Dirty Holographic Superconductors

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Holographic Methods for Strongly Coupled Systems
Firenze, Italia

Based on arXiv:1308.1920, arXiv:1407.7526
Work in Progress, D. Areán, A. Farahi, LPZ, I. Salazar, A. Scardicchio

Outline

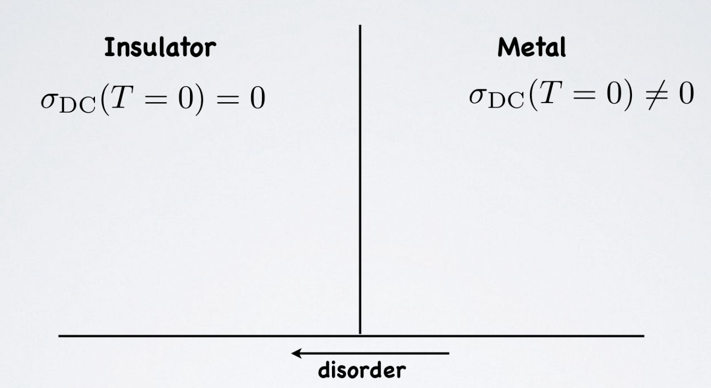
- Motivation: Anderson localization, Many Body Localization, Dirty Superconductors.
- A numerical approach to disorder in AdS/CFT.
- Disordered Holographic s- and p-wave Superconductor.
- Enhancement of superconductivity for mild disorder and a universailty in response.
- Part II: Aspects of conductivity into the physics of MBL holographically

Motivation: Anderson Localization

- Problem: Transport and translational symmetry/absence of momentum dissipation.
- Anderson Localization: The conductivity can be completely suppressed by quantum effects
- In the model originally introduced and discussed by Anderson (1958), a single electron moves on a regular lattice, e.g. a hypercubic lattice, where each lattice point carries a random on-site potential V_i

$$H = \sum_{i,j,\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + \sum V_i n_{i\sigma} \tag{1}$$

- $c^{\dagger}_{i\sigma}, c_{i\sigma}$ create/annihilate a particle of spin σ on site i.
- $n_{i\sigma} = c^{\dagger}_{i\sigma} c_{i\sigma}$ is the number operator.
- The random energies V_i are characterized by their distribution $P(V_i)$ (Uniform, Gaussian, etc.).


Insulator
$\sigma_{\mathrm{DC}}(T=0)=0$
Metal
$\sigma_{\mathrm{DC}}(T=0)\neq 0$
disorder

Motivation: Many Body Localization

- The rapidly emerging field of many-body localization is concerned with the fate of the Anderson insulator under electron-electron interaction and the characterization of the possible resulting many-body localized phase.
- Basko-Aleiner-Altshuler ('06): Presented compelling evidence in favor of a many body localized phase, based on an analysis of the perturbation theory in electron-electron interaction to all orders

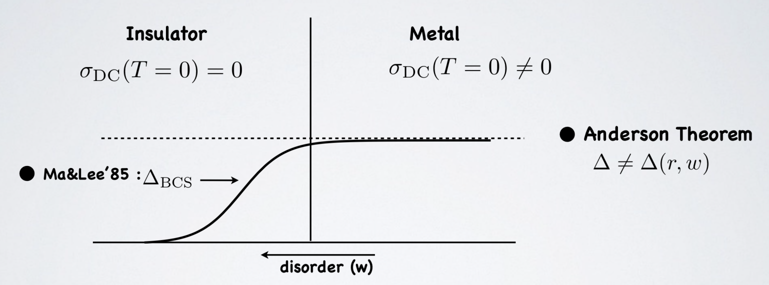
$$H = \sum_{\alpha} \xi_\alpha c_\alpha^\dagger c_\alpha + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta \tag{2}$$

- $c_\alpha^\dagger |\Psi_k\rangle$ Single particle state above a certain eigenstate of the interacting system.

