Superconformal Quantum Mechanics and Emerging Holographic QCD

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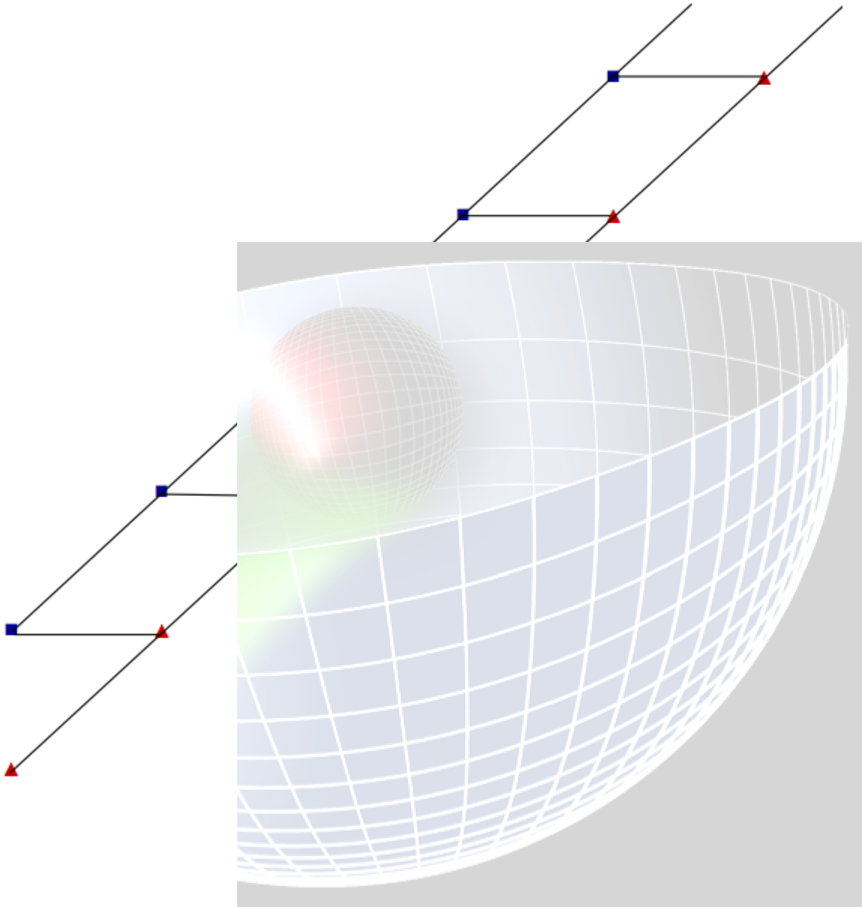

Gauge/Gravity Duality 2015

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In collaboration with Stan Brodsky and Hans G. Dosch

Quest for a semiclassical approximation to describe bound states in QCD

1. Semiclassical approximation to QCD in the Light-Front (LF): Reduction of QCD LF Hamiltonian leads to a relativistic LF wave equation, where complexities from strong interactions are incorporated in effective potential U

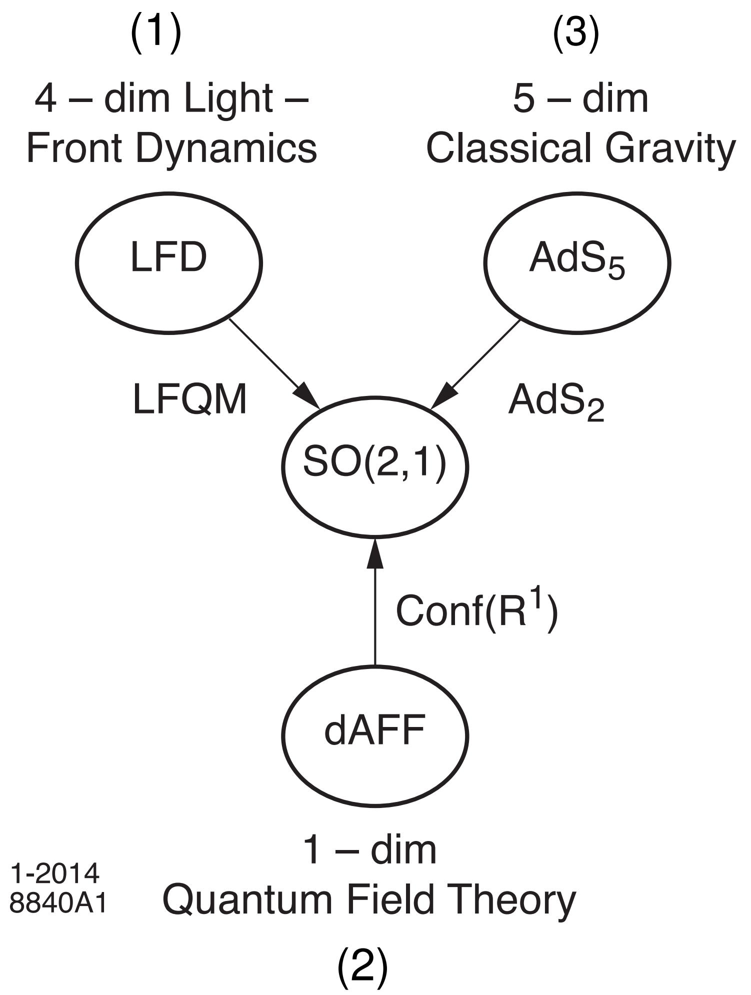
2. Construction of LF potential U: Since the LF semiclassical approach leads to a one-dim QFT, it is natural to extend conformal QM to the light front since it gives important insights into the confinement mechanism and the emergence of a mass scale

3. Correspondence between equations of motion for arbitrary spin in AdS space and relativistic LF bound-state equations in physical space-time: Embedding of LF wave equations in AdS leads to extension of LF potential U to arbitrary integer spin

4. Superconformal extension of conformal QM to describe baryons in complete analogy to mesons

5. Superconformal connection between mesons and baryons: Supersymmetry is broken since the ground state, the pion, is massless (in the chiral limit) and is not paired.

Hadronic triality



Conformal and Superconformal Quantum Mechanics

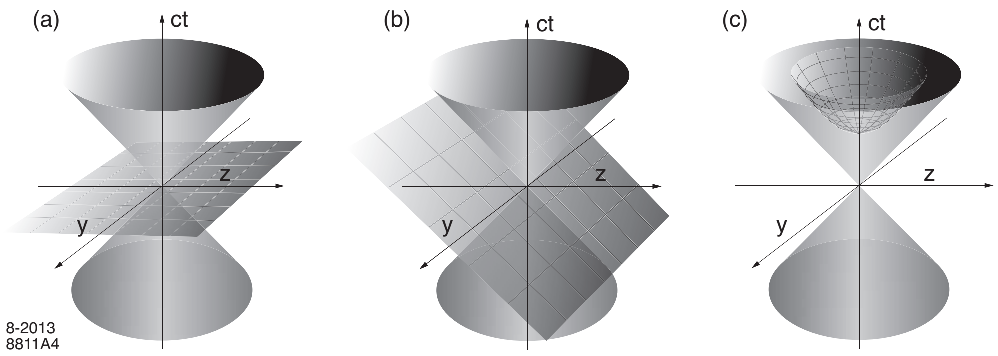
[de Alfaro, Fubini and Furlan (1976, Fubini and Rabinovici (1984)]

Isomorphism *Conf*$(R^1) \sim SO(2,1) \sim \mathrm{AdS}_2$

(1) Semiclassical approximation to QCD in the light front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Dirac Forms of Relativistic Dynamics [Dirac (1949)]



- (a) Instant form $x^0 = 0$, (b) Front form $x^0 + x^3 = 0$, (c) Point Form $x^2 = \kappa^2$
- LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = M^2 |\phi\rangle$ is a LF wave equation for ϕ

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right) \phi(\zeta) = M^2 \phi(\zeta)$$

- Invariant variable in impact space $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$ (N partons: LF cluster decomposition)
- Critical value $L = 0$ corresponds to lowest possible stable solution: ground state of the LF Hamiltonian
- Relativistic and frame-independent LF Schrödinger equation: U is instantaneous in LF time and comprises all interactions, including those with higher Fock states.

