Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

FROGS
FRont Of pro-Galician Scientists

# Holographic Charged Impurities
# [Islands and conductivities]

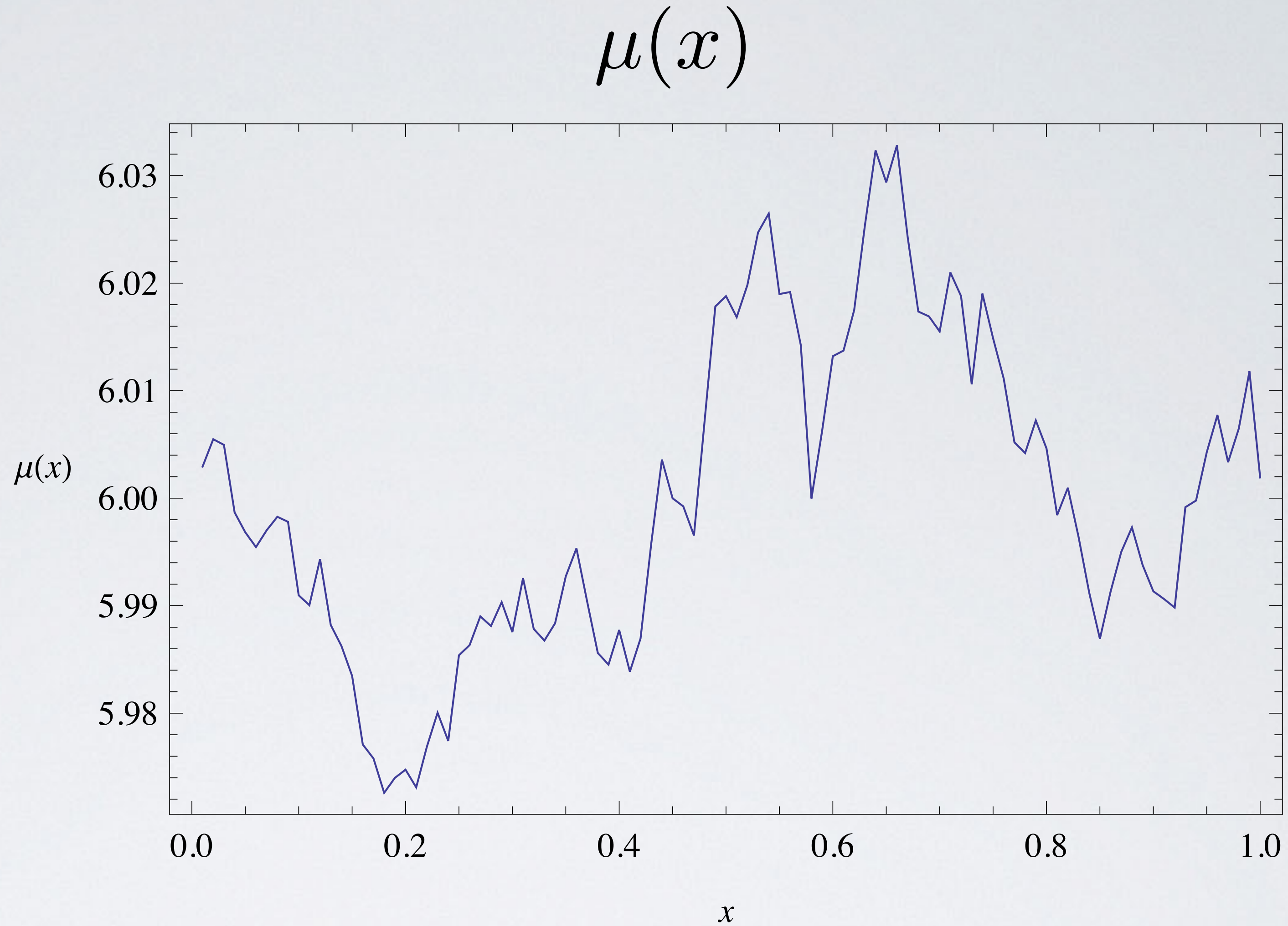
[....work in progress....]


with L.A. Pando-Zayas (Michigan, USA)
I. Salazar Landea (La Plata, Argentina)
A. Scardicchio (ICTP, Italy)

Daniel Areán
paradise-Firenze, April 2015

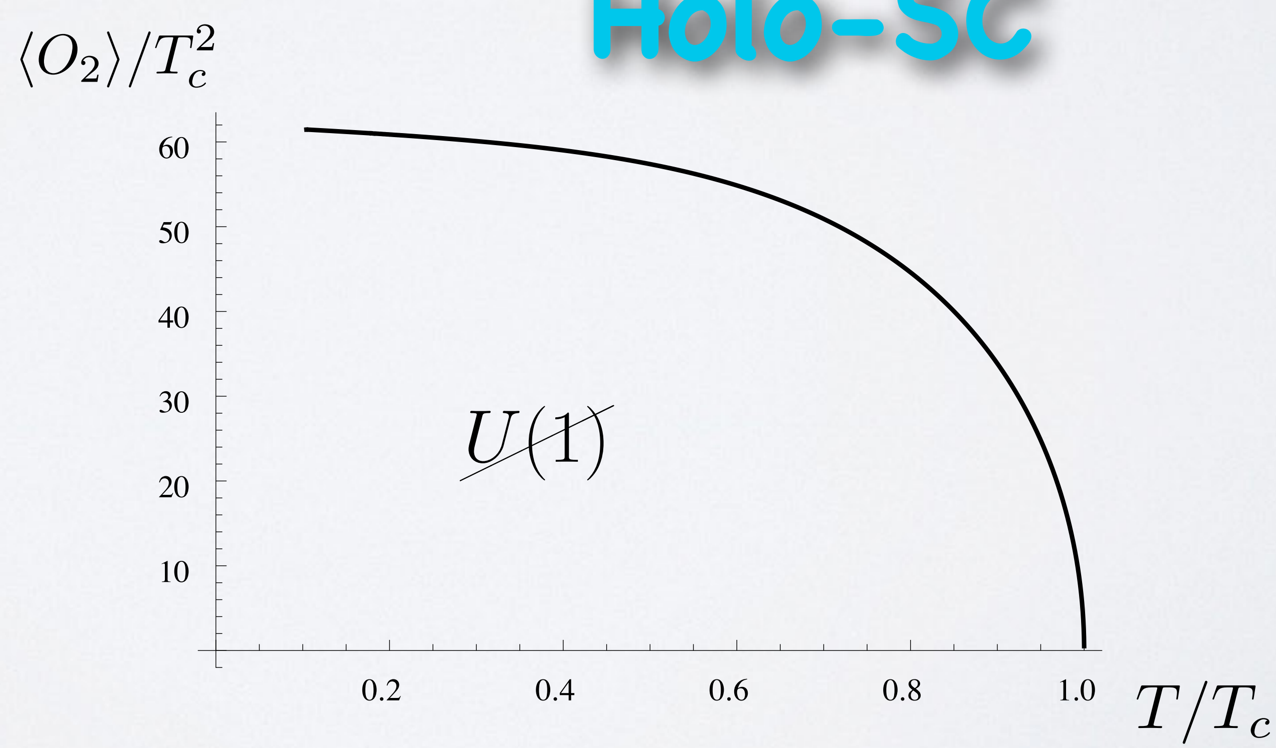
# Noise

[charged impurities]

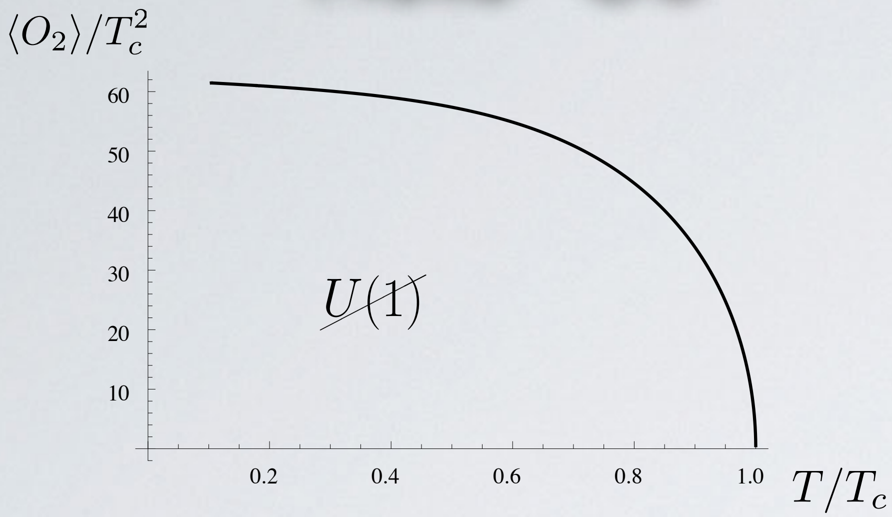



# in holography

# Holo-SC

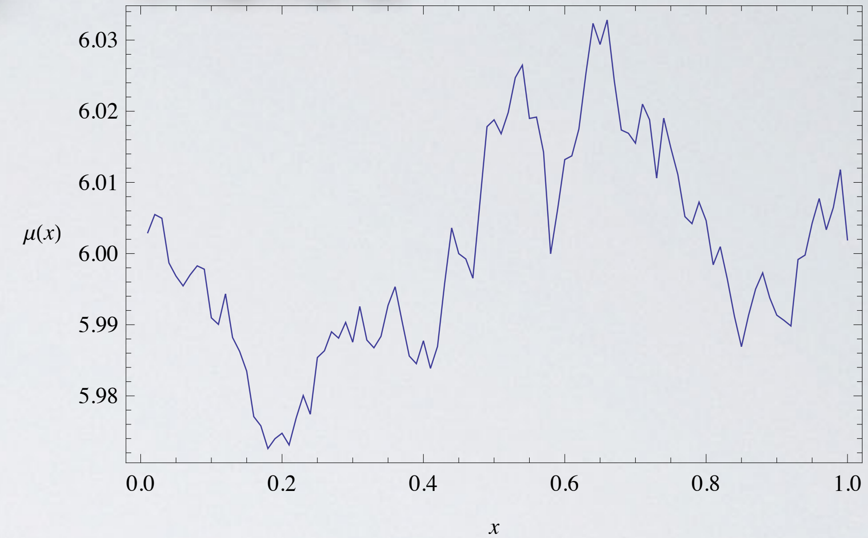


# Holo-SC



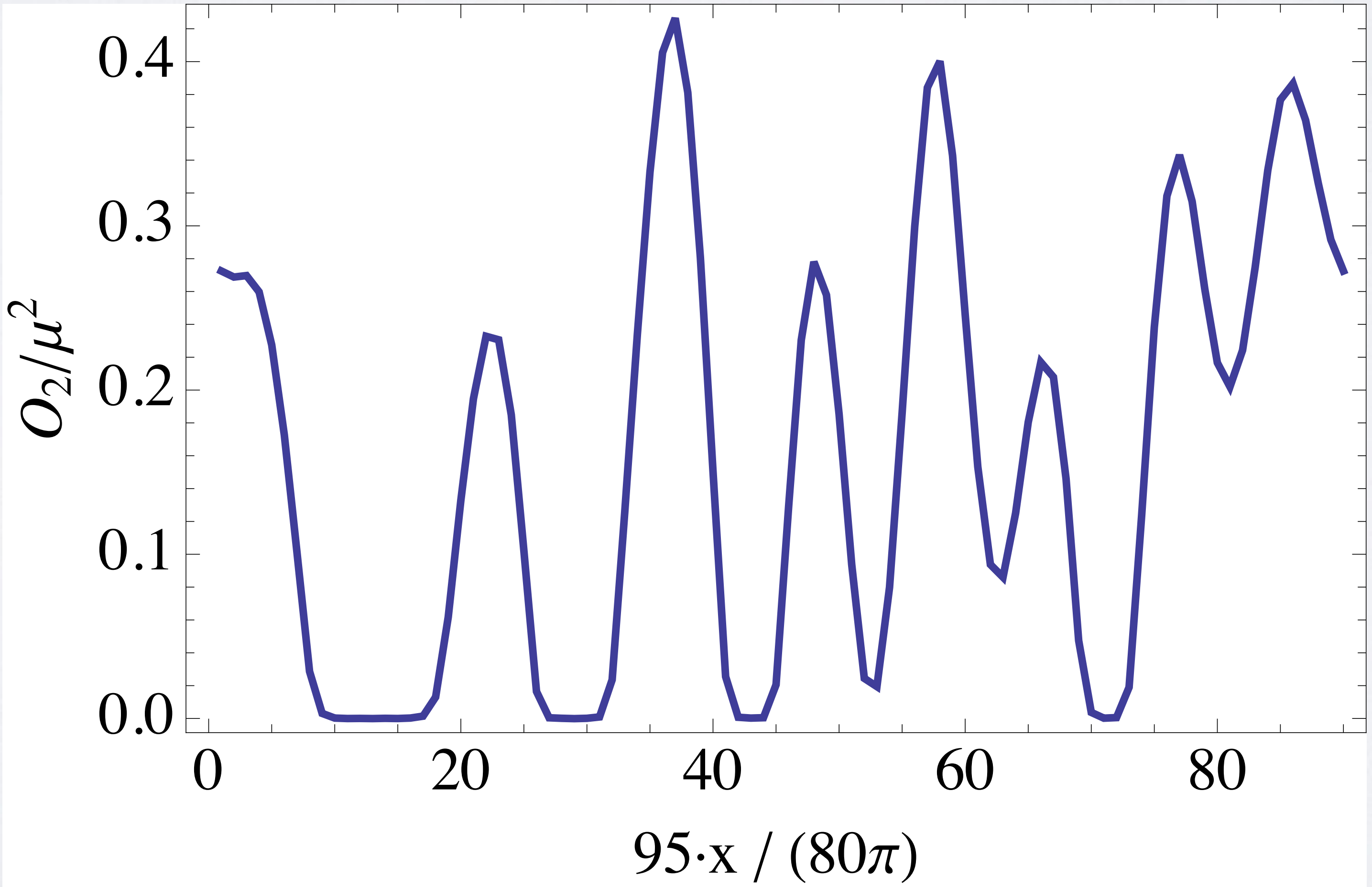

+

# Noise $\mu(x)$

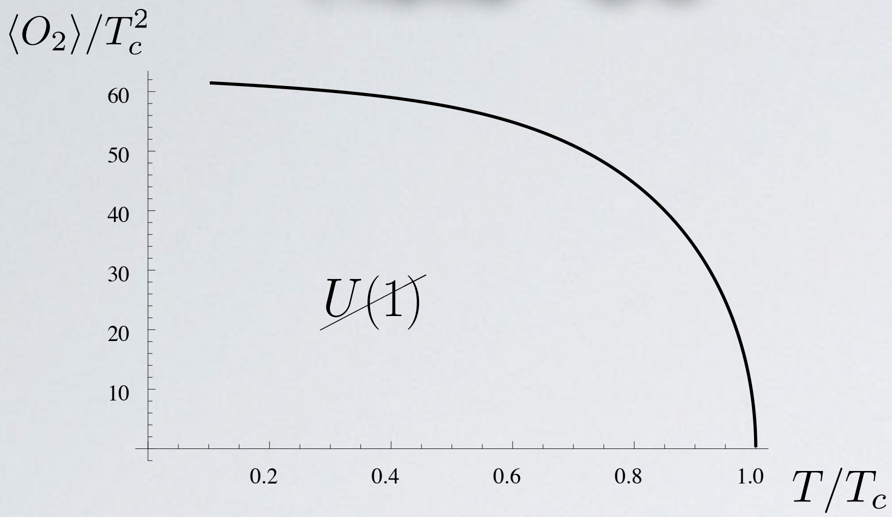



[Now taking 'uncorrelated' noise and large systems]

