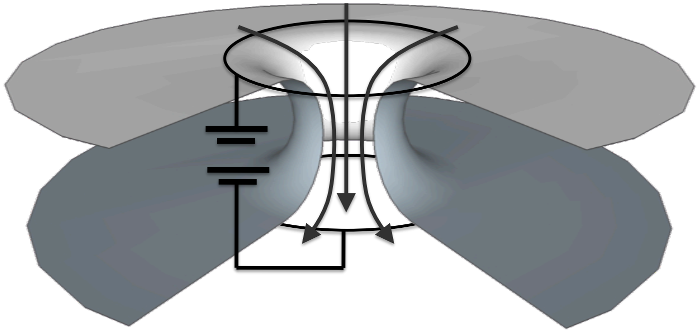
Electric fields and quantum wormholes

Nabil Iqbal

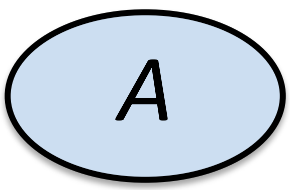
University of Amsterdam



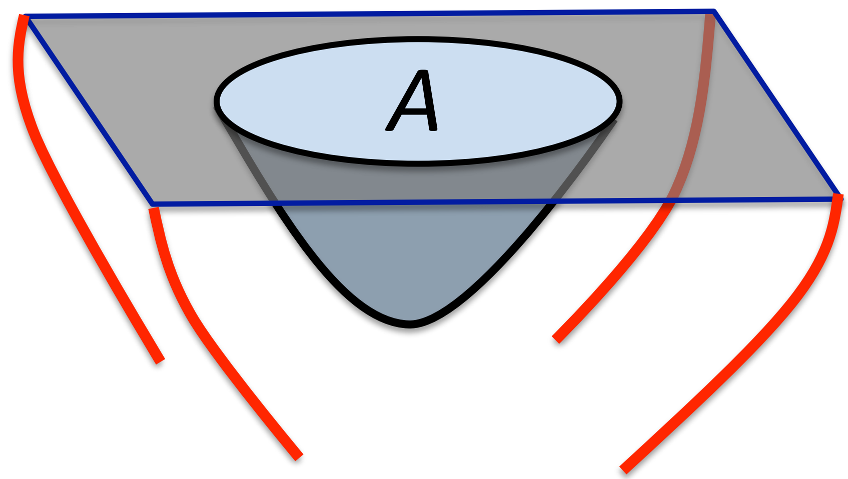
Work in progress with Dalit Engelhardt, Ben Freivogel.

Entanglement and geometry

Increasingly clear: entanglement and geometry are related in some way. Current most precise relation between these two ideas is in the context of AdS/CFT.



$$S_A = -\mathrm{Tr}\, \rho_A \log \rho_A$$



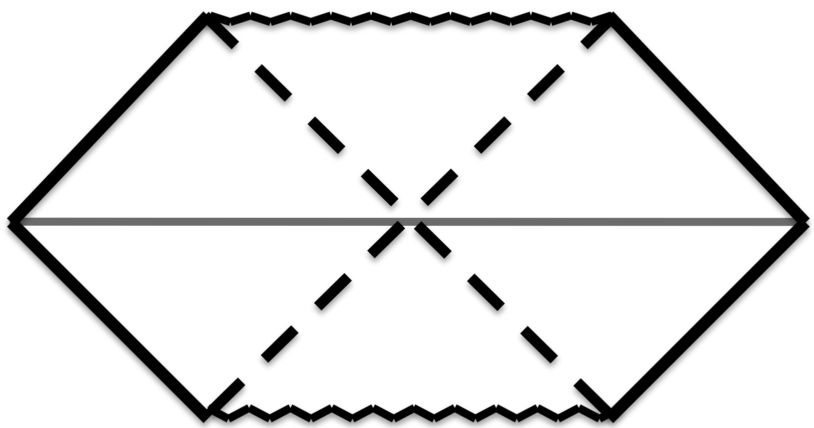
Ryu-Takayanagi relates entanglement entropy to a minimal area. Thus, in AdS/CFT, entanglement is dual to geometry.

However: in this talk, I will not be discussing AdS/CFT.

Entanglement and geometry

Recently, a stronger statement has been made (Maldacena, Susskind).

Consider the following two objects, all in the bulk:



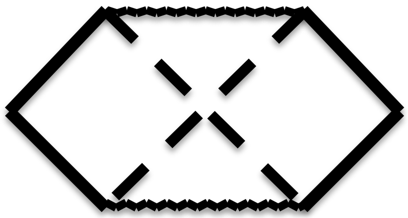
$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right)$$

Einstein-Rosen bridge connecting two black holes in asymptotically flat space.

Two electrons in an EPR entangled state, very far from each other.

Entanglement and geometry

Maldacena and Susskind (motivated by considerations of black hole information):



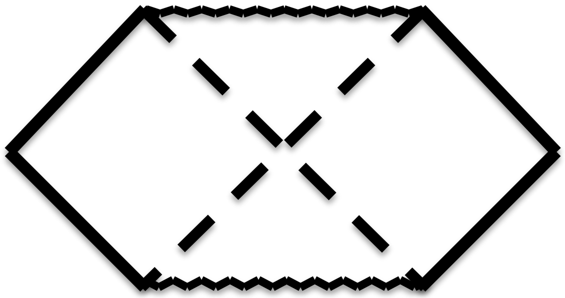
$$= \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$$

In a theory of quantum gravity, these are "the same."

