

# Analogue holographic correspondence in optical metamaterials

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*UNC-Chapel Hill, Physics & Astronomy*

# Outline

- Holographic conjecture (broadly defined) and its status
- Emergent metrics and analogue gravity in optical metamaterials
- Metamaterial-based implementations of analogue holography
- Take-home messages

DVK

EPL, v.109, p.61001 (2015), (arXiv:1411.1693)

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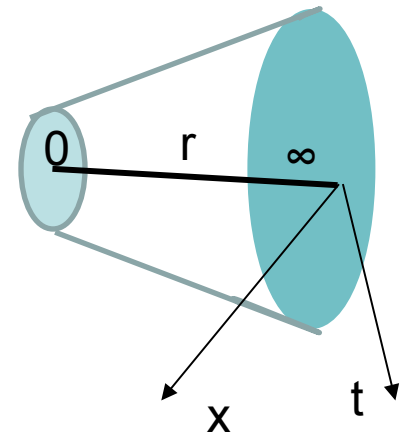
EPL, v.109, p.61001 (2015), (arXiv:1411.1693);

EPL, v.104, p.47002 (2013), (arXiv:1305.6651);

arXiv:1404.7000,

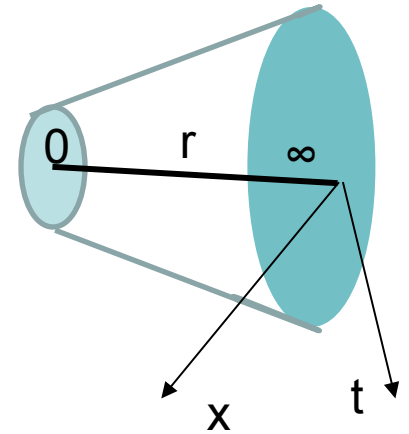
arXiv:1502.03375

# Holographic correspondence:conjecture



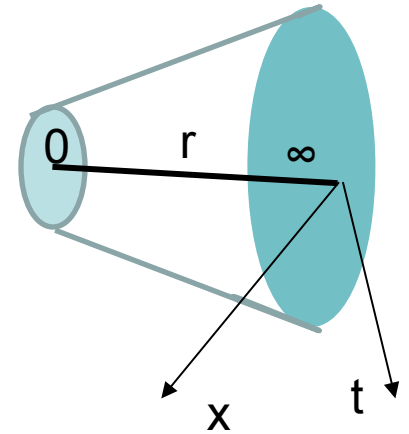
# Holographic correspondence:conjecture

- Original ('AdS/CFT'):  
d=4 N=4 SYM  $\leftrightarrow$  type-IIB superstrings (d=5 supergravity)  
(J.Maldacena; E.Witten, S.Gubser, I.Klebanov, A.Polyakov,...)
- SUSY,
- multi-component ( $N \gg 1$ ),
- Lorentz and scale-invariant,
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  - SUSY,
  - multi-component ( $N \gg 1$ ),
  - Lorentz and scale-invariant,
  - boundary theory: very strongly interacting
- Broadly defined ('non-AdS/non-CFT'):
  - non-SUSY,
  - only a few components ( $N \sim 1$ ),
  - Lorentz , scale, translationally, and/or rotationally non-invariant,
  - boundary theory: only moderately interacting ( $T \sim U$ )



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black holes, quantum information, and Hawking radiation,  
entanglement and tensor network states ('MERA'),  
hydrodynamics of quark-gluon plasma, cold atomic gases,...



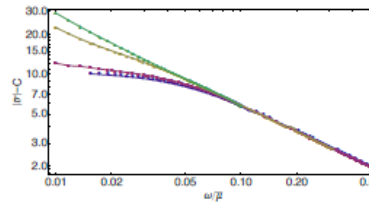
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- Data fitting:

Optical conductivity in cuprates  
(non-SUSY,  $N \sim 1$ ,  $T \sim U$ )



G, Horowitz and J. Santos,  
1302.6586

$$2 < \omega T < 8$$

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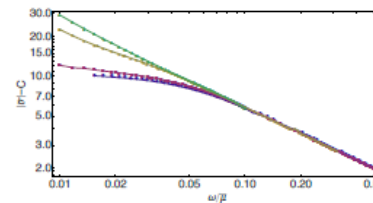
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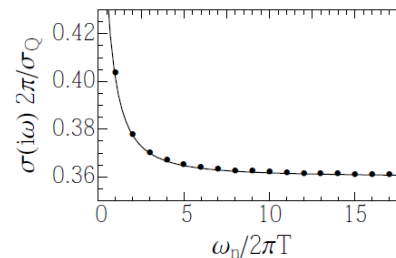
- (Almost) exact methods (MC):