= ⋆ **Islands**

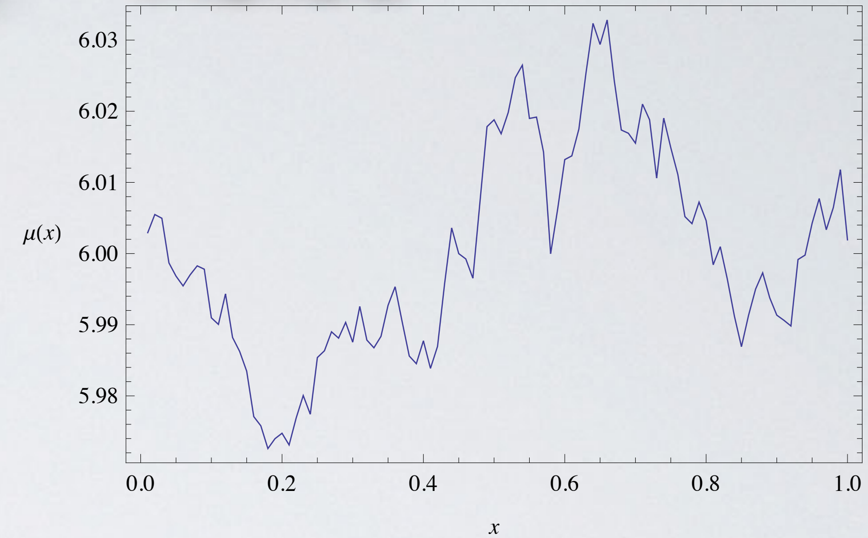


# Holo-SC

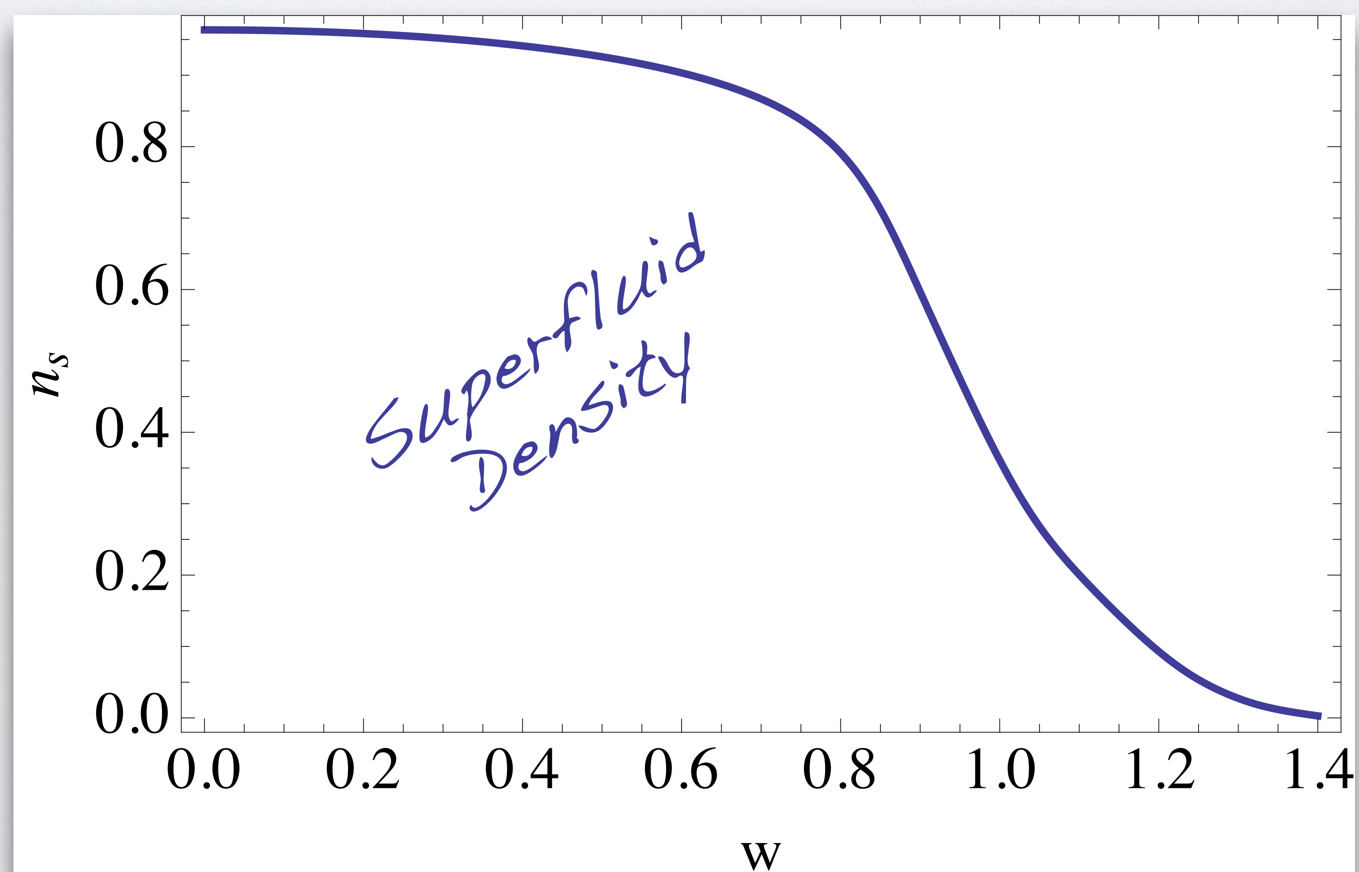



+

# Noise $\mu(x)$




= ★ **Disorder can suppress the Superconductivity**




**disorder**

# OUTLINE

> **Previously:** Correlated disorder, enhanced SC, spectrum renormalization ...

> **Motivation:** Disorder-induced SIT

> **Setup:** holo SC w/ flat noise

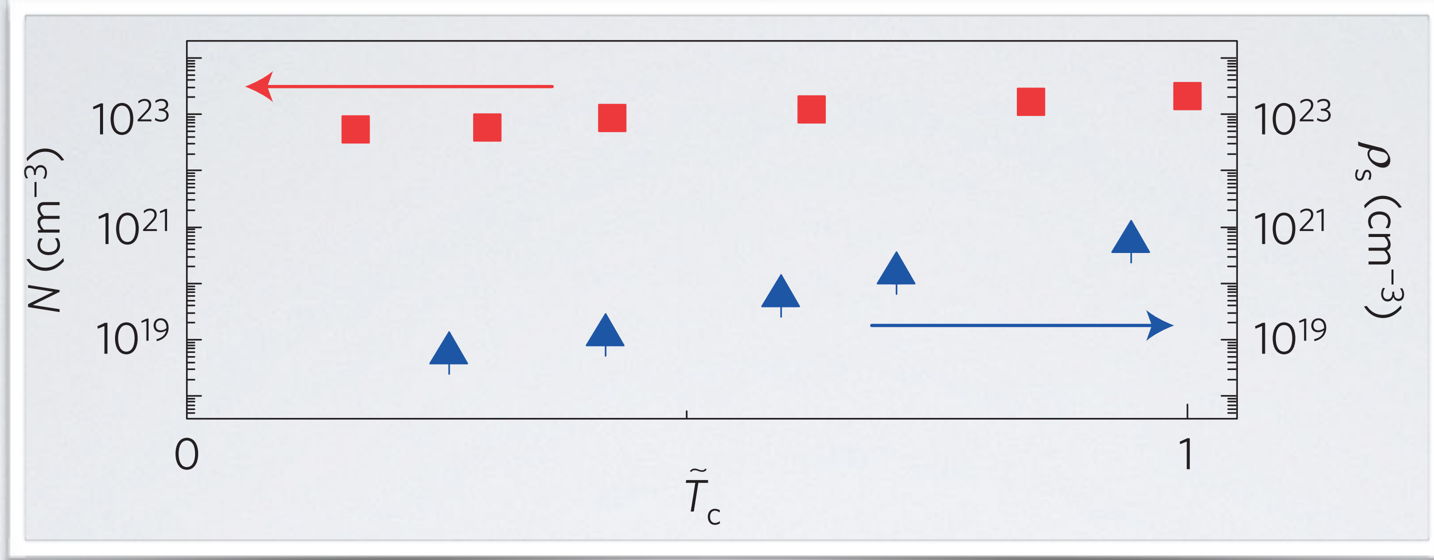
> **Results:** Islands (phase diagram), Conductivity.

> **Future:** Thin Films, backreaction (insulator?), . . .

# > SC to insulator disorder induced phase transition

## > Experiment

[Sherman et al, Nature Phys 11, 188–192 (2015)]

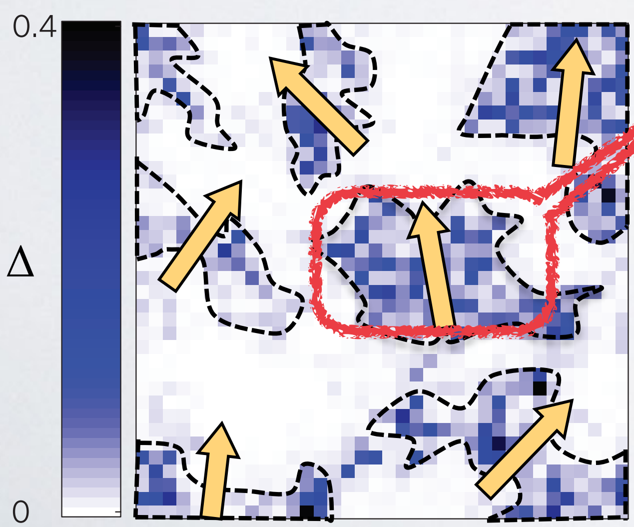



▲ Superfluid Density

disorder

## > Theory (quantum Montecarlo)

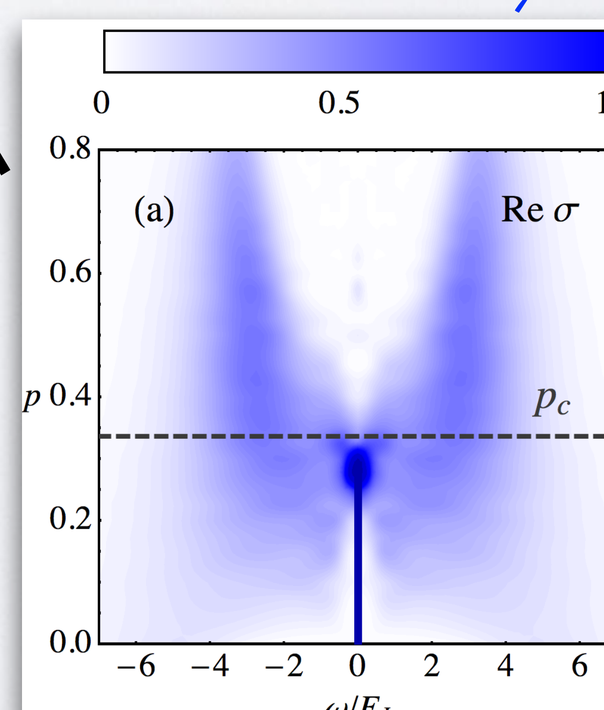
'Condensate'




Islands

Conductivity

disorder



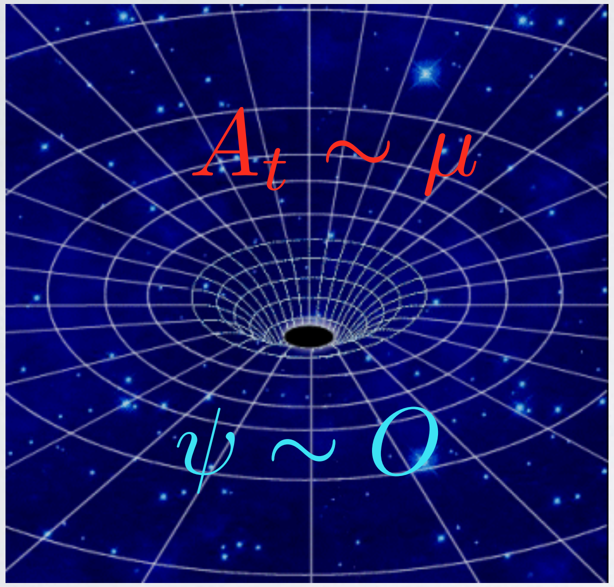

INSULATOR

SUPERCONDUCTOR

[Swanson et al, 1310.1073]

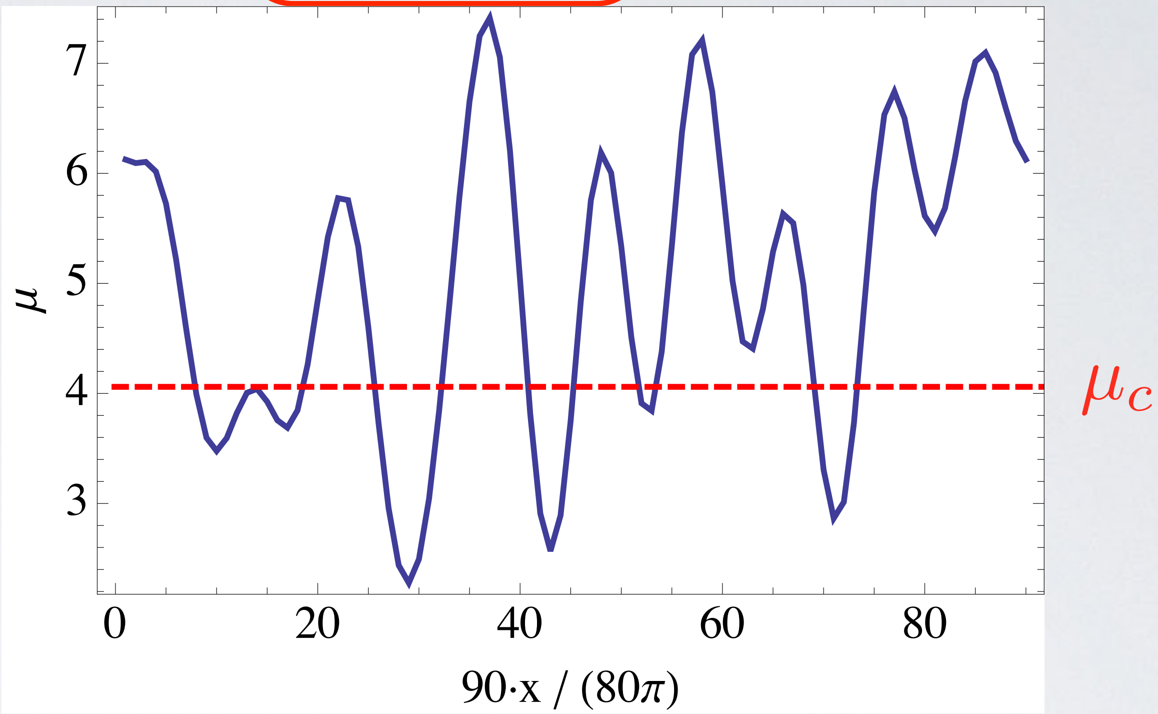
# > SETUP: Dirty Holo (s-wave) Superconductor

**Holo SC**




**+**

**Disordered** $\mu(x)$




- **Action (probe limit)** $S = \int d^4x\,\sqrt{-g}\left(-\frac{1}{4}F_{ab}\,F^{ab} - (D_\mu\Psi)(D^\mu\Psi)^\dagger - m^2\Psi^\dagger\Psi\right)$

- **Geometry: Sch-AdS BH** $ds^2 = \frac{1}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^2 + dy^2\right)\,, \quad f(z) = 1 - z^3$