Entanglement of ordinary perturbative quanta creates a very small Planckian Einstein-Rosen bridge between them: a "quantum wormhole".

ER "=" EPR

ER = EPR?



$$\stackrel{?}{=} \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right)$$

For now, this is just a definition of a quantum wormhole.

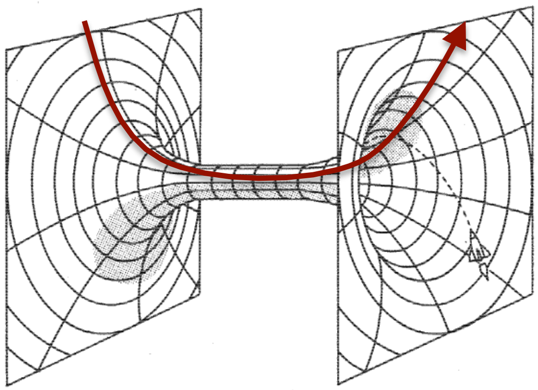
quan•tum worm•hole, *n.* (kwän-təm wərm-hōl):

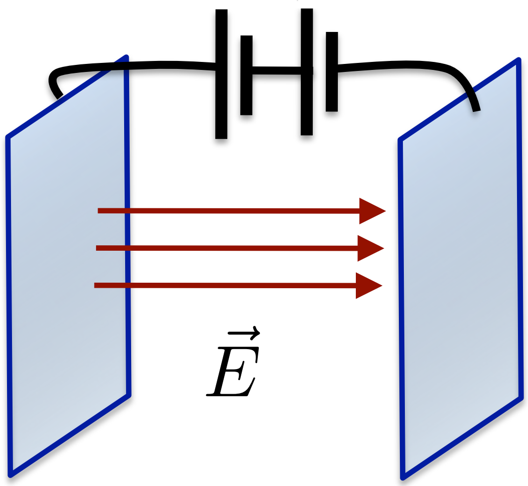
An entangled state of ordinary perturbative matter in two spatially separated lumps.

Challenge: are quantum wormholes anything like classical ones?

Classical wormholes

What good is a classical wormhole?





Changes the topology of space: non-shrinkable S^2 means that you can send an electric field through it.

$$E = \frac{2\mu}{\Delta x}$$

Can we send an electric field through a quantum wormhole?

1. Motivation
2. Electric fields and classical wormholes
3. Electric fields and quantum wormholes
4. Discussion

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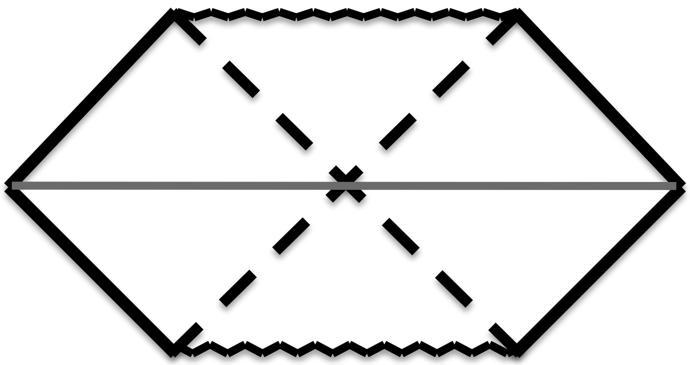
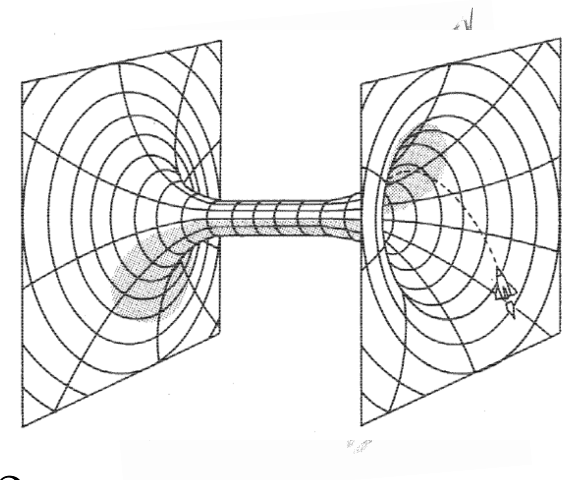
Classical wormholes

We should first understand precisely what it means to pass electric flux through a classical wormhole.

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} - \frac{1}{4g_F^2}F^2\right)$$

Usual eternal Schwarzschild solution:

