Chaos in the matrix model, and formation and evaporation of a black hole

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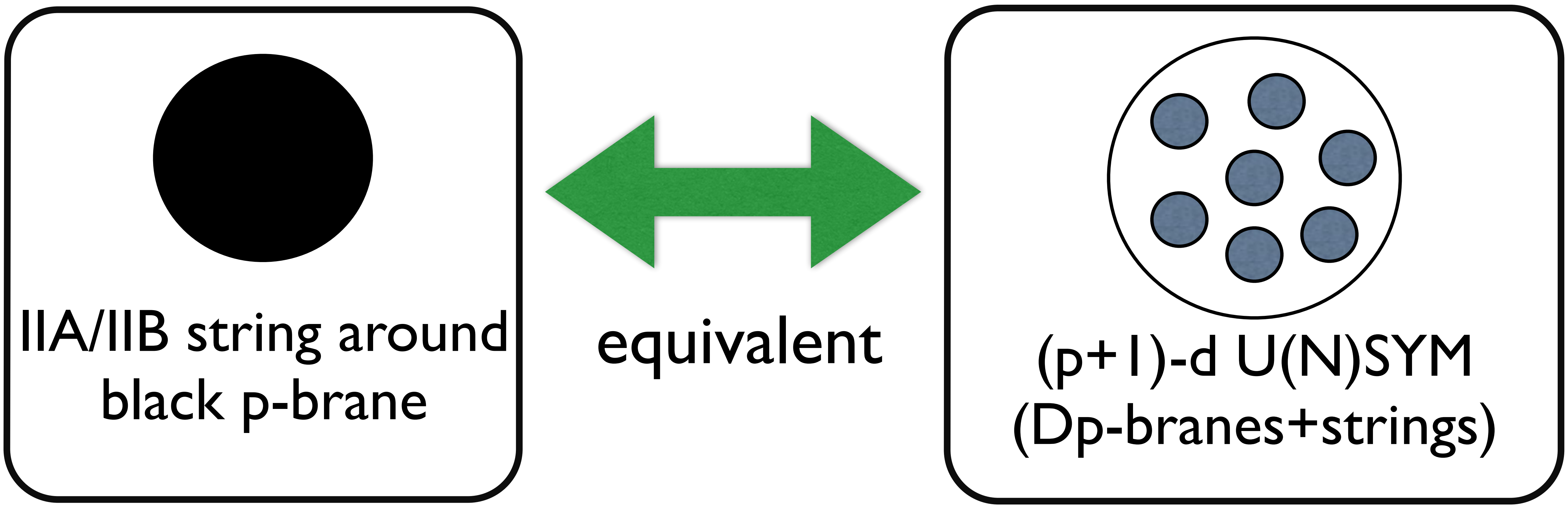
Aoki-M.H.-Iizuka, 1503.05562[hep-th]
Gur Ari-M.H.-Shenker, to appear
+ work in progress with Berkowitz, Gur Ari, Maltz and Shenker

16 April 2015 @ GGI, Florence

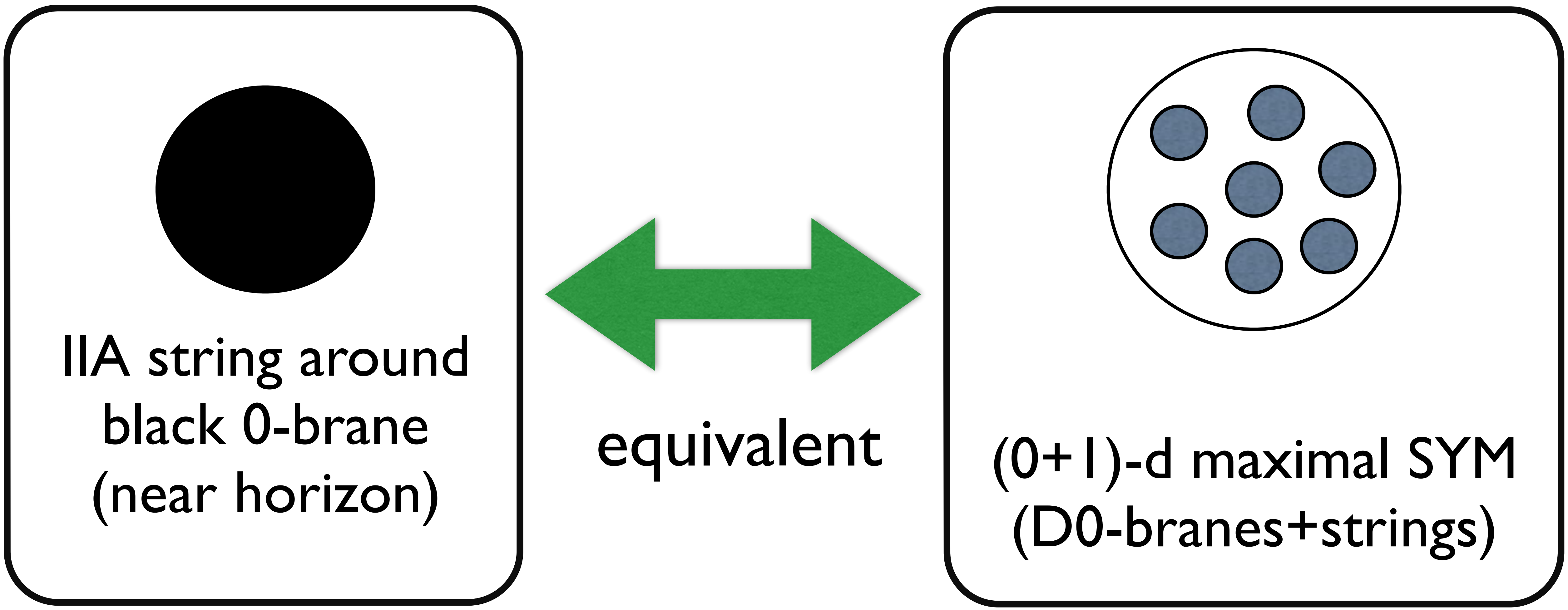
Gauge/Gravity Duality



We can learn about quantum gravity and BH by solving gauge theory.

But SYM is hard! → numerical calculation.

Numerically easiest example



(Maldacena1997, Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

Matrix model of super-membrane (de Wit-Hoppe-Nicolai, 1988)
Matrix model of M-theory (Banks-Fischler-Shenker-Susskind, 1996)

D0-brane quantum mechanics

$$S = \frac{N}{\lambda}\int_0^{\beta=1/T} dt\; Tr\Big\{\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\bar{\psi} D_t \psi - \frac{1}{2}\bar{\psi}\gamma^i [X_i, \psi]\Big\}$$

(dimensional reduction of 4d N=4 SYM)

It should reproduce thermodynamics of black 0-brane.