- **Field content**

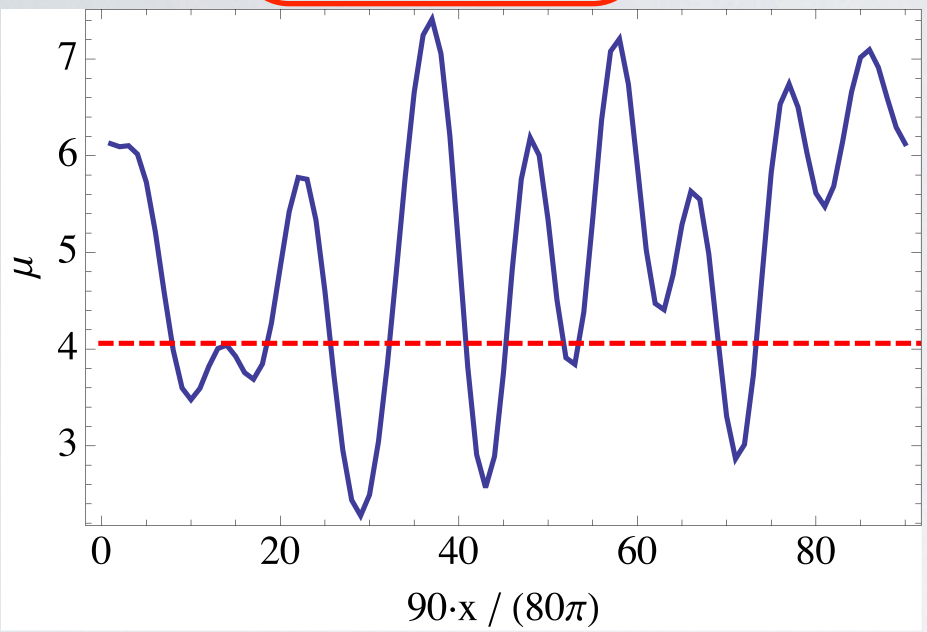
$$\Psi(x,z) = \psi(x,z)\,, \quad \psi(x,z) \in \mathbb{R} \qquad \sim \langle O(x)\rangle$$

$$A = \phi(x,z)\,dt \qquad \sim \quad \mu(x)$$

[Hartnoll et al'08]

# > SETUP: Dirty Holo (s-wave) Superconductor

Disordered $\mu(x)$




Flat Noise

$$\mu(x) = \mu_0 + w\,\mu_0 \sum_{k=k_0}^{k_*} \cos(k\,x + \delta_k)$$

- $w$ Noise strength
- $k_0 \sim$ 1/(System Size). [IR Scale]
- $k_* \sim$ 1/Correlation length [UV Scale]

EoMs ⇒ $\boxed{\text{2 Coupled PDEs}}$ with...

- UV (z=0) Boundary Conditions

$$\phi(x,z) = \mu(x) - \rho(x)\,z + \ldots$$

$$\psi(x,z) = \cancel{\psi^{(1)}(x)}\,z + \langle O(x)\rangle\, z^2 + \ldots$$

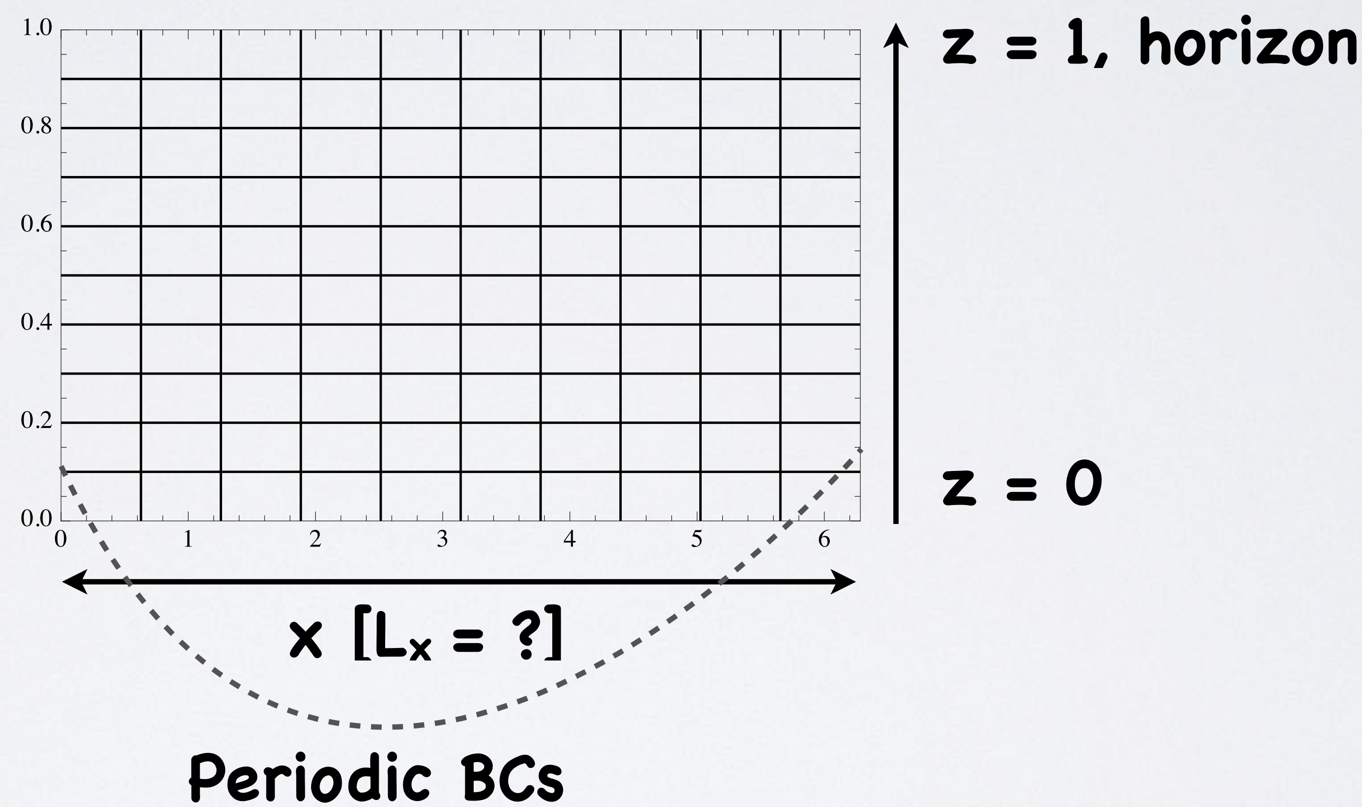
# > SETUP: Solving the background ..................

Flat Noise

$$\mu(x) = \mu_0 + w\,\mu_0 \sum_{k=k_0}^{k_*} \cos(k\,x + \delta_k)$$

- $w$ Noise strength
- $k_0 \sim$ 1/(System Size). [IR Scale]
- $k_* \sim$ 1/Correlation length [UV Scale]

- System on a grid




- With (ideally): $\boxed{L_x \gg 1 \; [\to \; k_0 \ll 1] \,, \; k_* \gg 1}$ 'Uncorrelated Noise'

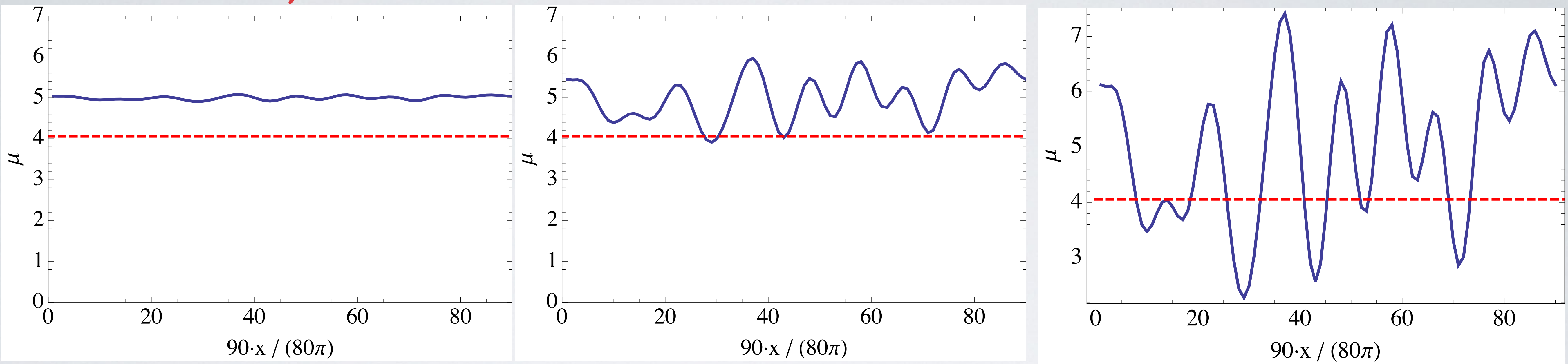
* [in units of temperature]

# > Results: The Inhomogeneous Condensate

$\mu = 5 \rightarrow T \sim 0.8\, T_c$

$L_x = 80\pi\ ,\ 9$ modes

## Chemical potential

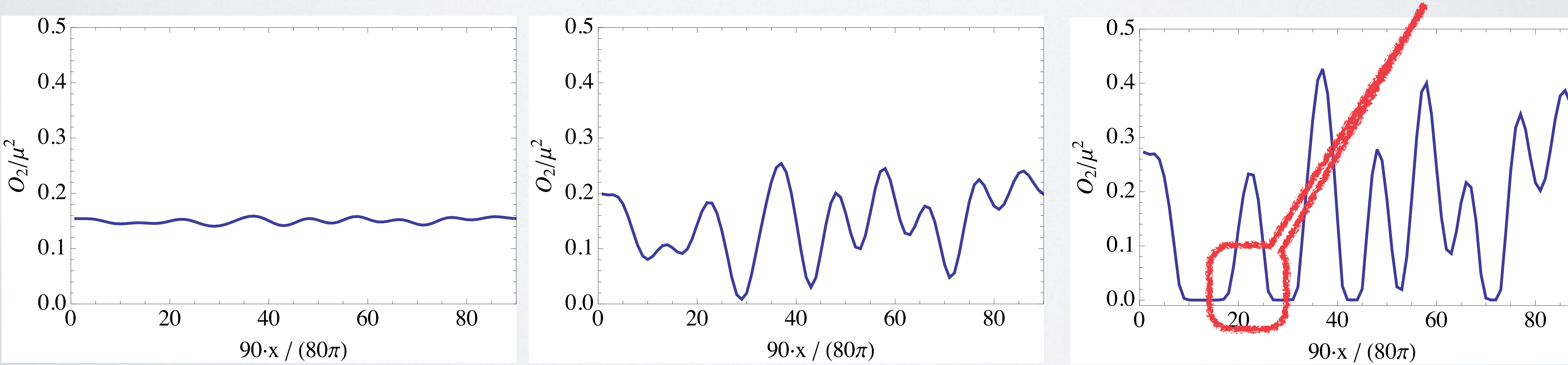



$w = 0.004$ $w = 0.048$ $w = 0.12$

**disorder** $\longrightarrow$

## Condensate

**Islands?**

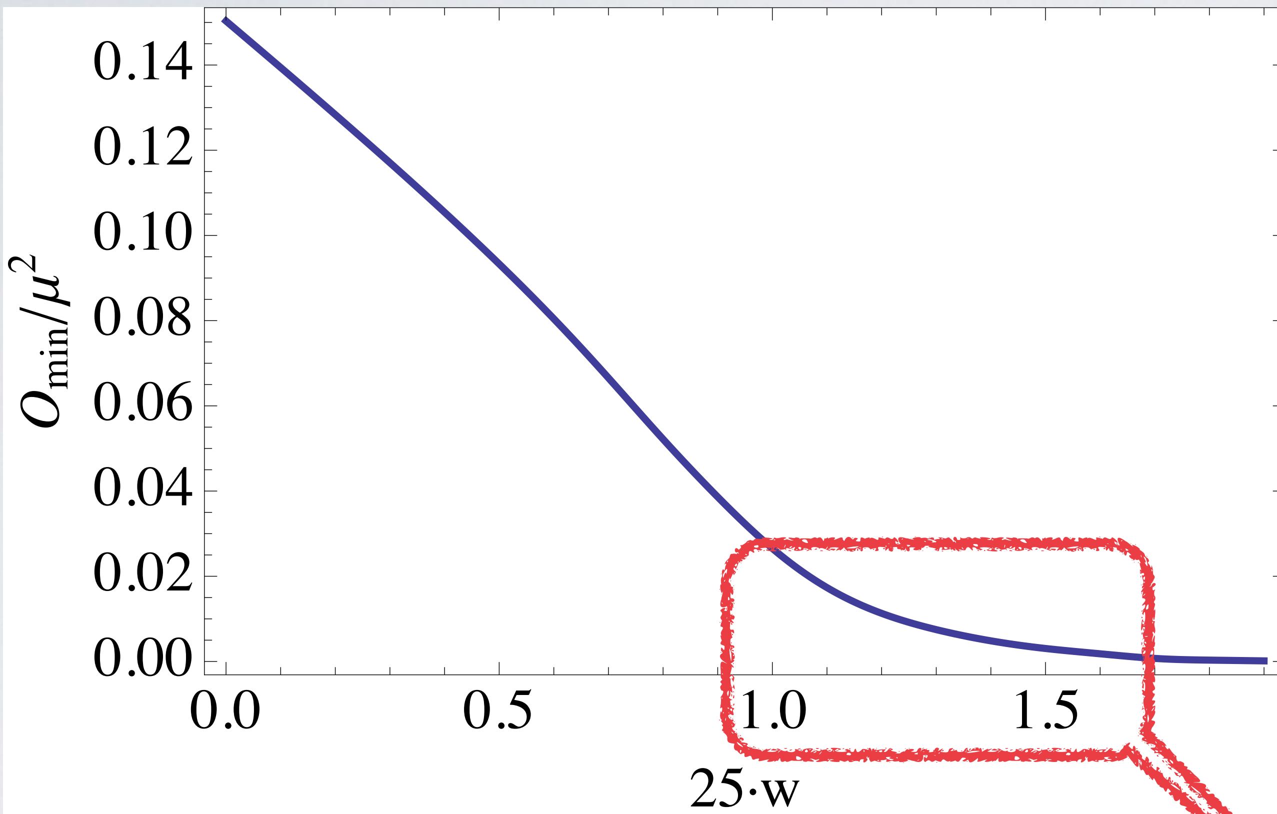


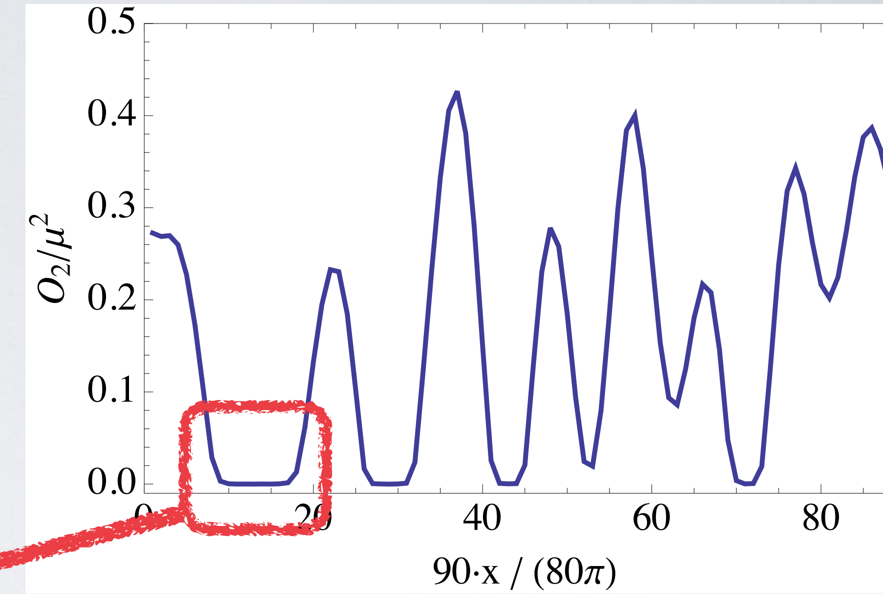
# > Results: ISLANDS?

