A more realistic thermalization scenario in holography

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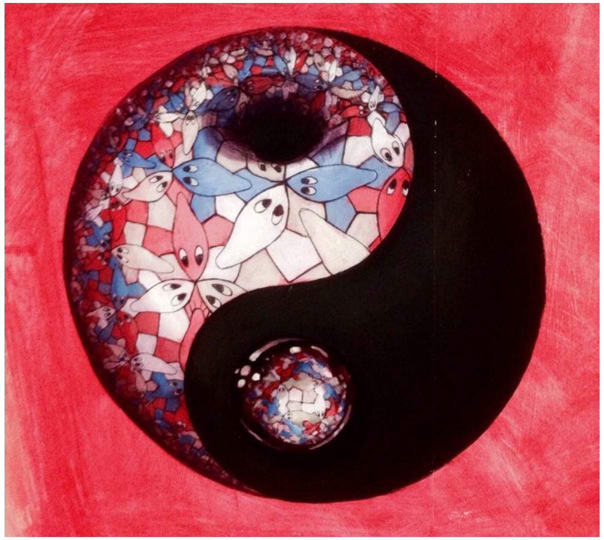


with Danny Hetharia, Petter Saeterskog, Jeroen van Gorsel

NWO
Netherlands Organisation for Scientific Research

FOM

Introduction

- AdS/CFT has the uniquely efficient ability to compute real time finite temperature/finite density physics in a single framework
  - Includes cross-over to hydrodynamics
  - Can give qualitative insights into previous inaccessible physics, even if "weakly coupled".

- Nested Einstein Equations

Chesler, Yaffe

$$ds^2 = -A dv^2 + \Sigma^2 \left[e^{B} dx_\perp^2 + e^{-2B} dx_\parallel^2 \right] + 2 dr dv$$

$$0 = \Sigma(\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2$$

$$0 = \Sigma(\dot{B})' + \frac{3}{2}\left(\Sigma'\dot{B} + B'\dot{\Sigma}\right)$$

$$0 = A'' + 3B'\dot{B} - 12\Sigma'\dot{\Sigma}/\Sigma^2 + 4$$

$$0 = \ddot{\Sigma} + \frac{1}{2}\left(\dot{B}^2\Sigma - A'\dot{\Sigma}\right)$$

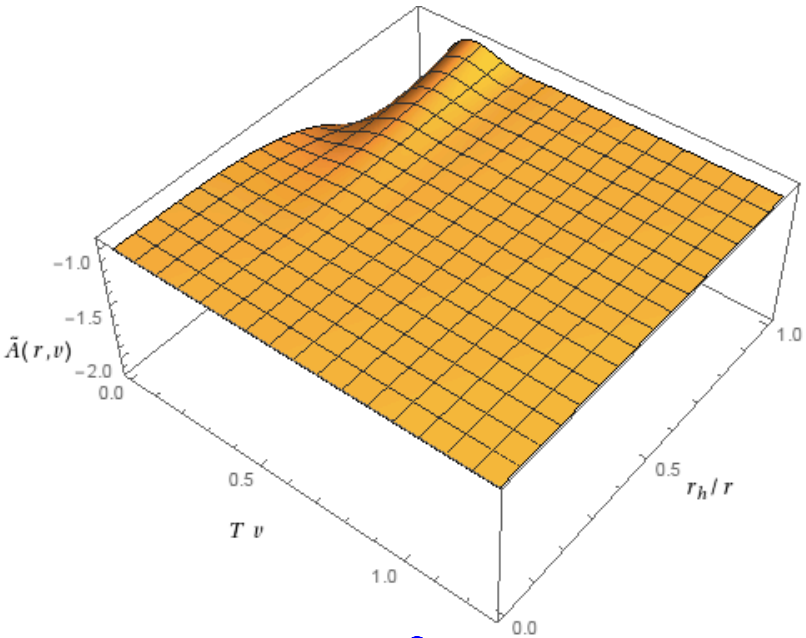
$$0 = \Sigma'' + \frac{1}{2}{B'}^2\Sigma$$

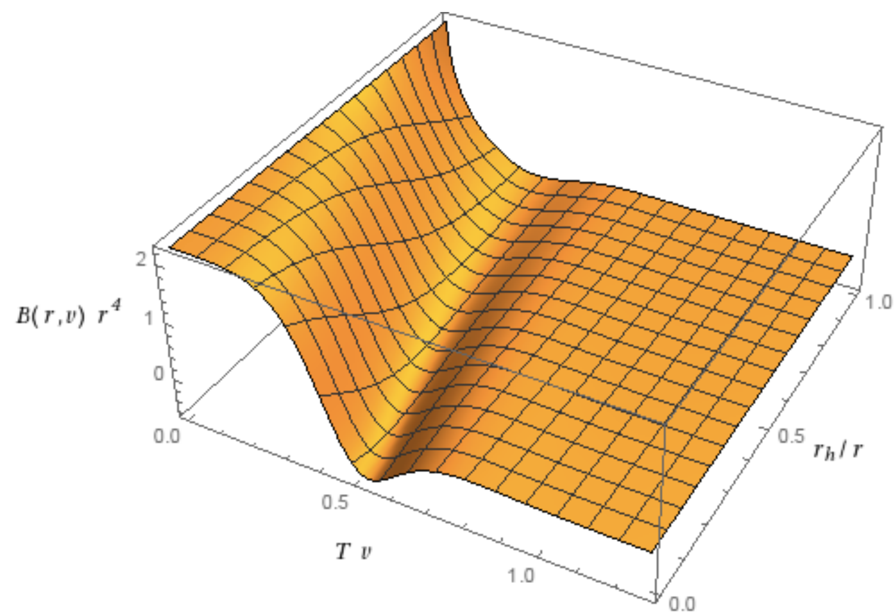
$$\dot{\Sigma} \equiv \partial_v \Sigma + \frac{1}{2} A \partial_r \Sigma \ , \quad \Sigma' = \partial_r \Sigma$$

A revolution in numerical GR

- Homogeneous, anisotropic plasma

Heller, Mateos, Triana, van der Schee





$$\hat{A} = \frac{(A - r^2)}{r^2}$$

$$\hat{B} = B r^4$$

- Initial conditions

$$\hat{B}(v=0, r) = \frac{9}{4}$$

$$\mathcal{E}(v=0) = \frac{3}{4} \quad \text{(units in which } \mathcal{E}_{BH} = \frac{3}{4}\pi^4 T^4\text{)}$$

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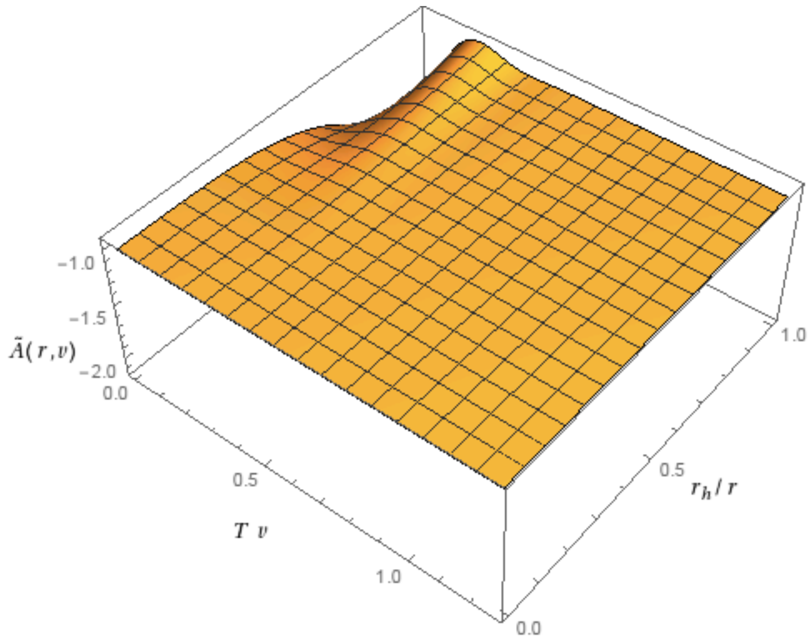
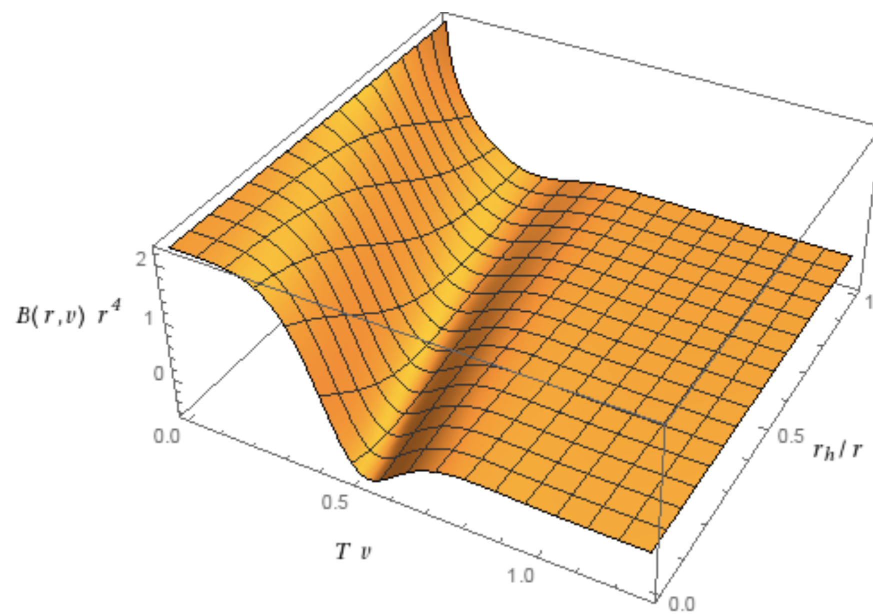
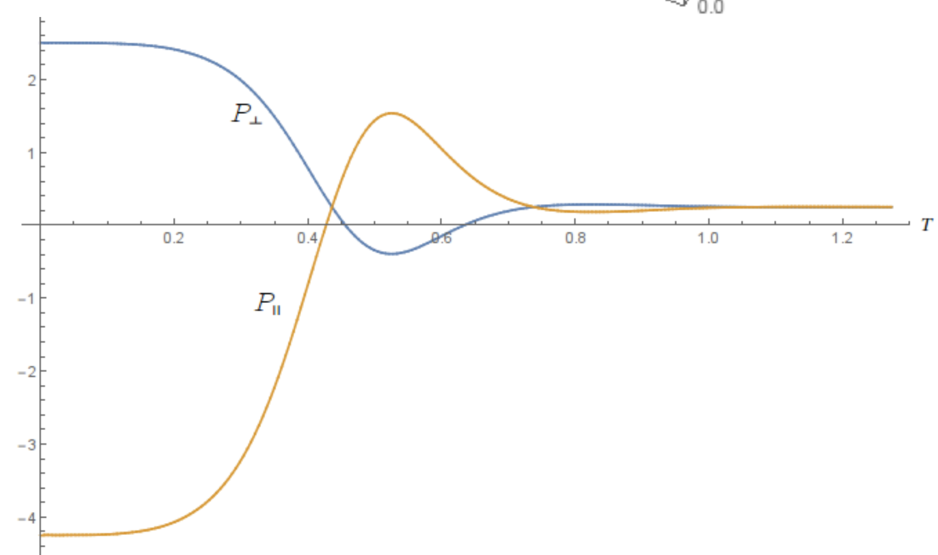




$$\lim_{r\to\infty} \hat{A} = -\frac{4}{3}\mathcal{E}$$

$$\lim_{r\to\infty} \hat{B} = \frac{1}{3}(P_\perp - P_\parallel)$$



Three issues with holographic thermalization

- The thermalization time tends to be faster than expected

- The thermalization is more UV-like than expected

- The thermalization process is captured by the linear approximation far earlier than expected