Anderson's Theorem

- Anderson's Theorem (58): stating that superconductivity is insensitive to perturbations that do not destroy time-reversal invariance (pair breaking), provided the central intuition.
- Critiques to Anderson's argument were raised, for example, in where the effects of strong localization were considered. A. Kapitulnik, G. Kotliar, Phys. Rev. Lett. 54, 473, (1985). G. Kotliar, A. Kapitulnik, Phys. Rev. B 33, 3146 (1986), M. Ma, P.A. Lee, Phys. Rev. B 32, 5658, (1985).
- More generally, the question of the role of interactions, in particular, the Coulomb interaction in dirty superconductors cannot be considered settled

Anderson's theorem



Disordered holographic s-wave superconductor

- The gravity model (Hartnoll-Herzog-Horowitz)

$$S=\int d^4x\,\sqrt{-g}\left(-\frac{1}{4}F_{ab}\,F^{ab}-(D_\mu\Psi)(D^\mu\Psi)^\dagger-m^2\Psi^\dagger\Psi\right)\,. \quad (3)$$

- The background Schwarzschild-AdS metric:

$$\begin{aligned} ds^2 &= \frac{1}{z^2}\left(-f(z)dt^2+\frac{dz^2}{f(z)}+dx^2+dy^2\right)\,, \\ f(z) &= 1-z^3\,, \end{aligned} \quad (4)$$

- The fields

$$\Psi(x,z)=\psi(x,z)\,,\quad \psi(x,z)\in\mathbb{R}\,, \quad (5)$$

$$A=\phi(x,z)\,dt\,. \quad (6)$$

Boundary Conditions: Spontaneously broken symmetry

- UV asymptotics ($z = 0$):

$$\phi(x,z) = \mu(x) + \rho(x)\, z + \phi^{(2)}(x)\, z^2 + o(z^3)\,, \tag{7}$$
$$\psi(x,z) = \psi^{(1)}(x)\, z + \psi^{(2)}(x)\, z^2 + o(z^3)\,, \tag{8}$$

- $\mu(x)$ and $\rho(x)$ are space-dependent chemical potential and charge density respectively.
- The functions $\psi^{(1)}(x)$ and $\psi^{(2)}(x)$ are identified, under the duality, with the source (vanishes) and VEV of an operator of dimension 2.
- IR regularity implies that A_t vanishes at the horizon ($z_h = 1$).

$$\phi(x,z) = (1-z)\, \phi_h^{(1)}(x) + (1-z)^2\, \phi_h^{(2)}(x) + \dots\,,$$
$$\psi(x,z) = \psi_h^{(0)}(x) + (1-z)\, \psi_h^{(1)}(x) + (1-z)^2\, \psi_h^{(2)}(x) + \dots\,,$$

Introducing disorder in the holographic s-wave superconductor

- What? Promote the chemical potential in the holographic superconductor to a random space-dependent function.
- Why? The chemical potential defines the local energy of a charged carrier placed at a given position x coupling with the particle number $n(x)$ locally. This choice of disorder replicates a local disorder in the on-site energy just as in the Anderson's model.
- Moreover, once disorder is introduced in such an interacting system, all observables will become disordered and, therefore, the physics is not expected to depend on the way disorder is implemented.

Disorder in details

- The noisy chemical potential:

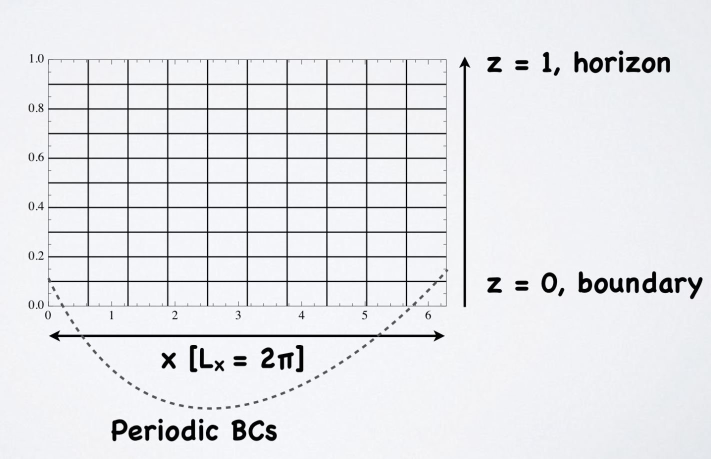
$$\mu(x) \;\;=\;\; \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\, \cos(k\, x + \delta_k)\,,$$