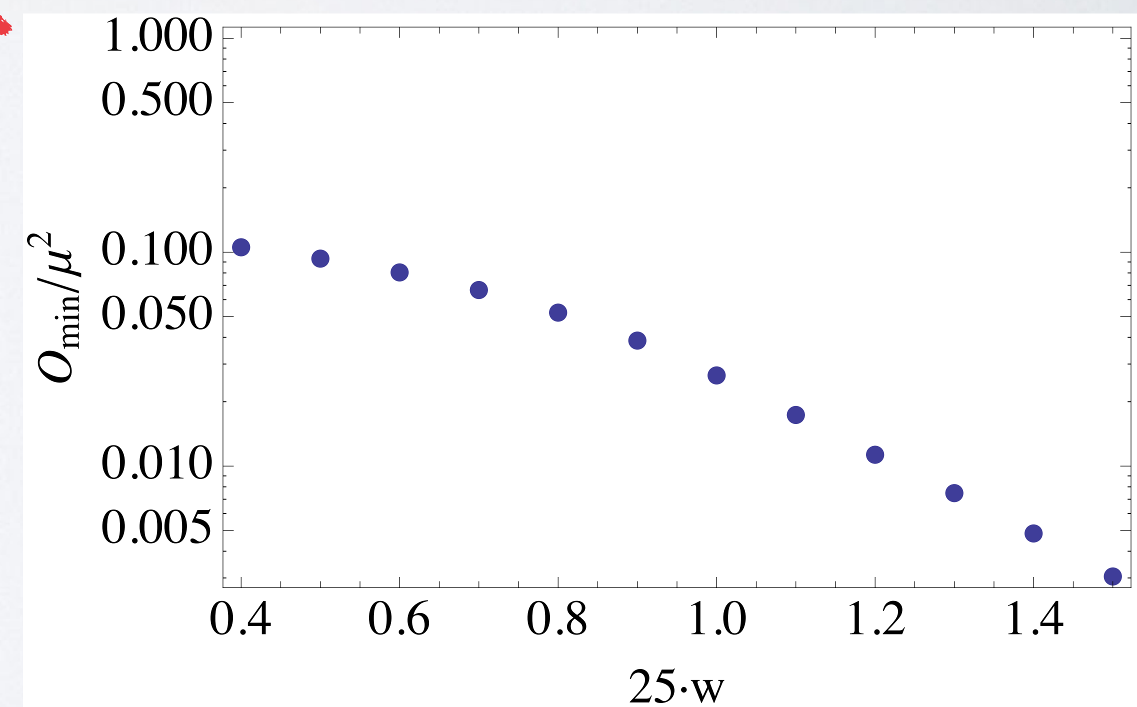
$\mu = 5 \to T \sim 0.8\, T_c$

$L_x = 80\pi\ ,\ 9$ modes

> Let's plot the minimum of the condensate




**The condensate dies away exp-like**

➡ *no critical noise?*

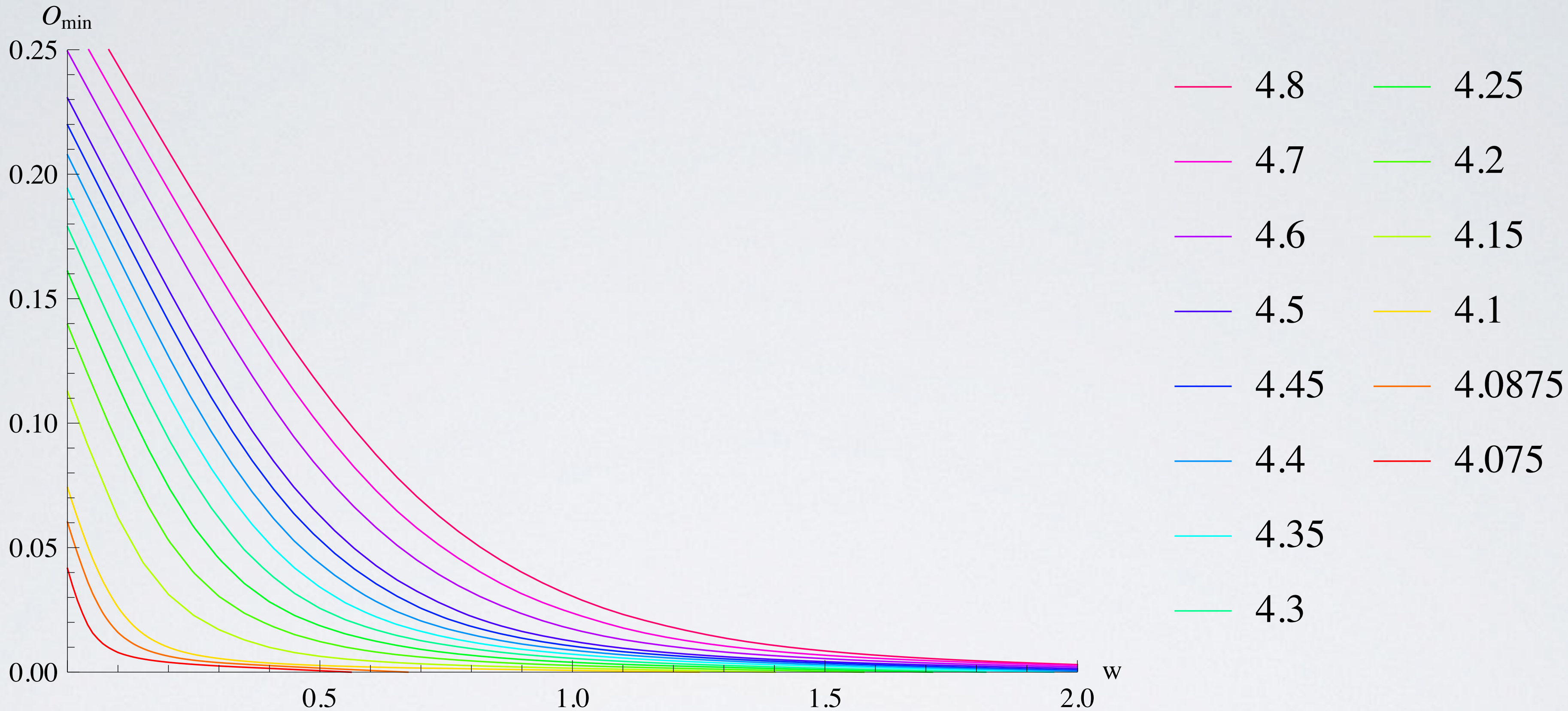
**Finite Size + Spec. Renorm. (kills higher modes)**

➡ *Condensate leaks!*

# > Results: Tentative Phase Diagram

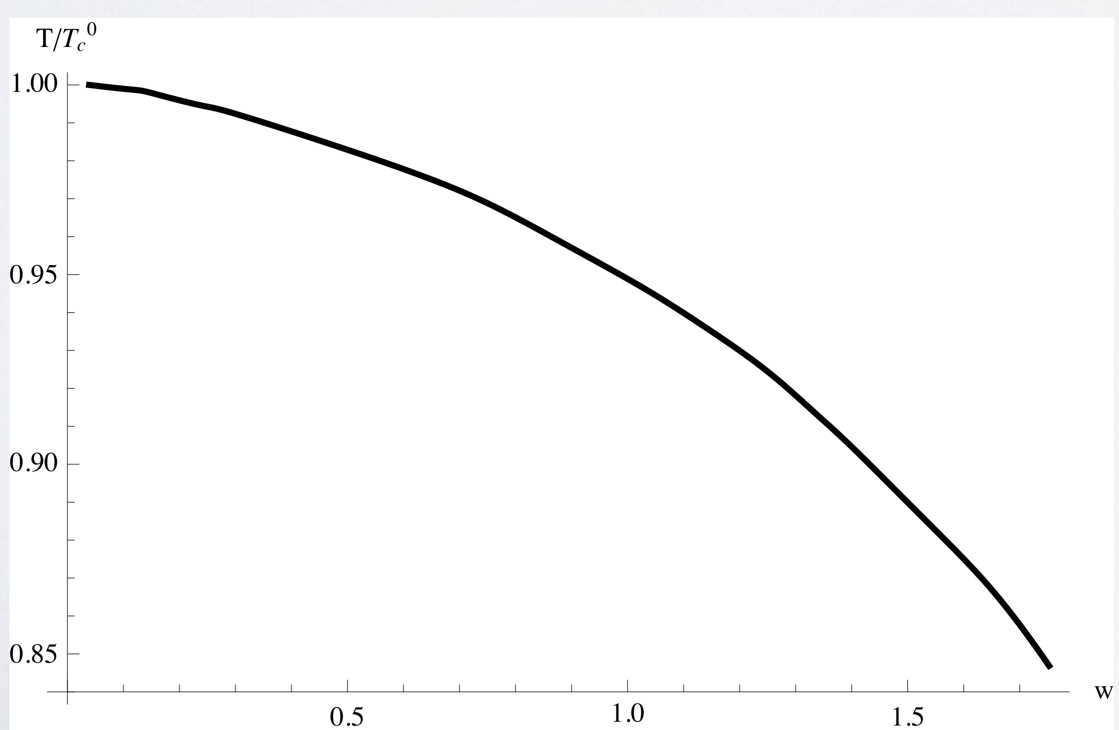
**Plotting the minimum vs noise for several values of $\mu$**

$O_{\text{min}}$




**disorder**

$$\frac{T_c}{T_c^{w=0}}$$



## > Computing the conductivity [⇨ Superfluid density]

$$\boxed{\sigma_x(\omega) = \frac{\langle j_x(x,\omega)\rangle}{E_x(\omega)}}$$

Study perturbations

$$\boxed{\delta A_x = a_x(z,x)\, e^{-i\omega\, t} \sim j_x(x,\omega)}$$

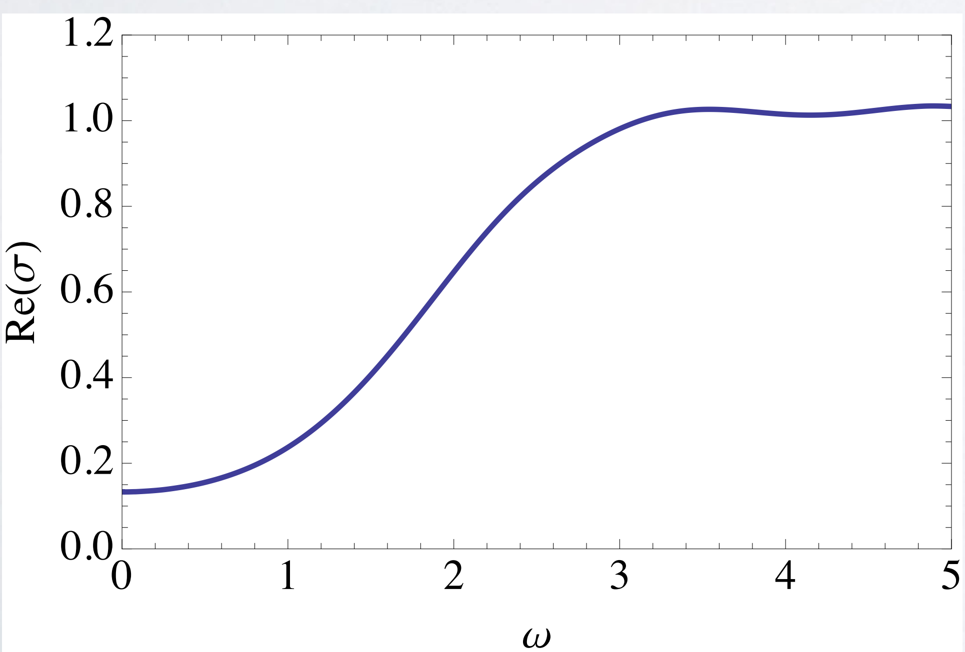
which couple to

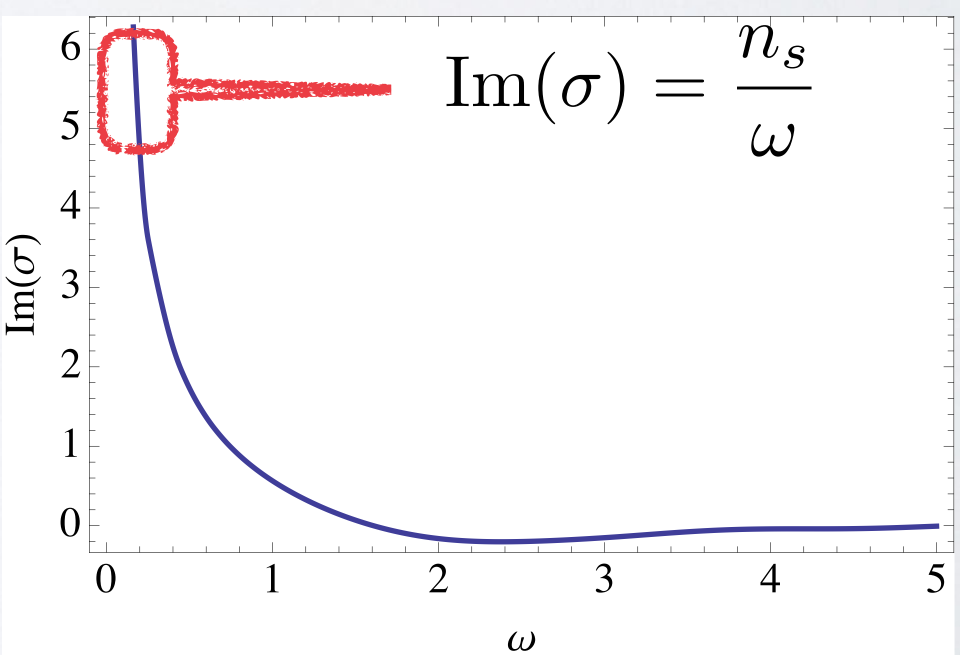
$$(\delta A_t\,,\,\delta\Psi) = (a_t(z,x)\,,\,\psi_r(z,x)\,,\,\psi_i(z,x))e^{-i\omega\, t}$$