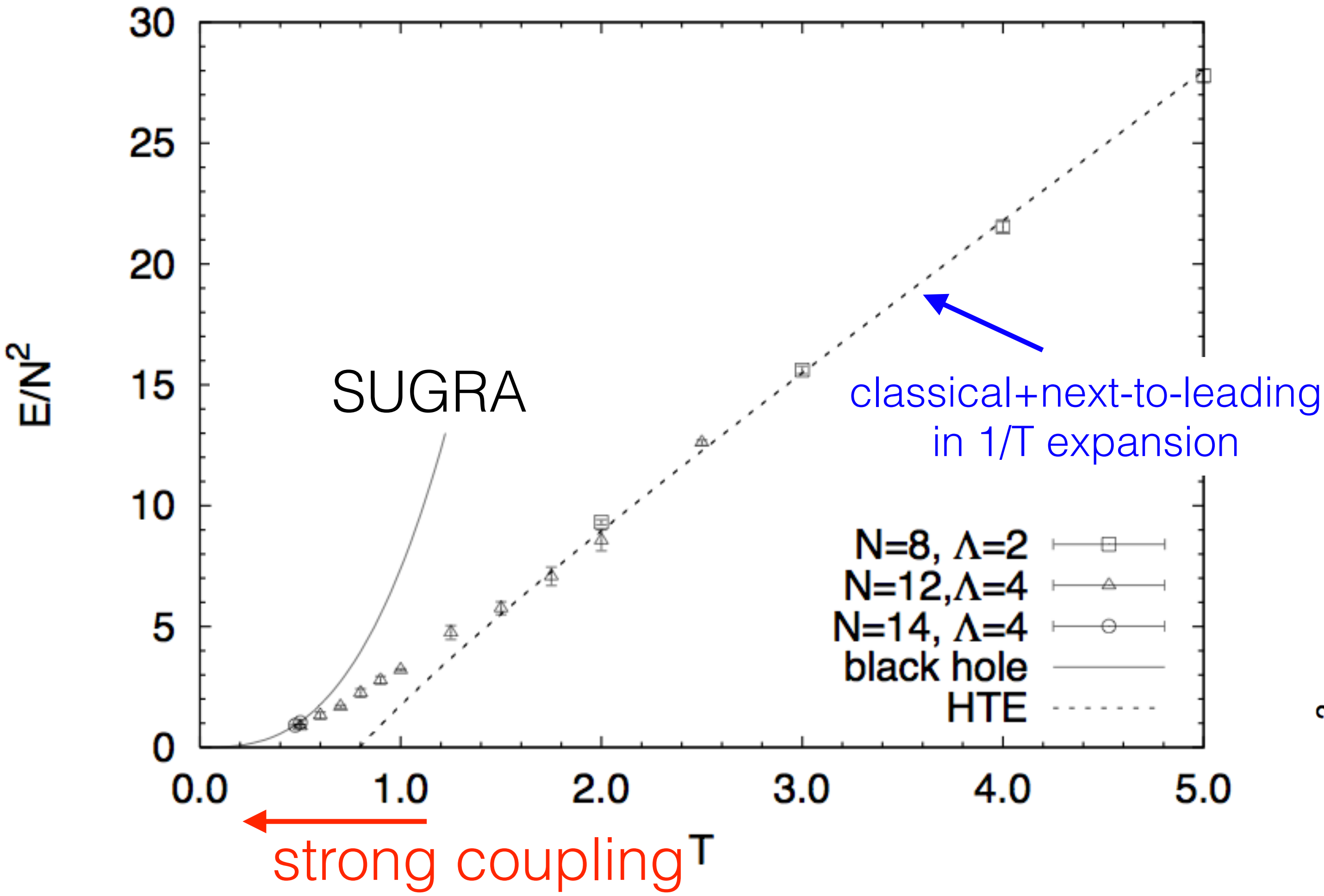
effective dimensionless temperature $T_{eff} = \lambda^{-1/3} T$

high-T = weak coupling = stringy (large α' correction)