- For $\alpha \neq 0$ the correlation length is proportional to $1/k_0$ which is the system size.
- $\delta_k \in [0, 2\pi]$ are random phases. Ensemble averages means averaging over these i.i.d. phases.
- We discretize the space, and impose periodic boundary conditions in the x direction. An IR scale k_0 and a UV scale $k_* = \frac{2\pi}{a}$.
- Our definition of $w \sim \epsilon/\mu_0$ corresponds, in the standard solid state notation, to $1/k_F l$, where k_F is the Fermi momentum and l is the mean free path.

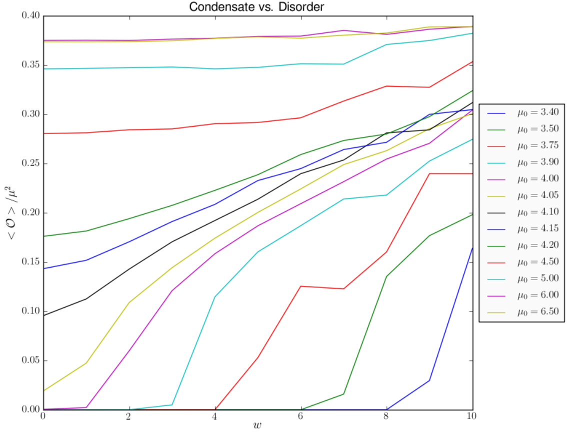
Numerics

- Most of the simulations were done independently in Mathematica and in Python. The latter ones ran in the University of Michigan Flux cluster.
- Our typical result contains a grid of 100×100 points but we have gone up to 200×200 to control issues of convergence and optimization.
- We used a relaxation algorithm to search for the solution and use an $\mathcal{L}_2$ measure for convergence which in most cases reached 10^{-16}. As the source of randomness we used $\mu(x)$ (sum of cosines) and also for uniform and Gaussian distributions.

Relaxation method

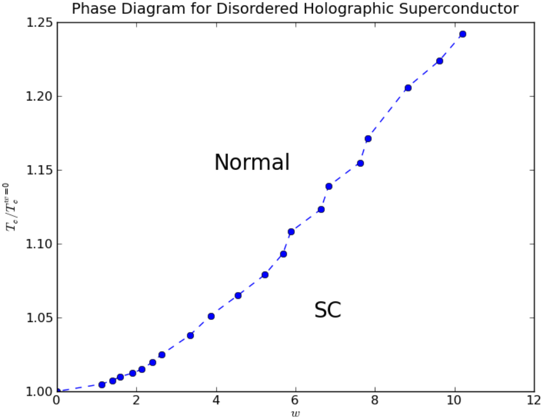


Condensate versus noise strength



- The value of the condensate grows with increasing disorder strength, w

Disordered phase diagram



- Phase diagram, dependence of the critical temperature on the strength of the noise.

Disorder renormalization?

- For highly discontinuous functions of the boundary value of the chemical potential, we find very smooth dependence of the condensate on the coordinate x. A typical form of $\mu(x)$ and its corresponding $\mathcal{O}(x)$:

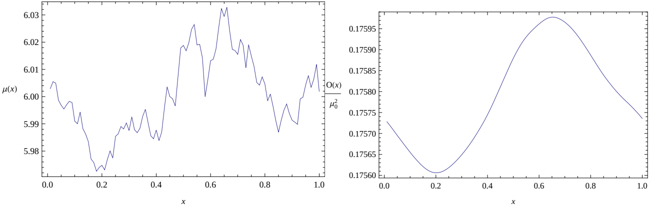


Figure: Initial chemical potential profile $\mu(x) = 6.0 + 0.1 \sum_{n=1}^{100} \frac{1}{2\pi n} \cos(2\pi nx + \delta_n)$ (left panel) and the corresponding condensate profile (right panel).