⇨ **4 Coupled linear PDEs**

## > Reminder: Homogeneous case

$$\mu = 5 \to T \sim 0.8\, T_c$$

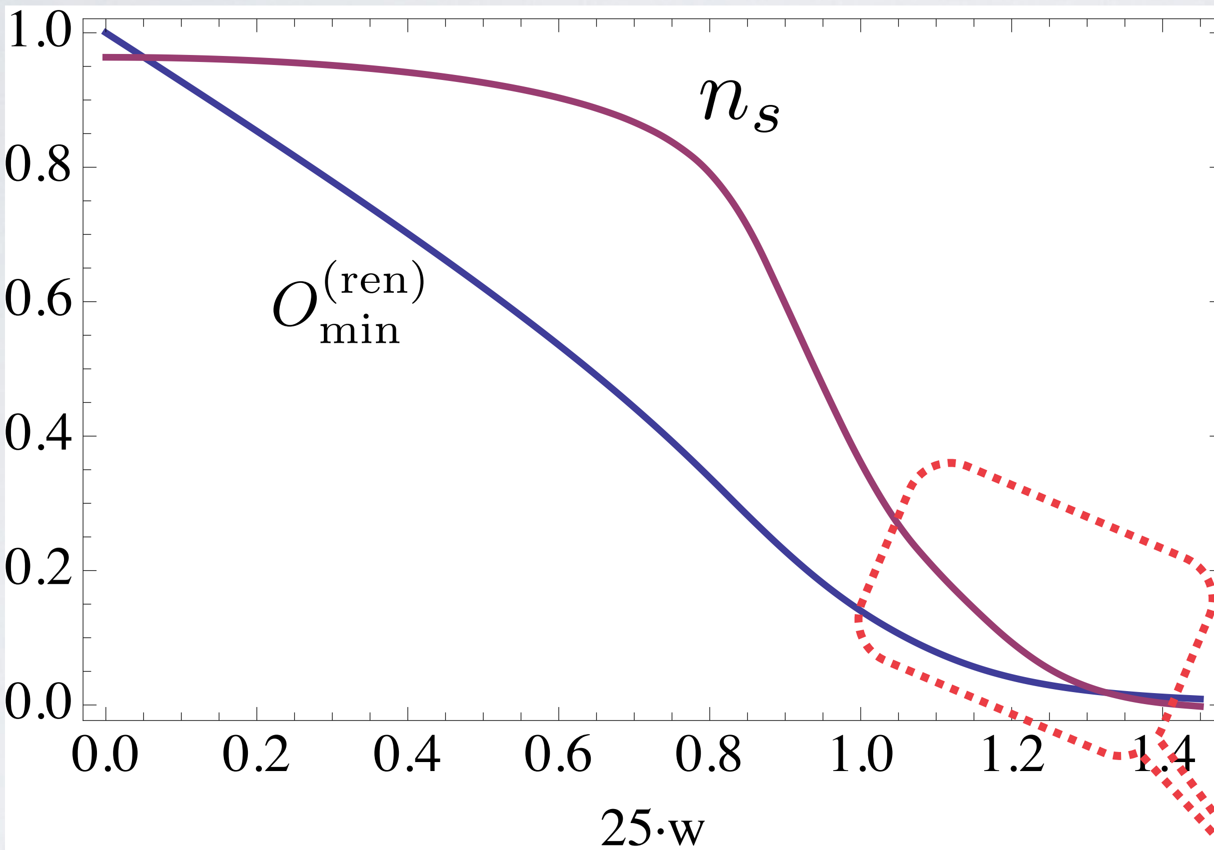


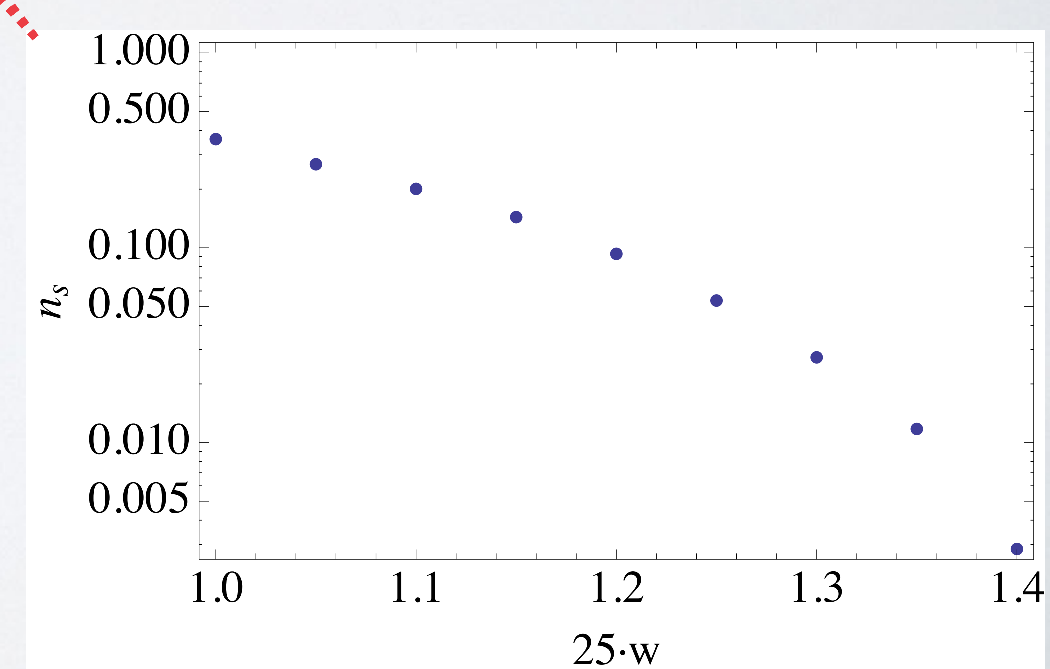
## > Noisy conductivity

$$\mu = 5 \to T \sim 0.8\, T_c$$
$$L_x = 80\pi\ ,\ 9 \text{ modes}$$

### > The superfluid density




disorder

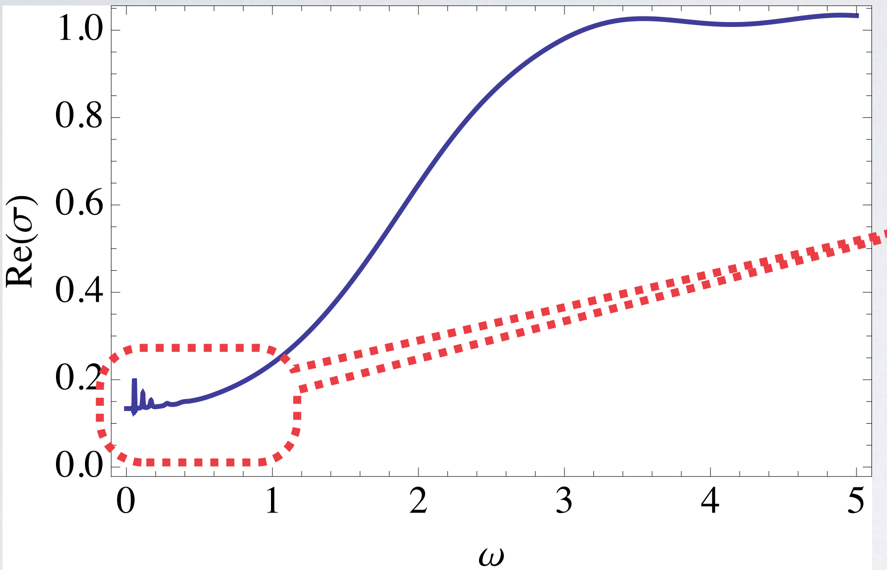


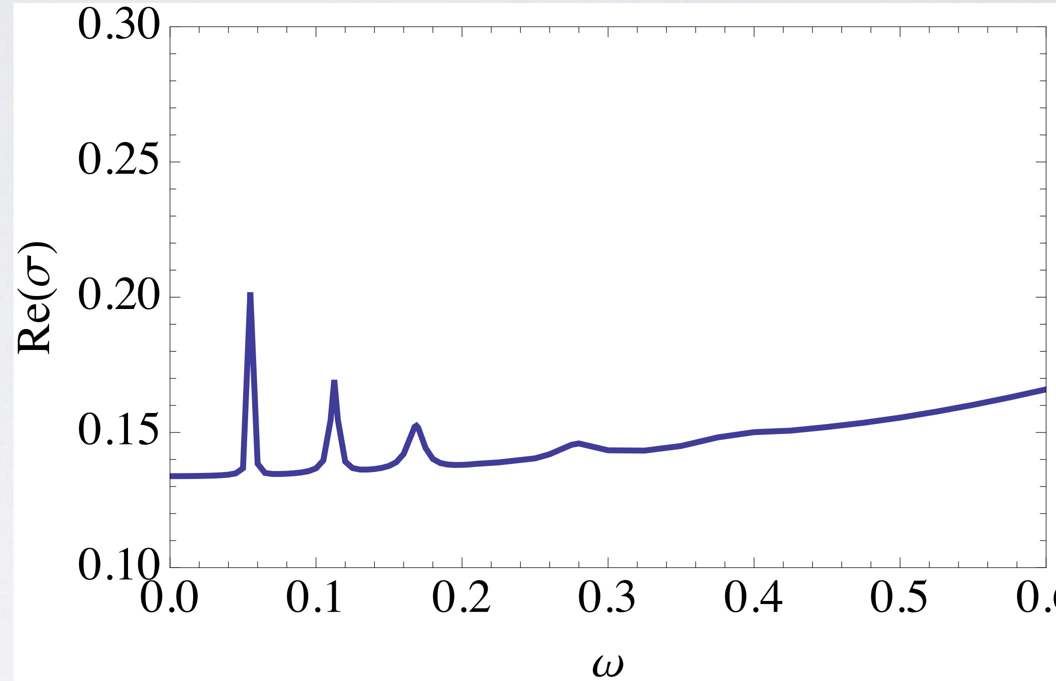

➡ Superfluid density follows $O_{min}$ and dies away (exp-like)

# > Noisy conductivity

$$\mu = 5 \to T \sim 0.8\, T_c$$
$$L_x = 20\pi\ ,\ 9 \text{ modes}$$

## > The Conductivity [averaged over x]

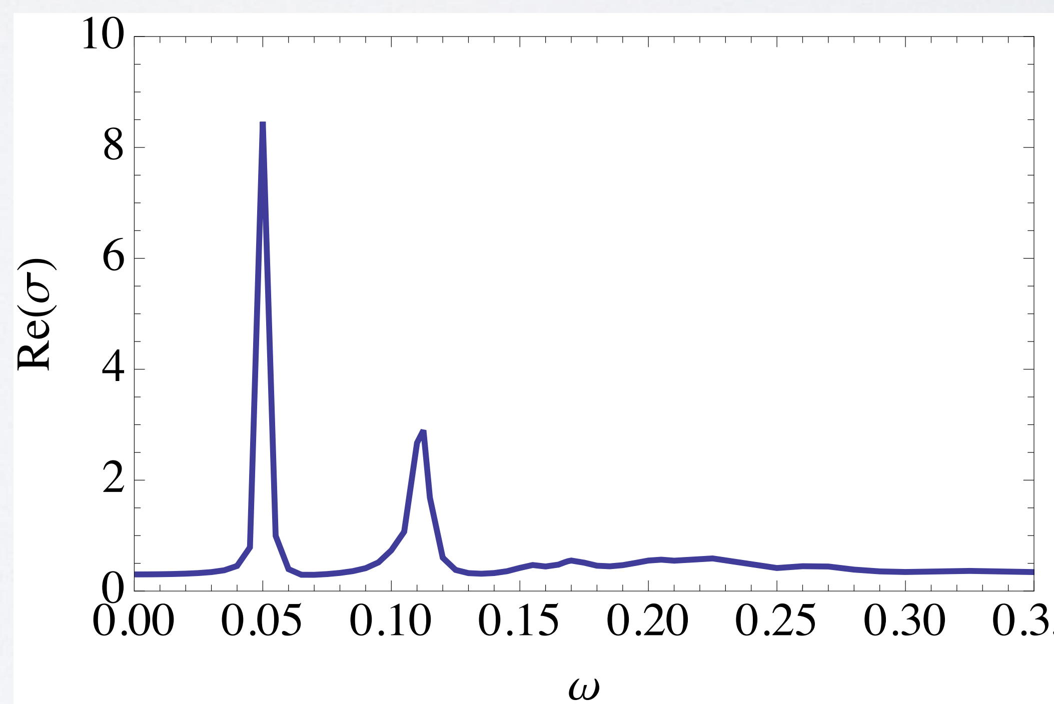



$w = 0.004$

disorder

$w = 0.04$




- Resonances appear
- 'Steal' spectral weight from the SC delta

[See also Donos&Gauntlett, 1409.6875]

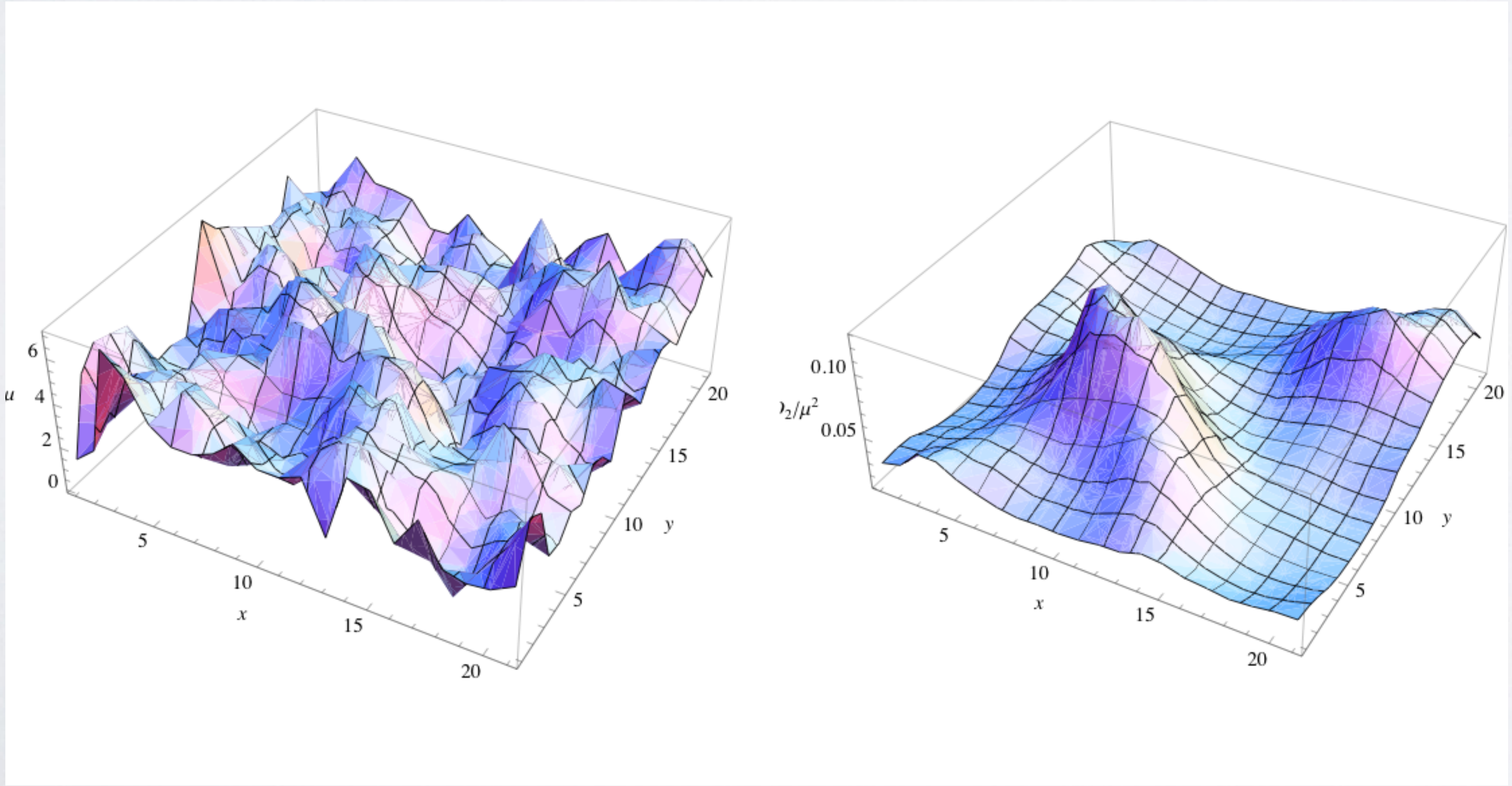
# > Outlook &To Do

>Disordered holo SCs: both s- and p-wave [1308.1920, 1407.7526]