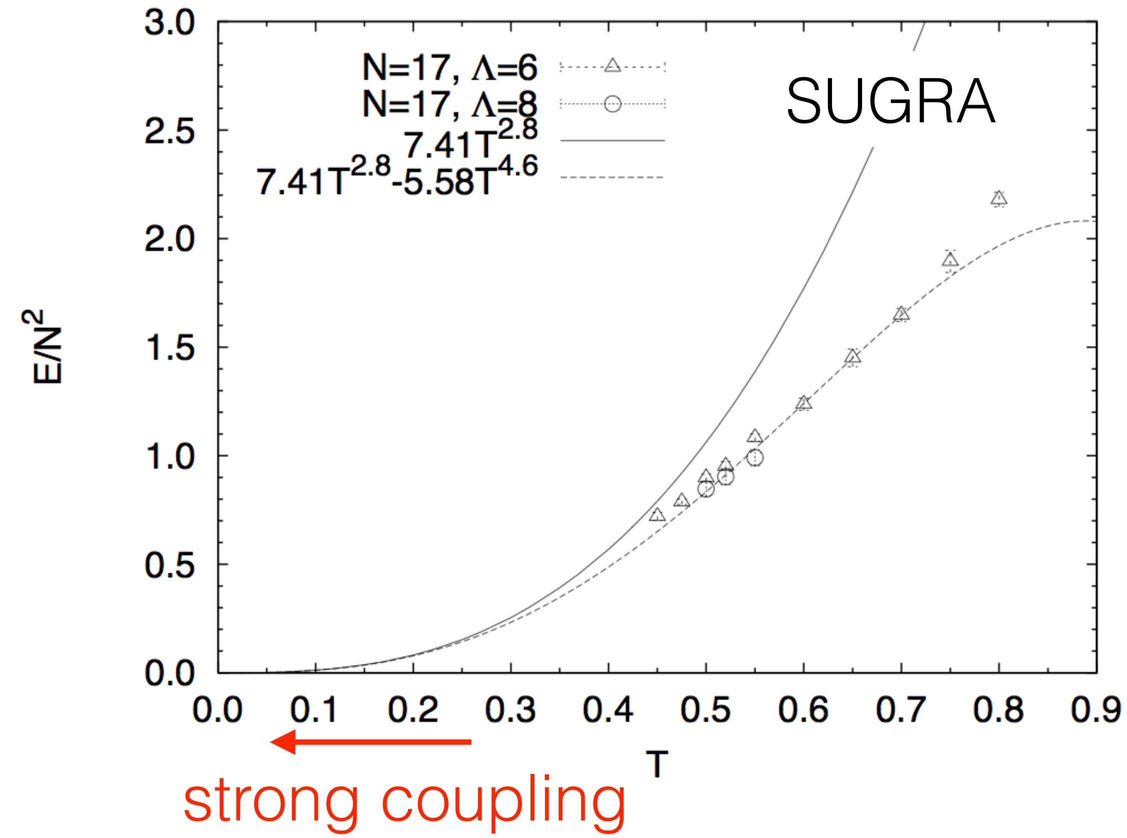
Thermodynamics

→ Successful so far

Energy of BH & MQM

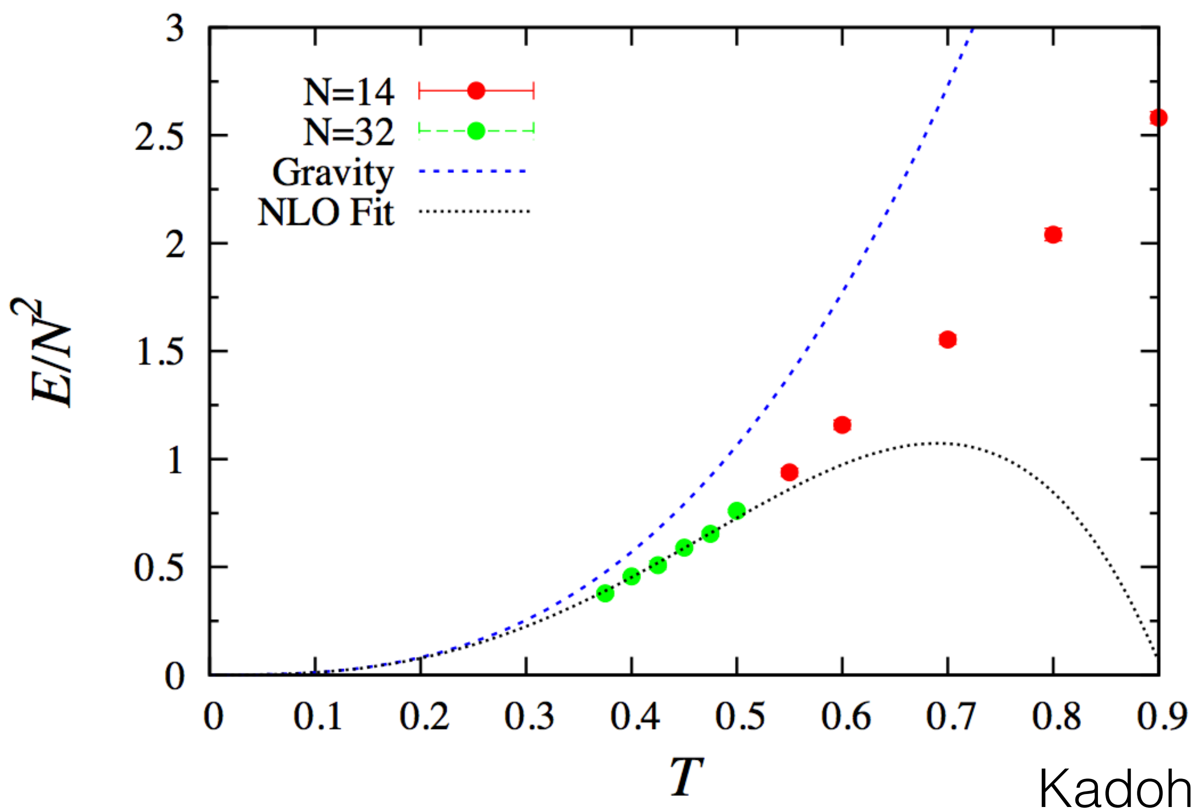


Anagnostopoulos-M.H.-Nishimura-Takeuchi, 2007



M.H.-Hyakutake-Nishimura-Takeuchi, 2008

Small deviation?
We will go closer to continuum.
(stay tuned.)



Kadoh-Kamata, 2015

If you want to know more, you can make plots by yourselves:

FREE simulation code for BFSS/BMN matrix models

RHMC algorithm + Fourier acceleration (with FFTW3)
Fortran 90/ Fortran 2003; MPI parallelized

Should be useful for learning about BH, M2 and M5.

$0!

Runs on supercomputer, cluster, and macbook

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Real-time study

Aoki-M.H.-Iizuka, 1503.05562[hep-th]
Gur Ari-M.H.-Shenker, to appear
+ work in progress with Berkowitz, Gur Ari, Maltz and Shenker

- **Full quantum study is impossible with current technology.**

stochastic quantization (complex Langevin)?
brute-force diagonalization?
quantum simulator? → experimental quantum gravity?

- **Strong coupling lattice gauge theory** (+improvement)

M.H.-Maltz-Susskind 2014

stringy d.o.f. is manifest; still numerically demanding, but should be possible in a few years.

- **Classical real time evolution**

i.e. just solve classical EOM

high temperature = weak coupling = highly stringy

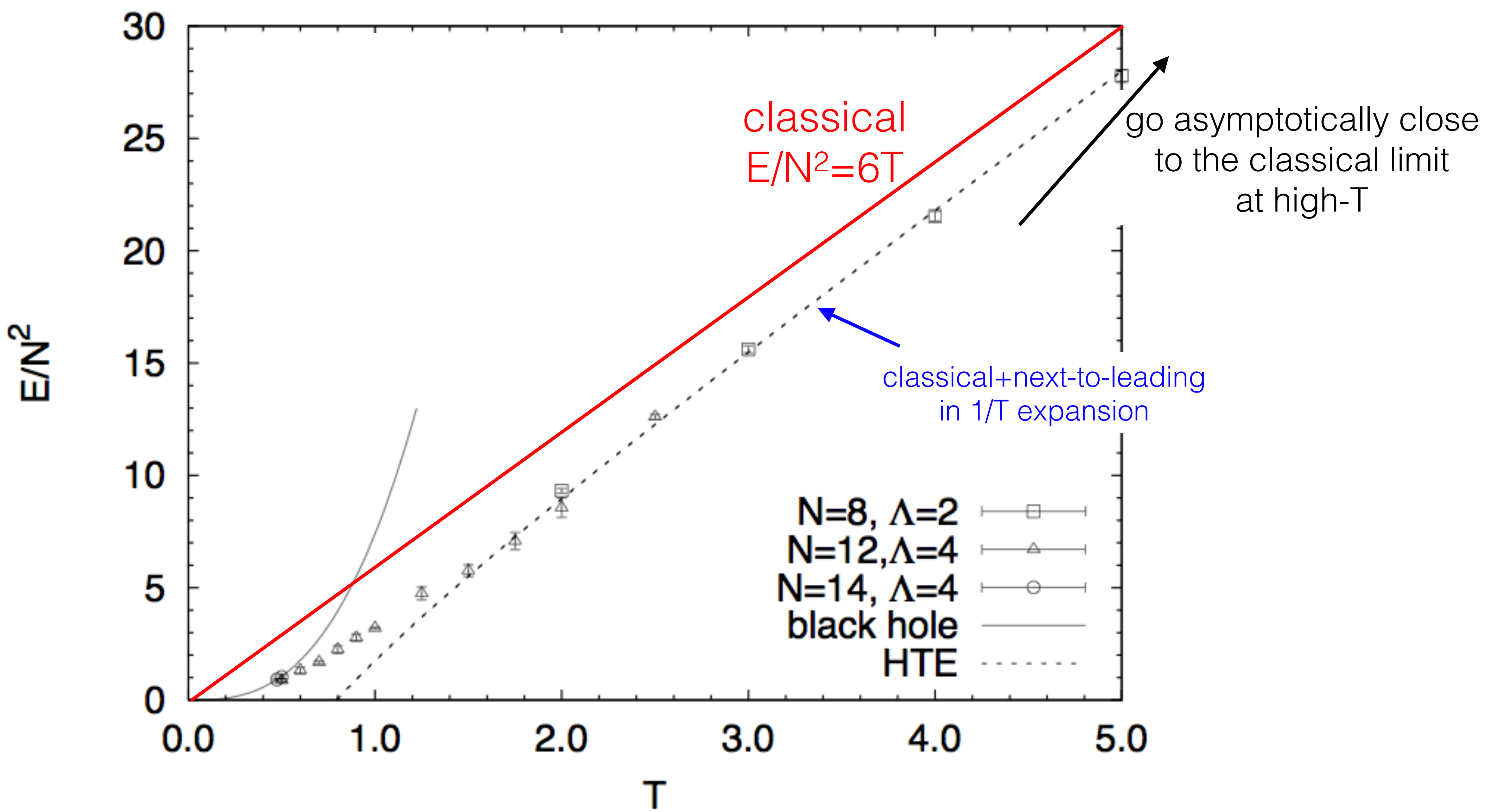
highly nonlinear & nonperturbative

“BH” = soliton (or resonance) of matrix model

We will see the formation & evaporation of “BH” in this limit.

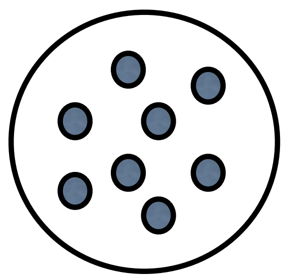
Remark

There is no phase transition between low- and high-T.