Thermalization times in holography

- Theoretically (QCD)

$$\tau = \frac{1}{g^4}\frac{1}{T}$$

- Theoretically (QCD)

$$\tau = \frac{1}{g^4}\frac{1}{T}\bigg|_{T=\Lambda_{QCD}} = \frac{1}{g^4}0.3\mathrm{fm/c} \simeq 3\mathrm{fm/c}$$

Thermalization times in holography

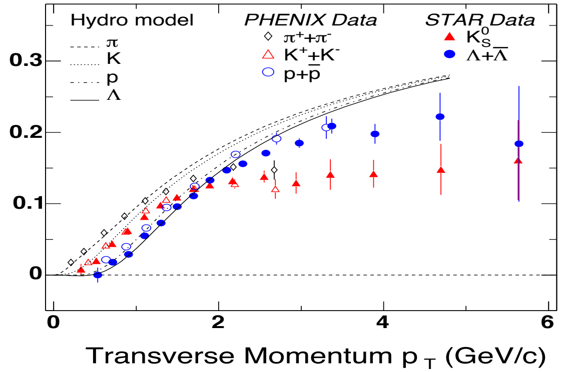
- Theoretically (QCD)

$$\tau = 3\text{fm/c}$$

- Experimentally

$$\tau = 0.6 - 1.0 \text{ fm/c}$$

Heinz, Kolb; ...



Thermalization times in holography

- Theoretically (QCD)

$$\tau = 3\mathrm{fm/c}$$

- Experimentally $\tau = 0.6 - 1.0\ \mathrm{fm/c}$

- Holographically
  - Instantaneous (local observables) — Bhattacharyya, Minwalla
  - Dimensional analysis (QNM) $\tau = 1/T = 0.3\mathrm{fm/c}$ — Friess, Gubser, Michalogiorgakis, Pufu
  - "Dimensional analysis"(Wilson loops) — Balasubramanian, Bernamonti, de Boer, Copland, Craps, Keski-Vakkuri, Mueller, Schaefer, Shigemori, Staessens

Thermalization times in holography

- Theoretically (QCD)

$$\tau = 3\mathrm{fm/c}$$

- Experimentally $\tau = 0.6 - 1.0 \ \mathrm{fm/c}$

- Holographically

$$\tau = 1/T = 0.3\mathrm{fm/c}$$

  - There is no small parameter/no other scale.

- Thermalization is more UV-like
  - Extreme: Vaidya metric of collapsing shell

$$ds^2 = -\left(r^2 - \frac{M(v)}{r^{d-2}}\right) dv^2 + r^2 dx^2 + 2drdv$$

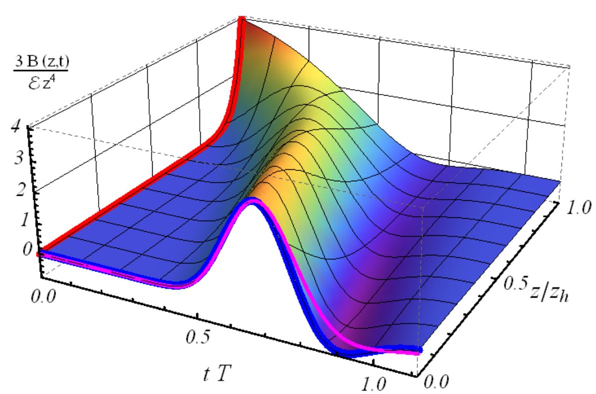
  - In general: all disturbances are sourced at the boundary (UV)

- Thermalization is more UV-like
  - Extreme: Vaidya metric of collapsing shell

$$ds^2 = -\left(r^2 - \frac{M(v)}{r^{d-2}}\right)dv^2 + r^2 dx^2 + 2drdv$$

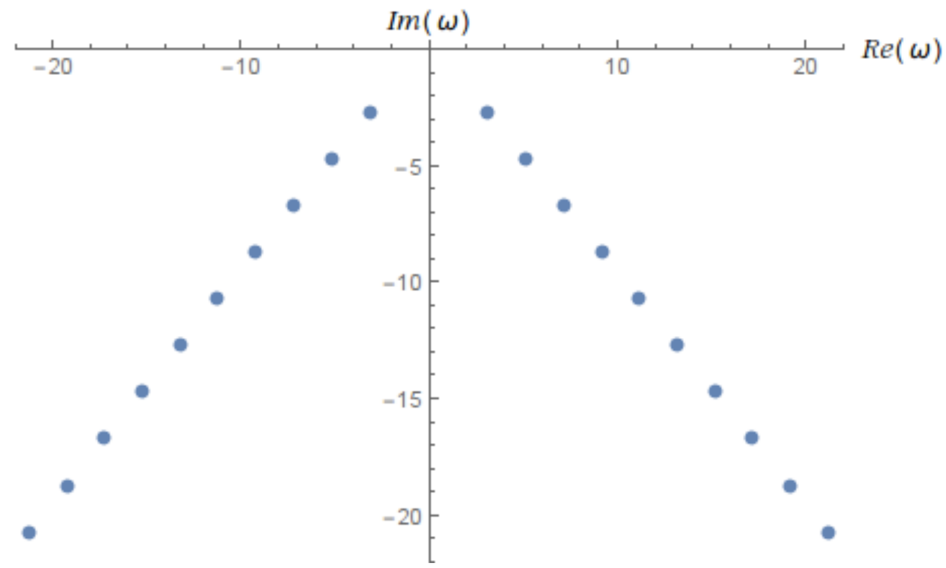
  - In general: all disturbances are sourced at the boundary (UV)
  - Counterexample: source in the IR

Heller, Mateos, Triana, van der Schee,



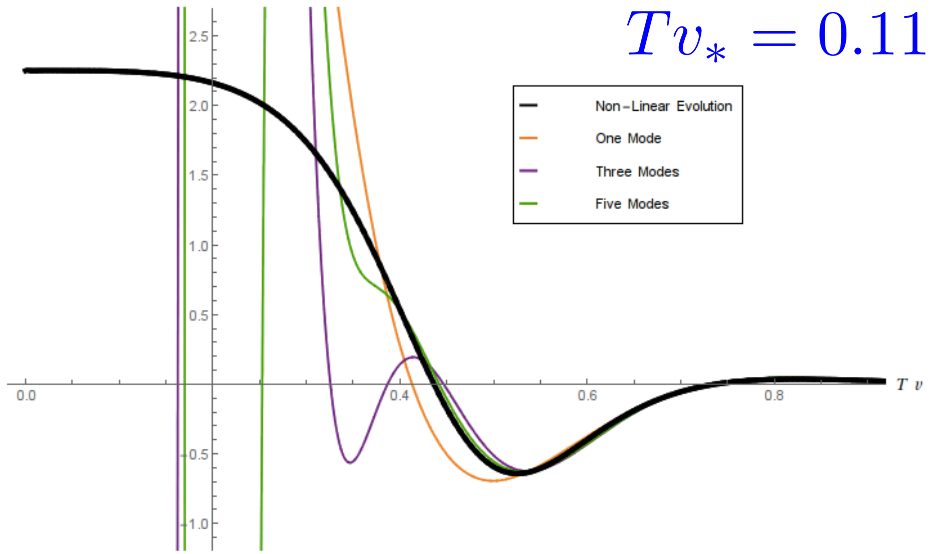
- Late time evolution is given by "ringing down" of quasi-normal modes (linear perturbations) around the black hole final state.

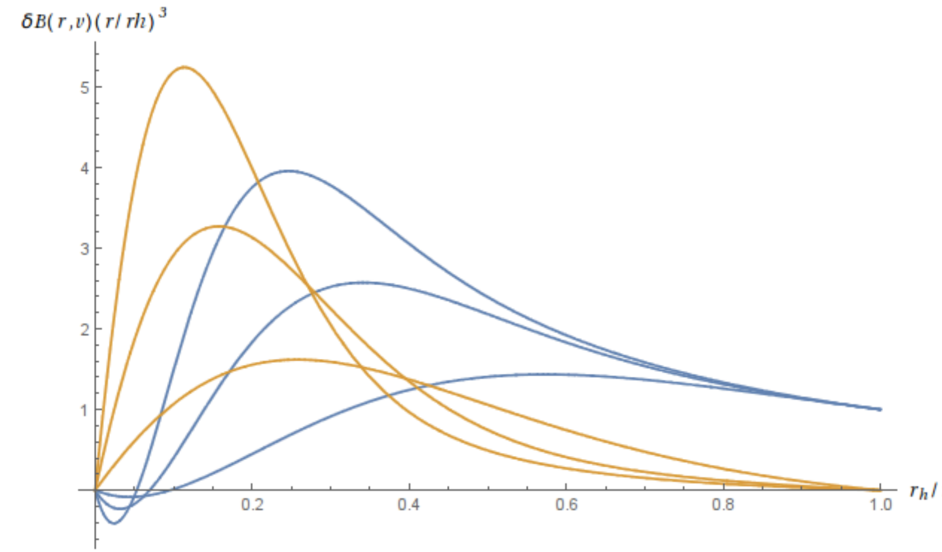
$$\delta A(v,r) = 0$$
$$\delta\Sigma(v,r) = 0$$
$$\delta B(v,r) = \mathrm{Re}\left[\sum_i c_i b_i(r) e^{i\omega_i v}\right]$$



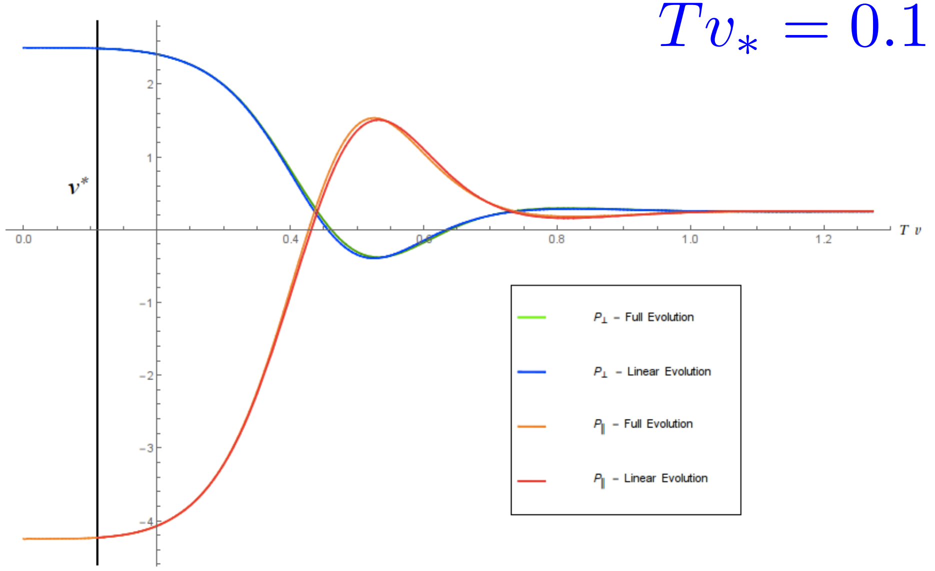
- Set QNM initial conditions by matching at Tv_*