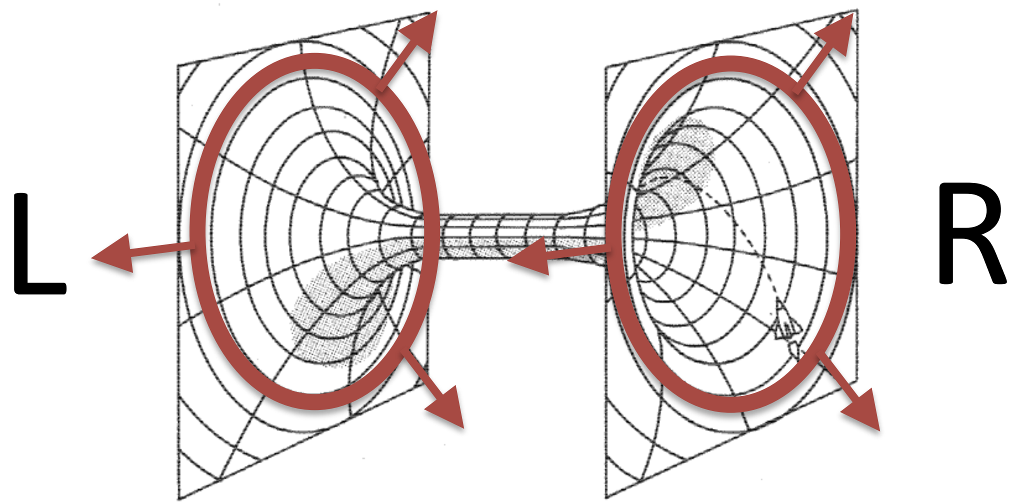




$$ds^2 = -\left(1-\frac{r_h}{r}\right)dt^2 + \frac{dr^2}{\left(1-\frac{r_h}{r}\right)} + r^2 d\Omega_2 \qquad r > r_h$$

Flux through the wormhole

Surround each horizon with a sphere of radius $a > r_h$.



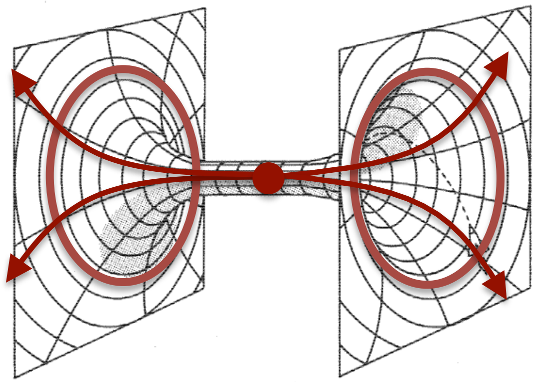
Consider the electric flux through each of these spheres:

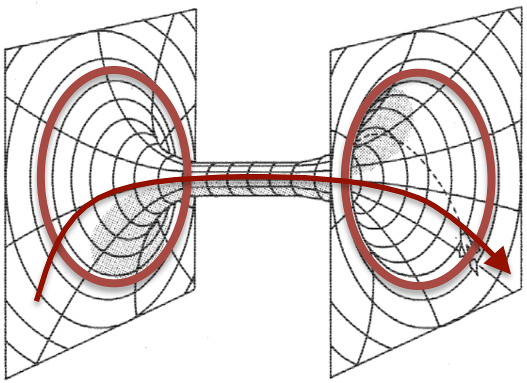
$$\Phi_{L,R} \equiv \frac{1}{g_F^2} \int_{S^2} d\vec{A} \cdot \vec{E}_{L,R}$$

Different kinds of fluxes

$$\Phi_\Sigma \equiv \Phi_L + \Phi_R$$

$$\Phi_\Delta \equiv \frac{1}{2}(\Phi_R - \Phi_L)$$





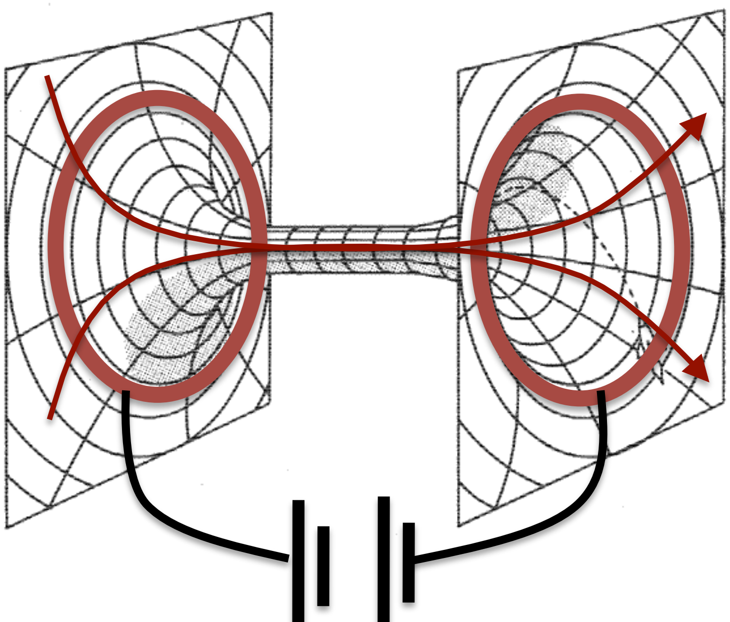
By Gauss's law: Counts the number of charged particles inside.
"Hard" to change.
Quantized: $\mathbb{Z}q$

Measures the field through the wormhole.
"Easy" to change.
Continuously tunable.

This distinction arises because the geometry is connected.

Fun with batteries

Consider setting up a potential difference across the two sides.



$V = 2\mu$

$$\nabla_\mu F^{\mu\nu} = 0$$

$$A_t(r = a_L) = -\mu$$
$$A_t(r = a_R) = +\mu$$

$$\boxed{\Phi_\Delta = \frac{1}{g_F^2}\left(4\pi r_h \mu\right)}$$

This illustrates that the flux through the wormhole is tunable.

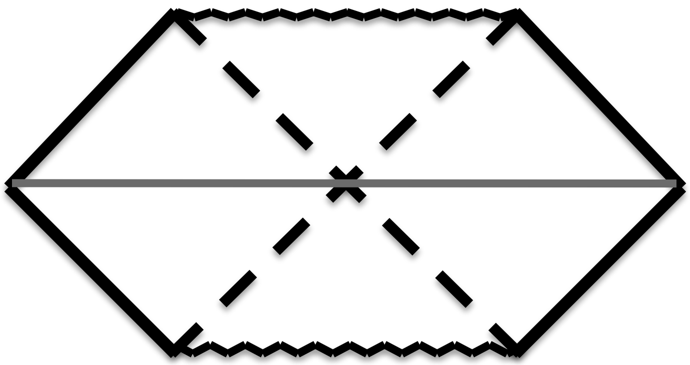
This is true, but it feels like a classical statement. Actually it has a precise meaning at the quantum level.