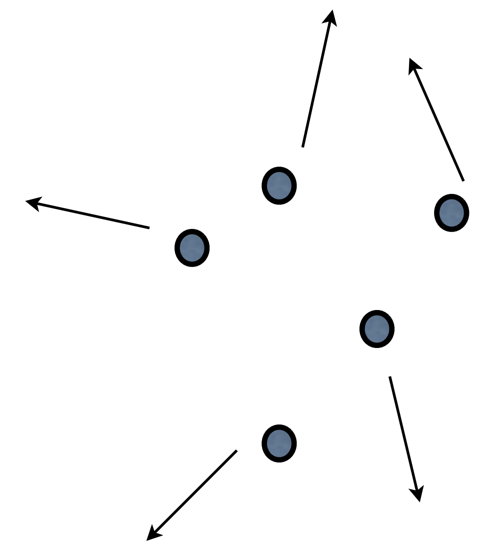


E/N^2 in BFSS vs 0-brane mass (0707.4454[hep-th])

‘eigenvalues’ = D0-branes



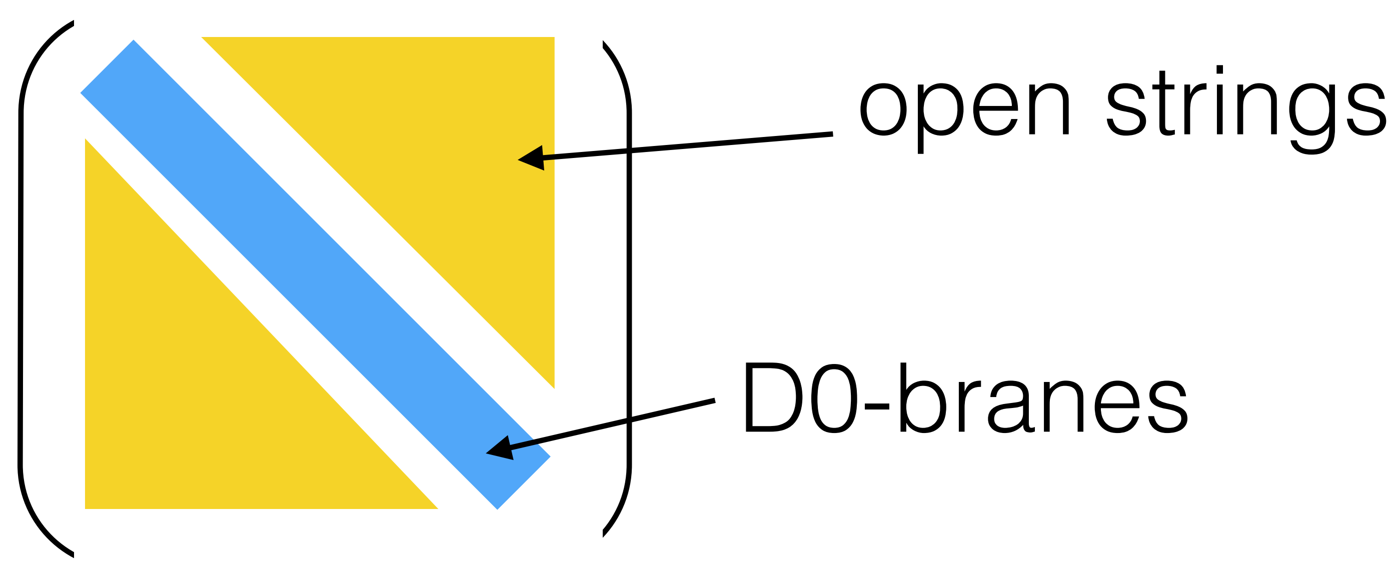
bound state of eigenvalues
= black hole



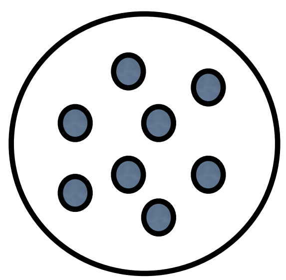
flat direction
= gas of D0-branes

This phase reproduced the dual BH thermodynamics.

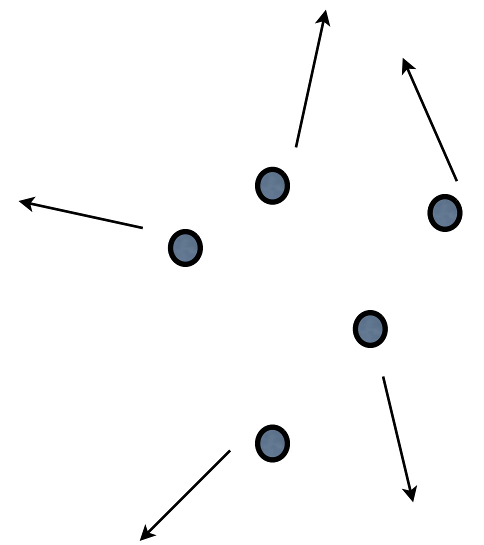
$X_M =$



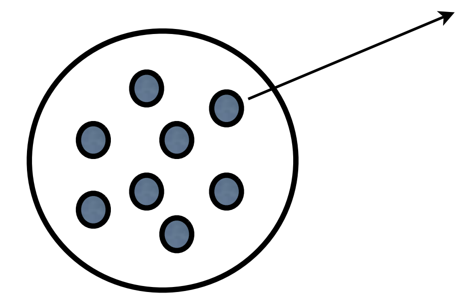
‘eigenvalues’ = D0-branes



bound state of eigenvalues
= black hole



flat direction
= gas of D0-branes



emission of eigenvalue
= evaporation of BH
(emission of D0)

This model can describe BH evaporation!

This evaporation is suppressed at $N=\infty$.

(The instability has been observed in imaginary time simulation.)

$$L = \frac{1}{2g_{YM}^2}\mathrm{Tr}\left(\sum_i (D_t X^i)^2 + \frac{1}{2}\sum_{i\neq j}[X_i, X_j]^2\right)$$

$$\longrightarrow \begin{cases} \dfrac{d^2 X^i}{dt^2} - \sum_j [X^j, [X^i, X^j]] = 0 \\ \sum_i \left[X^i, \dfrac{dX^i}{dt}\right] = 0 \end{cases}$$

(A=0 gauge)

discretize & solve it numerically.
(straightforward.)

It takes only 15 - 30 minutes for average graduate students to write C or Fortran codes.
[cf) Monte Carlo code for thermodynamics ➞ a few months ∼1 year for good students]

Remark

$$\frac{d^2 X^i}{dt^2} - \sum_j [X^j, [X^i, X^j]] = 0$$

Invariant under the scaling $t \to t/\alpha,\ X_M \to \alpha X_M$

All values of the energy (or 'temperature') are equivalent.

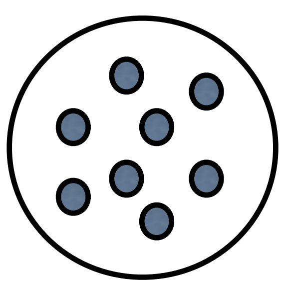
E, T → α^4E, α^4T

Formation & thermalization of BH

- This system is chaotic. (Savvidy, 1984; Berenstein et al, 2012)
- Almost all initial conditions end up with

‘typical’ matrix configurations — BH.

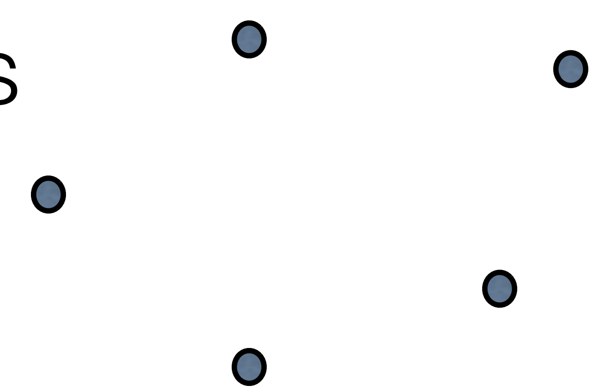
BH



open strings
(off-diagonal elements)
are excited

entropy $\sim N^2$

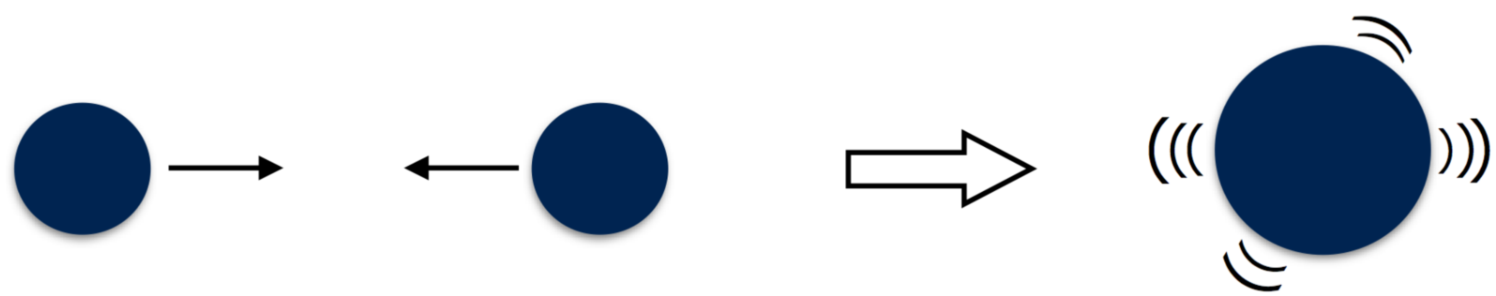
gas



open strings are suppressed

entropy $\sim N$

Example:
Collision of 2 BHs



Formation & Thermalization of “BH” can be seen.