$Tv_* = 0.11$





- Set QNM initial conditions by matching at Tv_*
  - Try to match as early as possible



- Set QNM initial conditions by matching at Tv_*
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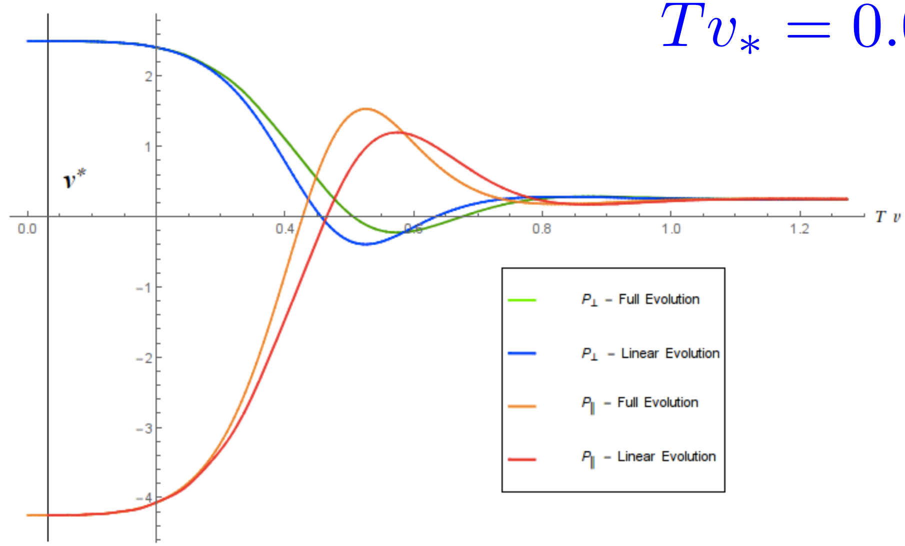
$Tv_* = 0.03$



- Set QNM initial conditions by matching at Tv_*
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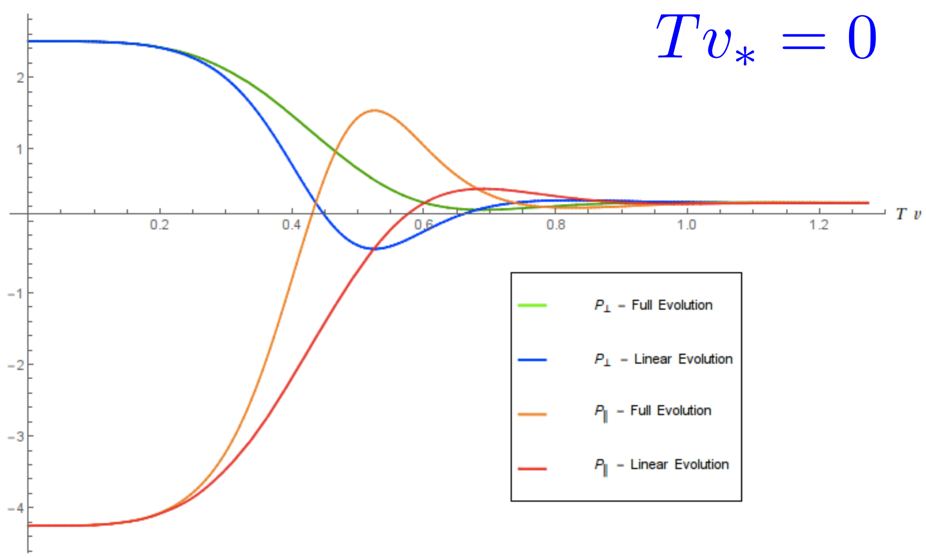
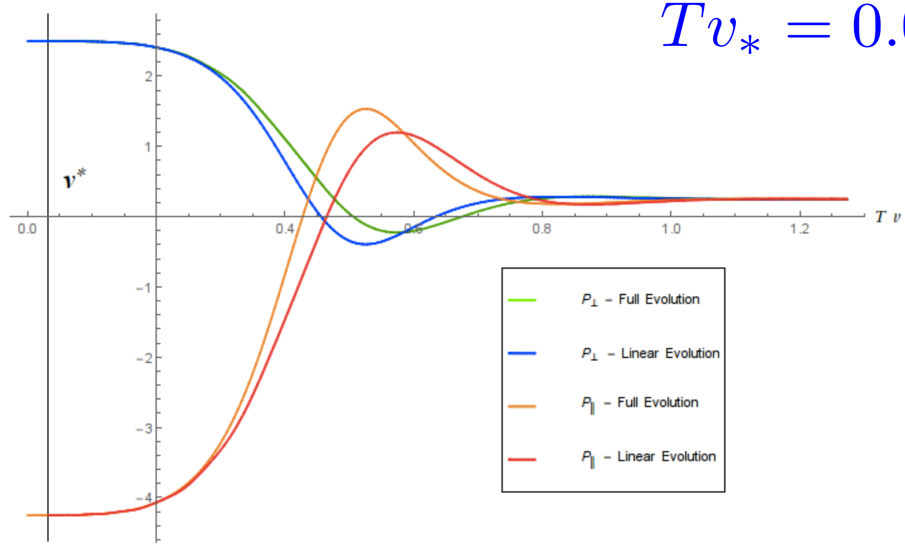
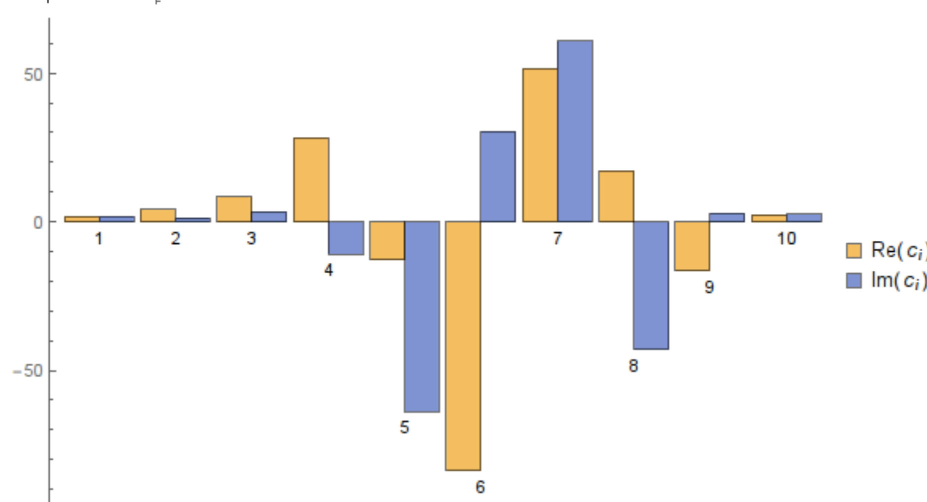


Holographic thermalization linearized

- Set QNM initial conditions by matching at Tv_*
  - Try to match as early as possible

$Tv_* = 0.03$





- This QNM linearization works far better than expected

Heller, Mateos, Triana, van der Schee,

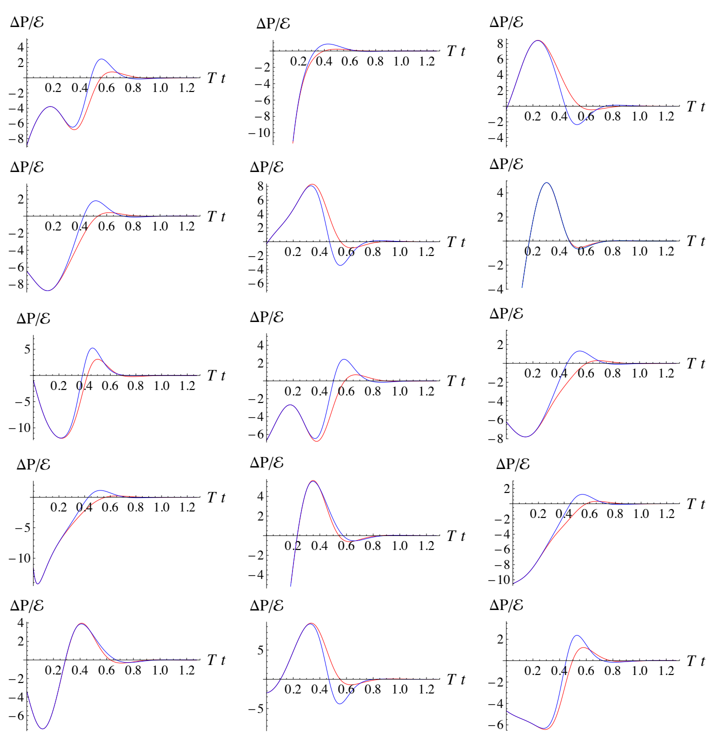
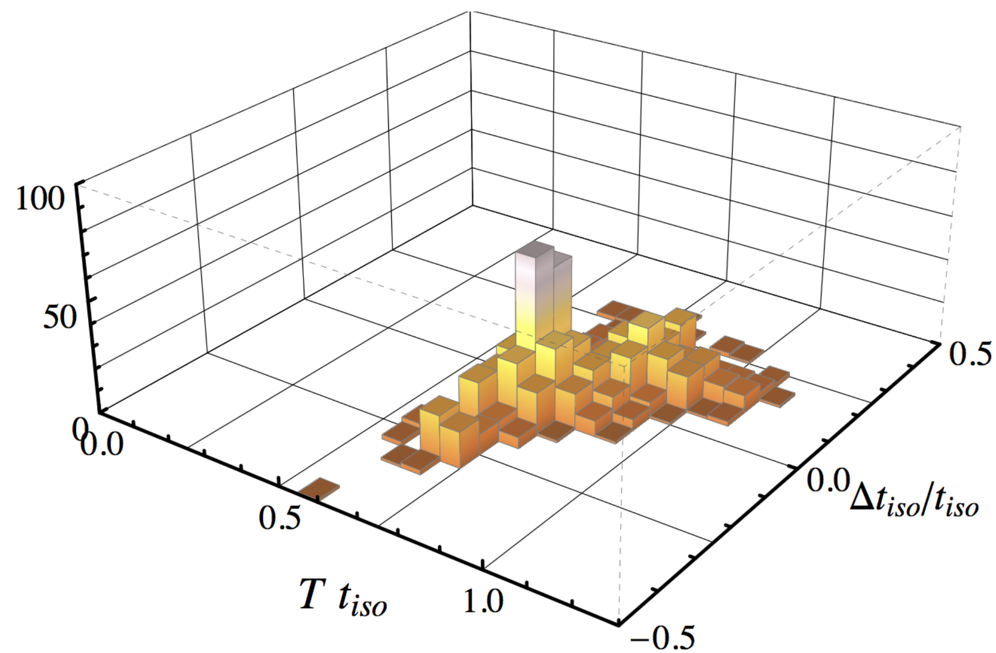


Figure 4. Comparison between the time evolution of the pressure anisotropy predicted by the linear equation (4.4) (red) and the full result (blue) for 15 different initial conditions. The leading order linearized Einstein's equations predict both qualitative and quantitative features of the dynamics of the dual stress tensor in our setup. A more thorough scan of the initial conditions (as shown in Fig. 5) did not reveal any instances in which the linearized approximation broke down.