>1D Islands of Superfluidity

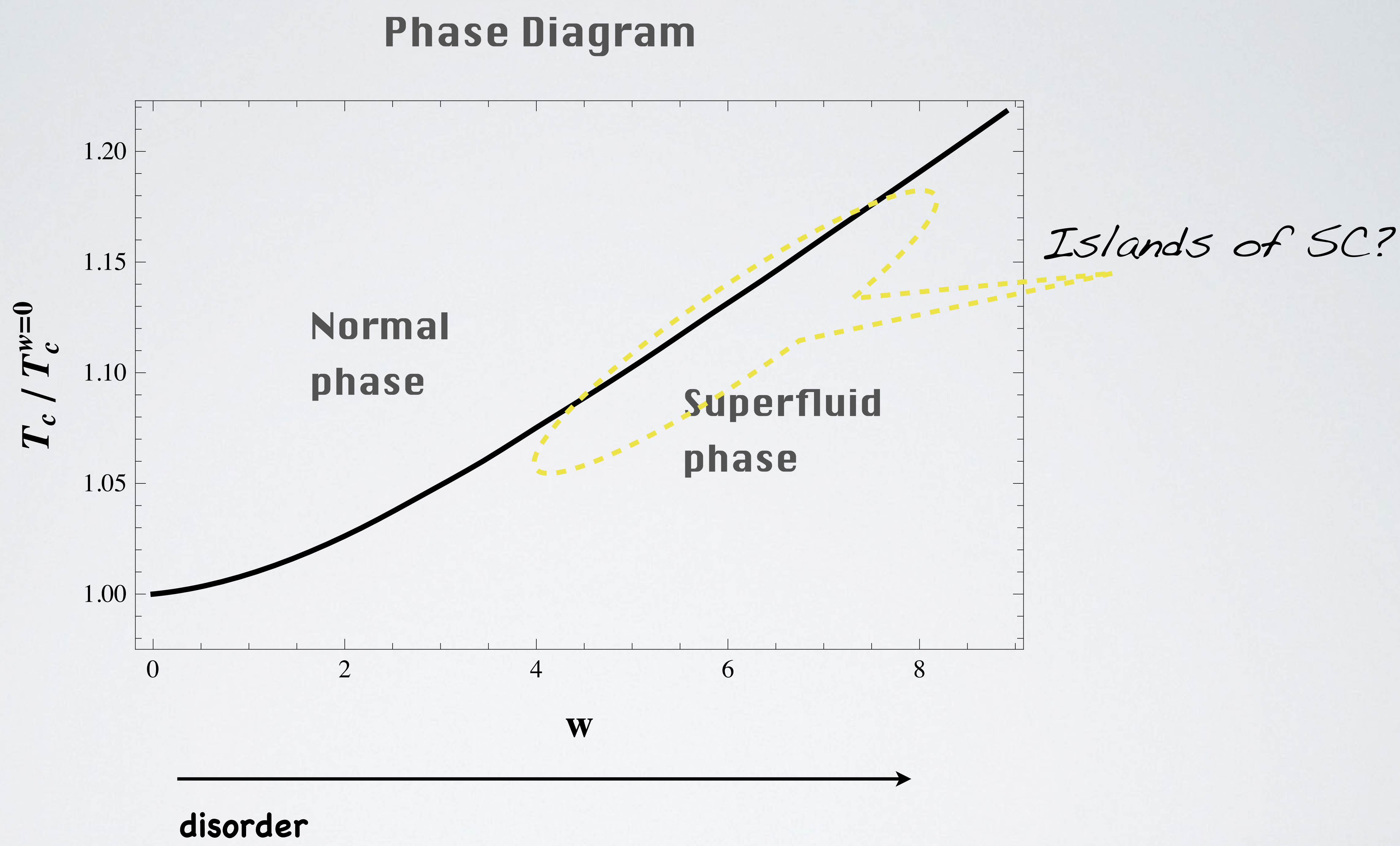
>Conductivity and superfluid density ⇨ phase diagram

>Future Thin Films, backreaction (insulator?), . . .



> And now, some additional slides...

## ⋆ Enhancement and the island menace




Seen before in CM (hard-core bosons)

- 'Disorder-induced superfluidity', Dang et al, Phys. Rev. B 79, 214529

★ **Spectrum 'renormalization'**

>>> **Noisy chemical potential**

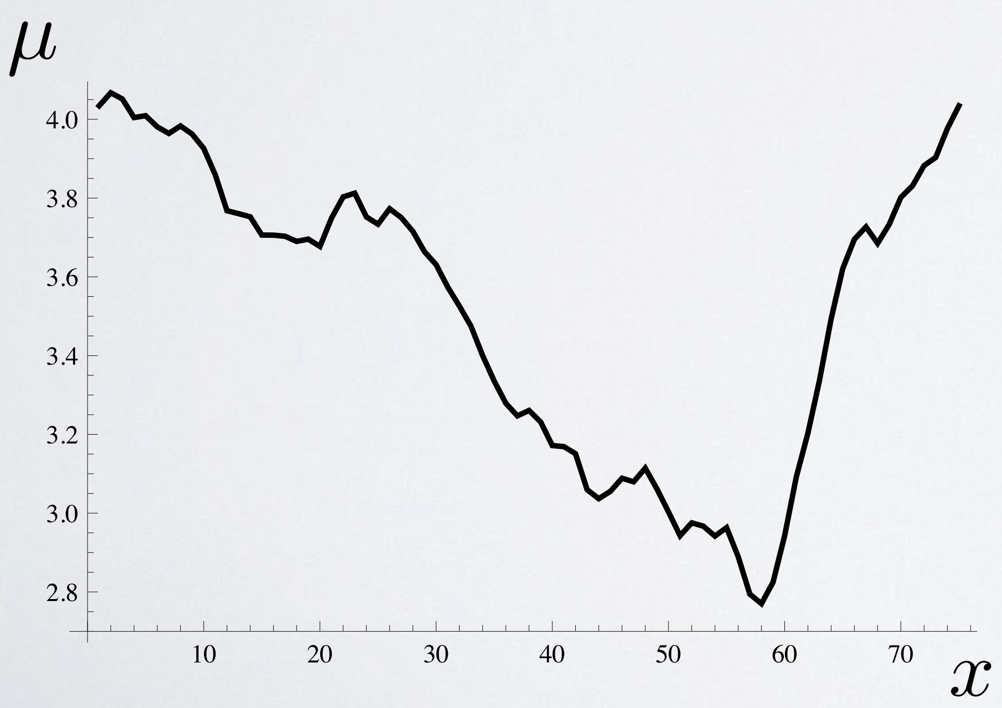
Random phases

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k} \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha} \cos(k\,x + \delta_k)$$

Power spectrum

Strength of noise

>**input spectrum** $\boxed{S_k = \frac{1}{k^{2\alpha}}}$




>**output spectra**

*Condensate*

*Charge density*

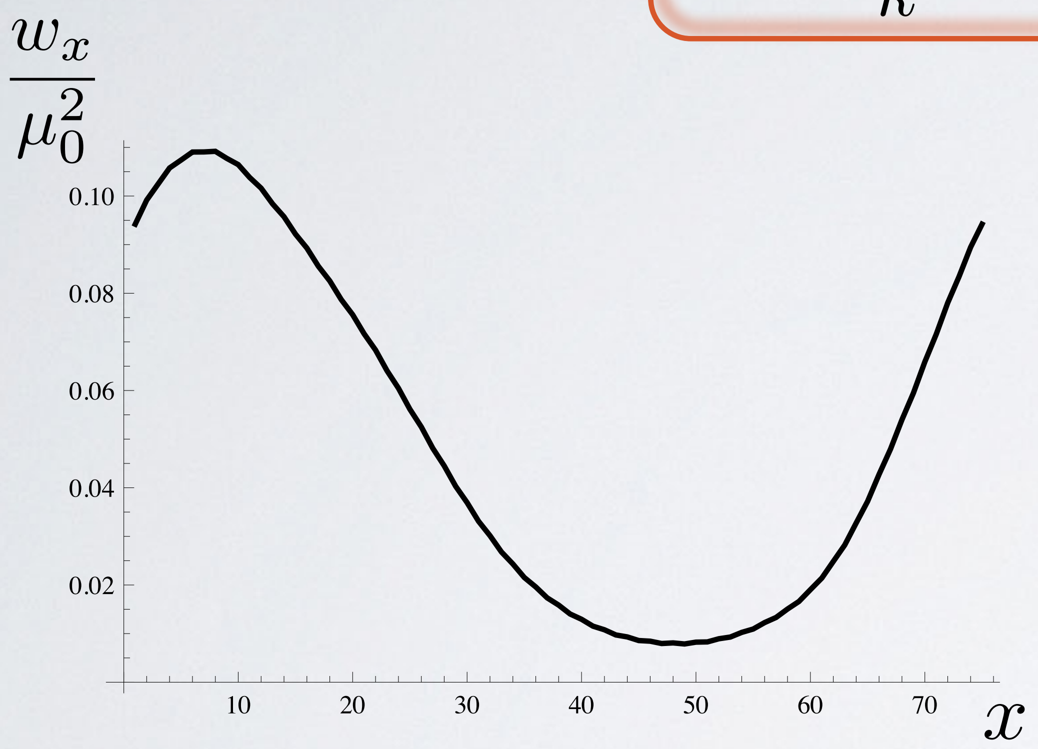
$$S_k = \frac{1}{k^\Gamma}$$ **?**

★ **Spectrum 'renormalization'**

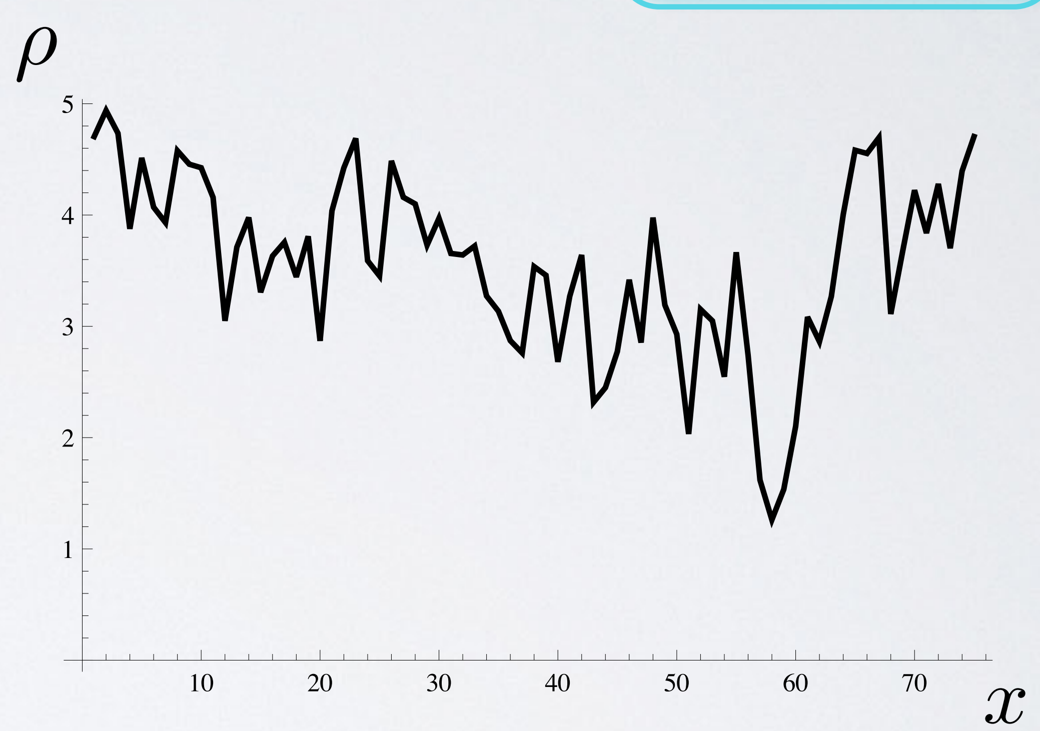
>input spectrum $S_k = \dfrac{1}{k^{2\alpha}}$

> OUTPUT

Condensate $S_k = \dfrac{1}{k^{2\alpha+4}}$




Charge density $S_k = \dfrac{1}{k^{2\alpha-2}}$

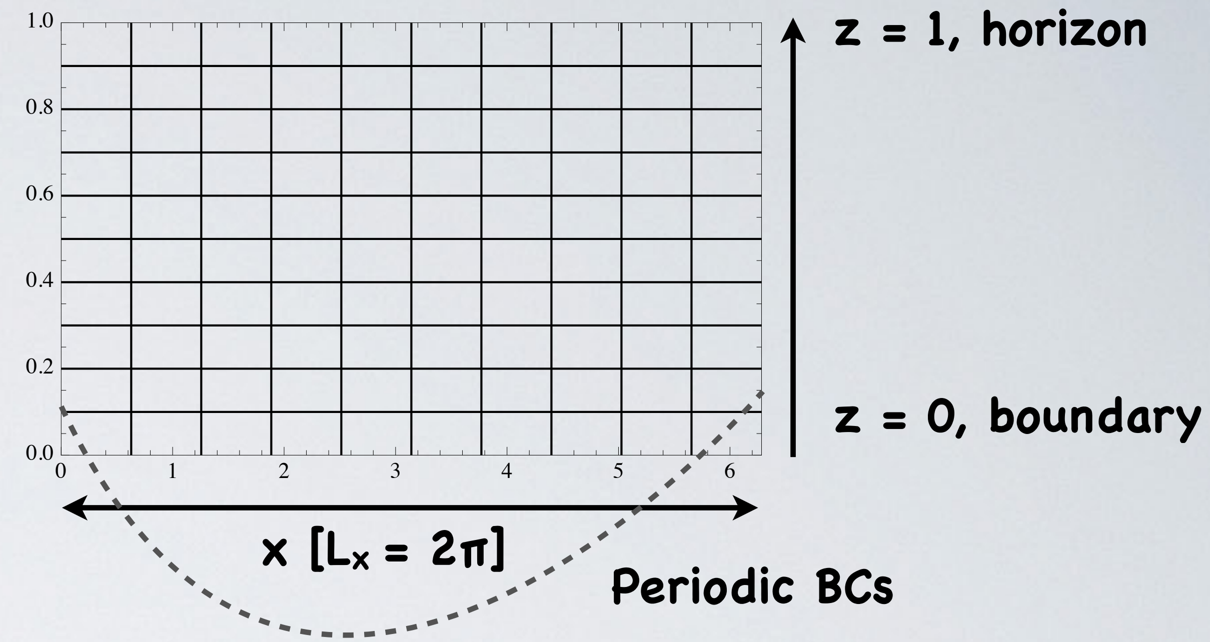



Hints of universality

- **S-wave** [1308.1920]
- [Hartnoll&Santos 1402.0872]
- **Fundamental matter (D3-D5)** [w/ M. Araújo, J. Lizana, I.S. Landea]
- **FT: noisy U(1) @ finite T** [D. Musso, I.S. Landea]

## > Noisy chemical potential

- GRID

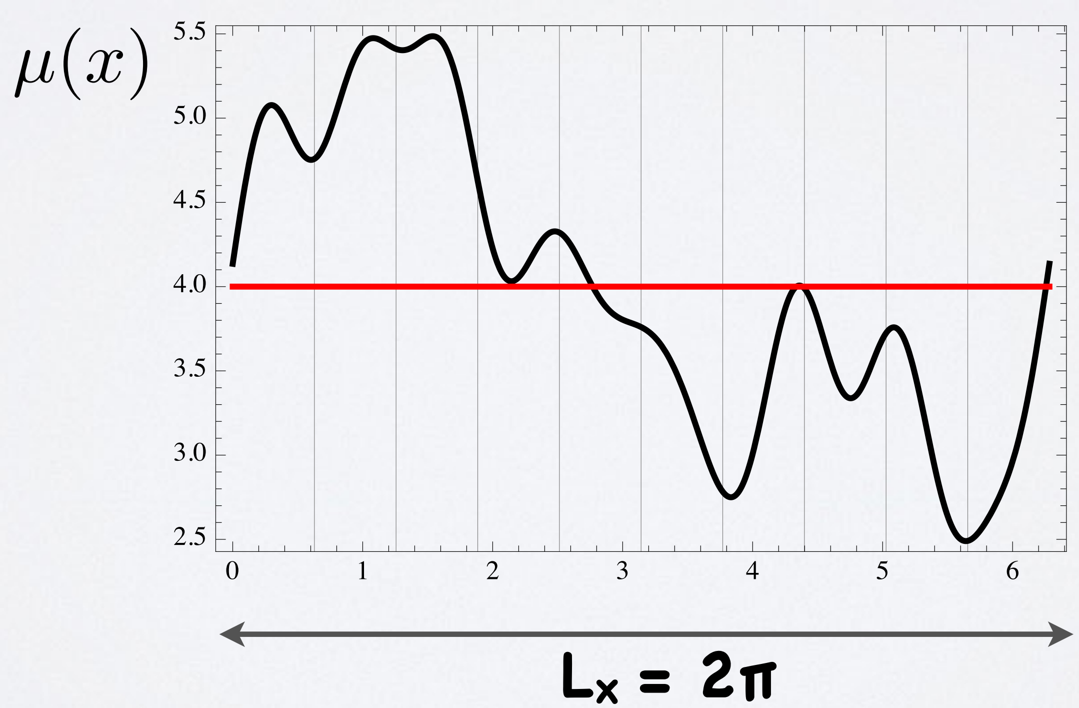



Random phases

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{\boxed{S_k}} \cos(k\,x + \boxed{\delta_k}) = \mu_0 + \boxed{\epsilon} \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha} \cos(k\,x + \delta_k)$$