After thermalization,

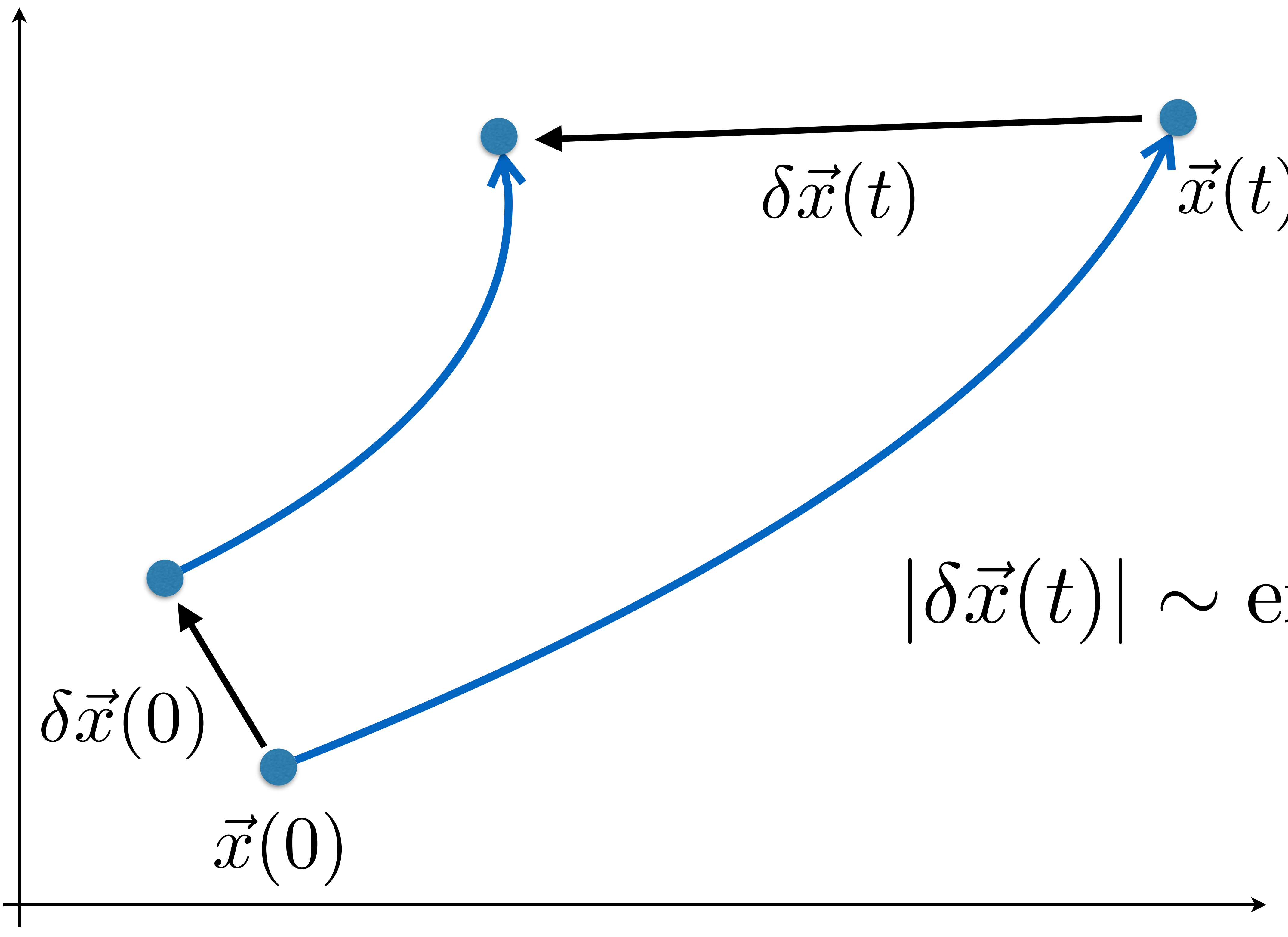
- (Tr X^2)/N, (Tr V^2)/N etc are t-independent at large-N.
- (X(t), V(t)) (→‘micro-state’) changes rapidly.

(I don’t have a time to explain the detail, sorry)

Fast scrambling

- Take a ‘micro-state’ (X,V) from a thermalized “BH.”
- Then add a small perturbation:
  $X \to X+\delta X$, $V \to V+\delta V$.

