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AdS_5 Solutions of Massive Type IIA Supergravity

based on arXiv:1502.06620 [hep-th] and arXiv:1502.06616 [hep-th]
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Gauge/Gravity Duality 2015

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Introduction: AdS_5 solutions in String/M-theory

A prominent instance of the gauge/gravity duality is the AdS_5/CFT_4 correspondence
Motive for the systematic study of AdS_5 backgrounds in String/M-theory

Type IIB

- Freund-Rubin backgrounds
  - ◇ $AdS_5 \times S^5$
  - ◇ $AdS_5 \times$ SE_5 ($T^{1,1}$, $Y^{p,q}$, $L^{a,b,c}$)
- Beyond Freund-Rubin
  - ◇ Pilch-Warner solution [Pilch, Warner '00]
  - ◇ analysis of general $\mathcal{N} = 1$ AdS_5 backgrounds* [Gauntlett, Martelli, Sparks, Waldram '05]

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*4 Q-supercharges

Introduction: AdS_5 solutions in String/M-theory

M-theory

- general $\mathcal{N} = 1$ AdS_5 backgrounds [Gauntlett, Martelli, Sparks, Waldram '04]
- general $\mathcal{N} = 2$ AdS_5 backgrounds [Lin, Lunin, Maldacena '04]

An interesting class arises from wrapping M5-branes on Riemann surfaces
[Maldacena, Núñez '00] [Gaiotto, Maldacena '09] [Bah, Beem, Bobev, Wecht '12] [Bah '13,'15]

Type IIA supergravity accessible via M-theory, but what about adding Romans mass?

Massive type IIA supergravity might hold surprises as shown recently by the discovery of a new AdS_7 solutions [Apruzzi, Fazzi, Rosa, Tomasiello '13]

Supersymmetric AdS_5 solutions

We want to perform a general analysis of supesymmetric $\mathrm{AdS}_5 \times M_5$ backgrounds of massive type IIA supergravity

geometry warped product of AdS_5 and a Riemannian manifold M_5

$$ds^2_{10} = e^{2A} ds^2_{\mathrm{AdS}_5} + ds^2_{M_5}$$

$\mathrm{SO}(2,4)$ symmetry

- warp factor A and dilaton ϕ functions on M_5
- fluxes H, F_0, F_2, F_4 forms on M_5

supersymmetry $\exists$ Spin$(1,9)$ Majorana spinor ϵ such that the gravitino and dilatino supersymmetry variations

$$\delta_\epsilon \psi = \delta_\epsilon \lambda = 0$$

Supersymmetric AdS_5 solutions

Existence of supersymmetry reduces the structure group

$$\mathrm{Spin}(5) \to G$$

where G is the stabilizer group of the supersymmetry parameter(s)

$\delta_\epsilon \psi = \delta_\epsilon \lambda = 0 \to$ system of differential constraints on the G-structure

We consider AdS_5 as a warped product of Mink_4 and $\mathbb{R}$

$$ds^2_{10} = e^{2A}\left(\frac{dr^2}{r^2} + r^2 ds^2_{\mathrm{Mink}_4}\right) + ds^2_{M_5}$$

and use the system of [Grãna, Minasian, Petrini, Tomasiello '05] for supersymmetric $\mathrm{Mink}_4 \times M_6$ backgrounds.

Supersymmetric AdS$_5$ solutions

We obtain a necessary and sufficient system of equations*

$$
\begin{aligned}
d_H\big(e^{3A-\phi}\mathrm{Re}\psi^1_+\big) + 2e^{2A-\phi}\mathrm{Im}\psi^1_- &= 0 \\
d_H\big(e^{4A-\phi}\psi^2_-\big) - 3ie^{3A-\phi}\psi^2_+ &= 0 \\
d_H\big(e^{4A-\phi}\mathrm{Re}\psi^1_-\big) &= 0 \\
d_H\big(e^{5A-\phi}\mathrm{Im}\psi^1_+\big) - 4e^{4A-\phi}\mathrm{Re}\psi^1_- &= \frac{1}{4}e^{5A} * \sum_p (-1)^{\left[\frac{p}{2}\right]} F_p
\end{aligned}
$$

$$
\psi^1 \equiv e^{-A}\eta_1 \otimes \eta_2^\dagger , \qquad \psi^2 \equiv e^{-A}\eta_1 \otimes (\eta_2^c)^\dagger
$$

where η_1, η_2 are Spin(5) Dirac spinors

ψ^1 and ψ^2 can be expressed as a sum of odd and even forms via application of Fierz expansion and the map

$$
\gamma^{m_1 \dots m_k} \to dx^{m_1} \wedge \dots \wedge dx^{m_k}
$$

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* $d_H = d - H\wedge$

Supersymmetric AdS$_5$ solutions

The stabilizer group G of η_1 and η_2

$$G = \begin{cases} \mathrm{SU}(2) \\ \mathrm{Id} \end{cases}$$

only $G = \mathrm{Id}$ is allowed (in contrast to type IIB)