(2) Conformal quantum mechanics and light-front dynamics

[S. J. Brodsky, GdT and H.G. Dosch, PLB **729**, 3 (2014)]

- Incorporate in 1-dim effective QFT the conformal symmetry of 4-dim QCD Lagrangian in the limit of massless quarks: Conformal QM [V. de Alfaro, S. Fubini and G. Furlan, Nuovo Cim. A **34**, 569 (1976)]

- Conformal Hamiltonian:

$$H = \tfrac{1}{2}\left(p^2 + \frac{g}{x^2}\right)$$

g dimensionless: Casimir operator of the representation

- Schrödinger picture: $p = -i\partial_x$

$$H = \tfrac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right)$$

- QM evolution

$$H|\psi(t)\rangle = i\frac{d}{dt}|\psi(t)\rangle$$

H is one of the generators of the conformal group *Conf*(R^1). The two additional generators are:

- Dilatation: $D = -\frac{1}{4}\left(px + xp\right)$

- Special conformal transformations: $K = \frac{1}{2}x^2$

- H, D and K close the conformal algebra

$$[H,D] = iH, \quad [H,K] = 2iD, \quad [K,D] = -iK$$

- dAFF construct a new generator G as a superposition of the 3 generators of *Conf*(R^1)

$$G = uH + vD + wK$$

and introduce new time variable τ

$$d\tau = \frac{dt}{u + vt + wt^2}$$

- Find usual quantum mechanical evolution for time τ

$$G|\psi(\tau)\rangle = i\frac{d}{d\tau}|\psi(\tau)\rangle \qquad H|\psi(t)\rangle = i\frac{d}{dt}|\psi(t)\rangle$$

$$G = \frac{1}{2}u\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right) + \frac{i}{4}v\left(x\,\frac{d}{dx} + \frac{d}{dx}\,x\right) + \frac{1}{2}wx^2.$$

- Operator G is compact for $4uw - v^2 > 0$, but action remains conformal invariant !
- Emergence of scale: Since the generators of *Conf*$(R^1) \sim SO(2,1)$ have different dimensions a scale appears in the new Hamiltonian G, which according to dAFF may play a fundamental role

Connection to light-front dynamics

- Compare the dAFF Hamiltonian G

$$G = \frac{1}{2}u\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right) + \frac{i}{4}v\left(x\,\frac{d}{dx} + \frac{d}{dx}\,x\right) + \frac{1}{2}wx^2.$$

with the LF Hamiltonian H_{LF}

$$H_{LF} = -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)$$

and identify dAFF variable x with LF invariant variable ζ

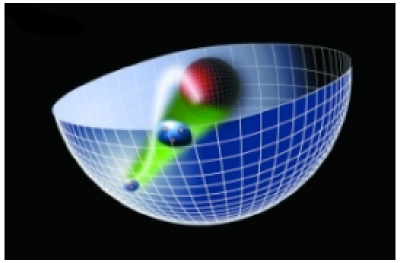
- Choose $u = 2, \quad v = 0$
- Casimir operator from LF kinematical constraints: $g = L^2 - \frac{1}{4}$
- $w = 2\lambda^2$ fixes the LF potential to harmonic oscillator in the LF plane $\lambda^2\,\zeta^2$

$$U \sim \lambda^2\zeta^2$$

- One can perform a level shift by adding an arbitrary constant to LF potential U: Not true for baryons !

(3) Higher integer-spin wave equations in AdS space

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]



- Description of higher spin modes in AdS space (Frondsal, Fradkin, Vasiliev, Metsaev . . .)
- Integer spin-J in AdS conveniently described by tensor field $\Phi_{N_1 \dots N_J}$ with effective action

$$S_{\mathit{eff}} = \int d^d x \, dz \, \sqrt{|g|} \, e^{\varphi(z)} \, g^{N_1 N_1'} \cdots g^{N_J N_J'} \Big(g^{M M'} D_M \Phi^*_{N_1 \dots N_J} \, D_{M'} \Phi_{N_1' \dots N_J'} - \mu^2_{\mathit{eff}}(z) \, \Phi^*_{N_1 \dots N_J} \, \Phi_{N_1' \dots N_J'} \Big)$$

D_M is the covariant derivative which includes affine connection and dilaton $\varphi(z)$ effectively breaks maximal symmetry of AdS$_{d+1}$

$$ds^2 = \frac{R^2}{z^2} \left(dx_\mu dx^\mu - dz^2 \right)$$

- Effective mass $\mu_{\mathit{eff}}(z)$ is determined by precise mapping to light-front physics
- Non-trivial geometry of pure AdS encodes the kinematics and additional deformations of AdS encode the dynamics, including confinement

- Physical hadron has plane-wave and polarization indices along $3{+}1$ physical coordinates and a profile wavefunction $\Phi(z)$ along holographic variable z

$$\Phi_P(x,z)_{\mu_1\cdots\mu_J} = e^{iP\cdot x}\Phi(z)_{\mu_1\cdots\mu_J}, \qquad \Phi_{z\mu_2\cdots\mu_J} = \cdots = \Phi_{\mu_1\mu_2\cdots z} = 0$$

with four-momentum P_μ and invariant hadronic mass $P_\mu P^\mu {=} M^2$

- Variation of the action gives AdS wave equation for spin-J field $\Phi(z)_{\nu_1\ldots\nu_J} = \Phi_J(z)\epsilon_{\nu_1\cdots\nu_J}(P)$

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1-2J}}\partial_z\right) + \left(\frac{mR}{z}\right)^2\right]\Phi_J = M^2\Phi_J$$

with

$$(m\,R)^2 = (\mu_{\mathit{eff}}(z)R)^2 - Jz\,\varphi'(z) + J(d-J+1)$$

and the kinematical constraints to eliminate the lower spin states $J-1,\, J-2, \cdots$

$$\eta^{\mu\nu}P_\mu\,\epsilon_{\nu\nu_2\cdots\nu_J} = 0, \quad \eta^{\mu\nu}\,\epsilon_{\mu\nu\nu_3\cdots\nu_J} = 0$$

- Kinematical constrains in the LF imply that m must be a constant

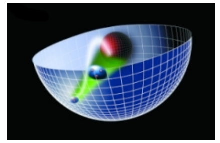
[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

Light-front mapping

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

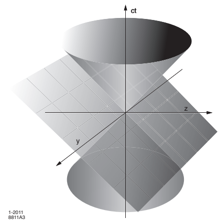
- Upon substitution $\Phi_J(z) \sim z^{(d-1)/2-J} e^{-\varphi(z)/2} \phi_J(z)$ and $z \to \zeta$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right] \Phi_J(z) = M^2 \Phi_J(z)$$



we find LFWE $(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$



with

$$U(\zeta) = \tfrac{1}{2}\varphi''(\zeta) + \frac{1}{4}\varphi'(\zeta)^2 + \frac{2J - 3}{2z}\varphi'(\zeta)$$

and $(\mu R)^2 = -(2 - J)^2 + L^2$

- Unmodified AdS equations correspond to the kinetic energy terms for the partons
- Effective confining potential $U(\zeta)$ corresponds to the IR modification of AdS space
- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

