

Generalized Global Symmetries

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Global vs. Local Symmetries

Global

- Intrinsic
- Can be accidental in IR – approximate
- Classify operators
- Can be spontaneously broken
- If unbroken can classify states
- Useful in classifying phases
- 't Hooft anomalies
- Not present in a theory of gravity

Local (gauge)

- Ambiguous – duality
- Can emerge in IR – exact
- All operators are invariant
- Not really a symmetry
- Hence it cannot be broken (Higgs description meaningful only at weak coupling)
- Cannot be anomalous
- Appears essential in formulating the Standard Model and in Gravity

Main point

- Will study global symmetries
 - Ordinary global symmetries act on local operators
 - The charged states are particles
- Will generalize
 - The charged operators are lines, surfaces, etc.
 - The charged objects are strings, domain walls, etc.
- It is intuitively clear and many people will feel that they have known it. We will make it more precise and more systematic.
 - The gauged version of these are common in physics and in mathematics
- We will repeat all the things that are always done with ordinary symmetries

Ordinary global symmetries

- Generated by operators associated with co-dimension one manifolds M

$$U_g(M)$$

$g \in G$ a group element

- The correlation functions of $U_g(M)$ are topological!
- Group multiplication $U_{g_1}(M)U_{g_2}(M) = U_{g_1g_2}(M)$
- Local operators $O(p)$ are in representations of G

$$U_g(M)O_i(p) = R_i^j(g)O_j(p)$$

where M surrounds p (Ward identity)

- If the symmetry is continuous,

$$U_g(M) = e^{i \int j(g)}$$

$j(g)$ is a closed form current (its dual is a conserved current).

q -form global symmetries

- Generated by operators associated with co-dimension $q + 1$ manifolds M (ordinary global symmetry has $q = 0$)

$$U_g(M)$$

$g \in G$ a group element

- The correlation functions of $U_g(M)$ are topological!
- Group multiplication $U_{g_1}(M)U_{g_2}(M) = U_{g_1g_2}(M)$.
Because of the high co-dimension the order does not matter and G is Abelian.
- The charged operators $V(L)$ are on dimension q manifolds L .
Representations of G – Ward identity

$$U_g(M)V(L) = R(g)V(L)$$

where M surrounds L and $R(g)$ is a phase.

q -form global symmetries

If the symmetry is continuous,

$$U_g(M) = e^{i \int j(g)}$$

$j(g)$ is a closed form current (its dual is a conserved current).

Compactifying on a circle, a q -form symmetry leads to a q -form symmetry and a $q - 1$ -form symmetry in the lower dimensional theory.

- For example, compactifying a one-form symmetry leads to an ordinary symmetry in the lower dimensional theory.

No need for Lagrangian

- Exists abstractly, also in theories without a Lagrangian
- Useful in dualities

q -form global symmetries

- Charged operators are extended (lines, surfaces)
- Charged objects are extended – branes (strings, domain walls)
 - In SUSY BPS bound
- Selection rules on amplitudes
- Couple to a background classical gauge field (twisted boundary conditions)
- Gauge the symmetry (sum over these background fields)
- The symmetry could be spontaneously broken
- There can be anomalies and anomaly inflow on walls

Example 1: $4d$ $U(1)$ gauge theory

Two global $U(1)$ one-form symmetries:

- Electric symmetry
 - Closed form currents: $\frac{2}{g^2} * F$ (measures the electric flux)
 - Shifts the gauge field A by a flat connection
- Magnetic symmetry
 - Closed form currents: $\frac{1}{2\pi} F$ (measures the magnetic flux)
 - Shifts the magnetic gauge field by a flat connection.

Example 1: 4d $U(1)$ gauge theory

The symmetries are generated by surface operators

$$U_{g_E=e^{i\alpha}, g_M=e^{i\eta}}(M) = e^{\frac{i\eta}{2\pi} \int F + \frac{2i\alpha}{g^2} \int *F}$$

- These are Gukov-Witten surface operators (rescaled α, η).
- They measure the electric and the magnetic flux through the surface M .

The charged objects are dyonic lines

$$W_n(L)H_m(L)$$

($W_n(L)$ are Wilson lines and $H_m(L)$ are 't Hooft lines)

with global symmetry charges n and m under the two global $U(1)$ one-form symmetries.