Flux sector of U(1) EM

Let us study the quantum theory. In compact U(1) EM electric flux operator on a nontrivial S^2 has a discrete spectrum.

$$\hat{\Phi} = q\mathbb{Z}$$

In thermal equilibrium, we are studying the Hartle-Hawking state. Takes the thermo-field form:



$$|\psi\rangle = \frac{1}{\sqrt{Z}}\sum_n |\mathrm{CPT}(n)\rangle_L |n\rangle_R \exp\left(-\frac{\beta E_n}{2}\right)$$

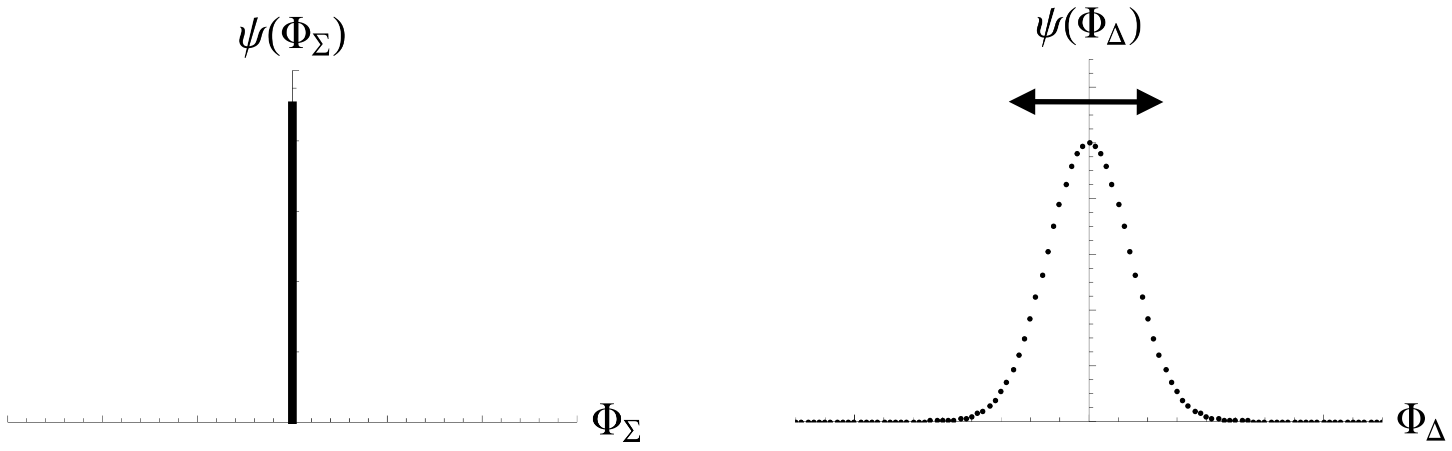
CPT flips the sign of the flux but leaves the energy invariant.

$$|\psi\rangle = \frac{1}{\sqrt{Z}}\sum_n |\cdots, E_n, -\Phi_n\rangle_L |\cdots, E_n, \Phi_n\rangle_R \exp\left(-\frac{\beta E_n}{2}\right)$$

$$\hat{\Phi}_\Sigma|\psi\rangle = 0 \qquad \hat{\Phi}_\Delta|\psi\rangle \neq 0$$

Wormhole susceptibility

The HH state is not an eigenstate of flux through the wormhole.



The intuitive difference between the two kinds of fluxes can be traced back to this wavefunction.

Now define the *wormhole susceptibility*:

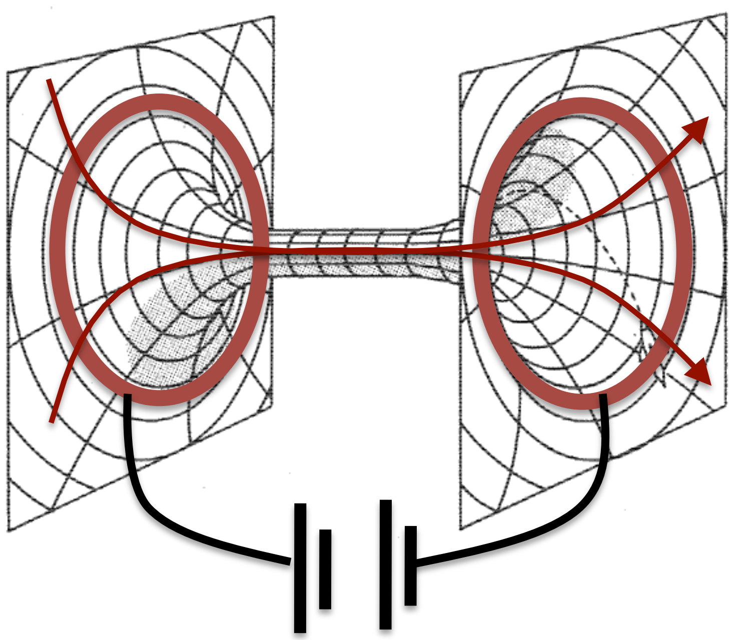
$$\boxed{\chi_\Delta \equiv \langle \Phi_\Delta^2 \rangle}$$

This is the object we will study for the remainder of the talk.