Meson spectrum

- Dilaton profile in the dual gravity model determined from one-dim QFTh (dAFF)

$$\varphi(z) = \lambda z^2, \qquad \lambda^2 = \tfrac{1}{2} w$$

- Effective potential: $U = \lambda^2 \zeta^2 + 2\lambda(J-1)$

- LFWE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z) = 1$

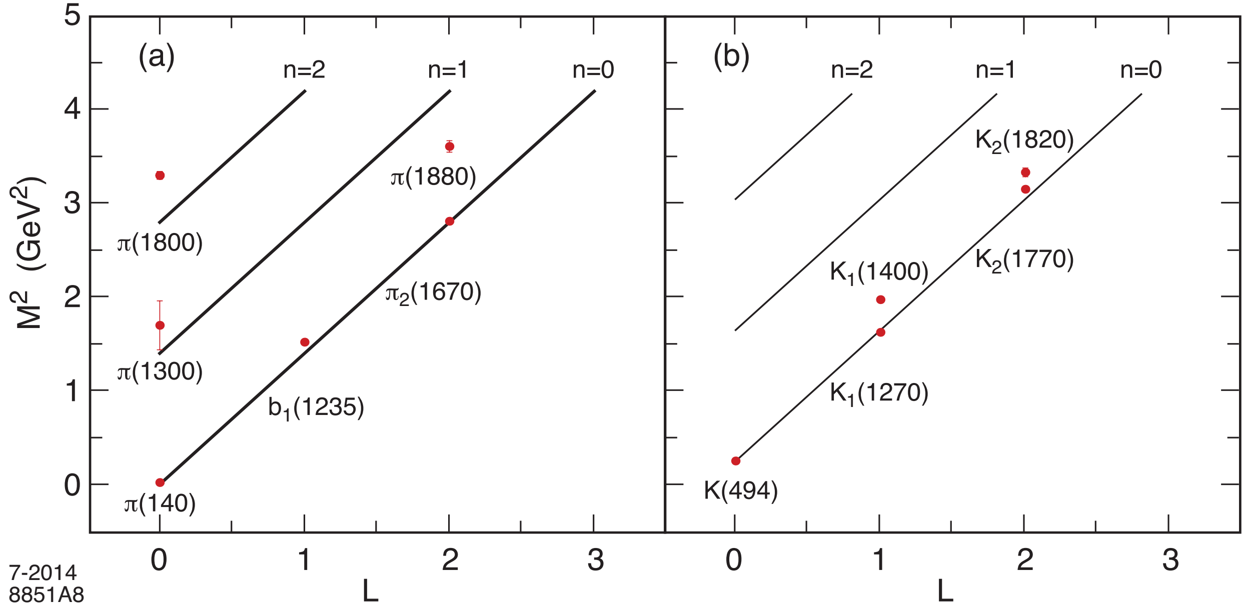
$$\phi_{n,L}(\zeta) = |\lambda|^{(1+L)/2} \sqrt{\frac{2n!}{(n+L)!}}\, \zeta^{1/2+L} e^{-|\lambda|\zeta^2/2} L_n^L(|\lambda|\zeta^2)$$

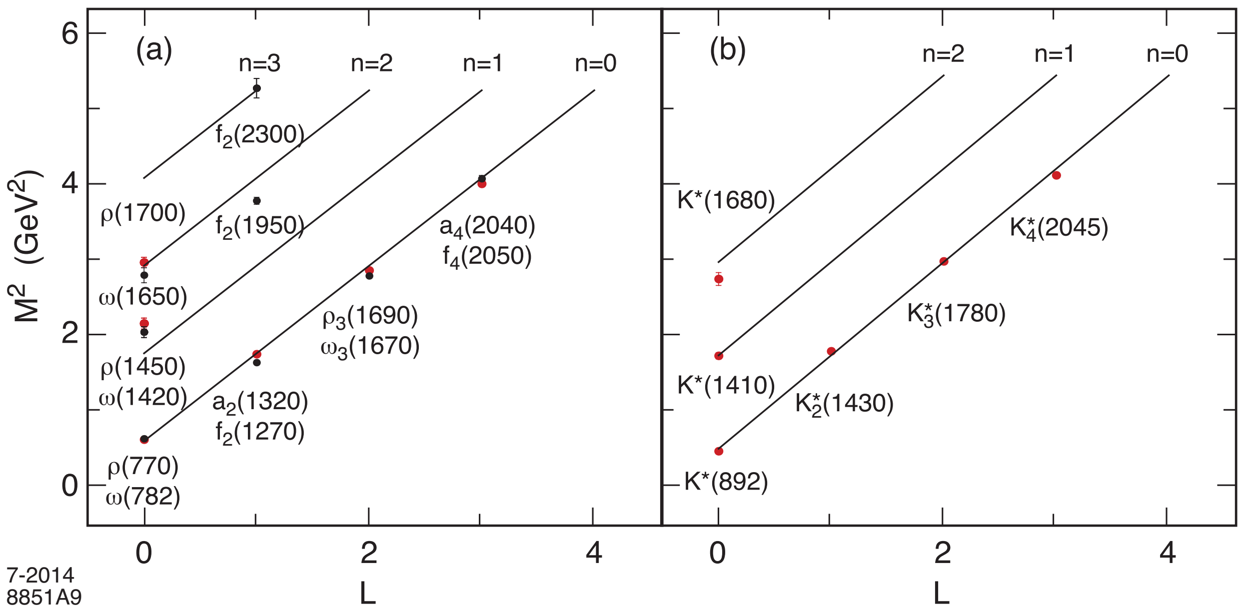
- Eigenvalues for $\lambda > 0$

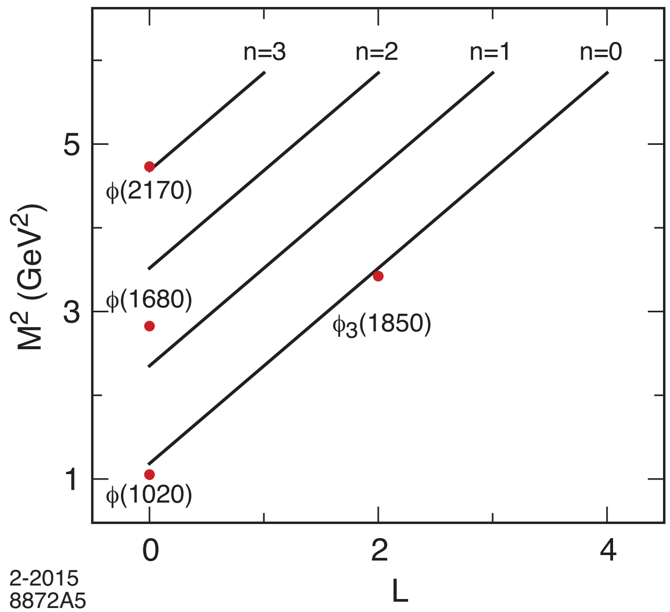
$$\mathcal{M}^2_{n,J,L} = 4\lambda\left(n + \frac{J+L}{2}\right)$$

- Results are easily extended to light quarks

- $\lambda < 0$ incompatible with LF constituent interpretation







Orbital and radial excitations for $\sqrt{\lambda} = 0.59$ GeV (pseudoscalar) and 0.54 GeV (vector mesons)

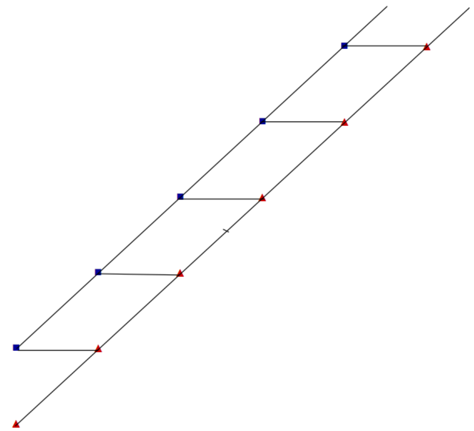
(4) Superconformal quantum mechanics and light-front dynamics

[GdT, H.G. Dosch and S. J. Brodsky, Phys. Rev. D **91**, 045040 (2015)]

- SUSY QM contains two fermionic generators Q and $Q^\dagger$, and a bosonic generator, the Hamiltonian H

  [E. Witten, NPB **188**, 513 (1981)]