Example 2: $4d$ $SU(N)$ gauge theory

- Electric \mathbf{Z}_N one-form symmetry
 - The Gukov-Witten operator is associated with a conjugacy class in $SU(N)$. When this class is in the center of $SU(N)$ the surface operator is topological.
 - It shifts the gauge field by a flat \mathbf{Z}_N connection.
 - It acts on the Wilson lines according to their representation under the $\mathbf{Z}_N \in SU(N)$ center.
- No magnetic one-form symmetry.
 - In this theory there are no 't Hooft lines – they are not genuine line operators – they need a surface.
 - An open surface operator, whose boundary is an 't Hooft line.

Example 3: $4d$ $SU(N)$ gauge theory with matter in N

The presence of the charged matter explicitly breaks the electric one-form \mathbf{Z}_N symmetry.

Hence, there is no global one-form symmetry.

Significance of these symmetries

- Consequence: selection rules, e.g. in compact space the vev of a charged line wrapping a nontrivial cycle vanishes [Witten].
- Dual theories must have the same global symmetries. (They often have different gauge symmetries.)
 - The one-form symmetries are typically electric on one side of the duality and magnetic on the other.
 - $4d$ $N = 1$ SUSY dualities respects the global symmetries. Check of non-BPS observables
 - The $SL(2, \mathbf{Z})$ orbit of a given $N = 4$ theory must have the same global symmetry.

Significance of these symmetries

- Twisted sectors by coupling to background gauge fields
 - An $SU(N)$ gauge theory without matter can have twisted boundary conditions – an $SU(N)/\mathbf{Z}_N$ bundle that is not an $SU(N)$ bundle – 't Hooft twisted boundary conditions.
- Gauging the symmetry by summing over twisted sectors – like orbifolds.
 - Discrete θ -parameters are analogs of discrete torsion.
- Can characterize phases of gauge theories by whether the global symmetry is broken or not...

Characterizing phases

- In a confining phase the electric one-form symmetry is unbroken.
 - The confining strings are charged and are classified by the unbroken symmetry.
- In a Higgs or Coulomb phase the electric one-form symmetry is broken.
 - Renormalizing the perimeter law to zero, the large size limit of $\langle W \rangle$ is nonzero – vev “breaks the symmetry.”
 - It is unbroken in “Coulomb” phase in $3d$ and $2d$.

Example 1: 4d $U(1)$ gauge theory

- There are two global $U(1)$ one-form symmetries.
- Both are spontaneously broken:

- The photon is their Goldstone boson

$$\langle 0|F_{\mu\nu}|\epsilon, p\rangle = (\epsilon_\mu p_\nu - \epsilon_\nu p_\mu)e^{i p x}$$

- Placing the theory on $\mathbf{R}^3 \times \mathbf{S}^1$, each one-form global symmetry leads to an ordinary global symmetry and a one-form symmetry.
- These ordinary symmetries are manifestly spontaneously broken – the moduli space of vacua is \mathbf{T}^2 parameterized by A_4 and the 3d dual photon.

Example 2: $4d$ $SU(N)$ gauge theory

In the standard confining phase the electric \mathbf{Z}_N one-form symmetry is unbroken.

- Charged strings
- Area law in Wilson loops
- When compactified on a circle an ordinary ($q = 0$) \mathbf{Z}_N , which is unbroken [Polyakov]

If no confinement, the global \mathbf{Z}_N symmetry is broken

- No charged strings
- Perimeter law in Wilson loops
- When compactified on a circle an ordinary ($q = 0$) \mathbf{Z}_N , which is broken [Polyakov]

Example 2: $4d$ $SU(N)$ gauge theory

Can also have a phase with confinement index t , where the global one-form symmetry is spontaneously broken $\mathbf{Z}_N \rightarrow \mathbf{Z}_t$.

- W has area law but W^t has a perimeter law [Cachazo, NS, Witten].
- In this case there is a $\mathbf{Z}_{N/t}$ gauge theory at low energies – long range topological order.

Example 3: $4d$ $SU(N)$ gauge theory with matter in N

No global one-form symmetry.

Hence we cannot distinguish between Higgs and confinement.

This is usually described as screening the loop [Fradkin, Shenker; Banks, Rabinovici].

From our perspective, due to lack of symmetry.

Conclusions

- Higher form global symmetries are ubiquitous.
- They help classify
 - extended objects (strings, domain walls, etc.)
 - extended operators/defects (lines, surfaces, etc.)
- As global symmetries, they must be the same in dual theories.
- They extend Landau's characterization of phases based on order parameters that break global symmetries.
 - Rephrase the Wilson/'t Hooft classification in terms of broken or unbroken one-form global symmetries.
- Anomalies
 - 't Hooft matching conditions
 - Anomaly inflow
 - Degrees of freedom on domain walls

Thank you for your attention