~800 different initial conditions

- Higher order gravitational QNM modes
  - Spin 2 excitations
  - These are "glueballs"

Csaki, Ooguri, Oz, Terning
de Teramond, Brodsky
Katz, Lewansowski, Schwartz

Imagine replacing the BH horizon with a hard/soft-wall.

  - They are artificially stable in the limit $N_c \to \infty$

- A more realistic scenario would account for the instability of the higher QNM
  - Note that these are quantum corrections. $(1/N_c)$
  - Classical corrections are in the full non-linear evolution. (G_N)

Including $1/N_c$ corrections

- Qualitative approach
  - The result of $1/N_c$ corrections should be that the higher order QNM decay into the lower QNM.
  - Introduce this into the evolution by hand

We are only interested in qualitative effects. This does introduce another scale/small parameter.

$$\frac{\Delta c_i}{\Delta v} = \sum_j \left[-i\omega_i \delta_{ij} + M_{ij}\right] c_j$$

Including $1/N_c$ corrections

$$\frac{\Delta c_i}{\Delta v} = \sum_j \left[-i\omega_i \delta_{ij} + M_{ij}\right] c_j$$

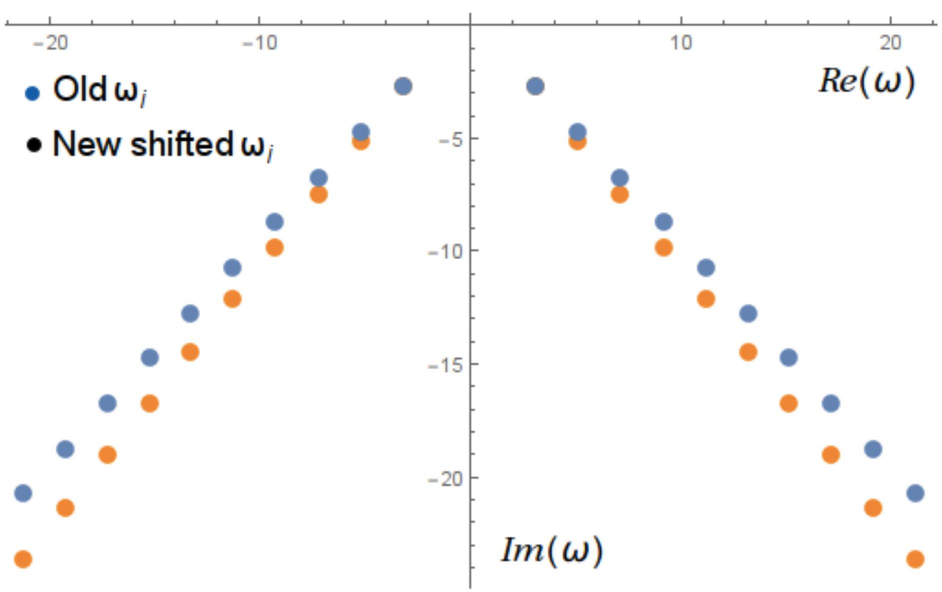
- The mixing is the same for the QNM's as for their complex conjugates. $\delta \hat{B}$ has to be real.
- Each mode has a corresponding energy $\epsilon_i = \mathrm{Re}\omega_i$. We let the higher energetic modes decay only into lower energetic modes with a rate proportional to their energy difference.
- The total energy of the modes has to be conserved during the mixing process. This introduces normalization factors Δ_i .

$$M_{ij} = \begin{pmatrix} -\Delta_1 & 0 & 0 & 0 \\ 1 - e^{-\frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2}} & -\Delta_2 & 0 & 0 \\ 1 - e^{-\frac{\epsilon_1 - \epsilon_3}{\epsilon_1 + \epsilon_3}} & 1 - e^{-\frac{\epsilon_2 - \epsilon_3}{\epsilon_2 + \epsilon_3}} & -\Delta_3 & 0 \\ 1 - e^{-\frac{\epsilon_1 - \epsilon_4}{\epsilon_1 + \epsilon_4}} & 1 - e^{-\frac{\epsilon_2 - \epsilon_4}{\epsilon_2 + \epsilon_4}} & 1 - e^{-\frac{\epsilon_3 - \epsilon_4}{\epsilon_3 + \epsilon_4}} & -\Delta_4 \end{pmatrix}$$

Including $1/N_c$ corrections

$$\frac{\Delta c_i}{\Delta v} = \sum_j \left[-i\omega_i \delta_{ij} + M_{ij}\right] c_j$$

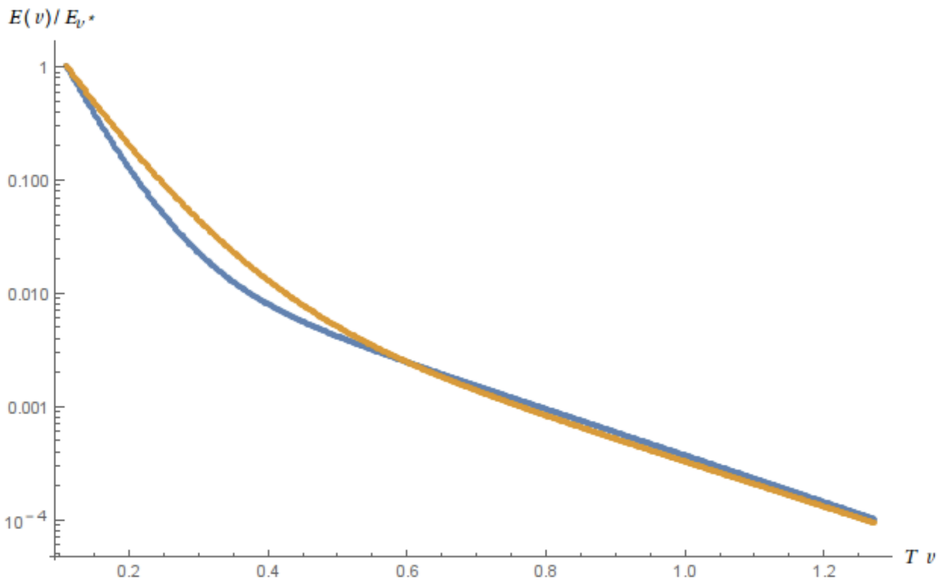
Effect: shift of the poles of the QNM (rediagonalize)