- Closure under the graded algebra $sl(1/1)$:



$$\frac{1}{2}\{Q, Q^\dagger\} = H$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$
$$[Q, H] = [Q^\dagger, H] = 0$$

  Note: Since $[Q^\dagger, H] = 0$ the states $|E\rangle$ and $Q^\dagger|E\rangle$ have identical eigenvalues E

- A simple realization is

$$Q = \chi\,(ip + W)\,, \qquad Q^\dagger = \chi^\dagger\,(-ip + W)$$

  where χ and $\chi^\dagger$ are spinor operators with anticommutation relation

$$\{\chi, \chi^\dagger\} = 1$$

- In a 2×2 Pauli-spin matrix representation: $\chi = \frac{1}{2}\,(\sigma_1 + i\sigma_2)\,, \;\; \chi^\dagger = \frac{1}{2}\,(\sigma_1 - i\sigma_2)$

$$[\chi, \chi^\dagger] = \sigma_3$$

- Following Fubini and Rabinovici consider a 1-dim QFT invariant under conformal and supersymmetric transformations [S. Fubini and E. Rabinovici, NPB **245**, 17 (1984)]

- Conformal superpotential (f is dimensionless)

$$W(x) = \frac{f}{x}$$

- Thus 1-dim QFT representation of the operators

$$Q = \chi \left(\frac{d}{dx} + \frac{f}{x} \right), \qquad Q^\dagger = \chi^\dagger \left(-\frac{d}{dx} + \frac{f}{x} \right)$$

- Conformal Hamiltonian $H = \frac{1}{2}\{Q, Q^\dagger\}$ in matrix form

$$H = \frac{1}{2} \begin{pmatrix} -\frac{d^2}{dx^2} + \frac{f(f-1)}{x^2} & 0 \\ 0 & -\frac{d^2}{dx^2} + \frac{f(f+1)}{x^2} \end{pmatrix}$$

- Conformal graded-Lie algebra has in addition to Hamiltonian H and supercharges Q and $Q^\dagger$, a new operator S related to generator of conformal transformations K

$$S = \chi\, x, \qquad S^\dagger = \chi^\dagger x$$

- Find enlarged algebra (Superconformal algebra of Haag, Lopuszanski and Sohnius (1974))

$$
\begin{aligned}
\tfrac{1}{2}\{Q, Q^\dagger\} &= H, \qquad \tfrac{1}{2}\{S, S^\dagger\} = K \\
\tfrac{1}{2}\{Q, S^\dagger\} &= \frac{f}{2} + \frac{\sigma_3}{4} + iD \\
\tfrac{1}{2}\{Q^\dagger, S\} &= \frac{f}{2} + \frac{\sigma_3}{4} - iD
\end{aligned}
$$

where the operators

$$
\begin{aligned}
H &= \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{f^2 - \sigma_3 f}{x^2}\right) \\
D &= \frac{i}{4}\left(\frac{d}{dx}x + x\frac{d}{dx}\right) \\
K &= \tfrac{1}{2}x^2
\end{aligned}
$$

satisfy the conformal algebra

$$[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK$$

- Following F&R define a supercharge R, a linear combination of the generators Q and S

$$R = \sqrt{u}\, Q + \sqrt{w}\, S$$

and consider the new generator $G = \frac{1}{2}\{R, R^\dagger\}$ which also closes under the graded algebra $sl(1/1)$

$$\frac{1}{2}\{R, R^\dagger\} = G \qquad \frac{1}{2}\{Q, Q^\dagger\} = H$$
$$\{R, R\} = \{R^\dagger, R^\dagger\} = 0 \qquad \{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$
$$[R, H] = [R^\dagger, H] = 0 \qquad [Q, H] = [Q^\dagger, H] = 0$$

- New QM evolution operator

$$G = uH + wK + \frac{1}{2}\sqrt{uw}\,(2f + \sigma_3)$$

is compact for $uw > 0$: Emergence of a scale since Q and S have different units

- Light-front extension of superconformal results follows from

$$x \to \zeta, \quad f \to \nu + \tfrac{1}{2}, \quad \sigma_3 \to \gamma_5, \quad 2G \to H_{LF}$$

- Obtain:

$$H_{LF} = -\frac{d^2}{d\zeta^2} + \frac{\left(\nu + \frac{1}{2}\right)^2}{\zeta^2} - \frac{\nu + \frac{1}{2}}{\zeta^2}\gamma_5 + \lambda^2\zeta^2 + \lambda(2\nu + 1) + \lambda\gamma_5$$

where coefficients u and w are fixed to $u = 2$ and $w = 2\lambda^2$

- Take the 'square root' of the LF Hamiltonian $H_{LF} = \{R, R^\dagger\}$

$$H_{LF}\, \psi = D_{LF}^2 \psi = M^2 \psi$$

with the linear Dirac equation

$$(D_{LF} - M)\, \psi = 0$$

- In a 2×2 component representation $\psi_\pm$

$$\begin{aligned} -\frac{d}{d\zeta}\psi_- - \frac{\nu + \frac{1}{2}}{\zeta}\psi_- - \lambda\,\zeta\psi_- &= M\psi_+ \\ \frac{d}{d\zeta}\psi_+ - \frac{\nu + \frac{1}{2}}{\zeta}\psi_+ - \lambda\,\zeta\psi_+ &= M\psi_- \end{aligned}$$

where the chiral spinors are defined by $\psi_\pm = \frac{1}{2}\left(1 \pm \gamma_5\right)\psi$

- Note: In a 4×4 Dirac-matrix representation the spinor operators χ and $\chi^\dagger$ satisfy the relations

$$\{\chi, \chi^\dagger\} = 1 \quad \text{and} \quad [\chi, \chi^\dagger] = \gamma_5$$

(3) Higher half-integer spin wave equations in AdS space

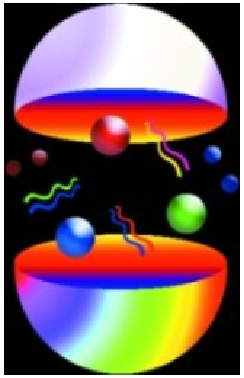


Image credit: N. Evans

[J. Polchinski and M. J. Strassler, JHEP **0305**, 012 (2003)]

[GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]

- Important similarities between spectra of mesons and baryons: similar slope and spacing of orbital and radial excitations, similar multiplicity
- Holographic embeddings in AdS also explains distinctive features, such as the absence of spin-orbit coupling for baryons
- Half-integer spin-J in AdS described by Rarita-Schwinger (RS) spinor field $\left[\Psi_{N_1 \cdots N_{J-1/2}}\right]_\alpha$ with effective action ($J = T + 1/2$)

$$S_{\textit{eff}} = \frac{1}{2} \int d^d x \, dz \, \sqrt{|g|} \, g^{N_1 \, N_1'} \cdots g^{N_T \, N_T'} \left[\overline{\Psi}_{N_1 \cdots N_T} \Big(i \, \Gamma^A \, e^M_A \, D_M - \mu - \rho(z)\Big) \Psi_{N_1' \cdots N_T'} + h.c.\right]$$

where covariant derivative D_M includes affine connection and spin connection

- e^A_M is the vielbein and Γ^A tangent space Dirac matrices $\quad \left\{\Gamma^A, \Gamma^B\right\} = \eta^{AB}$

- Dilaton term does not lead to confinement: introduce effective interaction $\rho(z)$ in AdS Dirac equation

  [Z. Abidin and C. E. Carlson, Phys. Rev. D **79**, 115003 (2009)]

- Baryons described by half-integer spin-J field in AdS

$$\Psi^{\pm}_{\nu_1\cdots\nu_{J-1/2}}(x,z) = e^{iP\cdot x}\, u^{\pm}_{\nu_1\cdots\nu_{J-1/2}}(P)\Psi^{\pm}_J(z)$$

  with invariant hadronic mass $P_\mu P^\mu = M^2$ and chiral spinors $u^{\pm} = \frac{1}{2}(1 \pm \gamma_5)u$ with polarization indices along physical coordinates