Power spectrum

Strength of noise



# > Noisy chemical potential

- Noise through random phases

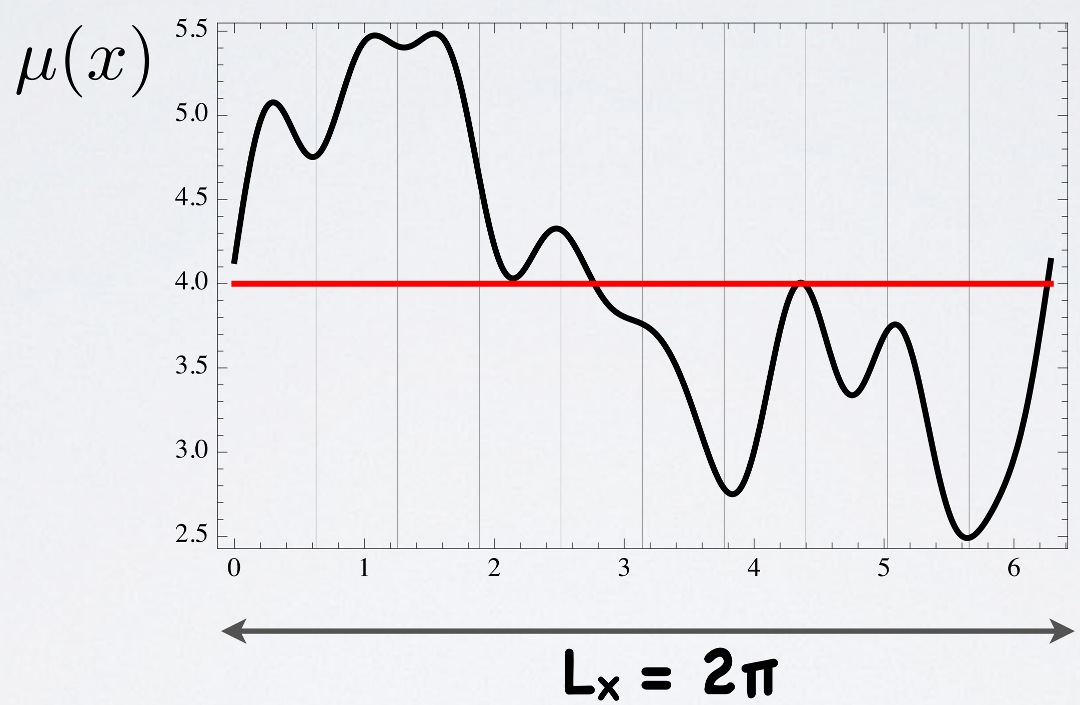
$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k} \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^{\alpha}} \cos(k\,x + \delta_k)$$

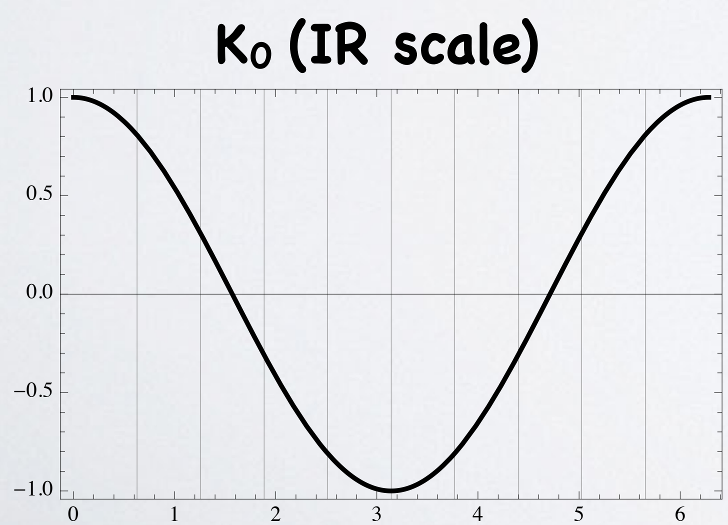
Random phases

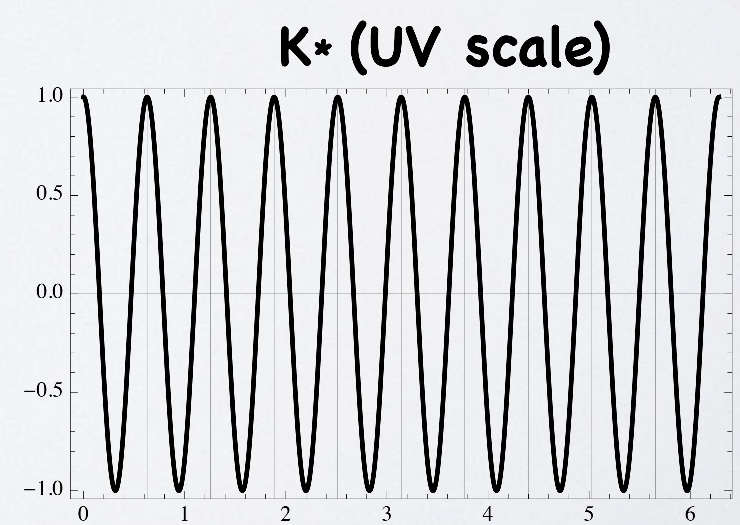
Power spectrum

Strength of noise

- System on a grid



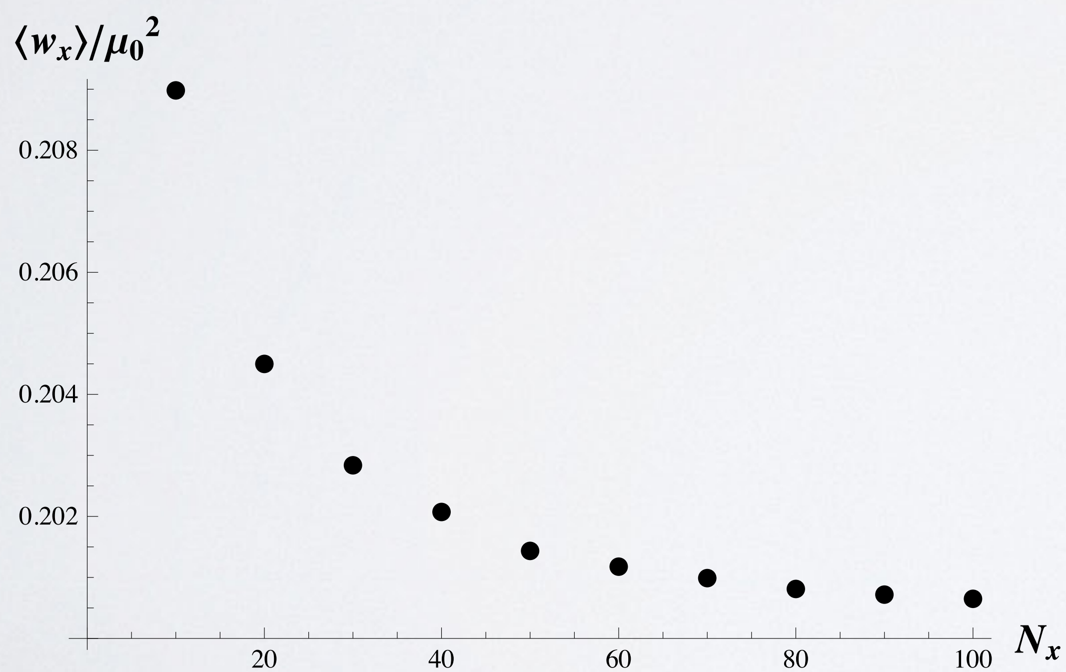



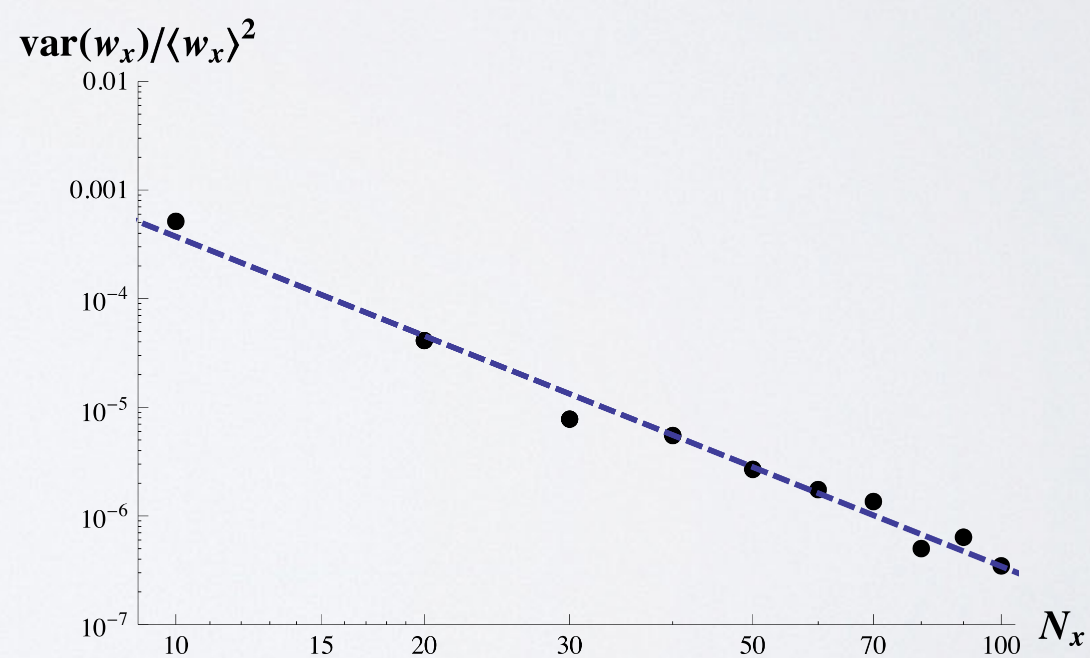
## > Thermodynamic limit

- **Thermo limit: Noise correlation length << System length**
  - > Flat spectrum noise: correlation length $\propto$ 1 / (grid size)

- **Condensate and Charge density are self-averaging in the thermo limit:**
  - > $X_n$ is self-averaging when $$\frac{< X_n^2 > - < X_n >^2}{< X_n >^2} \to 0$$

Condensate




$$\boxed{\log(\mathrm{var}(w_x)/\langle w_x \rangle^2) = -0.90 - 3.03\,\log(N_x)}$$

# > Thermodynamic limit

- **Thermo limit: Noise correlation length << System length**
  - > Flat spectrum noise: correlation length $\propto$ 1 / (grid size)

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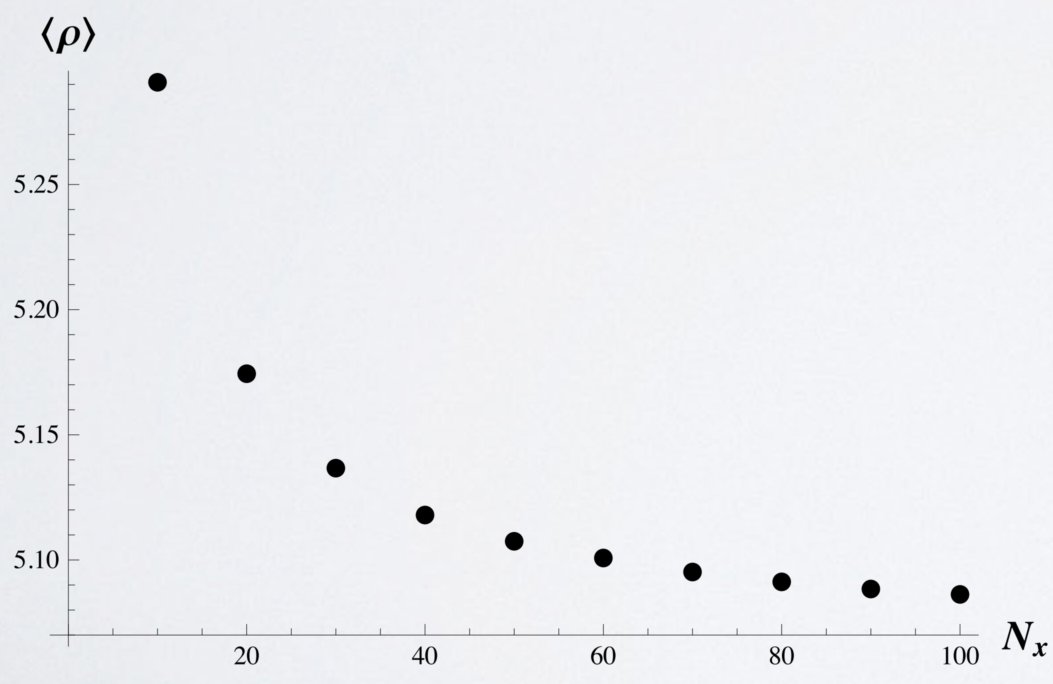
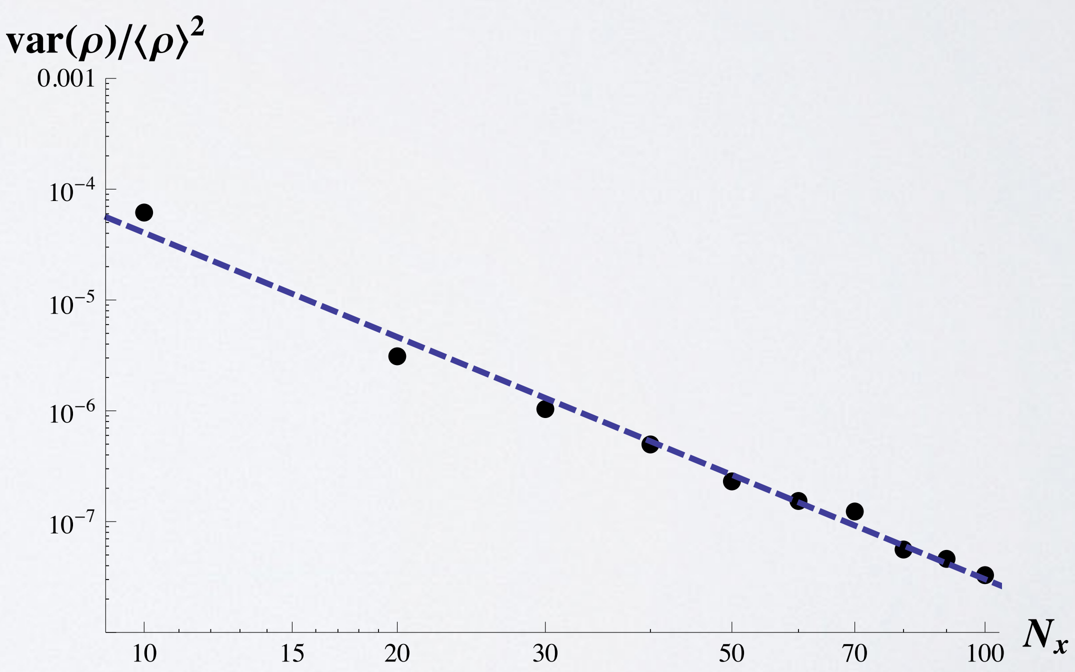
Charge density