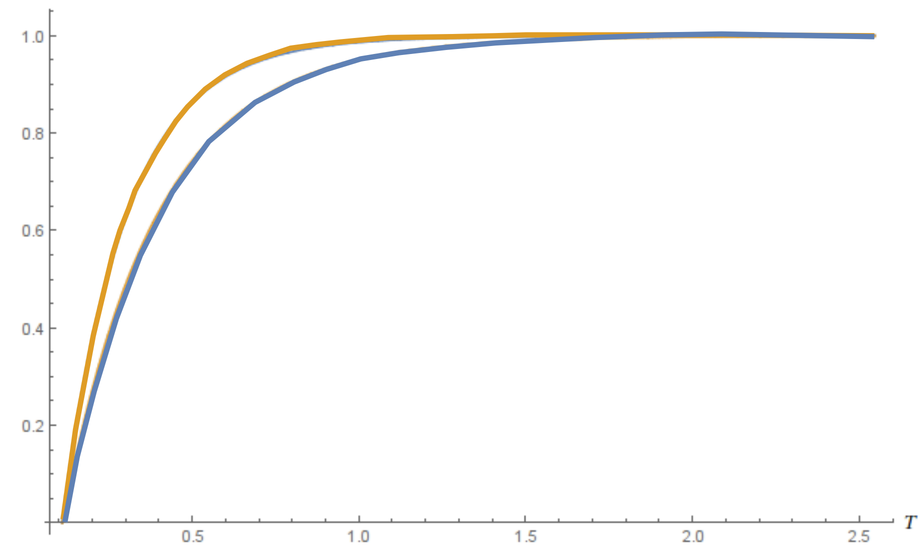


Energy Check

- Energy flux through the horizon

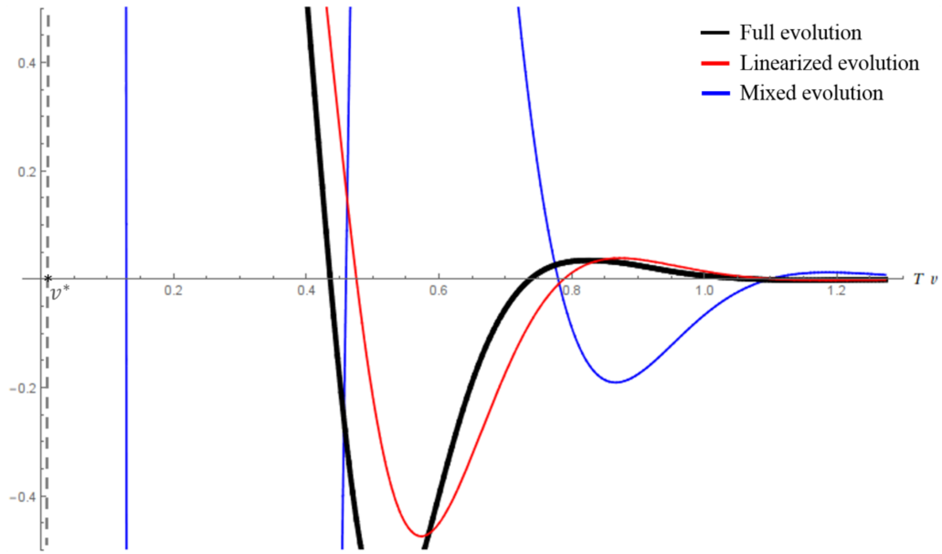
No loss introduced





$$\int_{v_*}^{v} \frac{E(v')}{E(v_*)} dv'$$

- Set mixed QNM initial conditions by matching at Tv_*



$$Tv_* = \frac{1}{10\pi}$$

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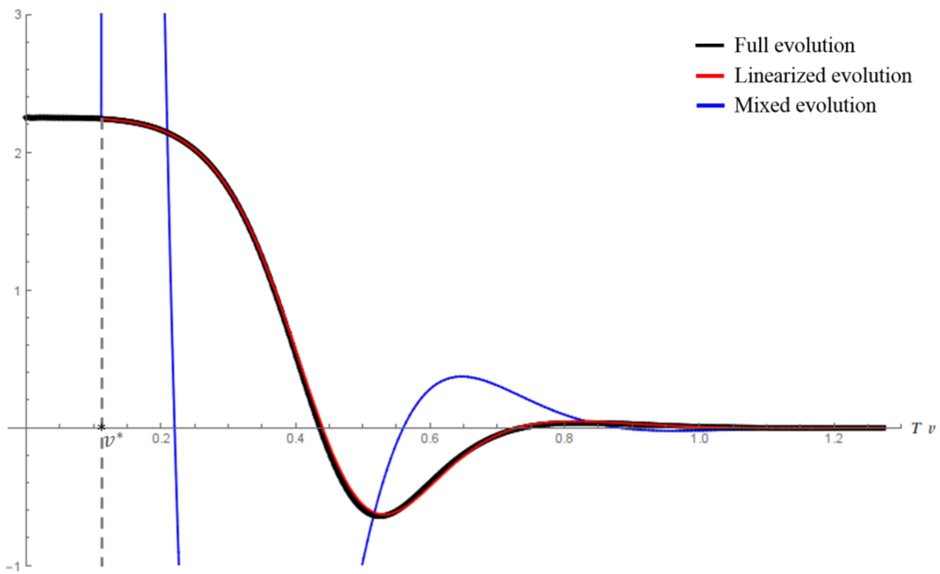
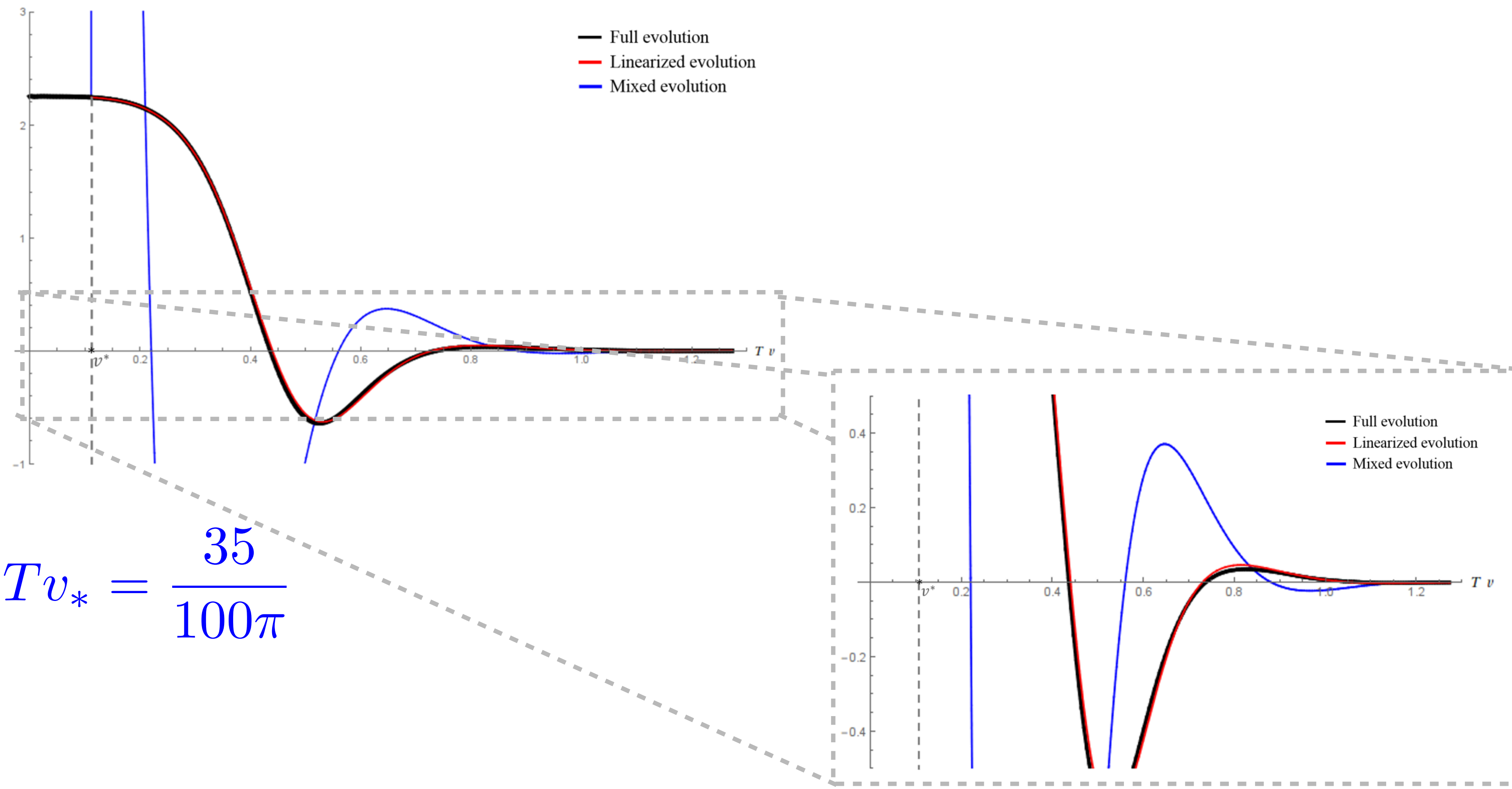


$$Tv_* = \frac{35}{100\pi}$$

- Set mixed QNM initial conditions by matching at Tv_*



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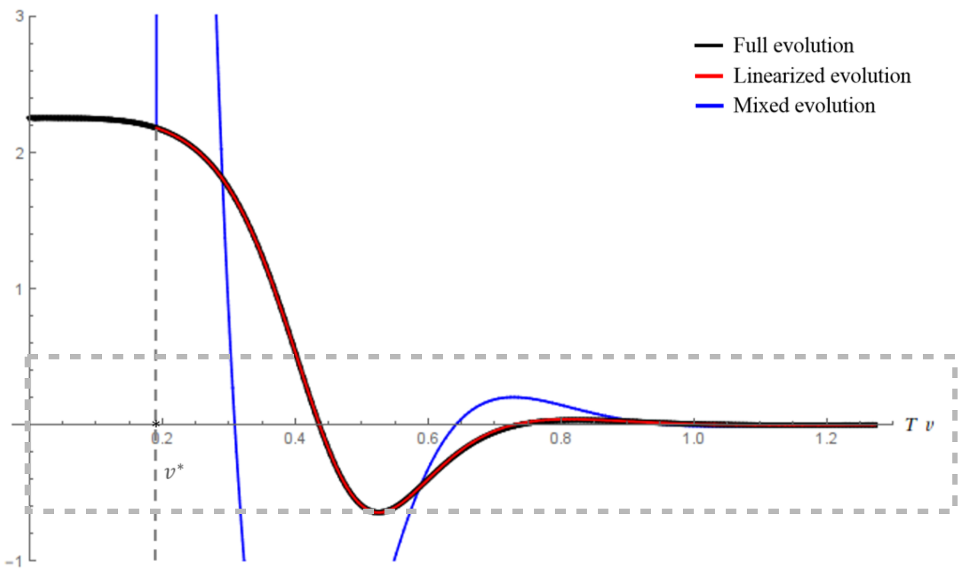
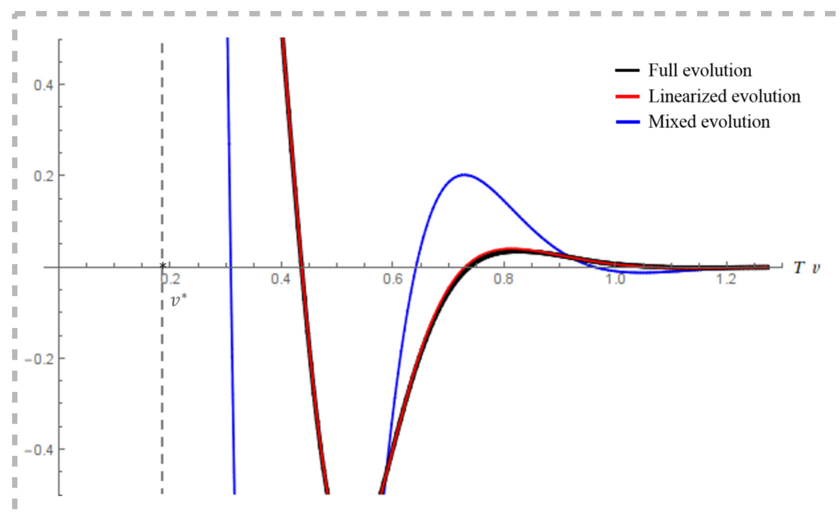


early linearization fails

$$Tv_* = \frac{6}{10\pi}$$



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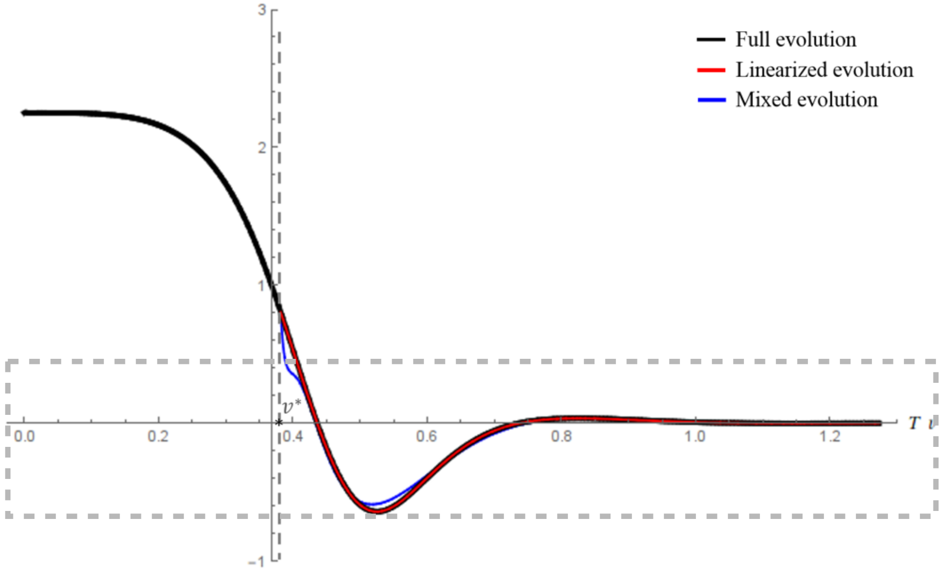
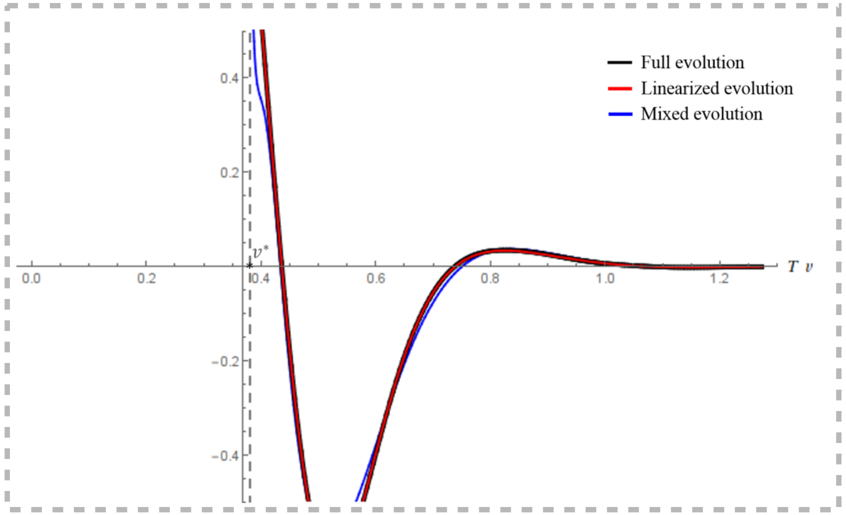