- Variation of the AdS action leads to Dirac equation $\big(V(z) = \frac{R}{z}\rho(z)\big)$

$$\left[-\frac{d}{dz} - \frac{\mu R}{z} - V(z)\right]\Psi^- \;=\; M\Psi^+$$

$$\left[\frac{d}{dz} - \frac{\mu R}{z} - V(z)\right]\Psi^+ \;=\; M\Psi^-$$

  and the Rarita-Schwinger condition in physical space-time

$$\gamma^\nu \Psi_{\nu\nu_2\,\ldots\,\nu_T} = 0$$

- Compare AdS Dirac equation for spin J

$$
\begin{aligned}
-\frac{d}{dz}\Psi_J^- - \frac{\mu R}{z}\Psi_J^- - V(z)\Psi_J^- &= M\Psi_J^+ \\
\frac{d}{dz}\Psi_J^+ - \frac{\mu R}{z}\Psi_J^+ - V(z)\Psi_J^+ &= M\Psi_J^-
\end{aligned}
$$

with LF SUSY Dirac equation

$$
\begin{aligned}
-\frac{d}{d\zeta}\psi_- - \frac{\nu+\frac{1}{2}}{\zeta}\psi_- - \lambda\,\zeta\psi_- &= M\psi_+ \\
\frac{d}{d\zeta}\psi_+ - \frac{\nu+\frac{1}{2}}{\zeta}\psi_+ - \lambda\,\zeta\psi_+ &= M\psi_-
\end{aligned}
$$

- Identifying holographic variable z with invariant LF variable ζ, map AdS into LF Dirac eq. $\Psi_J \to \psi$

- AdS mass is related to parameter ν by $\mu R = \nu + \frac{1}{2}$ and

$$V(\zeta) = \lambda\,\zeta$$

a J-independent potential – No spin-orbit coupling along a given trajectory !

Baryon spectrum

- In 2×2 block-matrix form

$$H_{LF} = \begin{pmatrix} -\frac{d^2}{d\zeta^2} - \frac{1-4\nu^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(\nu+1) & 0 \\ 0 & -\frac{d^2}{d\zeta^2} - \frac{1-4(\nu+1)^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda\nu \end{pmatrix}$$

- The light-front eigenvalue equation $H_{LF}|\psi\rangle = M^2|\psi\rangle$ has eigenfunctions

$$\begin{aligned} \psi_+(\zeta) &\sim \zeta^{\frac{1}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^\nu(\lambda\zeta^2) \\ \psi_-(\zeta) &\sim \zeta^{\frac{3}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu+1}(\lambda\zeta^2) \end{aligned}$$

and eigenvalues

$$M^2 = 4\lambda(n + \nu + 1)$$

identical for both plus and minus eigenfunctions

- In contrast with mesons, we observe in the light-baryon spectrum a spin-J degeneracy for states with the same orbital angular momentum

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

- Lowest possible state $n = 0$ and $\nu = 0$: orbital excitations $\nu = 0, 1, 2 \cdots = L$

$$M^2 = 4\lambda(n + L + 1)$$

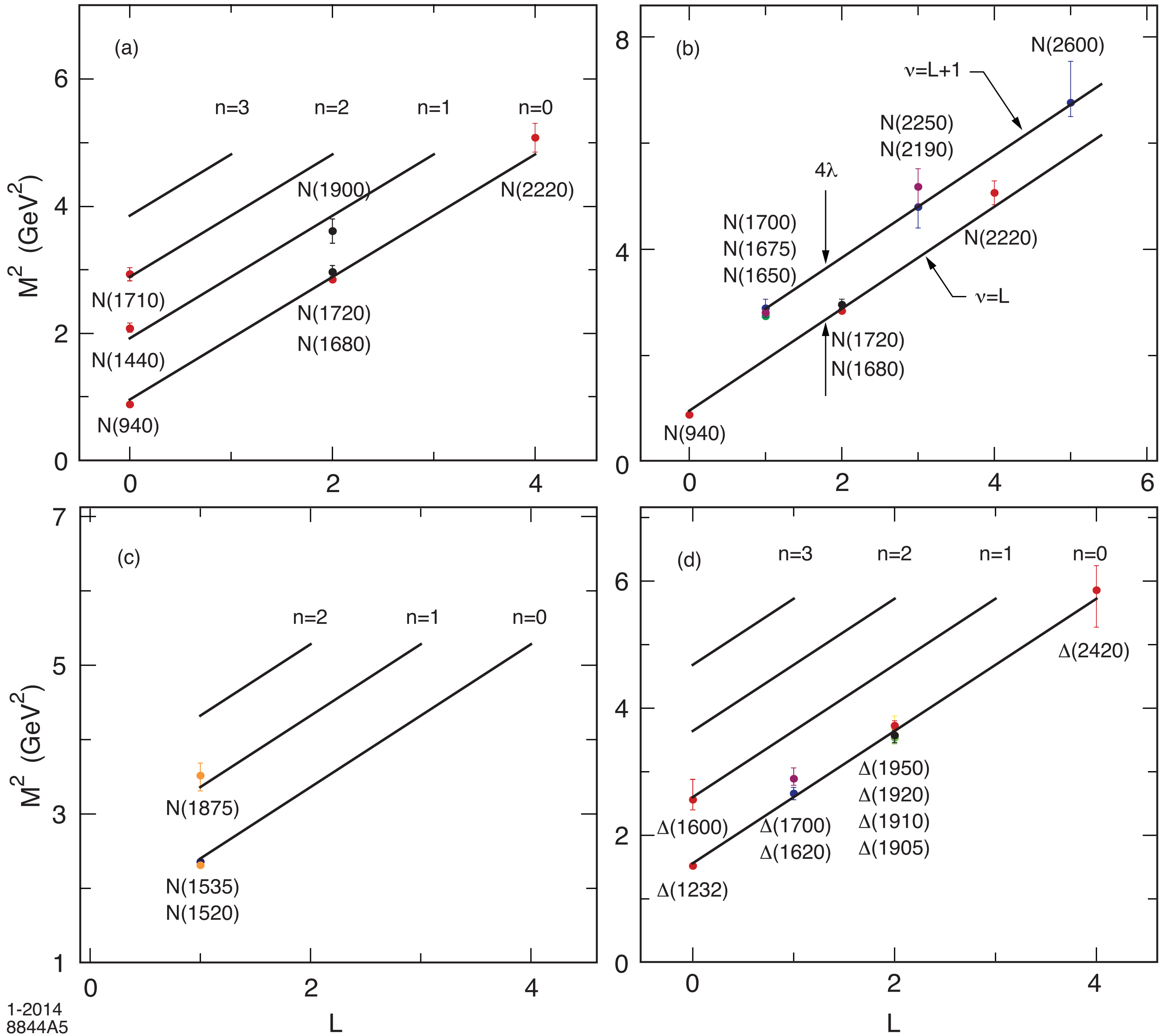
- L is the relative LF angular momentum between the active quark and spectator cluster
- In general ν depends on internal spin and parity