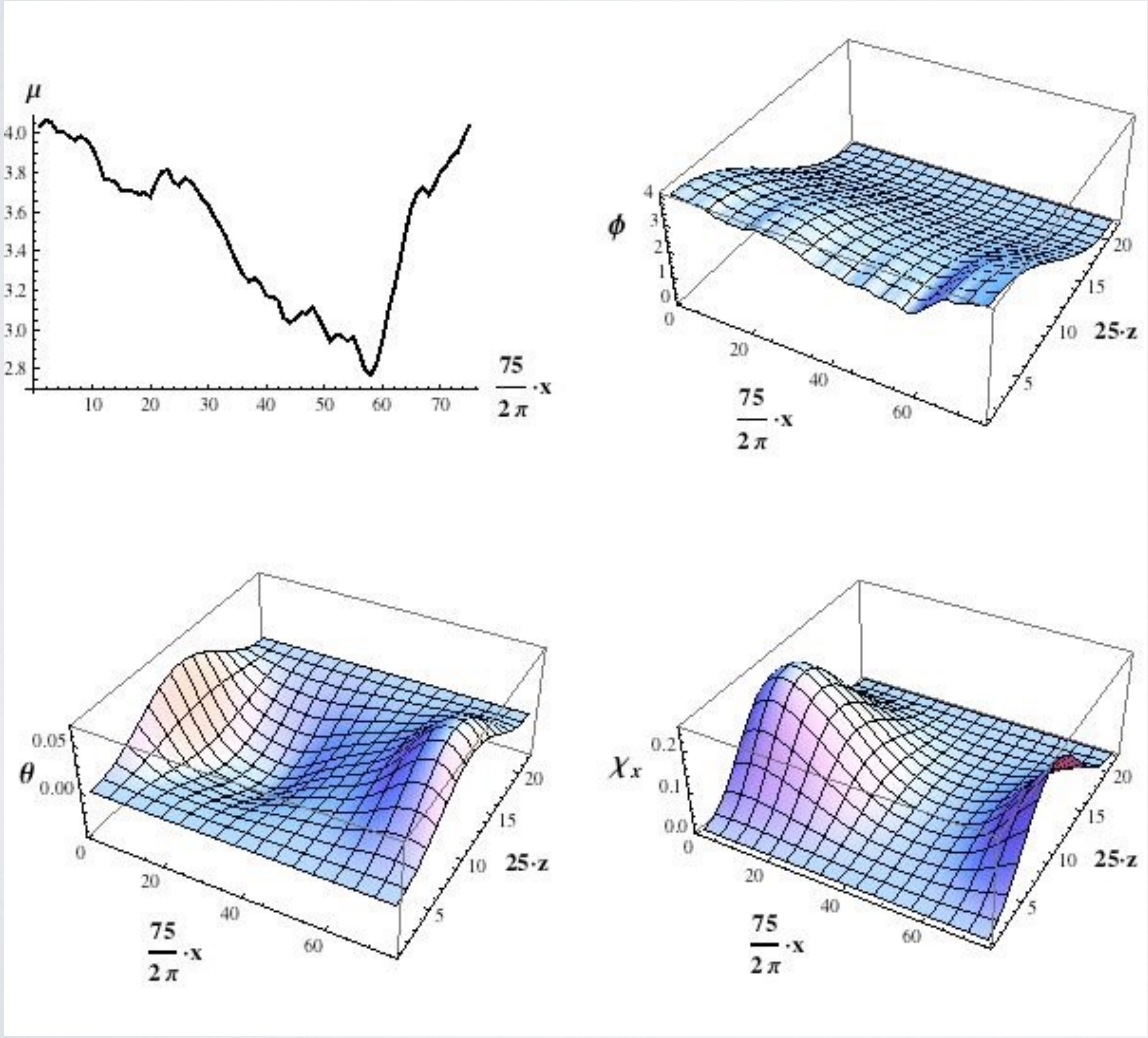

$$\boxed{\log(\mathrm{var}(\rho)/\langle\rho\rangle^2) = -2.92 - 3.13\,\log(N_x)}$$

## > Simulation #1

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\, \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\, \cos(k\,x + \delta_k)$$

$$w = 25\epsilon/\mu_0$$

- $\mu_0 = 3.50\,,\ \alpha = 1.50\,,\ w = 3.50 \quad [\mu_0 < \mu_c = 3.66]$




$L_x = 2\pi \rightarrow K_0 = 1$

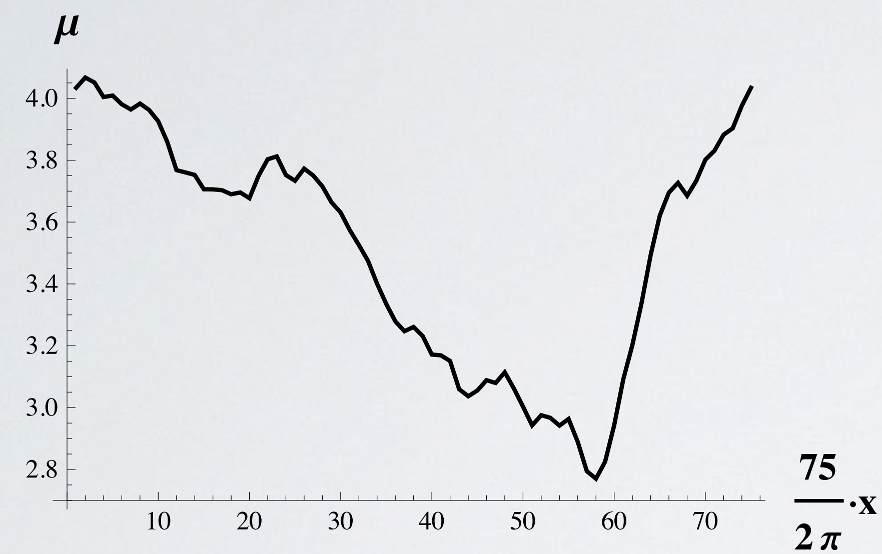
$N_z \times N_x = 25 \times 75$

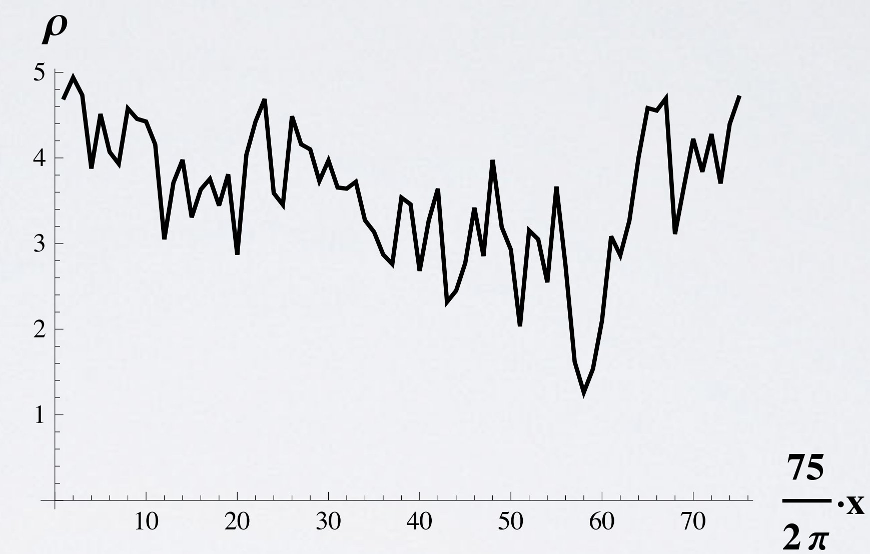
# > Simulation #1

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\, \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\, \cos(k\,x + \delta_k)$$

$$w = 25\epsilon/\mu_0$$

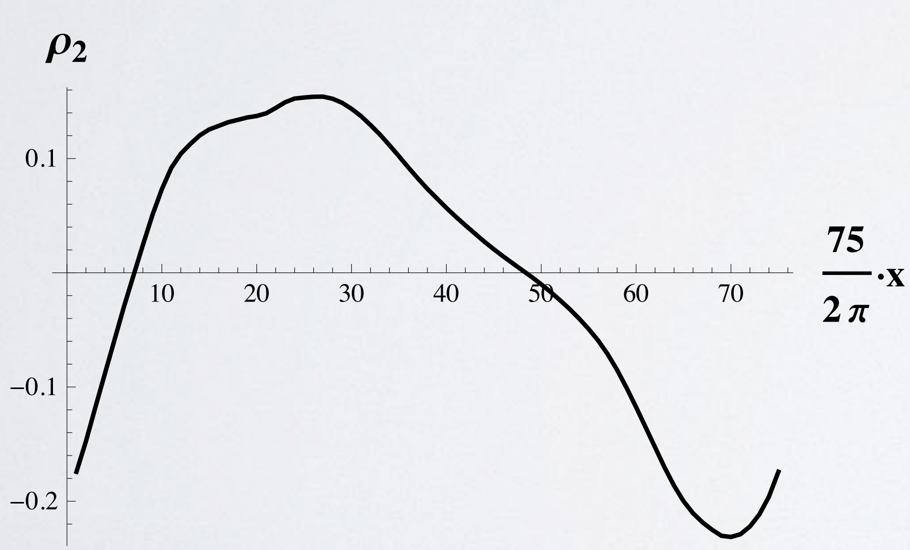
- $\mu_0 = 3.50\,,\ \alpha = 1.50\,,\ w = 3.50 \quad [\mu_0 < \mu_c = 3.66]$

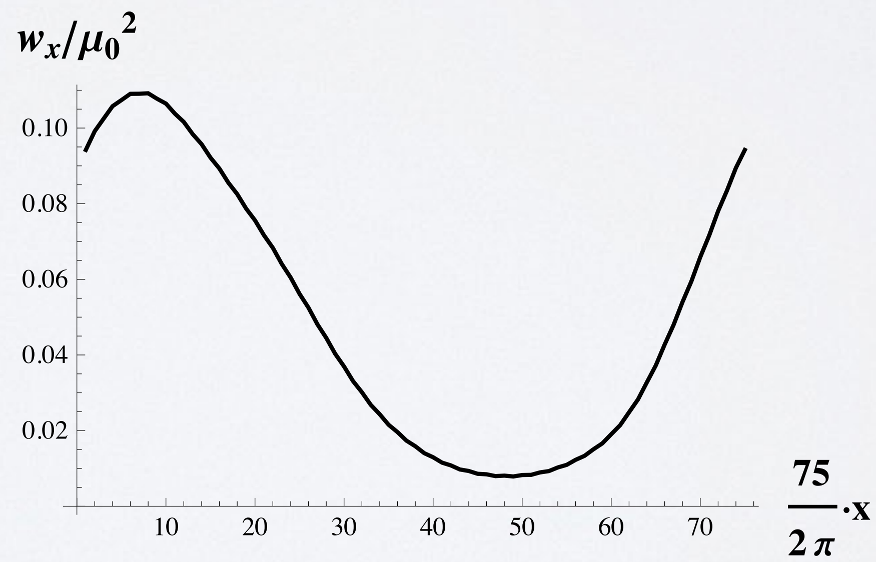



$L_x = 2\pi \rightarrow K_0 = 1$

$N_z \times N_x = 25 \times 75$



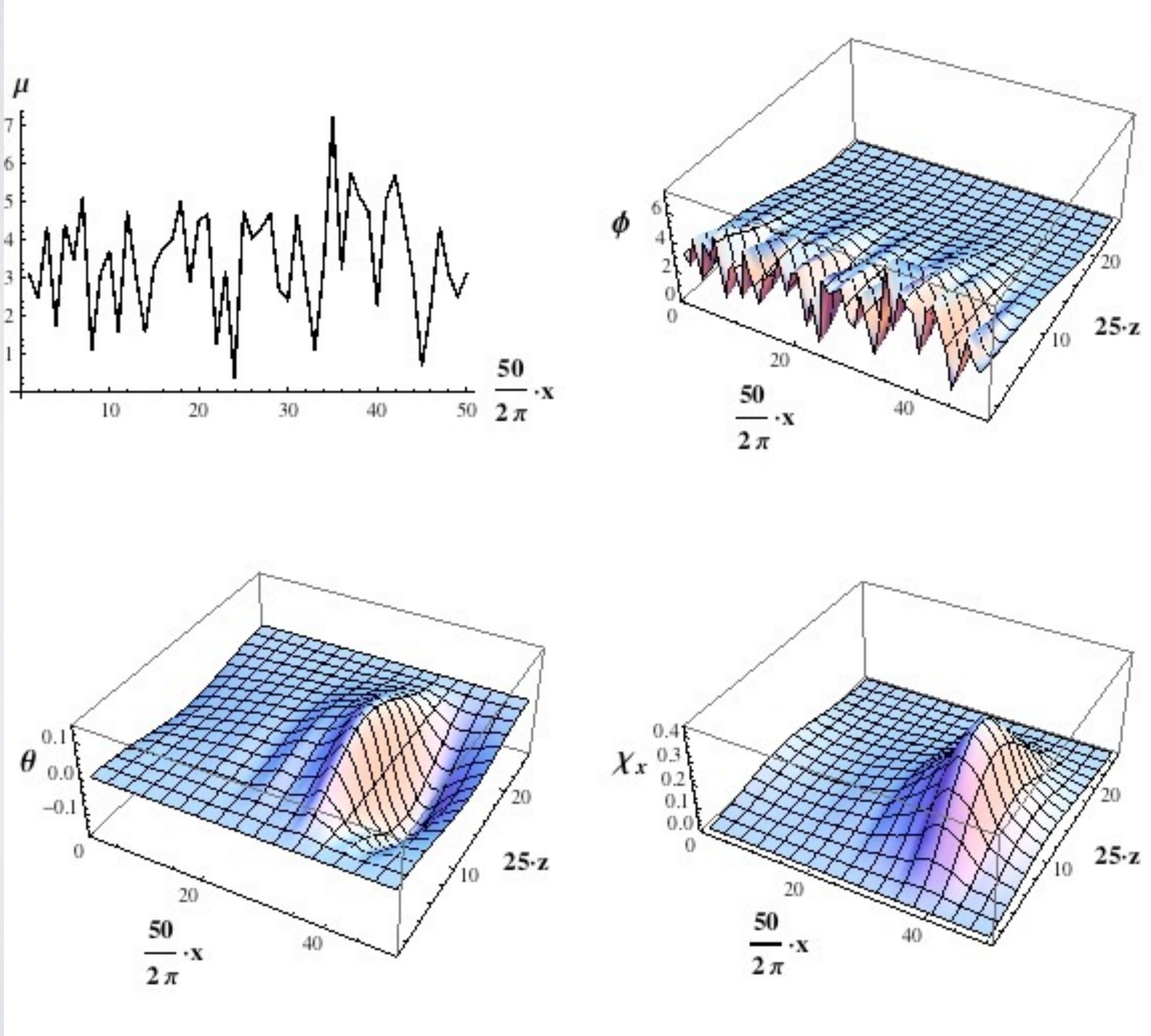
## > Simulation #2

Flat Noise

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\, \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\, \cos(k\,x + \delta_k)$$

$$w = 25\epsilon/\mu_0$$

- $\mu_0 = 3.50\,,\ \alpha = 0\,,\ w = 3.50\ \ [\mu_0 < \mu_c = 3.66]$




$L_x = 2\pi \rightarrow K_0 = 1$

$N_z \times N_x = 25 \times 75$

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