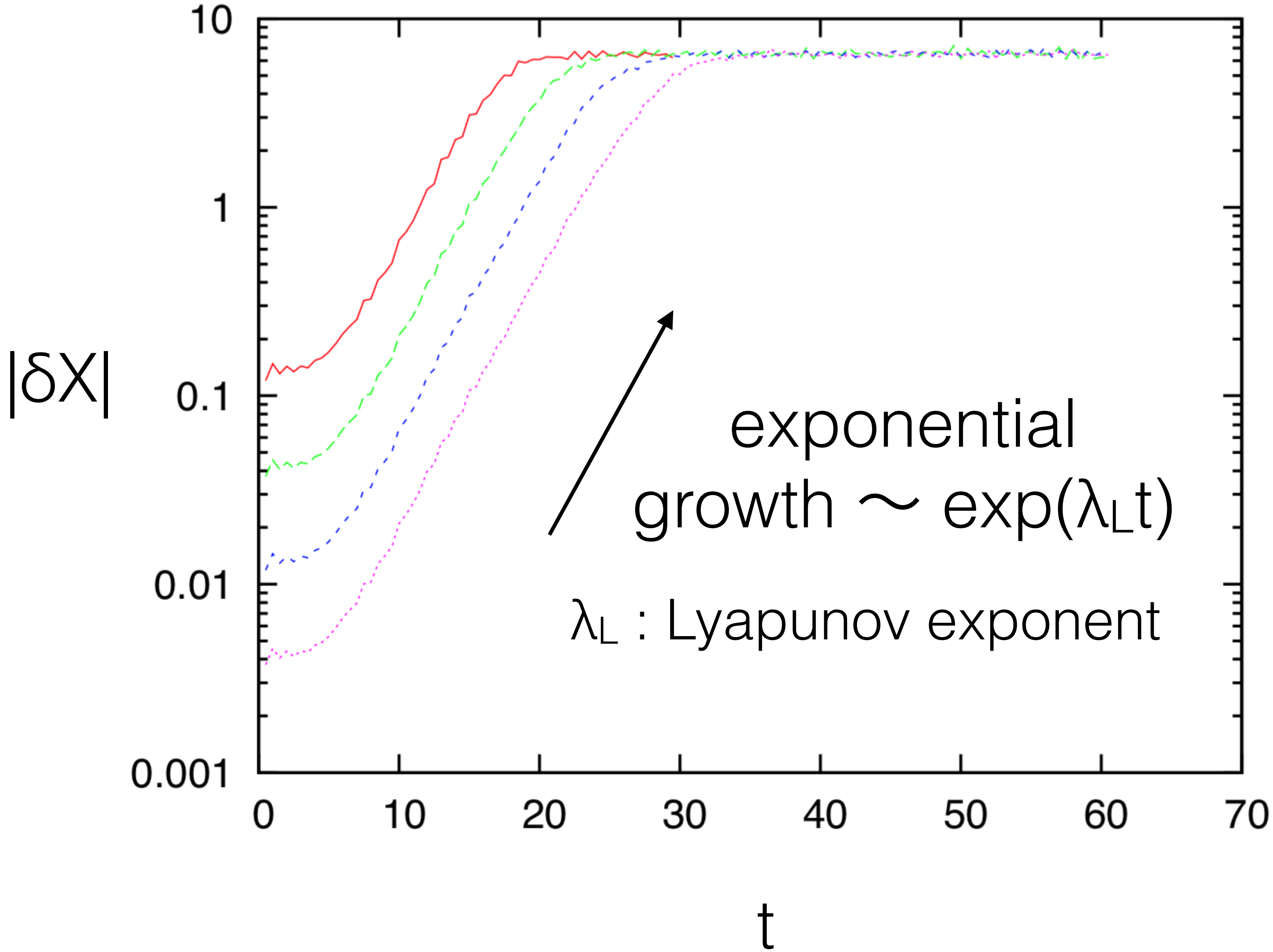
Lyapunov Exponent



$$|\delta\vec{x}(t)| \sim \exp(\lambda_L t)$$

Fast scrambling

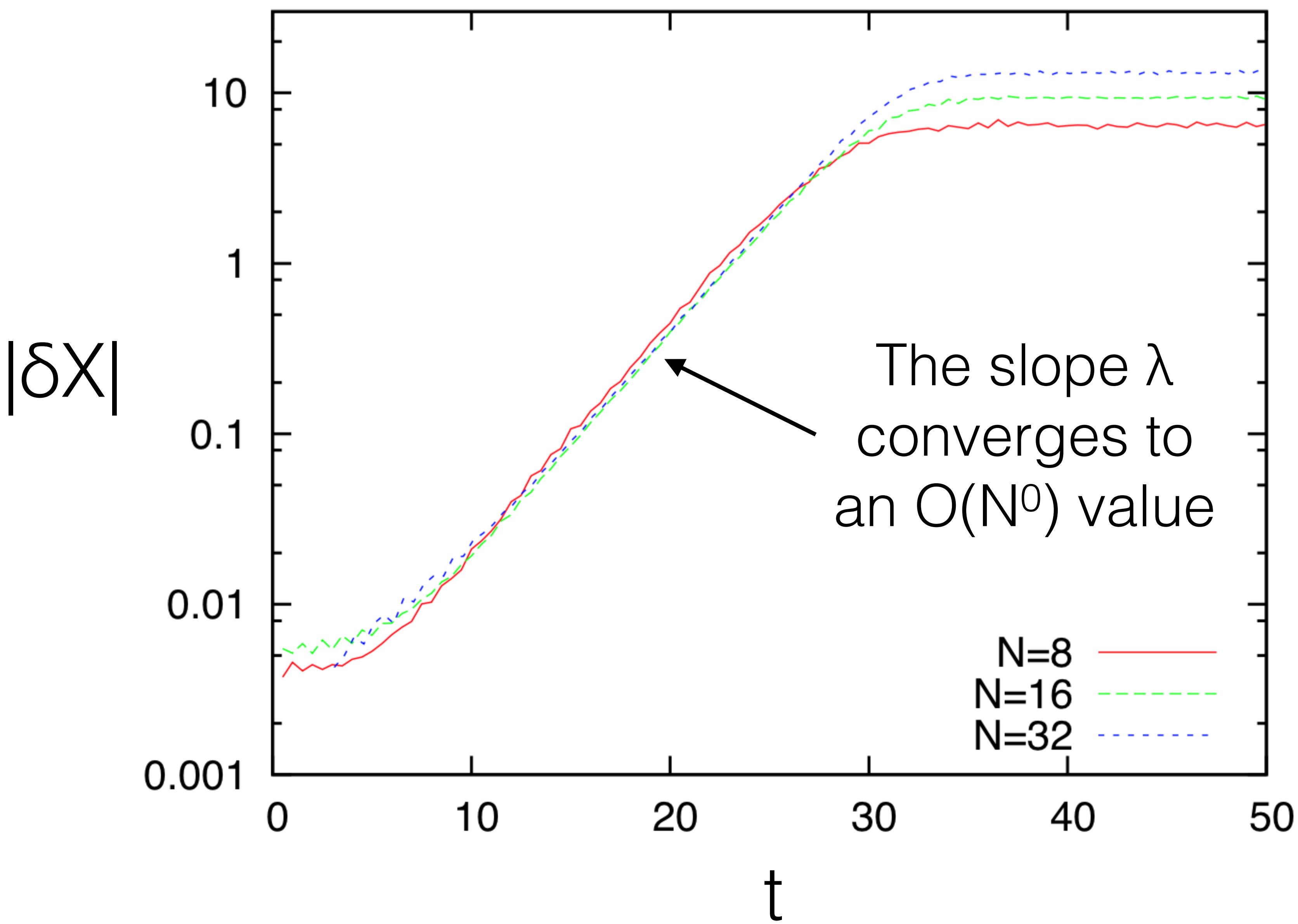
- Take a 'micro-state' (X, V) from a thermalized "BH."
- Then add a small perturbation:
  $X \to X+\delta X$, $V \to V+\delta V$.
- δX and δV grows quickly, i.e. information of the initial state is scrambled.
- 'scrambling time' $t_s \sim \log N$. (Sekino-Susskind, 2008; Shenker-Stanford 2013, 2014; Maldacena-Shenker-Stanford 2015)
- Let's test this conjecture.



10
1
0.1
0.01
0.001
|δX|
0
10
20
30
40
50
60
70
t
exponential
growth ∼ exp(λLt)
λL : Lyapunov exponent