  The assignment

	$S = \frac{1}{2}$	$S = \frac{3}{2}$
P = +	$\nu = L$	$\nu = L + \frac{1}{2}$
P = -	$\nu = L + \frac{1}{2}$	$\nu = L + 1$

  describes the full light baryon orbital and radial excitation spectrum

$SU(6)$	S	L	n	Baryon State
56	$\frac{1}{2}$	0	0	$N\,\frac{1}{2}^+(940)$
	$\frac{3}{2}$	0	0	$\Delta\,\frac{3}{2}^+(1232)$
56	$\frac{1}{2}$	0	1	$N\,\frac{1}{2}^+(1440)$
	$\frac{3}{2}$	0	1	$\Delta\,\frac{3}{2}^+(1600)$
70	$\frac{1}{2}$	1	0	$N\,\frac{1}{2}^-(1535)$ $N\,\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	0	$N\,\frac{1}{2}^-(1650)$ $N\,\frac{3}{2}^-(1700)$ $N\,\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	0	$\Delta\,\frac{1}{2}^-(1620)$ $\Delta\,\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	0	2	$N\,\frac{1}{2}^+(1710)$
	$\frac{1}{2}$	2	0	$N\,\frac{3}{2}^+(1720)$ $N\,\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	0	$\Delta\,\frac{1}{2}^+(1910)$ $\Delta\,\frac{3}{2}^+(1920)$ $\Delta\,\frac{5}{2}^+(1905)$ $\Delta\,\frac{7}{2}^+(1950)$
70	$\frac{3}{2}$	1	1	$N\,\frac{1}{2}^-$ $N\,\frac{3}{2}^-(1875)$ $N\,\frac{5}{2}^-$
	$\frac{3}{2}$	1	1	$\Delta\,\frac{5}{2}^-(1930)$
56	$\frac{1}{2}$	2	1	$N\,\frac{3}{2}^+(1900)$ $N\,\frac{5}{2}^+$
70	$\frac{1}{2}$	3	0	$N\,\frac{5}{2}^-$ $N\,\frac{7}{2}^-$
	$\frac{3}{2}$	3	0	$N\,\frac{3}{2}^-$ $N\,\frac{5}{2}^-$ $N\,\frac{7}{2}^-(2190)$ $N\,\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	0	$\Delta\,\frac{5}{2}^-$ $\Delta\,\frac{7}{2}^-$
56	$\frac{1}{2}$	4	0	$N\,\frac{7}{2}^+$ $N\,\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	0	$\Delta\,\frac{5}{2}^+$ $\Delta\,\frac{7}{2}^+$ $\Delta\,\frac{9}{2}^+$ $\Delta\,\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	0	$N\,\frac{9}{2}^-$ $N\,\frac{11}{2}^-$
	$\frac{3}{2}$	5	0	$N\,\frac{7}{2}^-$ $N\,\frac{9}{2}^-$ $N\,\frac{11}{2}^-(2600)$ $N\,\frac{13}{2}^-$



Baryon orbital and radial excitations for $\sqrt{\lambda} = 0.49$ GeV (nucleons) and 0.51 GeV (Deltas)

(5) Superconformal baryon-meson symmetry and LF holographic QCD

[H.G. Dosch, GdT, and S. J. Brodsky, Phys. Rev. D **91**, 085016 (2015)]

- Previous application: positive and negative chirality components of baryons related by supercharge R

$$R^\dagger|\psi_+\rangle = |\psi_-\rangle$$

with identical eigenvalue M^2 since $[R, G] = [R^\dagger, G] = 0$

- Conventionally supersymmetry relates fermions and bosons

$$R^\dagger|\text{Baryon}\rangle = |\text{Meson}\rangle \quad \text{or} \quad \text{R}\,|\text{Meson}\rangle = |\text{Baryon}\rangle$$

- If $|\phi\rangle_M$ is a meson state with eigenvalue M^2, $G\,|\phi\rangle_M = M^2|\phi\rangle_M$, then there exists also a baryonic state $R\,|\phi\rangle_M = |\phi\rangle_B$ with the same eigenvalue M^2:

$$G\,|\phi\rangle_B = G\,R|\phi\rangle_M = R\,G\,|\phi\rangle_M = M^2|\phi\rangle_B$$

- For a zero eigenvalue M^2 we can have the trivial solution

$$|\phi(M^2 = 0)\rangle_B = 0$$

Special role played by the pion as a unique state of zero energy: same role as the unique vacuum state in supersymmetric quantum field theory

Superpartner of the nucleon trajectory

$$|\phi\rangle = \begin{pmatrix} \phi_{\text{Baryon}} \\ \phi_{\text{Meson}} \end{pmatrix}$$

- Compare superconformal equations with LFH nucleon (leading twist) and pion wave equations:

$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f + \lambda + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2}\right)\phi_{\text{Baryon}} = M^2 \phi_{\text{Baryon}}$$

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2 \zeta^2 + 2\lambda_B(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2}\right)\psi^+_{L_B} = M^2 \psi^+_{L_B}$$

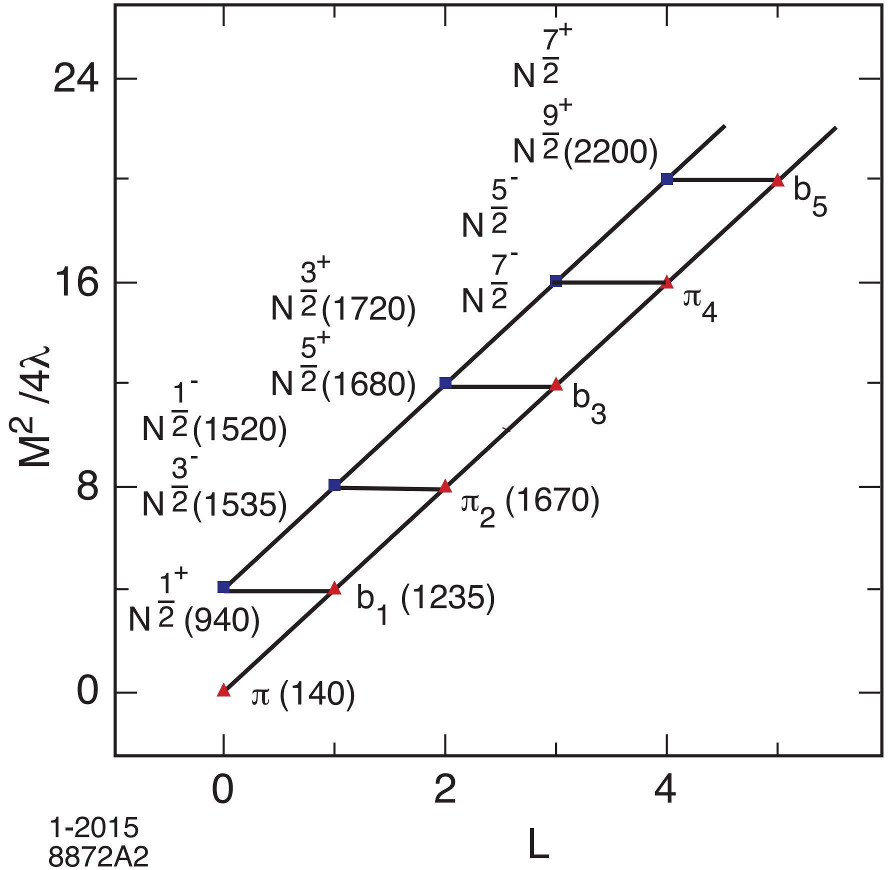
$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f - \lambda + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2}\right)\phi_{\text{Meson}} = M^2 \phi_{\text{Meson}}$$

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_M^2 \zeta^2 + 2\lambda_M(L_M - 1) + \frac{4L_M^2 - 1}{4\zeta^2}\right)\psi_{L_M} = M^2 \psi_{L_M}$$