An identity structure is characterized by an orthonormal frame

supersymmetry equations → constraints on the frame

The fluxes are expressed in terms of the identity structure

Equations of motion implied by

1. Supersymmetry Equations
2. Bianchi identities $dH = 0$, $d_H F = 0$ $(F = \sum F_p)$

Supersymmetric AdS$_5$ solutions

We have determined the local form of the metric

$$ds^2_{M_5} = ds^2_{\mathcal{C}} + \frac{1}{9} b^2 D\psi^2 + \frac{e^{-4A+2\phi}}{b^2}\left[b^2(e^{-4A}dx^2 + dy^2) + (a_2 e^{-2A}dx + a_1 dy)^2\right]$$

$$b^2 = 1 - a_1^2 - a_2^2, \quad a_1 = -\frac{1}{2}e^{-3A+\phi}y, \quad D\psi = d\psi + \rho$$

Properties

- $\xi = 3\partial_\psi$ generates the U(1) R-symmetry
- S^1 fibered over $\mathcal{C}$ – the curvature of the connection ρ determines the curvature of $\mathcal{C}$.

Solutions characterized by three functions $\{a_2, \phi, A\}$ of four variables obeying six partial differential equations

Hard to solve...

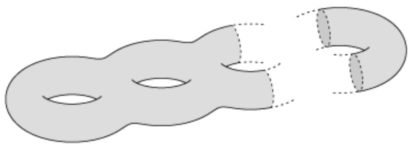
Supersymmetric AdS_5 solutions

Assumption

1. $ds^2_{10} = e^{2A}[ds^2_{\mathrm{AdS}_5} + ds^2_\Sigma(x_1, x_2)] + ds^2_{M_3}$
2. no dependence on x_1, x_2

Consequences

- Σ has constant curvature $\rightarrow$ compact Riemann surface Σ_g



- The PDEs reduce to ODEs for a single function $\beta(y)$!

  For a subclass of massive ($F_0 \neq 0$) cases

$$\left(\frac{16y^2\beta}{\beta'^2}\right)' = \frac{2}{9}F_0$$

Supersymmetric AdS_5 solutions

$F_0 = 0$ recovered

- Maldacena–Núñez
- Bah–Beem–Bobev–Wecht
- Itsios–Núñez–Sfetsos–Thompson [Itsios, Núñez, Sfetsos, Thompson '13] (non-abelian T-dual of $AdS_5 \times T^{1,1}$ – M-theory lift fits in the Bah–Beem–Bobev–Wecht class)

$F_0 \neq 0$

- a couple of globally problematic solutions
- a new infinite class of massive solutions

$$\beta = \frac{y_0^3}{b_2^3 F_0}\left(\sqrt{\hat{y}} - 6\right)^2 \left(\hat{y} + 6\sqrt{\hat{y}} + 6b_2 - 72\right)^2 , \quad \hat{y} \equiv 2b_2\left(\frac{y}{y_0} - 1\right) + 36$$

Σ_g of genus $g > 1$ – realised as $\mathbb{H}^2/\Gamma$

A simple example

$$ds^2_{M_3} = \sqrt{-\frac{y_0}{8F_0}}\left(\frac{d\tilde{y}^2}{(1-\tilde{y})\sqrt{\tilde{y}+2}} + \frac{4}{9}\frac{(1-\tilde{y})(\tilde{y}+2)^{3/2}}{2-\tilde{y}}Ds^2_{S^2}\right)\,, \quad \tilde{y} \equiv \frac{y}{y_0} \in [-2,1]$$

$$ds^2_{M_3} = \begin{cases} dr^2 + r^2 Ds^2_{S^2} & y = 1 \qquad \text{regular endpoint} \\ \frac{1}{\sqrt{r}}\left(dr^2 + r^2 Ds^2_{S^2}\right) & y = -2 \qquad \text{D6 brane} \end{cases}$$