$$Tv_* = \frac{12}{10\pi}$$



Three issues with holographic thermalization

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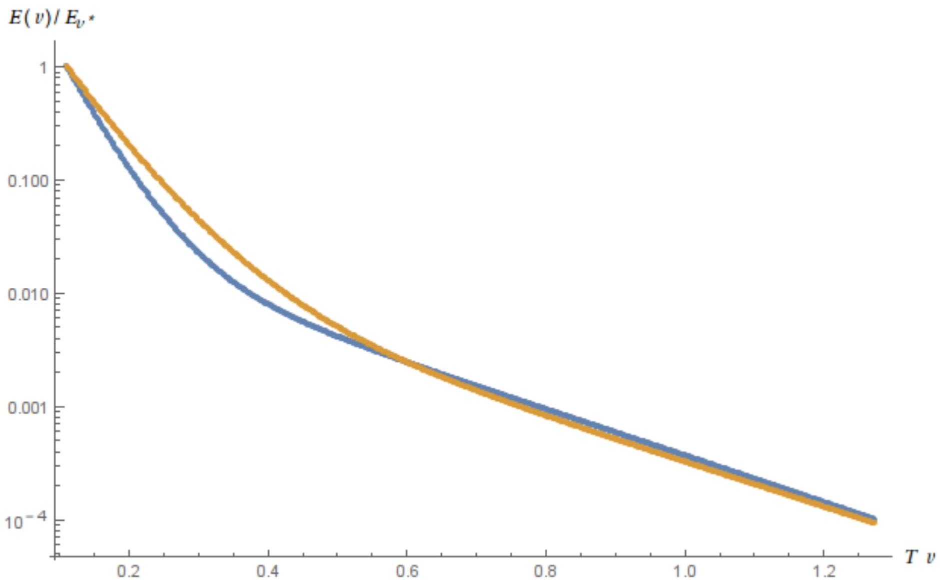
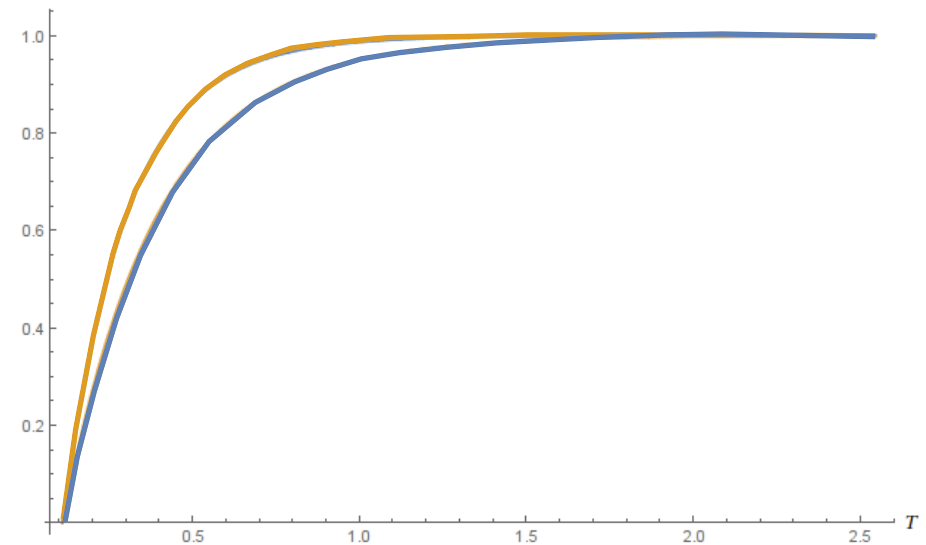
early linearization fails with $1/N_c$ corrections

Thermalization is delayed

- Energy flux through the horizon





$$\int_{v_*}^{v} \frac{E(v')}{E(v_*)} dv'$$

Thermalization is delayed

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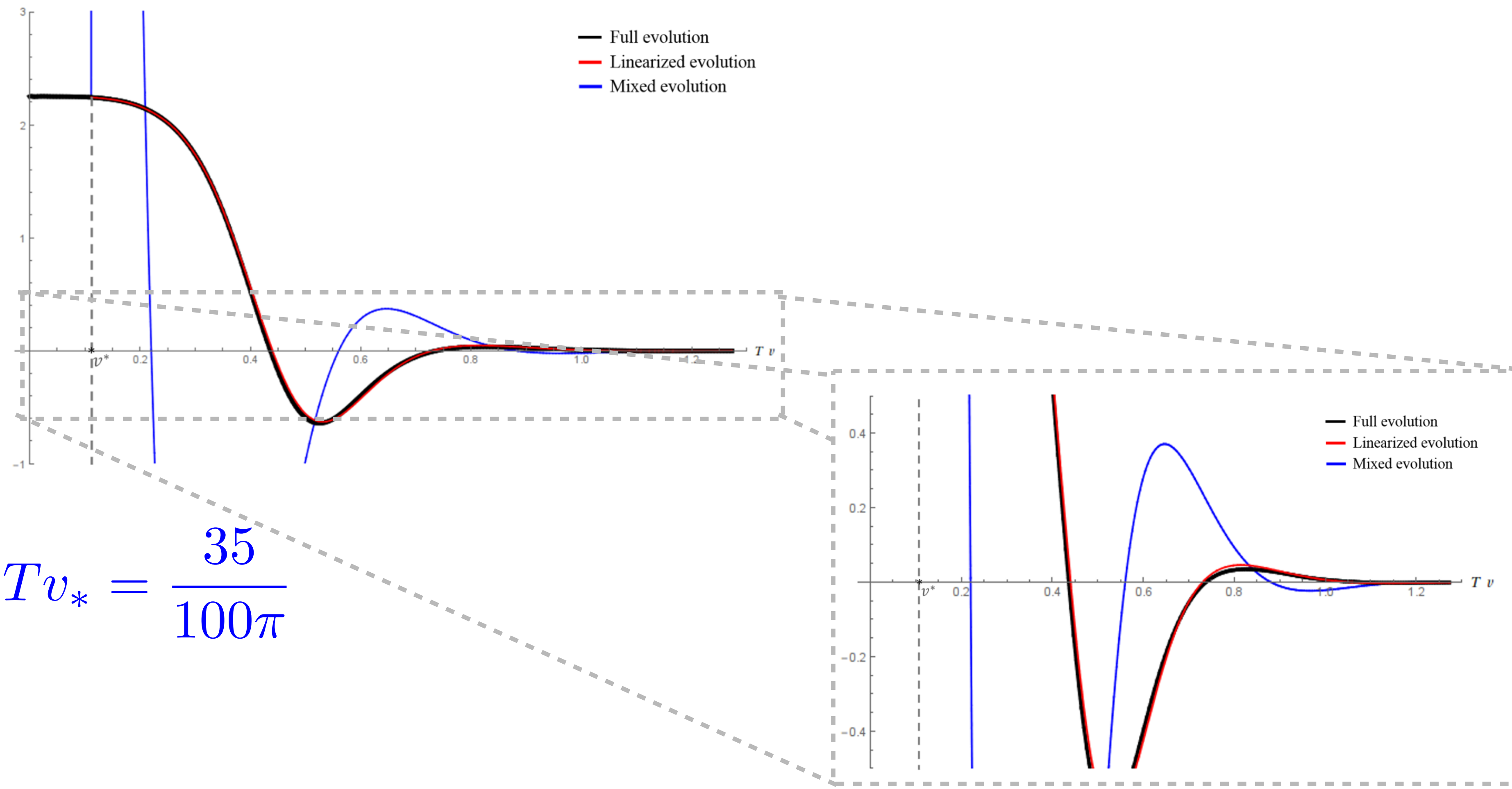

$$Tv_* = \frac{35}{100\pi}$$

Three issues with holographic thermalization

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  thermalization slows with $1/N_c$ corrections

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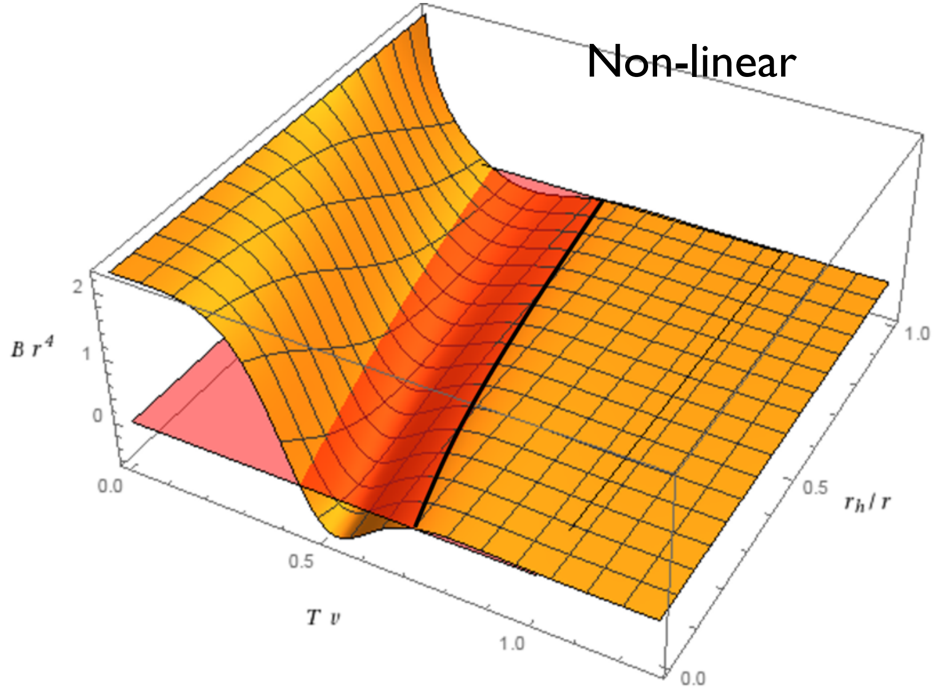
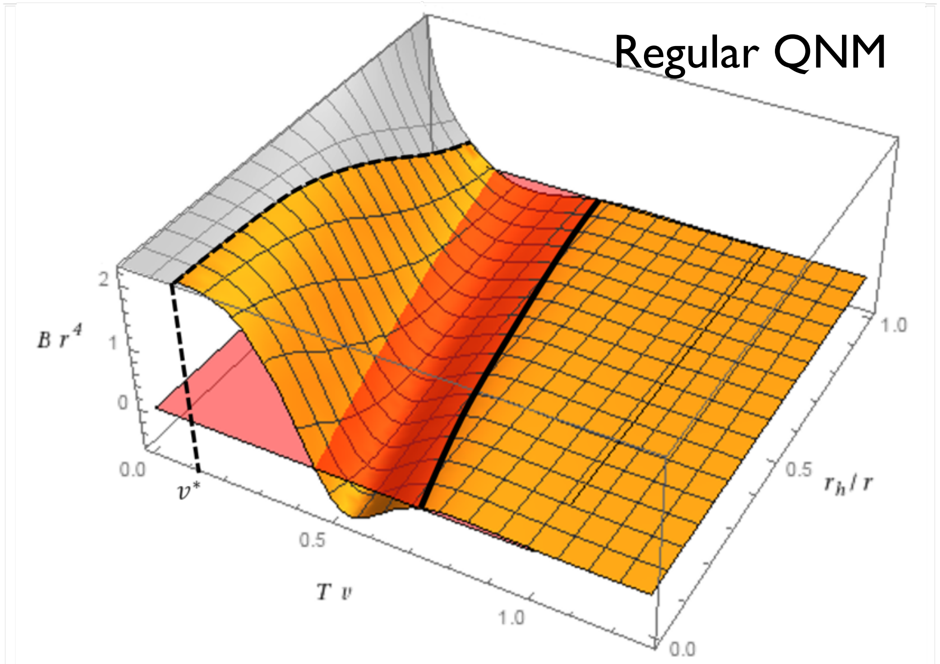
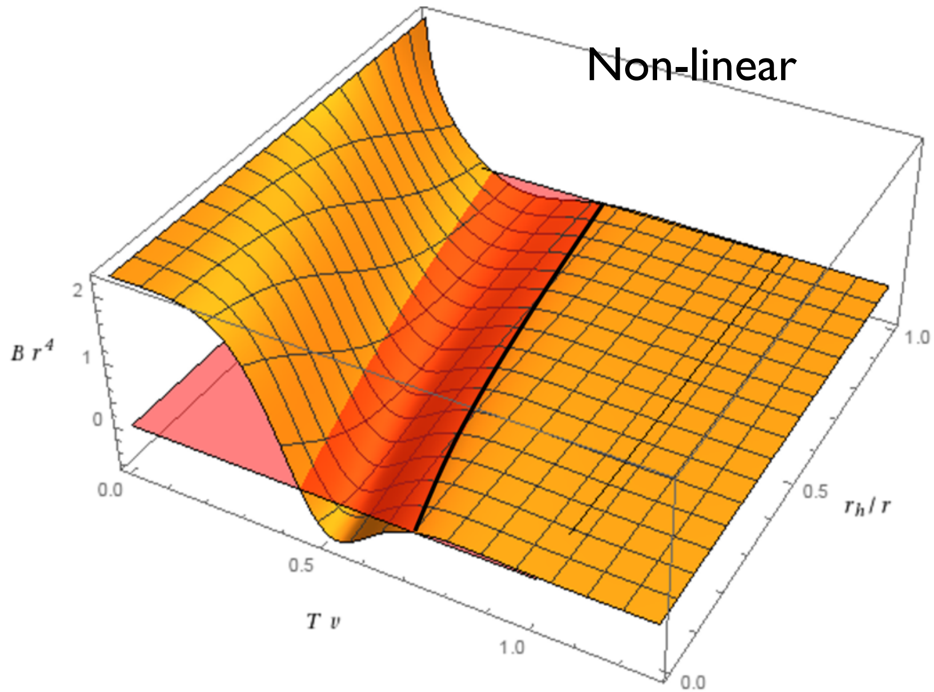
UV vs IR Thermalization