2d Bose-Hubbard model



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E. Katz et al, 1409.3841

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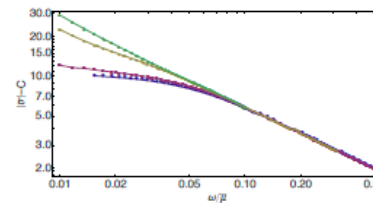
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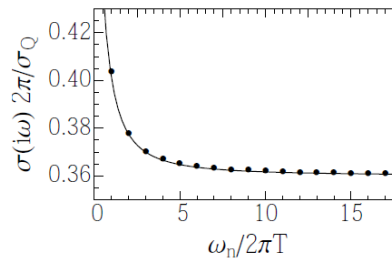
- Experiment:

$\frac{\eta}{s}$  : ratio from ARPES in cuprates



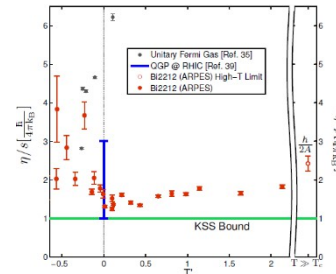
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J. Rameau et al, 1409.5820

# Holographic correspondence: outstanding questions

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- General:

DVK, arXiv:1404.7000

- What is the TRUE status of the (broadly defined) holographic conjecture?

- If it is indeed valid, then WHY?

- Do ALL strongly coupled systems have duals (or only a precious FEW)?

- How much SYMMETRY is enough and is LARGE N really necessary?

- Has the (non)AdS/(non)CFT dictionary been established in its FINAL form?

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- Do the boundary theories of the previously studied gravity duals have physical realizations among the known materials?

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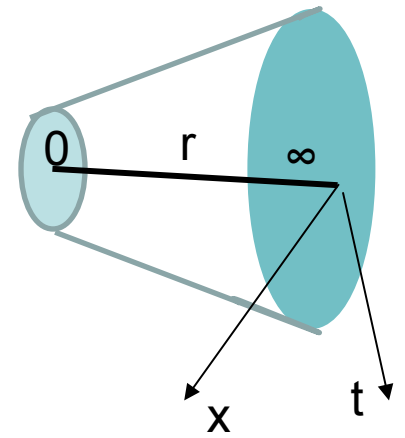
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- What are the gravity duals of the documented 'strange metals'?

- Can anything at all be tested systematically in the lab?**

# Holographic correspondence: physical origin

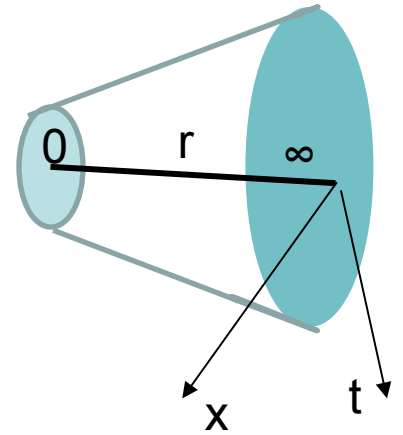




# Holographic correspondence: physical origin

- Emergent extra dimension:

Dynamical renormalization (energy/length/information) scale  
or a mere 'spectator' coordinate?



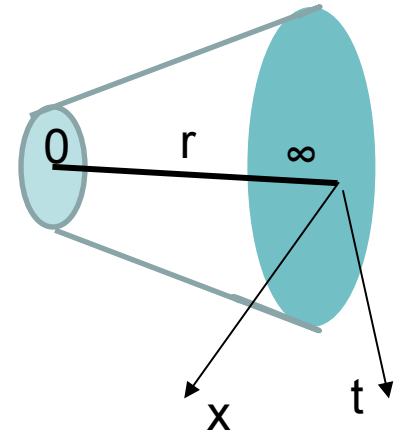
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Full fledged fluctuating (quantum) gravity in the bulk  
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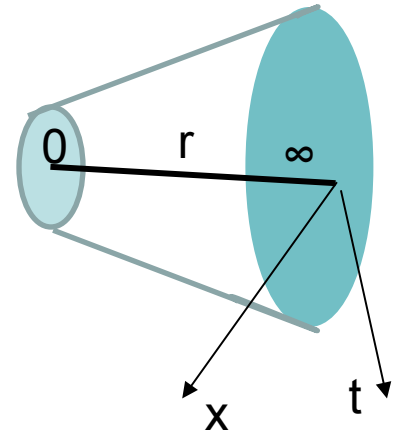
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Berry connections in crystal band structure, dynamical time evolution,  
Quantum Hall and other topological states (Fubini/Study),...