Linear response

Consider now turning on the chemical potential:

$$|\psi(\mu)\rangle = \frac{1}{\sqrt{Z}} \sum_n |\mathrm{CPT}(n)\rangle_L |n\rangle_R \exp\left(-\frac{\beta}{2}(E_n - \boxed{\mu\Phi_n)}\right)$$



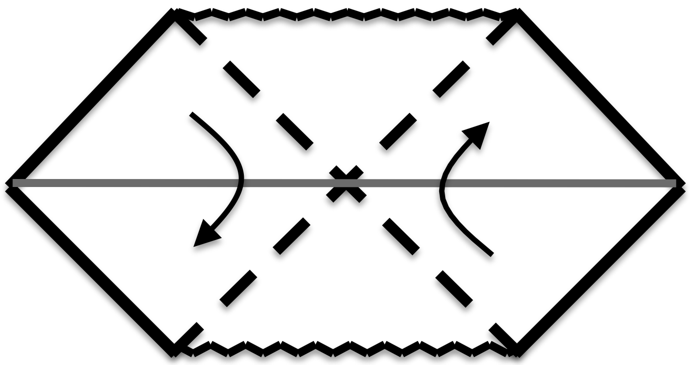
$$\langle \Phi_\Delta(\mu) \rangle = \chi_\Delta(\beta\mu)$$

$$\chi_\Delta^{ER} = \frac{1}{g_F^2}$$

In this case, wormhole susceptibility is nonzero because of geometric connection. Can we understand the full wave-function?

EM flux sector on the BH background

Can be more explicit: because the S^2 never shrinks, the flux on each side is a quantum degree of freedom with an effective Hamiltonian with respect to Schwarzschild time.



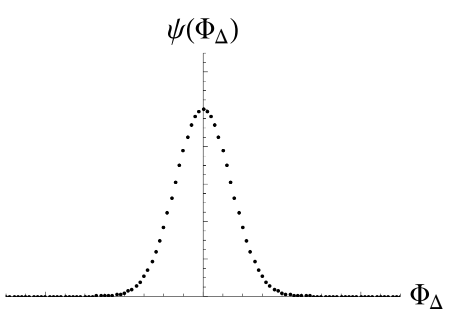
$$-\frac{1}{g_F^2}\int d^4x\sqrt{-g}F^2$$

$$\downarrow$$

$$H = \frac{g_F^2}{8\pi r_h}\Phi^2$$

Can now explicitly work out reduced wavefunction from thermofield state.

$$\psi(\Phi_\Delta) = \exp\left(-\frac{g_F^2}{4}\Phi_\Delta^2\right)$$



1. Motivation
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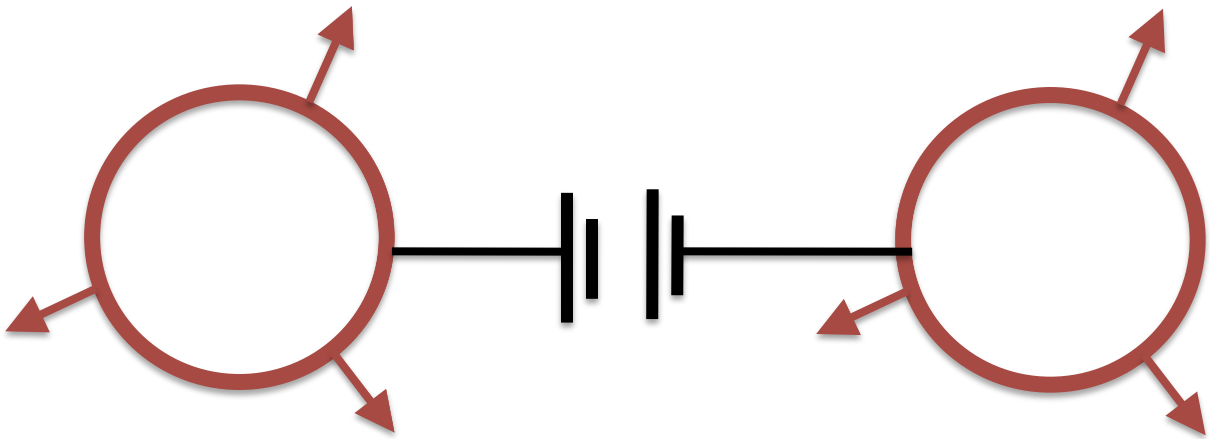
Quantum wormholes

We now want to try the same thing for a quantum wormhole.

$$S = \int d^4x \left(-\frac{1}{4g_F^2} F^2 + \cdots \right)$$

Take two disconnected spherical boxes of radius *a* in flat space.



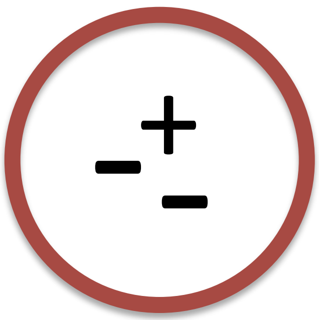
$$\Phi_L = \Phi_R = 0$$

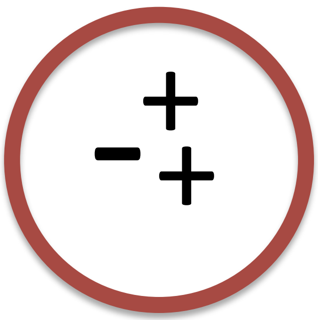
Does not matter how much you entangle them: because of the trivial topology, looks bleak. This is a useless quantum wormhole.