N=8

$E/N^2=10$



$\sim\sqrt{N}$

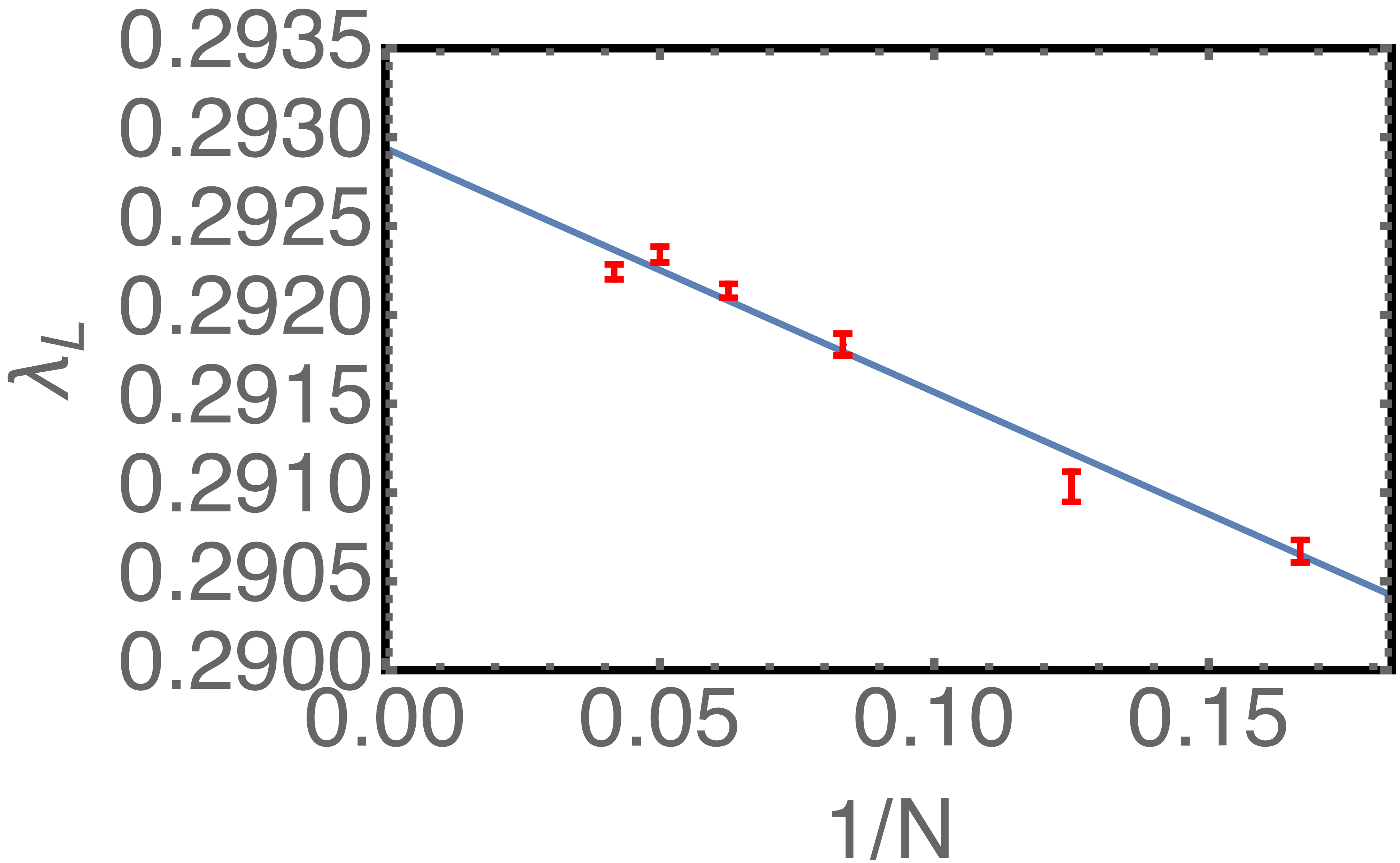
N=8,16,32
$E/N^2=10$

$\exp(\lambda_L t_s)\sim\sqrt{N}$

→ "scrambling time" $t_s = (\log N)/\lambda_L \sim \log N$

Fast scrambling!

1/N Behavior



T=1
$E/N^2=6T$

$$\lambda_L = 0.293 - \frac{0.014}{N} + O(1/N^2) \qquad (N = 8, \ldots, 24)$$

$$\lambda_L = 0.293\,(\lambda_{\text{'t Hooft}} T)^{1/4}$$

$(E/N^2 = 6T)$

$$\frac{d^2 X^i}{dt^2} - \sum_j [X^j, [X^i, X^j]] = 0$$

↑

Invariant under the scaling

$$t \to t/\alpha,\ X_M \to \alpha X_M$$

$E, T, \lambda \to \alpha^4 E, \alpha^4 T, \alpha\lambda$

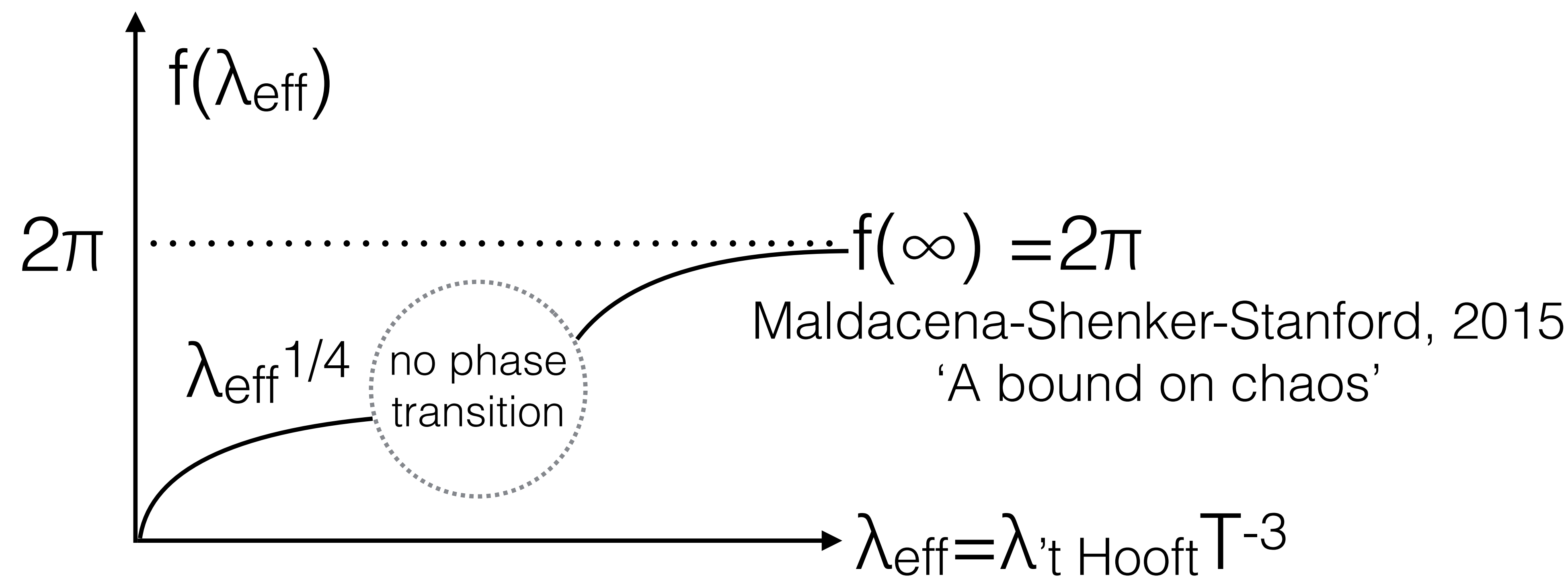
strong coupling vs. weak coupling

effective dimensionless temperature $T_{eff} = (\lambda_{\text{'t Hooft}})^{-1/3} T$

effective dimensionless 't Hooft coupling $\lambda_{eff} = \lambda_{\text{'t Hooft}} T^{-3}$

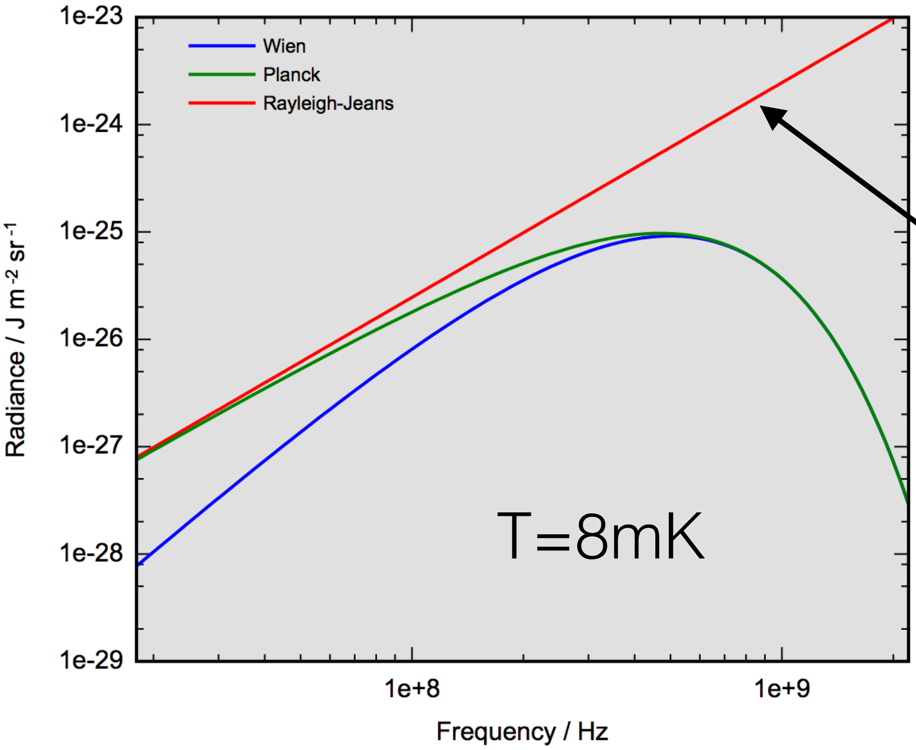
$$\lambda_L = 0.293\,(\lambda_{\text{'t Hooft}} T)^{1/4} = (0.293\,\lambda_{eff}^{1/4})\,T$$

$$\lambda_L = f(\lambda_{eff})\,T$$



Classical Yang-Mills theory?