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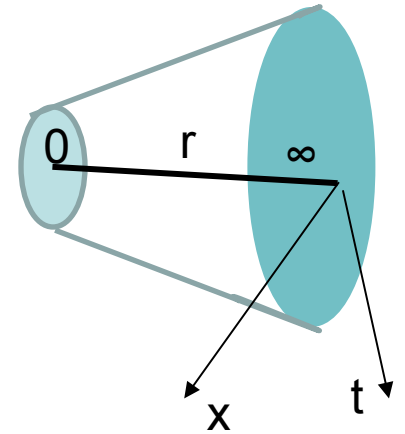
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So... '**Just do it!**' (HEP) or '**Holographic WHAT?!**' (CMT)



# 'Holography light': analogue holography

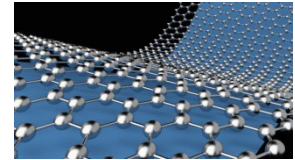
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Explains certain apparent holography-like features

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- Desktop realizations:
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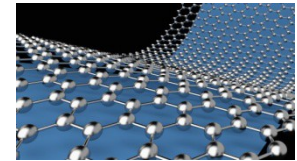
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DVK, EPL v.104, p.47002 (2013)

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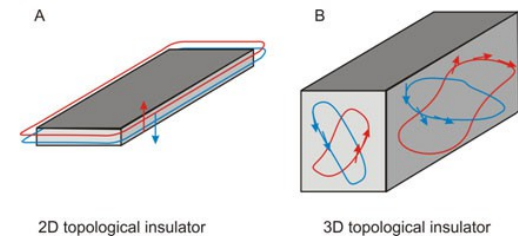
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- Topological insulators/superconductors

Problematic:

- Curved 3d space
- Fermi liquid on a 2d boundary is more robust than in 1d

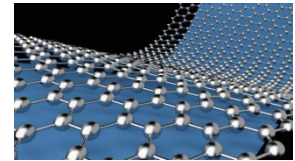


[ssrl.slac.stanford.edu](http://ssrl.slac.stanford.edu)

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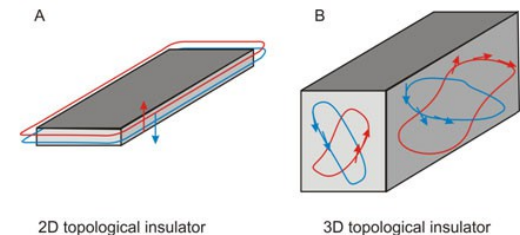
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2D topological insulator

3D topological insulator

- **Hyperbolic optical/IR metamaterials**

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# Metamaterials: analogue gravity

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- Artificial metric in electrically and/or magnetically active media

$$\gamma_{ij} = g_{ij}/|g_{\tau\tau}| = \epsilon_{ij}/\det\hat{\epsilon} = \mu_{ij}/\det\hat{\mu}$$

$$\epsilon_{ij} = \mu_{ij} = \sqrt{-\hat{g}}g_{ij}/|g_{\tau\tau}|$$

W.Lu et al,'10,  
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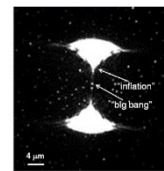
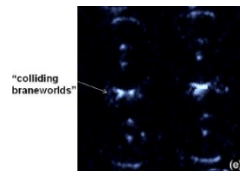
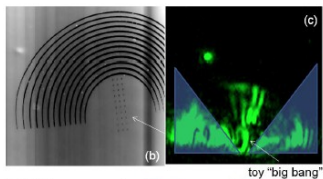
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W.Lu et al,'10,  
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- Hyperbolic metamaterials
  - Rindler and event horizons, black/white/worm-holes,
  - inflation, Big Bang, Rip, and Crunch,
  - metric signature transitions, end-of-time, multiverse,...