Quantum wormholes

Add now a scalar field of charge q:

$$S = \int d^4x \left(-\frac{1}{4g_F^2} F^2 - |D\phi|^2 - m^2|\phi|^2 \right)$$





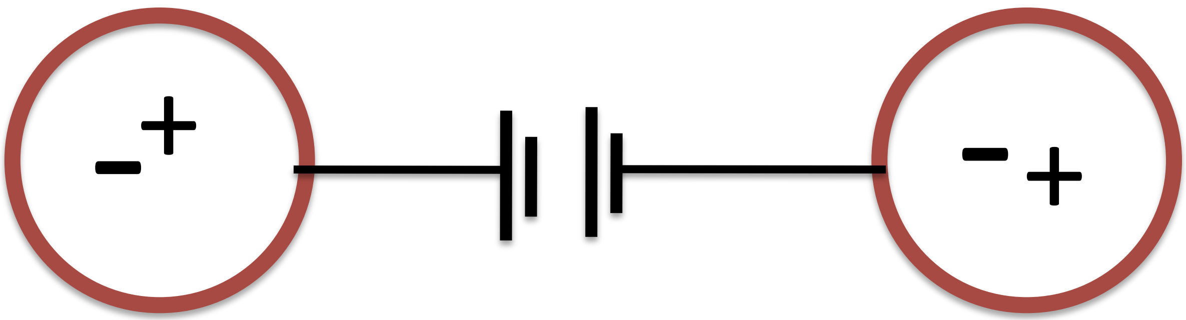
$$\frac{1}{g_F^2} \vec{\nabla} \cdot \vec{E} = \rho \qquad \hat{\Phi}_L = \hat{Q}_L \qquad \hat{\Phi}_R = \hat{Q}_R$$

Each flux just counts the number of particles in the appropriate box.
Looks like no difference between Φ_Σ and Φ_Δ.

Quantum wormholes

Consider again the entangled thermofield state with the battery:

$$|\psi(\mu)\rangle = \frac{1}{\sqrt{Z}} \sum_n |\mathrm{CPT}(n)\rangle_L |n\rangle_R \exp\left(-\frac{\beta}{2}(E_n - \mu\Phi_n)\right)$$

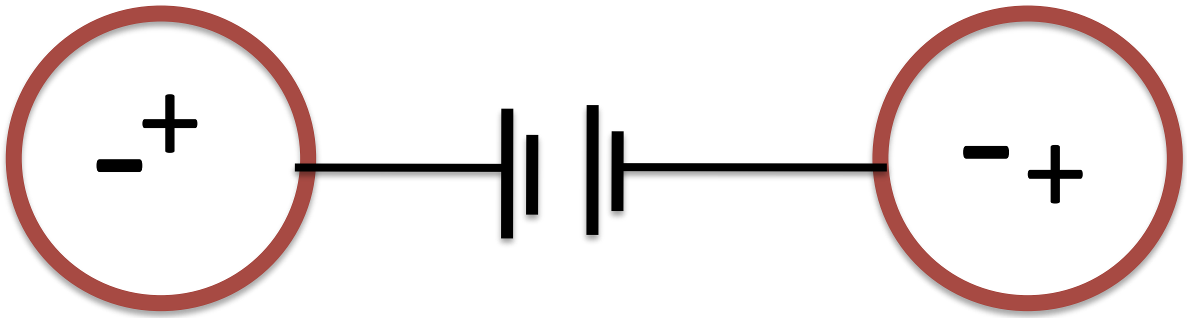


The scalar field sector now looks like:

$$|\psi(\mu)\rangle_\phi = \frac{1}{\sqrt{Z}} \sum_n |\cdots, E_n, -Q_n\rangle_L |\cdots, E_n, Q_n\rangle_R \exp\left(-\frac{\beta}{2}(E_n - \mu Q_n)\right)$$

The scalar field "feels" a chemical potential $+\mu$ on right and $-\mu$ on left.

More fun with batteries



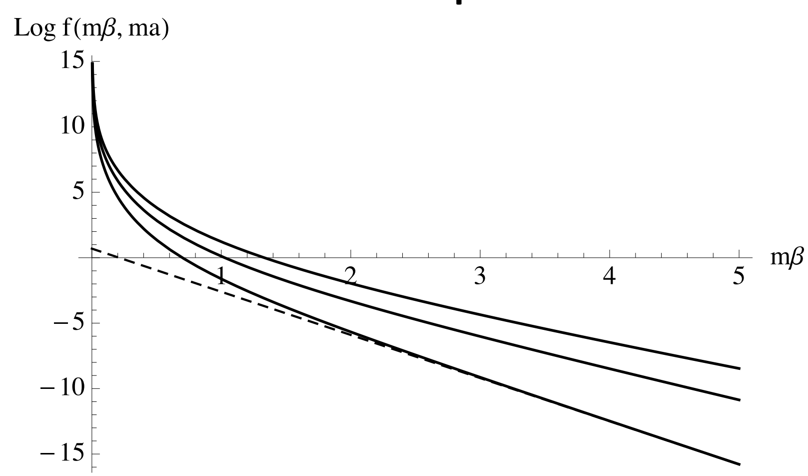
Compute the "flux through":

$$\langle\psi(\mu)|\hat{\Phi}_\Delta|\psi(\mu)\rangle = \frac{1}{2}\langle\psi(\mu)|\hat{Q}_R - \hat{Q}_L|\psi(\mu)\rangle$$