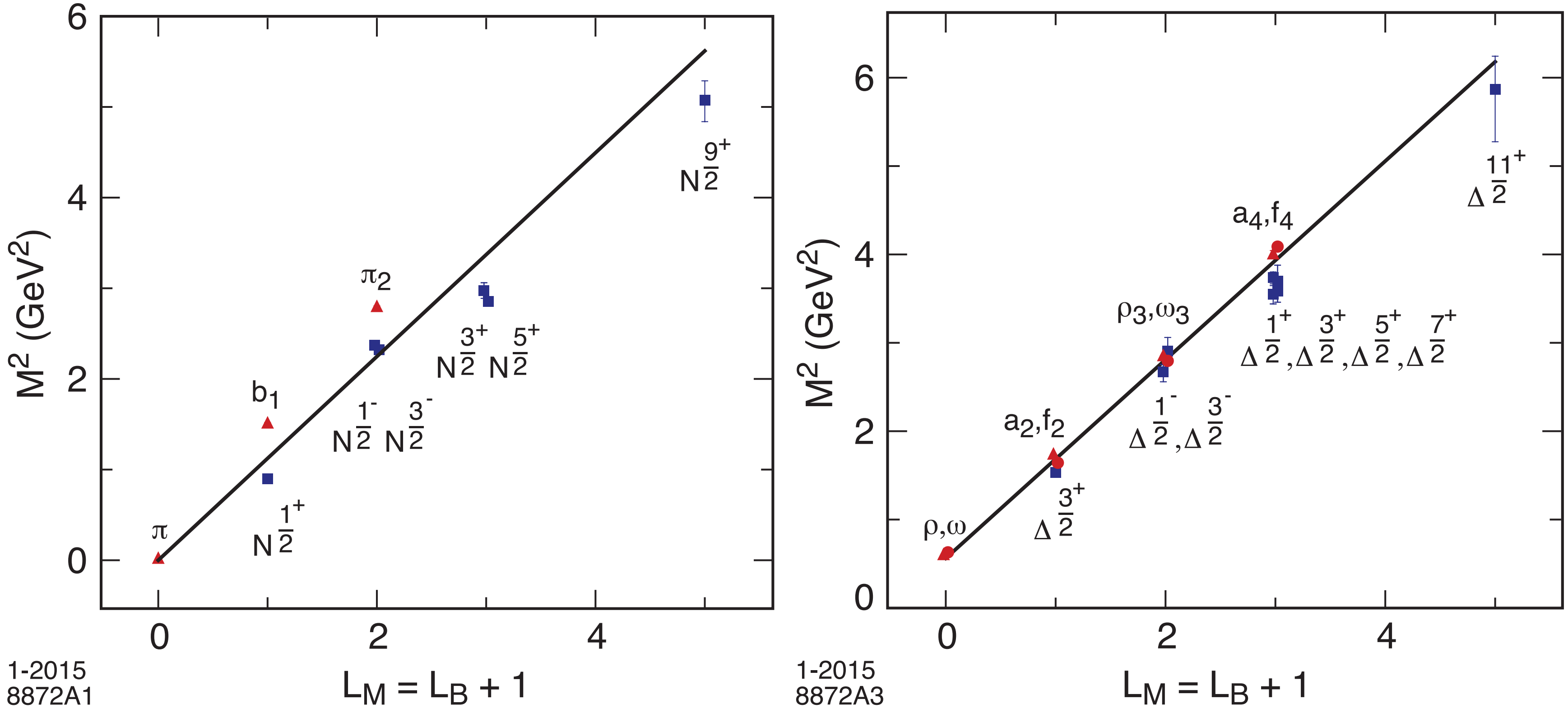
- Find: $\lambda = \lambda_M = \lambda_B, \quad f = L_B + \frac{1}{2} = L_M - \frac{1}{2} \quad \Rightarrow \quad \boxed{L_M = L_B + 1}$

- The LF angular momentum L_M of the meson is by one unit larger than its baryonic partner
- Identical confinement mechanism for mesons and baryons: introduction of a scale inside the algebra
- Equality $\lambda_M \simeq \lambda_B$ dictated by SCQM
- Constant term in LF potential for mesons from SCQM: pion is massless in the chiral limit !
- Should be regarded as a zero order approximation: Phenomenologically:

$$\sqrt{\lambda_N} = 0.49\,\text{GeV} \quad \sqrt{\lambda_\pi} = 0.59\,\text{GeV}$$
$$\sqrt{\lambda_\Delta} = 0.51\,\text{GeV} \quad \sqrt{\lambda_\rho} = 0.54\,\text{GeV}$$

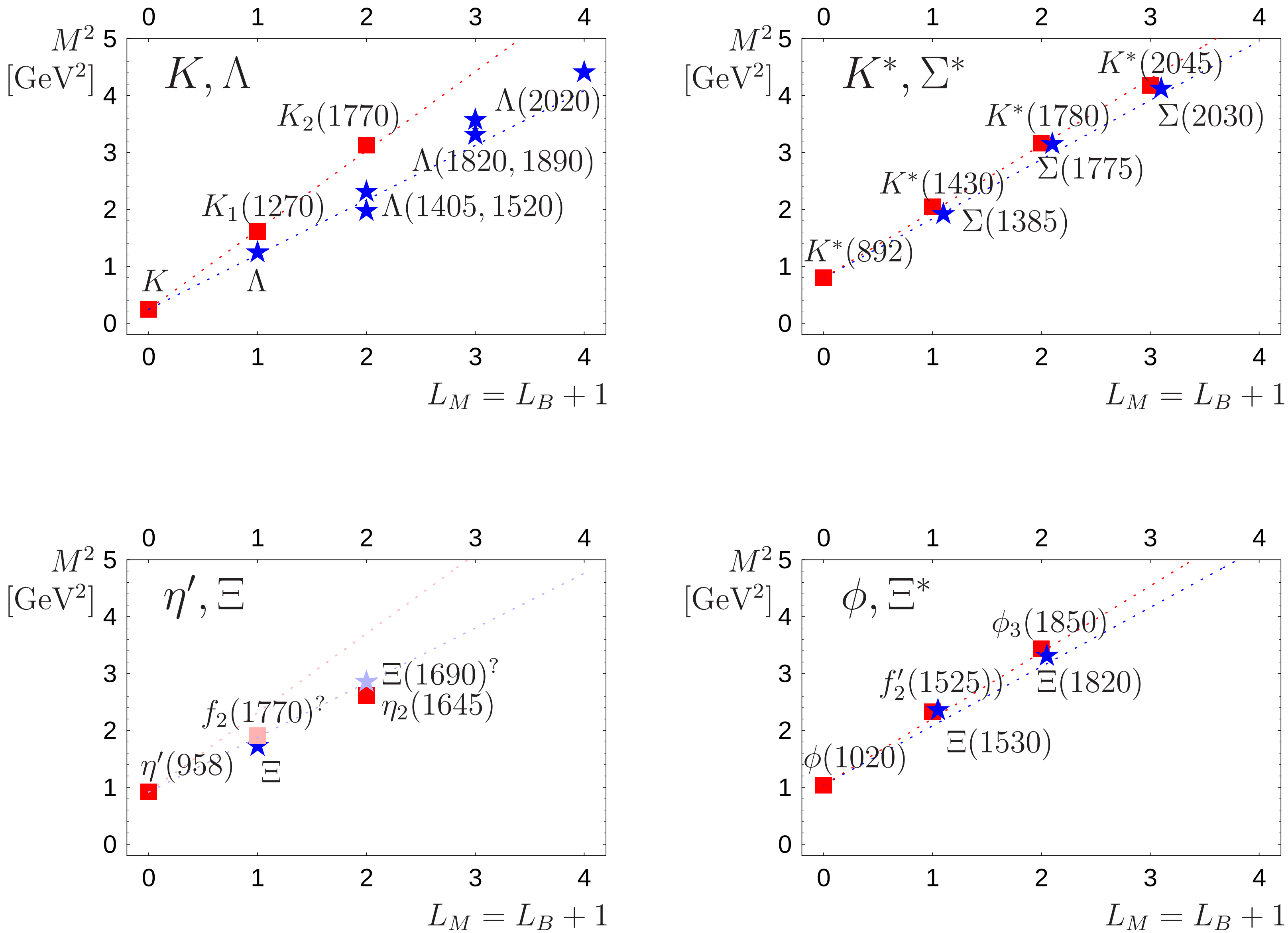


- For the ρ – Δ no complete SCQM–LFHQCD correspondence: Spectra agree, wave functions differ (Source of the problem: half-integer twist of Δ)