I.Smolyaninov et al,  
1111.3300, 1201.5348

# **String holography meets its optical namesake: analogue correspondence in optical metamaterials**

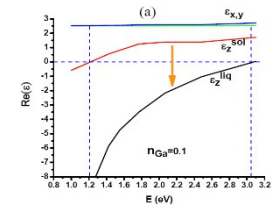
# String holography meets its optical namesake: analogue correspondence in optical metamaterials

- Dispersion of extraordinary waves

$$\frac{\omega^2}{c^2} \vec{D}_\omega = \vec{\nabla} \times \vec{\nabla} \times \vec{E}_\omega \quad \text{and} \quad \vec{D}_\omega = \vec{\epsilon}_\omega \vec{E}_\omega$$

$$\omega^2 = k_z^2 / \epsilon_{xy} + k_{xy}^2 / \epsilon_{zz}$$

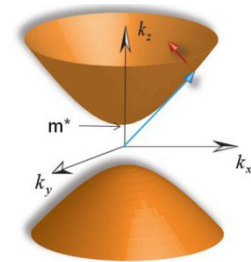
$$ds^2 = -\epsilon_{xy} dz^2 - \epsilon_{zz} (dx^2 + dy^2)$$



I. Smolyaninov, E. Narimanov, '09...

$$\epsilon_{xy} > 0$$

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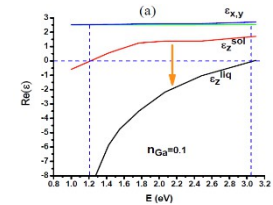
- Attainable 2+1 geometries

$$ds^2 = \frac{d\tau^2}{u^{2\alpha}} + R^2 \frac{du^2}{u^{2\beta}} + \frac{dx^2}{u^{2\gamma}}$$

$$ds^2 = u^{2\theta/d} \left( \frac{d\tau^2}{u^{2\zeta}} + \frac{L^2 du^2 + dx^2}{u^2} \right)$$

$$\zeta = \frac{1 - \beta + \alpha}{1 - \beta + \gamma}, \quad \theta = \frac{1 - \beta}{1 - \beta + \gamma}$$

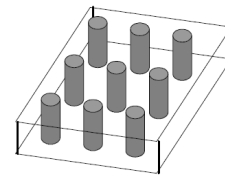
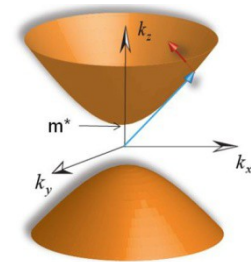
Hyperscaling-violation metrics



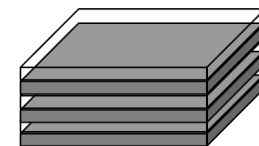
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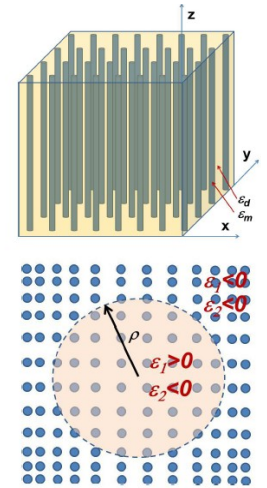


$n(x,y)$



$n(z)$

# Axially-symmetric setup



$n(x,y)$  - density  
of the  $m$ -component

# Axially-symmetric setup

- Radially-dependent density of the metallic component

$$ds^2 = -\epsilon_{xy}dz^2 - \epsilon_{zz}(dx^2 + dy^2)$$

$$g_{\tau\tau} = -\epsilon_{xy}, \quad g_{rr} = g_{\phi\phi}/r^2 = -\epsilon_{zz}$$

$$\epsilon_{zz}(r) = \epsilon_m n + \epsilon_d(1 - n)$$

$$\epsilon_{xy}(r) = \epsilon_d \frac{\epsilon_m(1 + n) + \epsilon_d(1 - n)}{\epsilon_m(1 - n) + \epsilon_d(1 + n)}$$

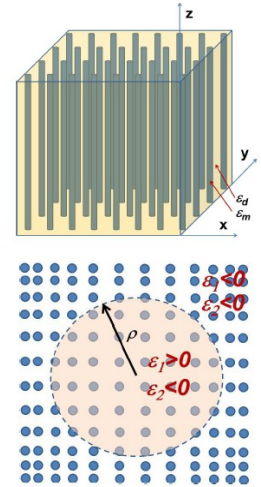
Zeros/poles:

$$n_1 = (\epsilon_m + \epsilon_d)/(\epsilon_d - \epsilon_m)$$

$$n_2 = \epsilon_d/(\epsilon_d - \epsilon_m)$$

$$\epsilon_d > 0$$

$$\epsilon_m < 0$$



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$$\epsilon_{xy}(r) = \epsilon_d \frac{\epsilon_m(1 + n) + \epsilon_d(1 - n)}{\epsilon_m(1 - n) + \epsilon_d(1 + n)}$$