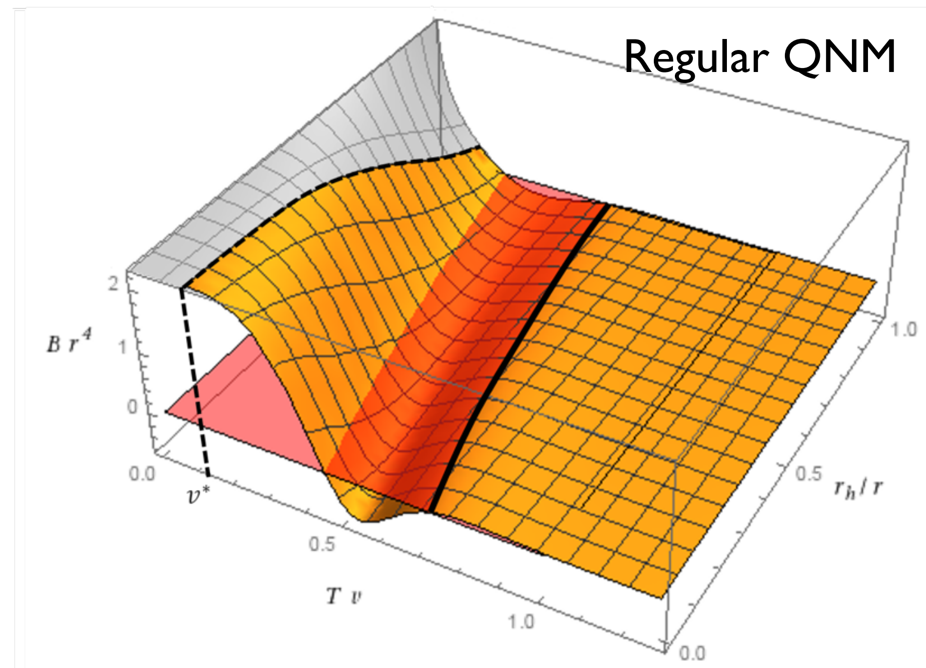


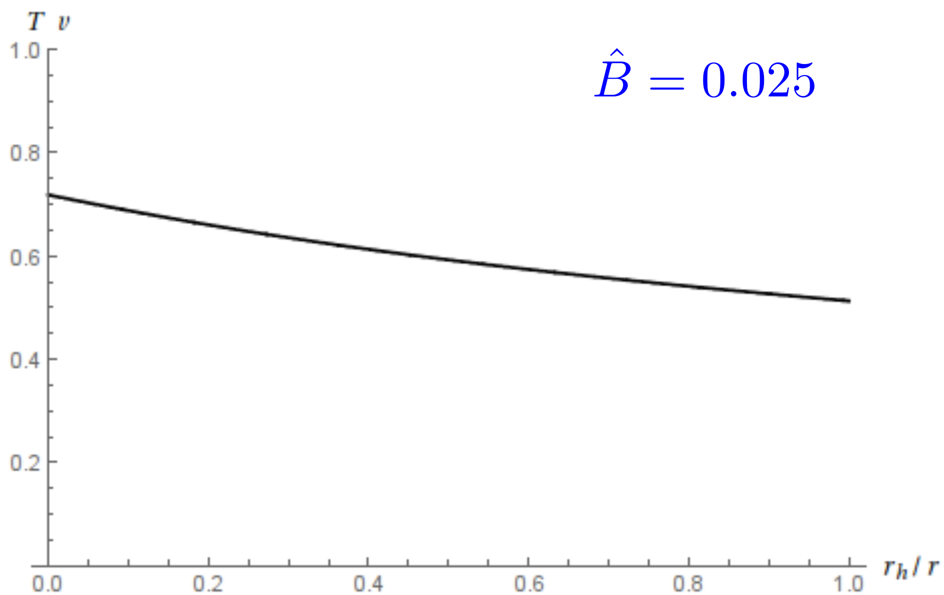


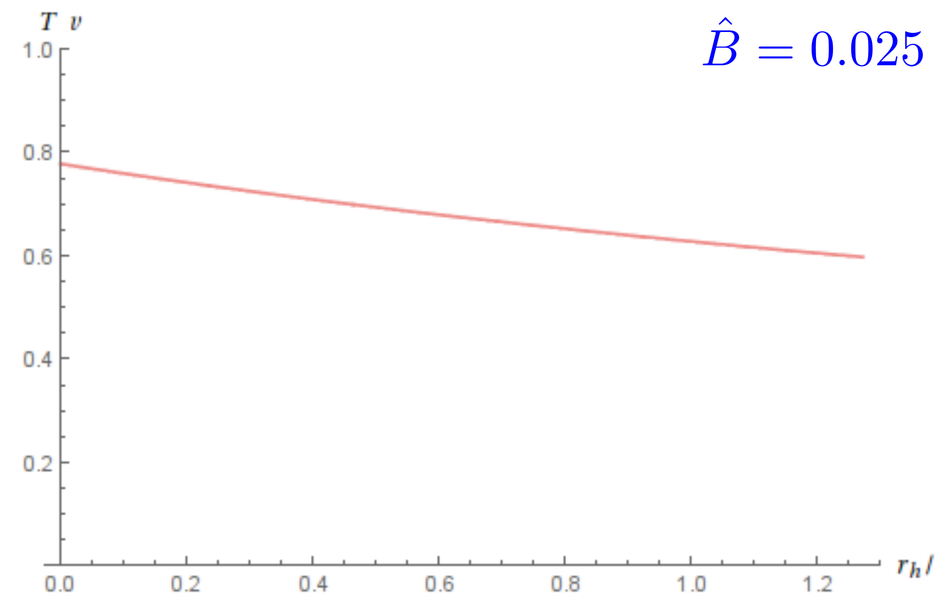
- Thermalization condition:
  anistropy less than 10% energy density $\hat{B} \leq 0.025$