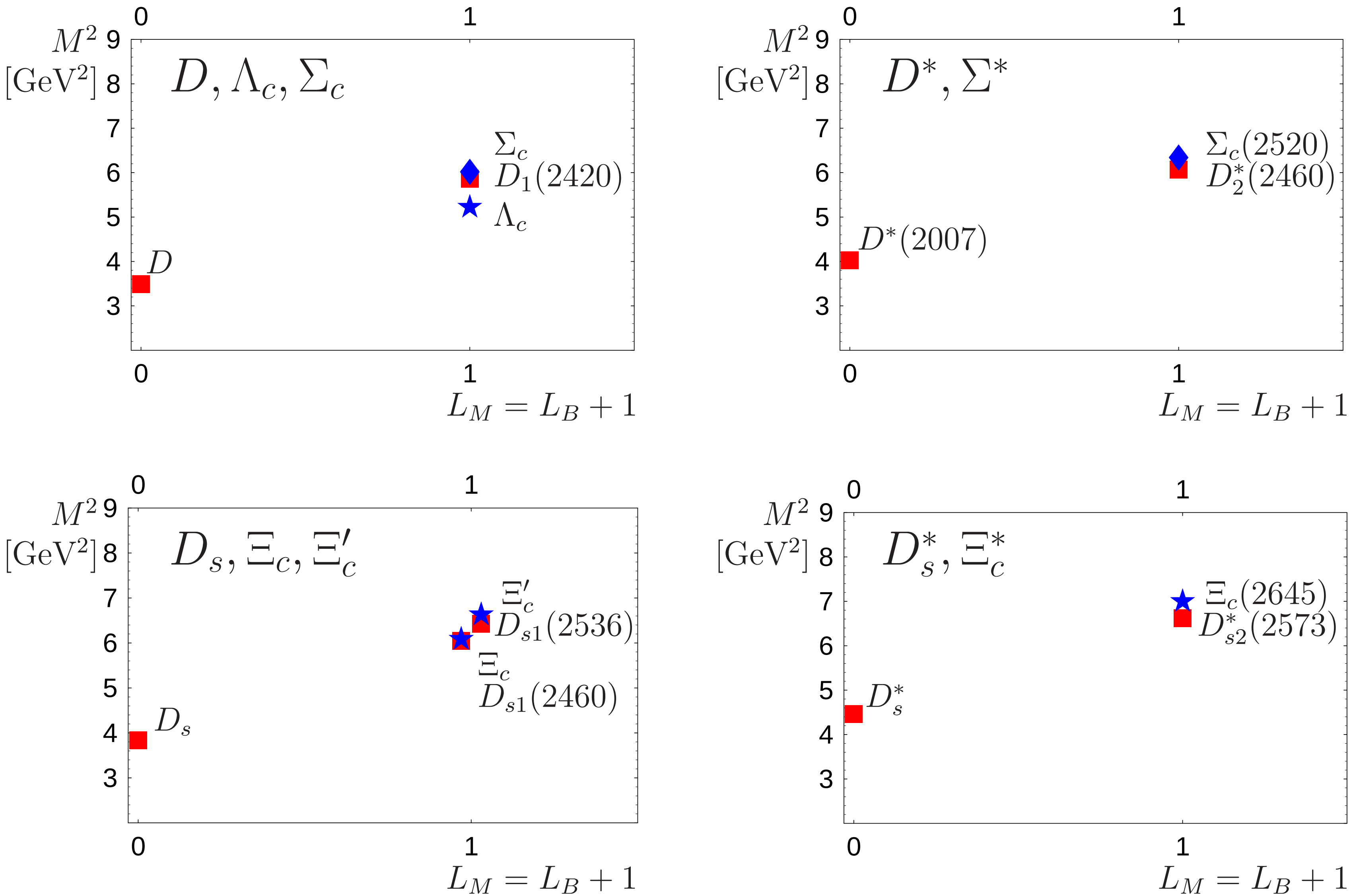
Superconformal meson-nucleon partners: solid line corresponds to $\sqrt{\lambda} = 0.53$ GeV

PRELIMINARY



Supersymmetry of strange mesons and baryons

PRELIMINARY



Supersymmetry of charmed mesons and baryons

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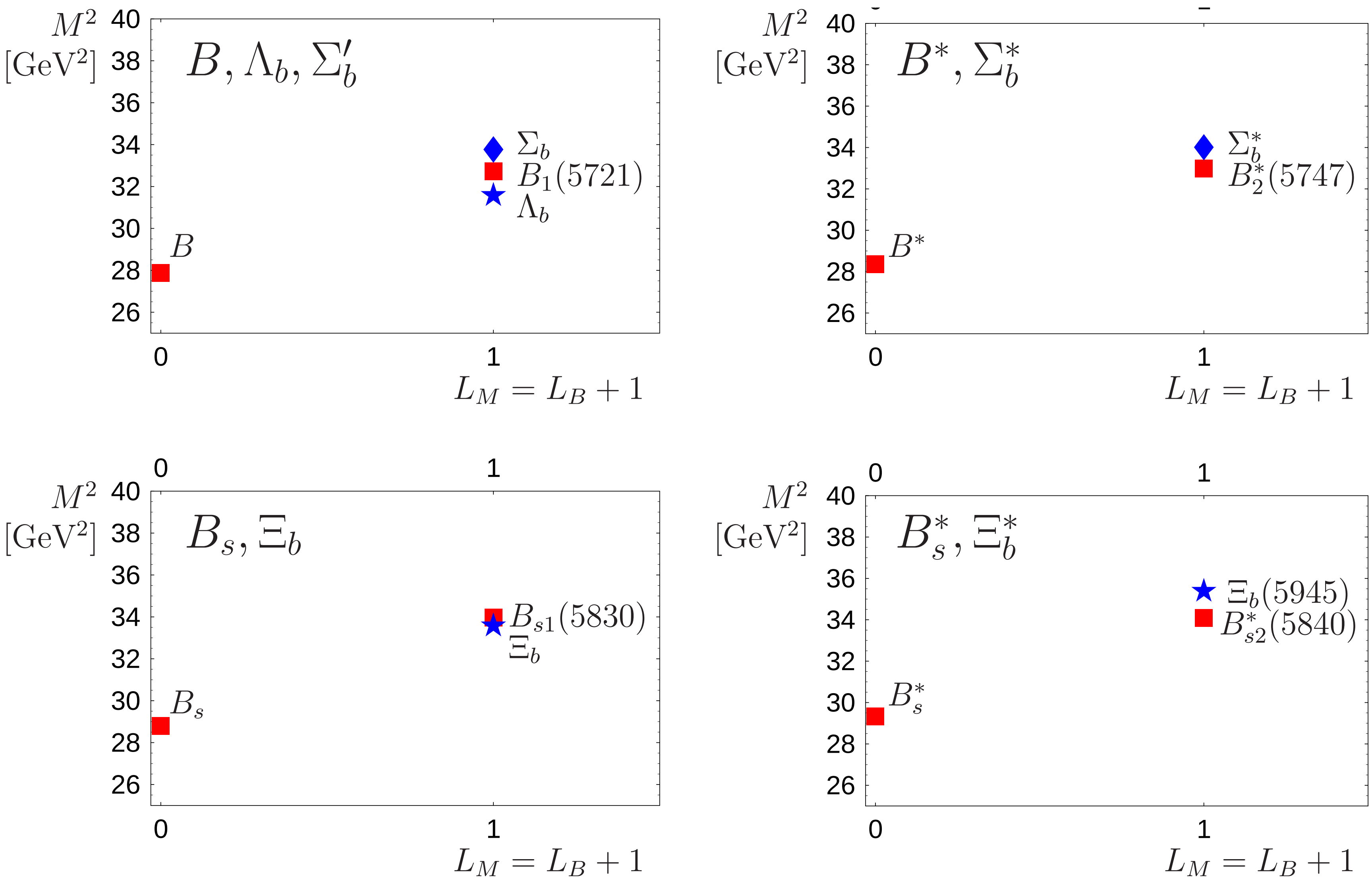

Supersymmetry of beautiful mesons and baryons

Thanks !

For a review: S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, arXiv:1407.8131 [hep-ph]

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