Disorder renormalization II

- A noisy chemical potential will translate into an even noisier charge density. A typical form of $\mu(x)$ and its corresponding $\rho(x)$:

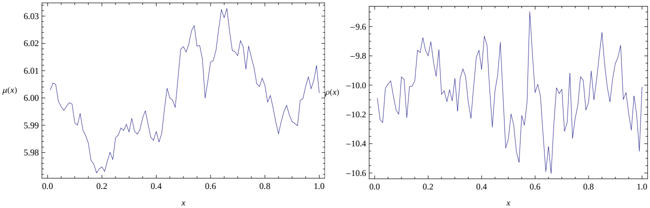
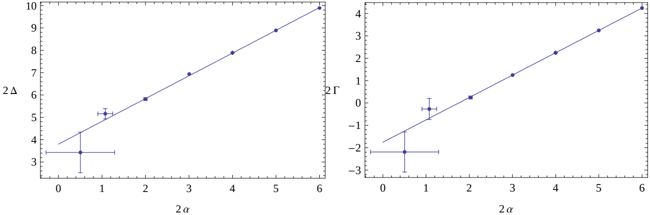


Figure: Initial chemical potential profile $\mu(x) = 6.0 + 0.1 \sum_{n=1}^{100} \frac{1}{2\pi n} \cos(2\pi n x + \delta_n)$ (left panel) and the corresponding charge density profile (right panel).

Universality?

- Power spectra: For a given random signal with power spectrum of the form $k^{-2\alpha}$ we study the power spectrum of the condensate $k^{-2\Delta(\alpha)}$ and of the charge density $k^{-2\Gamma(\alpha)}$



- Renormalization of the disorder: Condensate $\Delta = 3.8 + 1.0\,\alpha$ (left panel) and charge density $\Gamma = -1.75 + 1.0\,\alpha$ (right panel).

Holographic p-wave sc

- What about p-wave? Anderson's theorem doesn't even apply!
- Action (Gubser):

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{4}F^c_{\mu\nu}F_c^{\mu\nu} + R - \Lambda\right). \tag{9}$$

- Field content:
  - $A_t^3(z) \sim \mu$ (Chemical potential), breaks $SU(2) \to U(1)$
  - $A_x^1(z) \sim < J_x^1 >$ (p-wave condensate) breaks $U(1)$ and rotational invariance.

Set up

$$A = \phi(x,z)\, dt\, T_3 + w_x(x,z)\, T_1\, dx + w_y(x,z)\, T_1\, dy + \theta(x,z)\, T_2\, dt\,, \quad (10)$$

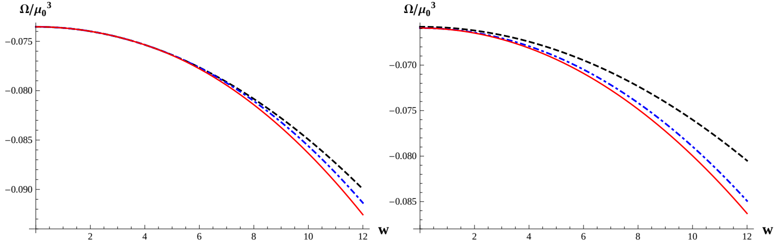
$$\phi(x,z) = \mu(x) - \rho(x)\, z + o(z^2)$$

- Disorder as before:

$$\begin{aligned} \mu(x) &= \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\, \cos(k\, x + \delta_k) \\ &= \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\, \cos(k\, x + \delta_k)\,, \end{aligned}$$