On each side, trace out the other. Normal QFT computation. Find:

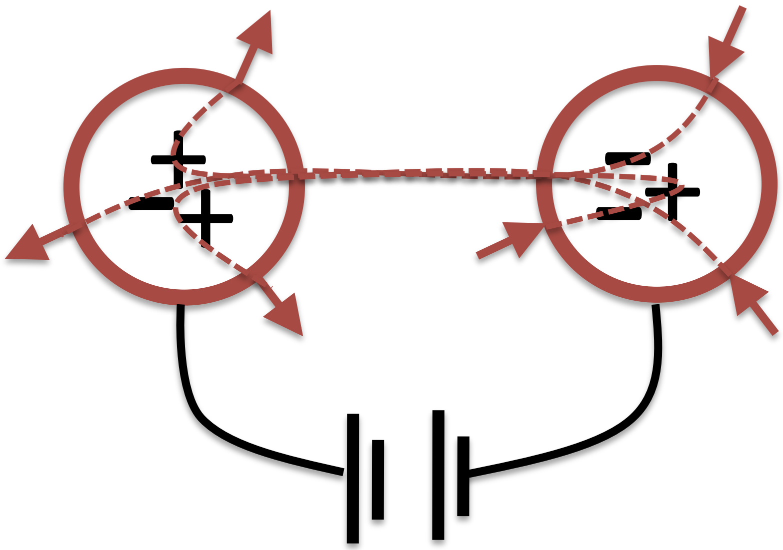
$$\langle\Phi_\Delta\rangle = (q^2\beta\mu)f(ma, m\beta)$$

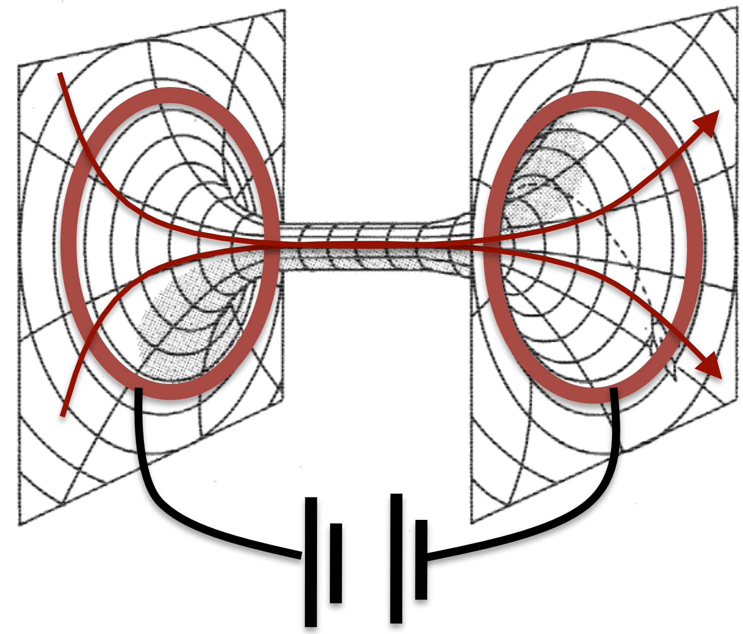
$$\boxed{\chi_\Delta^{EPR} \neq 0}$$



Flux through a quantum wormhole

What just happened?





From the point of view of electric field response, the black hole and entangled matter behave qualitatively the same. There is an electric field through the quantum wormhole.

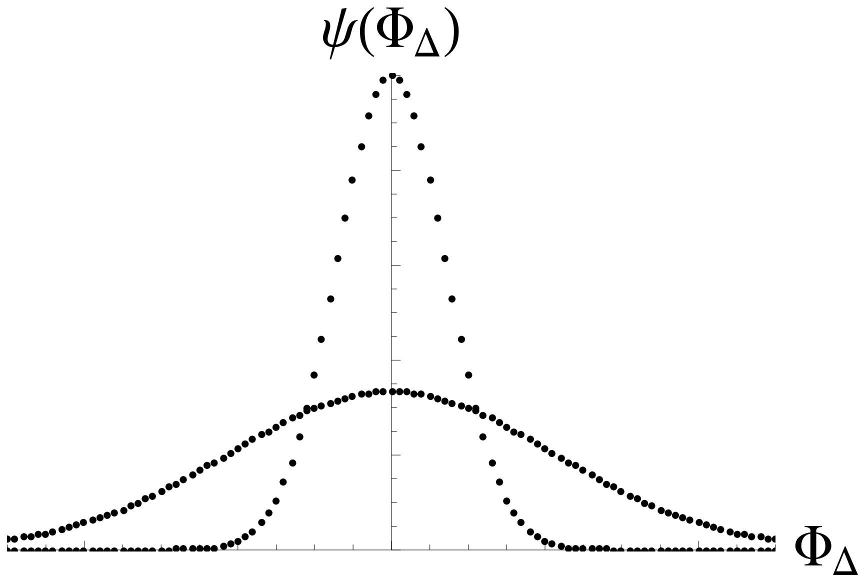
The entanglement has tricked the gauge field into thinking there is a connection.