- Mimicking the metric

$$ds^2 = \frac{d\tau^2}{u^{2\alpha}} + R^2 \frac{du^2}{u^{2\beta}} + \frac{dx^2}{u^{2\gamma}}$$

$$u = R/r$$

$$n(r) = 1/3 + c(R/r)^{2\alpha}$$

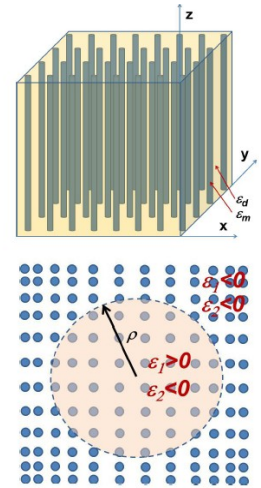
$$x = R\phi$$

$$\beta = 2 - \alpha \quad \gamma = 1 - \alpha$$

$$c \ll 1$$

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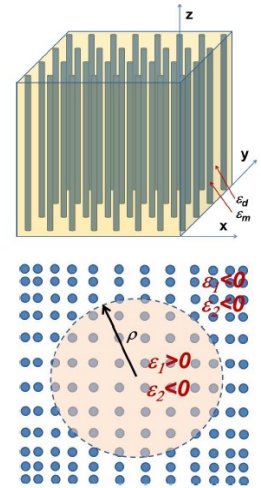
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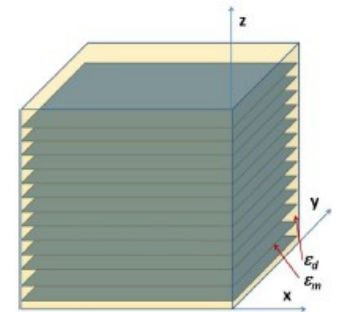


$n(x,y)$  - density of the m-component

- HV geometry:  $\theta, z \rightarrow \infty \quad \theta/\zeta = (\alpha-1)/(2\alpha-1)$

$$\gamma = 0, \quad \alpha = \beta = 1 \quad AdS_2 \times R$$

# Layered setup



$n(z)$

# Layered setup

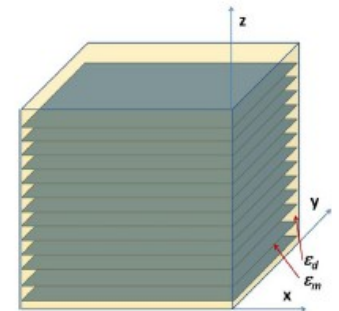
- z-dependent density of the metallic component

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$$\epsilon_{zz}(z) = \frac{\epsilon_m \epsilon_d}{\epsilon_m(1-n) + \epsilon_d n}$$

$$\epsilon_{xy}(z) = \epsilon_m n + \epsilon_d(1-n)$$



$n(z)$

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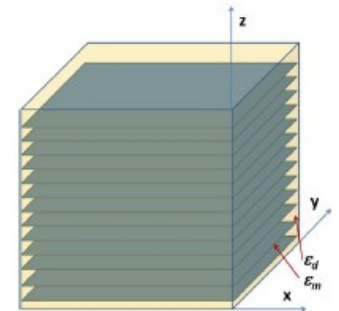
- Inhomogeneous dielectric function

$$n(z) = 1/2 + c(z/L)^{2\alpha}$$

$$ds^2 = \frac{d\tau^2}{u^{2\alpha}} + R^2 \frac{du^2}{u^{2\beta}} + \frac{dx^2}{u^{2\gamma}}$$

$$\alpha = -\beta = \gamma.$$

$$u = z$$



$n(z)$

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$$ds^2 = -\epsilon_{xy} dz^2 - \epsilon_{zz} (dx^2 + dy^2)$$