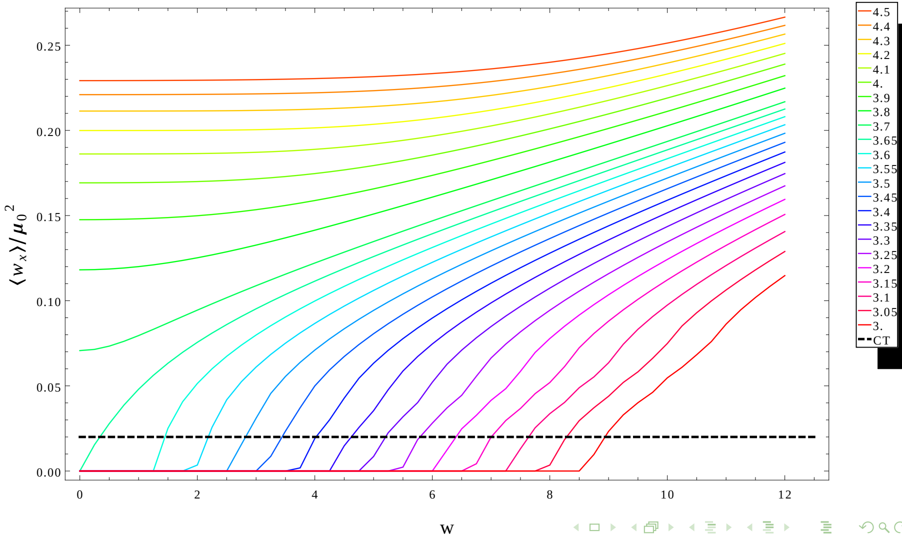
Free energy and competing solutions

$$\Omega = -\frac{T S_{\rm on-shell}}{L_y\, L} =$$
$$= l - \frac{1}{4L}\int_0^L dx\,\mu\,\rho + \frac{1}{4L}\int_0^L dx \int_0^1 dz\,\frac{1}{f}\left[(\theta^2+\phi^2)\,(w_x^2+w_y^2) + w_x(\phi\,\partial_x\theta - \theta\,\partial_x\phi)\right],$$



- The left panel corresponds to $\mu_0 = 3.4 < \mu_c$, and the right one to $\mu_0 = 3.8 > \mu_c$ (both with $\alpha = 1.50$). Black dashed line – the normal phase solution, the blue dot-dashed line – Y solution, red solid line – X solution.

Condensate versus disorder



Disordered p-wave phase diagram

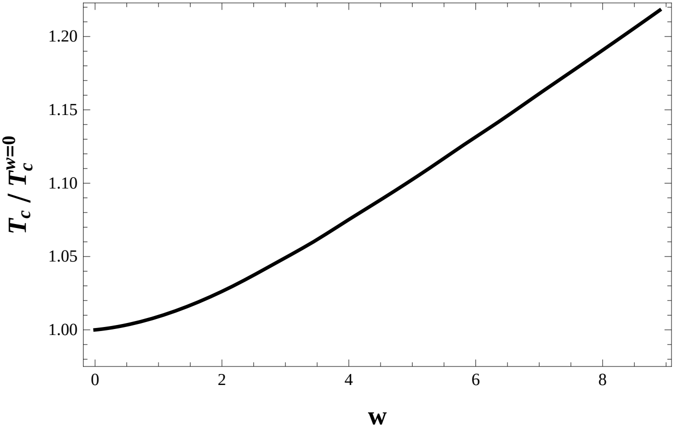


Figure: Enhancement of T_c with the noise strength w ($T_c^{w=0}$ stands for the critical temperature in the absence of disorder).

Conclusions

- Implementation of disorder in AdS/CFT for a s- and p-wave holographic superconductor.
- Better understanding of the x-dependent behavior.
- Disordered thin film superconductors.
- Toward a disordered phase transition and critical exponents in MBL.

Quantum theory of friction following the textbook QFT of Many-Body Systems

- Dissipation (friction) in a quantum system can be simulated by coupling to a heat bath (a collection of harmonic oscillators).
- Coupling to the environment.

$$\begin{aligned} Z &= \text{constant} \times \int \mathcal{D}[x(t)]\mathcal{D}[h_i(t)] \times \\ & \exp i \int dt \left(\frac{1}{2} m\dot{x}^2 + \sum_i \frac{1}{2}\dot{h}_i^2 - \frac{1}{2}\left[\Omega_i h_i + g_i x\right]^2 \right). \end{aligned}$$

- x coordinate of particle, h_i a collection of oscillators.

- Classical friction γ with external force $f(t)$:

$$m\ddot{x} = -Kx - \gamma\dot{x} + f(t). \tag{11}$$

- $$\langle x(t_b)x(t_a)\rangle = \int \mathcal{D}[f(t)]\mathcal{D}[x(t)]x(t_b)x(t_a)P[f(t)]\delta\big[x(t) - x^f(t)\big],$$

- x^f classical solution, $P[f(t)]$ is the probability distribution (Gaussian) of $f(t)$.