Flux quantization $\int_{S^2}(F_2 - F_0 B) = n_{\mathrm{D6}}$ fixes $y_0 = -\frac{3}{8}\frac{n^2_{\mathrm{D6}}}{F_0}$

In the general case the end-points can be

- regular points
- D6 brane
- O6 singularity $ds^2_{M_3} = \sqrt{r}\left(dr^2 + r_0^2 Ds^2_{S^2}\right)$

One can also add D8 branes

AdS_7 origin

The form of the new solutions suggests a higher dimensional origin

[Apruzzi, Fazzi, Rosa, Tomasiello '13] new AdS_7 solutions of massive type IIA supergravity

$$ds^2_{10} = e^{2A(r)} ds^2_{\mathrm{AdS}_7} + dr^2 + e^{2A(r)} v(r)^2 ds^2_{S^2}$$

Dual to 6d (1,0) SCFTs engineered by NS5-D6-D8 brane intersections [Gaiotto, Tomasiello '14] [Hanany, Zaffaroni '97] [Brunner, Karch '97] The isometry of S^2 corresponds to the R-symmetry

see A. Tomasiello's talk

v, A and the dilaton ϕ which characterize the solution satisfy a set of ODEs solved initially numerically

AdS$_7$ origin

How do the AdS$_5$ solutions arise? There is a one-to-one map between the ODEs AdS$_7$ solutions now known analytically

The AdS$_5$ solution can be written in the form

$$ds^2_{10} = \left(\frac{3}{4}\right)^{\frac{1}{2}} \left[\frac{3}{4} e^{2A}(ds^2_{\text{AdS}_5} + ds^2_{\Sigma_g}) + dr^2 + \frac{e^{2A} v^2}{1-4v^2} Ds^2_{S^2}\right]$$

An analogous “compactification" to AdS$_4$ was found [Rota, Tomasiello '15]

$$ds^2_{10} = \left(\frac{5}{8}\right)^{\frac{1}{2}} \left[\frac{5}{8} e^{2A}(ds^2_{\text{AdS}_4} + \tfrac{4}{5} ds^2_{\Sigma_3}) + dr^2 + \frac{e^{2A} v^2}{1-6v^2} Ds^2_{S^2}\right]$$

Σ_3 has negative constant curvature; the $SU(2)$ symmetry of S^2 is twisted by the $SU(2)$ rotations of $T\Sigma_3$

AdS_7 origin

$$ds^2_{10} = X^{\frac{3}{2}} e^{2A} ds^2_7 + X^{\frac{1}{2}} \left(dr^2 + \frac{e^{2A} v^2}{w} Ds^2_{S^2} \right) \,, \qquad w \equiv X 16 v^2 + (1 - 16 v^2)$$

$$ds^2_7 = \begin{cases} ds^2_{\mathrm{AdS}_7} \\ ds^2_{\mathrm{AdS}_5} + ds^2_{\Sigma_g} \\ ds^2_{\mathrm{AdS}_4} + \frac{4}{5} ds^2_{\Sigma_3} \end{cases} \qquad X = \begin{cases} 1 \\ \frac{3}{4} \\ \frac{5}{8} \end{cases}$$

Conjecturally dual to compactifications of 6d (1,0) SCFTs

Free Energy simple solution

$$\mathcal{F}_{0,4} = \left(\frac{3}{4}\right)^3 \pi (g-1) \mathcal{F}_{0,6} \,, \quad \mathcal{F}_{0,3} = \left(\frac{5}{8}\right)^4 \mathrm{Vol}(\Sigma_3) \mathcal{F}_{0,6}$$

$$\mathcal{F}_{0,6} = \frac{512}{45} \pi^4 n^2_{\mathrm{D6}} N^3 \,, \quad N = \frac{1}{4\pi^2} \int H$$

Future directions

Holographic RG flows

A strategy is to construct a consistent reduction to a 7d gauged supergravity

- introduce $SU(2)$ gauge fields A^i, $i = 1, 2, 3$ gauging the SU(2) R-symmetry
- promote X to a scalar field in seven dimensions

candidate theory $\mathcal{N} = 1$ SU(2) gauged supergravity [Townsend, van Nieuwenhuizen '83]. Comprises a scalar, an SU(2) gauge field and a three-form potential Shown to arise as a consistent reduction of M-theory on S^4 [Lü, Pope '99].

work in progress

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