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$$\epsilon_{zz}(z) = \frac{\epsilon_m \epsilon_d}{\epsilon_m (1 - n) + \epsilon_d n}$$

$$\epsilon_{xy}(z) = \epsilon_m n + \epsilon_d (1 - n)$$

- Inhomogeneous dielectric function

$$n(z) = 1/2 + c(z/L)^{2\alpha}$$

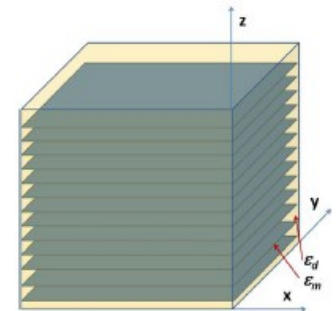
$$ds^2 = \frac{d\tau^2}{u^{2\alpha}} + R^2 \frac{du^2}{u^{2\beta}} + \frac{dx^2}{u^{2\gamma}}$$

$$\alpha = -\beta = \gamma.$$

- HV geometry:  $\zeta = 1$   $\theta = (1 + \alpha)/(1 + 2\alpha)$

$$\alpha = -1 \quad AdS_3$$

$$u = z$$



$n(z)$

# Boundary propagator

# Boundary propagator

- WKB solutions  $\psi_{\pm}(r, \omega, k) \sim \frac{1}{V^{1/4}(r)} e^{\mp \int_r^R dr' \sqrt{V}(r')}$   $mR \gg 1$
- Asymptotic behavior of the propagator  $G_{\omega}(\tau, x) \sim \exp[-S(\tau, x)]$
- Action  $S(\tau, x) = L\omega \int du \sqrt{g_{uu} + g_{\tau\tau} \left(\frac{d\tau}{du}\right)^2 + g_{xx} \left(\frac{dx}{du}\right)^2}$



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- Extremal value  $S(\tau, x) = L\omega^2 \int_{u_0}^{u_t} du \sqrt{\frac{g_{uu}}{r(u)}}$

$$r(u) = \omega^2 - k_x^2/g_{xx}(u) - k_{\tau}^2/g_{\tau\tau}(u)$$

- Geodesics  $\tau = Lk_{\tau} \int_{u_0}^{u_t} \frac{du}{g_{\tau\tau}} \sqrt{\frac{g_{uu}}{r(u)}}, \quad x = Lk_x \int_{u_0}^{u_t} \frac{du}{g_{xx}} \sqrt{\frac{g_{uu}}{r(u)}}$

$$u_t = (\omega / \sqrt{k_{\tau}^2 + k_x^2})^{1/\alpha} \quad |x| = \sqrt{\tau^2 + x^2}$$

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 $u_t = (\omega/\sqrt{k_{\tau}^2 + k_x^2})^{1/\alpha}$   $|x| = \sqrt{\tau^2 + x^2}$
- Layered setup:  $G_{\omega}(x) \sim \exp[-\sqrt{c}L\omega|x/cL|^{\theta/\zeta}]$   $1/2 < \theta/\zeta = (1 + \alpha)/(1 + 2\alpha) < 1$
- Axially-symmetrical:  $S(\tau, 0) \sim \tau^{\theta/\zeta}$ ,  $S(\tau, 0) \sim \ln \tau$   $\alpha = 1$   
 $0 < \theta/\zeta = (\alpha - 1)/(2\alpha - 1) < 1/2$   $S(0, x) \sim x$

# Prospective 'boundary dual': fluctuating 2d membrane

- Fluctuating elastic membrane:  
(coupled in- and out-of-plane modes)

$$F = \int d^d \mathbf{x} \left[ \frac{\kappa}{2} (\nabla^2 h)^2 + \mu v_{\alpha\beta}^2 + \frac{\lambda}{2} v_{\alpha\alpha}^2 \right]$$

$$v_{\alpha\beta} = \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha + \partial_\alpha h \partial_\beta h$$

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$$\Delta F \sim \int d^d \mathbf{k} k^{4-\eta} |h_{\mathbf{k}}|^2$$

$$\eta_c \sim 1$$

2d membrane crumpling transition:

$$S_{boundary} = \frac{1}{2\nu} \int d^2 \mathbf{k} k^{2+\theta/\zeta} |\phi_{\mathbf{k}}|^2 \quad \nu \sim L^{1-\theta/\zeta} \omega$$

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- Practical realizations: Co nanoparticles in kerosene, PMMA on gold,

*InGaAs* (m) / *AlGaAs/GaAs* (d), ...

# Conclusions

- The holographic conjecture still remains to be verified, especially in its broadly generalized 'non-CFT/non-AdS' form.
- The bottom-up approach can be prone to confusing the genuine 'bona fide' holography with some apparent 'analogue' one.
- In every specific case, the physical nature of the latter can be elucidated without invoking any new physical principles.
- In optical metamaterials, the simulated (pseudo)holographic behavior can be systematically studied with such established experimental techniques as holographic and speckle interferometry, as well as the other optical field correlation probes.