UV vs IR Thermalization

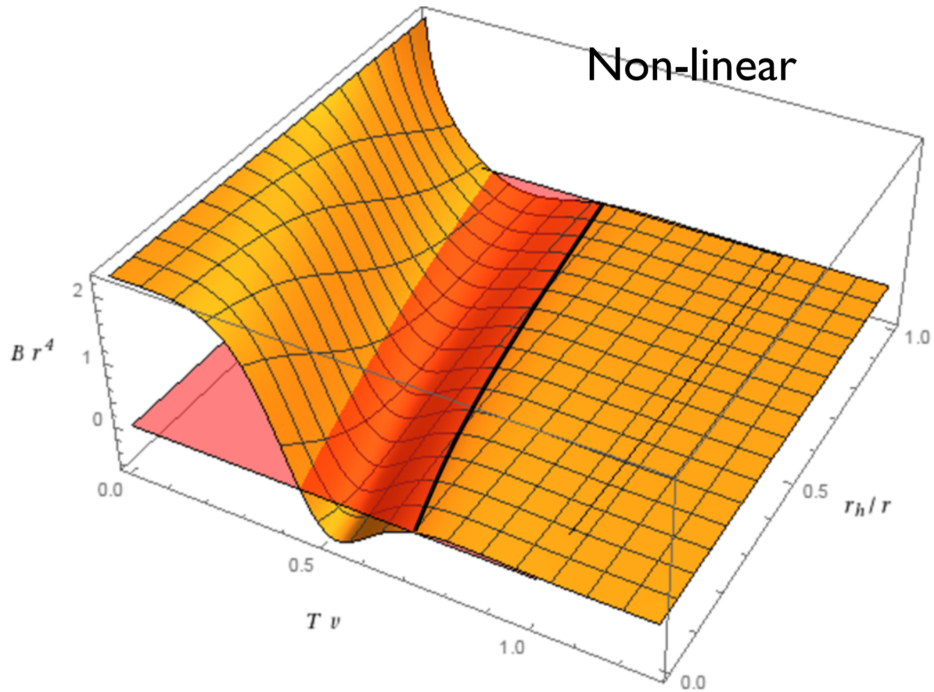


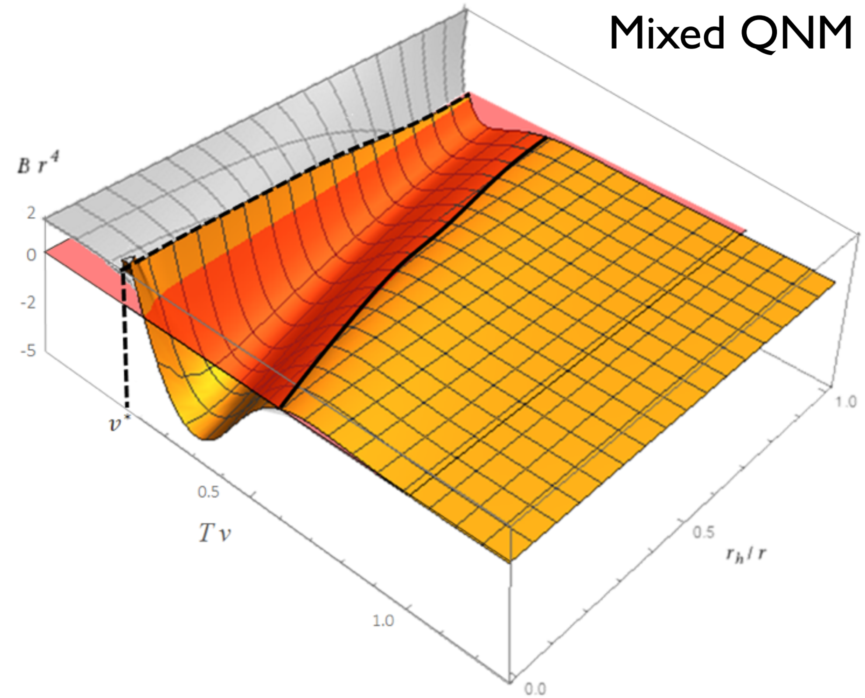


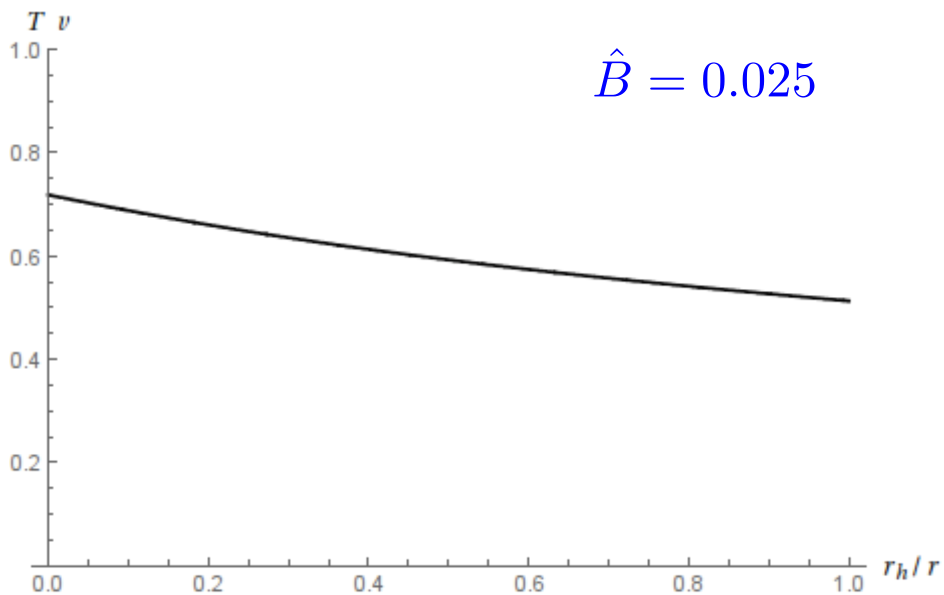


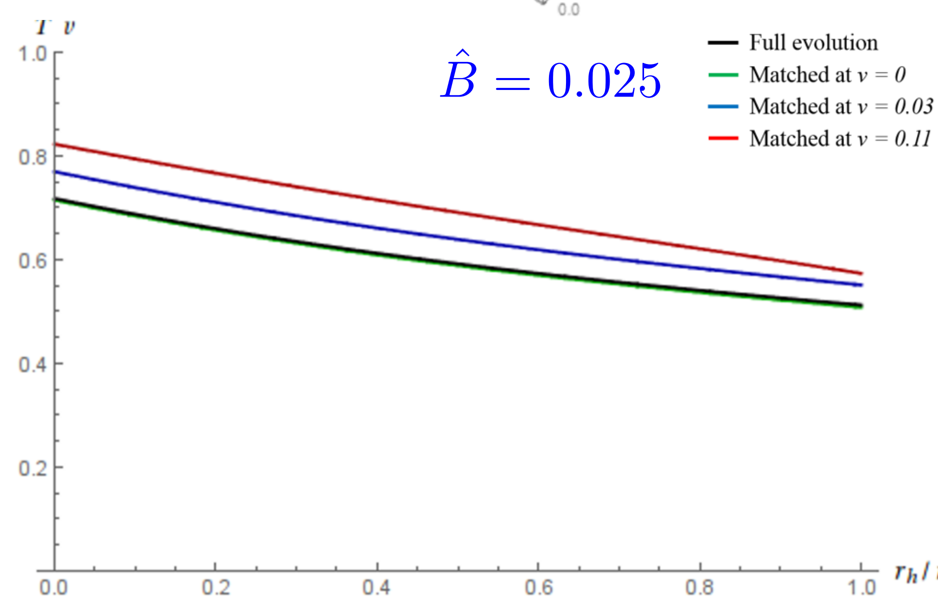


UV vs IR Thermalization with mixing









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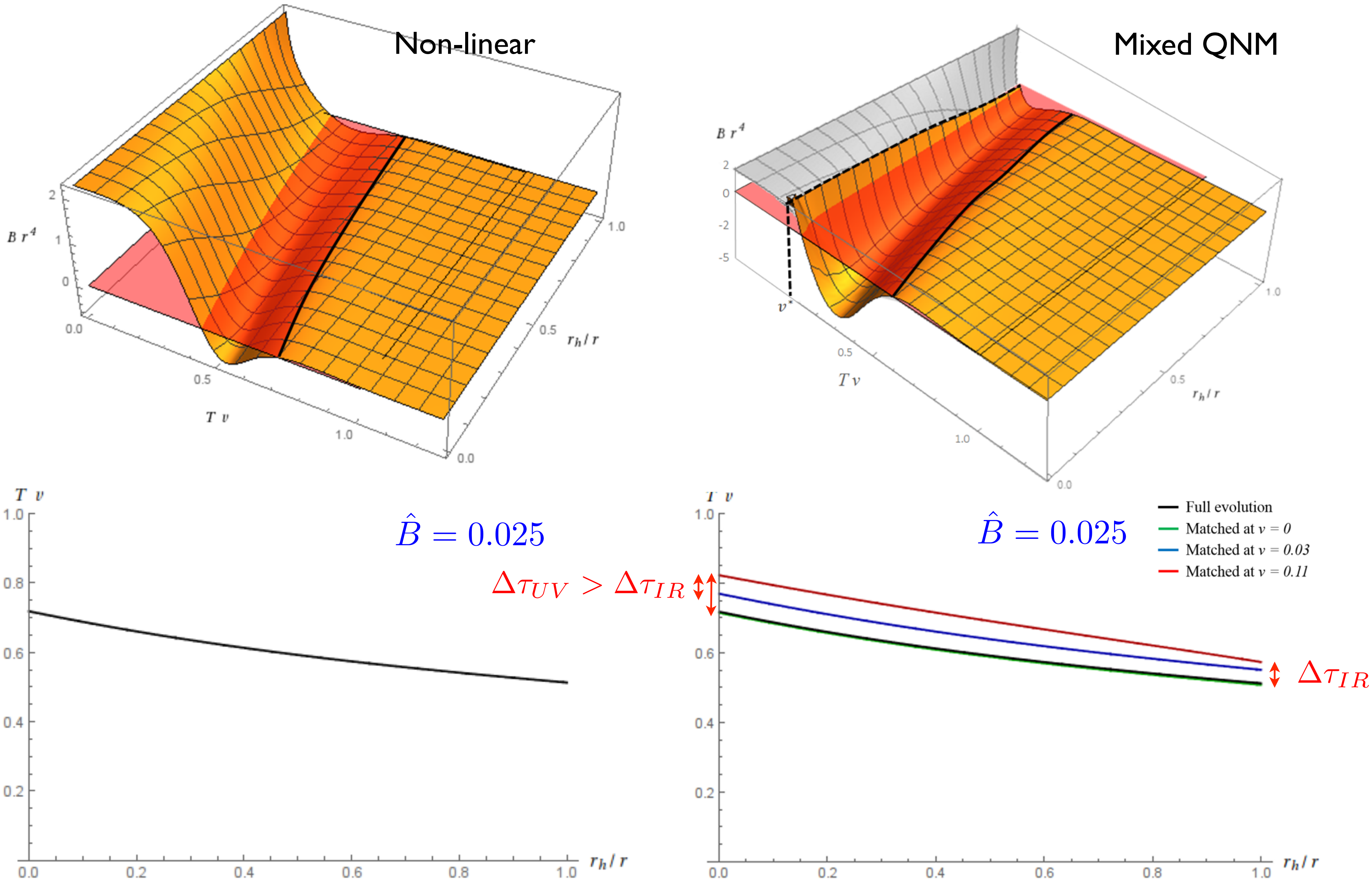

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thermalization slows with $1/N_c$ corrections

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hints that thermalization becomes more IR-
with $1/N_c$ corrections

- The thermalization process is captured by the linear approximation far earlier than expected

early linearization fails with $1/N_c$ corrections

Conclusions

- There are no stable* higher order QNM in any real system
  - Taking this into account improves our qualitative insights from holography.
  - Early linearization (QNM) is an artifact of the large N_c approximation.
  - Including $1/N_c$ corrections will slow down thermalization and make it more IR-like.
  - Is there a way to mimic $1/N_c$ corrections classically?
    (Can there be an extra scale/small parameter that affects "universal" BH physics?)

Thank you

Bosons and Fermions together

- AdS Einstein-Maxwell, scalars, fermions, Yukawa

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |(\partial_\mu - 2iqA_\mu)\phi|^2 - m_\phi^2|\phi|^2$$
$$-i\bar{\Psi}(\Gamma^\mu(\partial_\mu - iqA_\mu) - m_\Psi)\Psi + \eta_5^*\bar{\phi}\bar{\bar{\Psi}}^C\Gamma^5\Psi + \eta_5\phi\bar{\Psi}\Gamma^5\Psi^C$$

$$q_b = 2q_f$$

- Backgrounds:

AdS-RN/ Non-Fermi liquid AdS_2 metal	Cooper instability (absent for NFL)	Hartman, Hartnoll
Holographic Superconductor	BCS gap in fermion spectral function	Faulkner, Horowitz, McGreevy, Roberts, Vegh
Holographic Fermi liquid	*BCS instability and resulting background*	

- The Holographic Fermi Liquid

$$\text{AdS/CFT dictionary}$$

$$\Psi = \mathrm{Tr}\psi\phi$$

The fermion is a composite of fundamental fields.

For energies $E \ll E_{bind}$ composite operator acts a fundamental field.

Familiar from neutron stars.

Following textbook CMT this Fermi-liquid should have a BCS instability

- Composite double trace operators

$$\mathcal{O}_{\rm pair} = \mathcal{O}_{\bar{\Psi}^C}\mathcal{O}_{\Psi} \quad = {\rm Tr}\phi\psi{\rm Tr}\phi\psi$$

- Higher order operators mixes with $\mathcal{O}_{\rm pair}$ under RG flow
- Postulate: Higher order moments in the particular solution should be seen as vevs of these higher order operators.

$$\phi(z) = \underbrace{\mathcal{H}_0 z^{d-\Delta_\phi} + \mathcal{H}_1 z^{\Delta_\phi} + \ldots}_{\text{Homogeneous solution}} + \underbrace{\mathcal{P}_1 z^{2\Delta_\Psi} + \mathcal{P}_2 z^{2\Delta_\Psi+1} + \mathcal{P}_3 z^{2\Delta_\Psi+2} + \ldots}_{\text{Particular solution}}$$