this solution stable?

- When m=0, the solution is planar Schwarzschild-AdS4.

- At non-zero m/T, we wrote down the simplest solution. Is it stable?

- We can check its stability to linear perturbations at various points in parameter space:

  – When T=0, the near-horizon geometry is AdS2xR^2. All perturbations satisfy the AdS2 BF bound for all (m,k).

  – When $m = \sqrt{8}\pi T$, there is a major symmetry enhancement such that all quasinormal modes can be analytically computed exactly. They are all stable.

The self-dual point

- At this special value of m, there is a reduction in the number of independent equations for perturbations of the metric + scalars:

$$G^R_{P^x P^x}(\omega, k)\, G^R_{P^y P^y}(\omega, k) = -\omega^2 \left(k^2 + 8\pi^2 T^2\right)^2$$

- Due to this, the AC heat conductivity is exactly constant:

$$\kappa(\omega) = 8\pi^2 T$$

- This is a gravitational analogue of electric/magnetic self-duality.

- In the field theory, it is the heat transport version of the very simple charge transport properties of certain zero density states.

Herzog, Kovtun, Sachdev, Son (2007)

Summary and conclusions

- The transport properties are qualitatively different between systems where momentum is almost conserved and where it is not.
- There are very simple toy models in holography of both cases.
- These can be understood via simple, non-holographic effective theories.

Further work:

- Does a similar approach work for charged systems?
- Are there any common properties of holographic systems with fast momentum dissipation?