Classical versus quantum

There is an important quantitative difference.

$$\chi_\Delta^{ER} = \frac{1}{g_F^2}$$

$$\chi_\Delta^{EPR} \sim O(1)$$

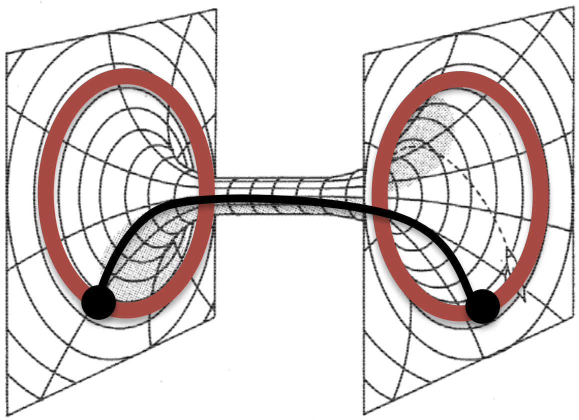


Similarity of wavefunctions means similar "universal" behavior: but much harder to pass an electric field through a quantum wormhole.

Can view susceptibility as defining the U(1) gauge coupling in wormhole region: quantum wormhole then has strongly coupled gauge field.

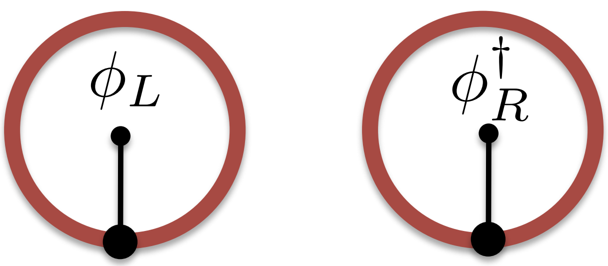
Wilson lines through the horizon

Imagine now threading a Wilson line through the black hole horizon:



$$W_{ER} = \left\langle \exp\left(iq \int_L^R A \right) \right\rangle \approx 1$$

For the quantum wormhole, no geometry, but there is an object with the same quantum numbers:



$$W_{EPR} = \left\langle e^{iq \int_L^0 A} \phi_L(0) \phi_R(0)^\dagger e^{iq \int_0^R A} \right\rangle$$

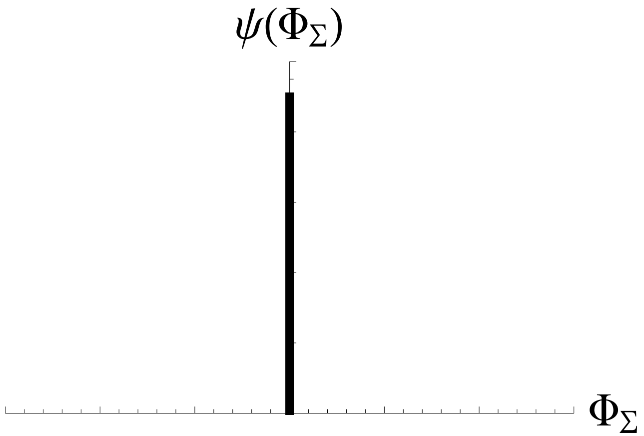
$$\sim \exp\left(-\frac{m\beta}{2} \right)$$

Much smaller: might say the gauge field in "wormhole" is fluctuating wildly.

Gauss's law in the quantum wormhole

Even if the gauge field is strongly coupled in the wormhole, it still satisfies Gauss's law:

$$\left(\hat{\Phi}_L + \hat{\Phi}_R\right)|\psi\rangle = 0$$



This is because we took a charge pairing of the form:

$$|\psi\rangle \sim \sum_q |-q\rangle_L |q\rangle_R$$

More general pairings (e.g. "generic" states) will not respect Gauss's law. No simple geometric interpretation (see Marolf, Polchinski; Balasubramanian, Berkooz, Ross, Simon), but somewhat similar to filling the quantum wormhole with a superconducting fluid.

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Future directions

Our results may be understood as studying the symmetry breaking pattern:

$$U(1)_L \times U(1)_R \to U(1)_{L+R}$$

We have been studying finite-volume systems. In infinite-volume systems such symmetry breaking is expected to result in Goldstone modes. Not entirely clear what this means for an excited state.

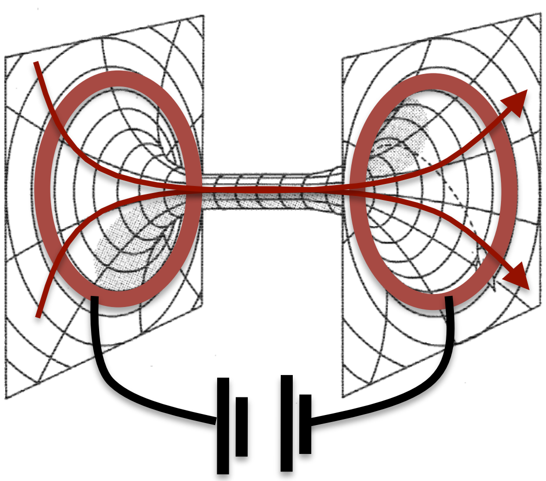
A similar story applies for gravitational fields, which also obey a Gauss's law.

$$\kappa_\Delta^{ER} = \frac{1}{8\pi G_N} \qquad \kappa_\Delta^{EPR} \sim O(1)$$

Here the corresponding susceptibility measures Newton's constant in the wormhole throat.

Summary

- It was suggested that we should view the entanglement of perturbative quanta as creating a "quantum wormhole."
- Classical wormholes admit electric field lines due to nontrivial topology.
- Quantum wormholes also admit electric field lines in a precise sense, measured by a wormhole susceptibility, mimicking the effects of nontrivial topology.
- May provide insight on how geometric structures can arise from entanglement.



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