(nonzero spatial dimensions)



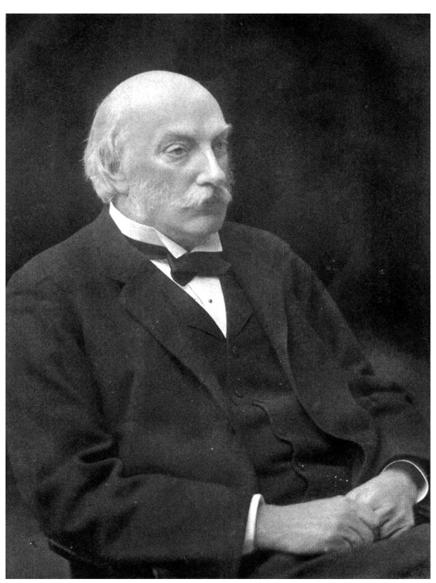
(wikipedia)

oops!

equipartition of energy
+ infinite d.o.f. in UV

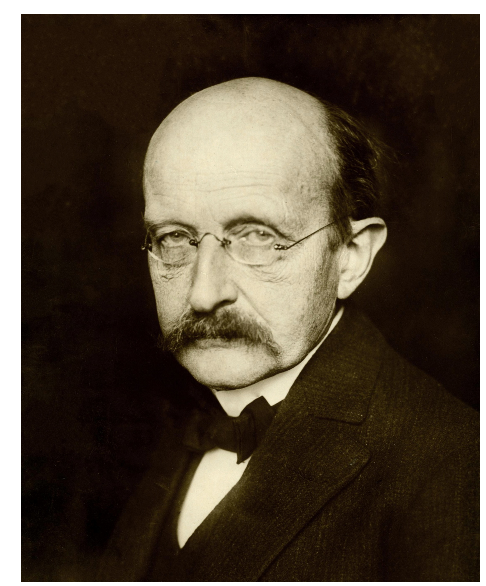
→ *UV catastrophe*

In classical YM, energy flows to UV;
thermal equilibrium is never reached.



Lord Rayleigh
1842-1919

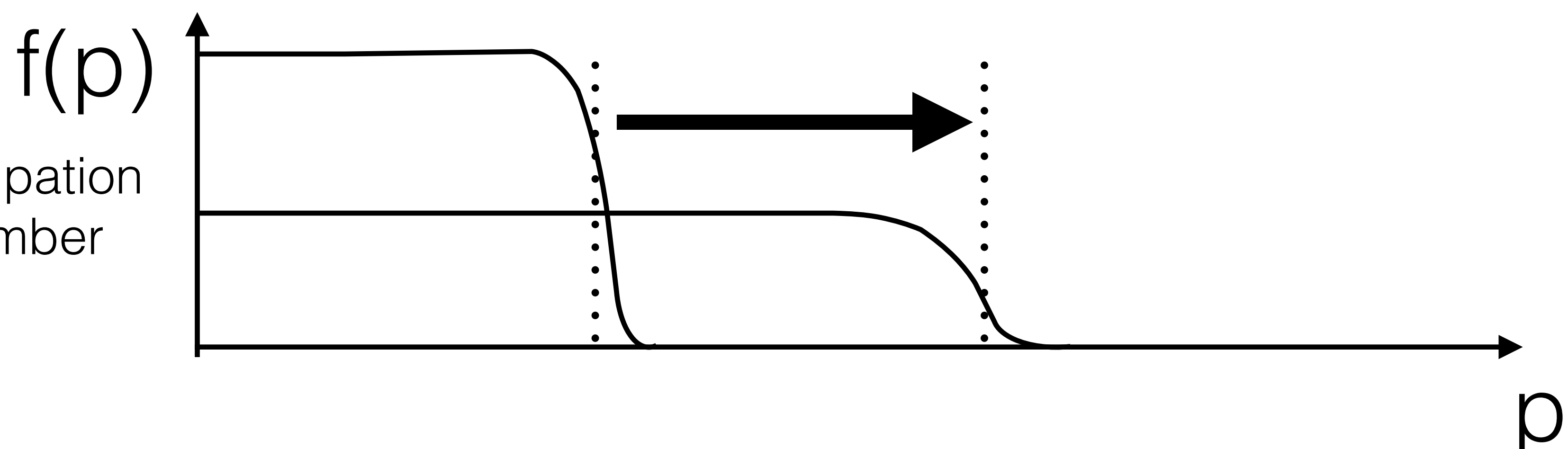
James Jeans
1877-1946



Max Planck
1858-1947

But UV catastrophe might not be so catastrophic

- Energy flow to UV is slow. (Kurkela-Moore, 2012)



$p_{max} \sim aQ$ @ $t \sim a^7/Q$

where $\varepsilon = Q^4N^2/\lambda_{\text{'t Hooft}}$: energy density

scrambling time $\sim$ (log N)/Q

no problem when (log N) << a^7

$\exp((3/2)^7)$
$\sim 2.6\times10^7$

'thermalization' at IR is achieved, then very slow flow to UV follows.

What can we do?

- Thermalization of black brane.
- Correlation functions.
- Scrambling in 2d, 3d and 4d theories; how perturbations grow in color space and in spatial dimensions.
- Black hole / black string topology change.
- What is the 'stringy effect' ?

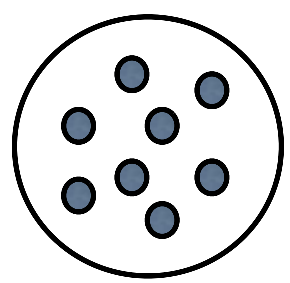
Similarity to & difference from
strong coupling limit (supergravity) ?

Evaporation

(in progress, still speculative)

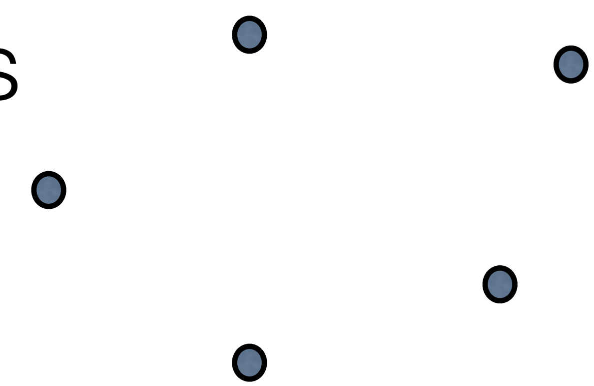
chaos (or ergodicity) + flat direction → evaporation

BH



entropy $\sim N^2$

gas

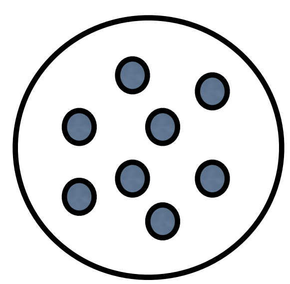


entropy $\sim N$

Exponentially suppressed,
but still can appear after long time.

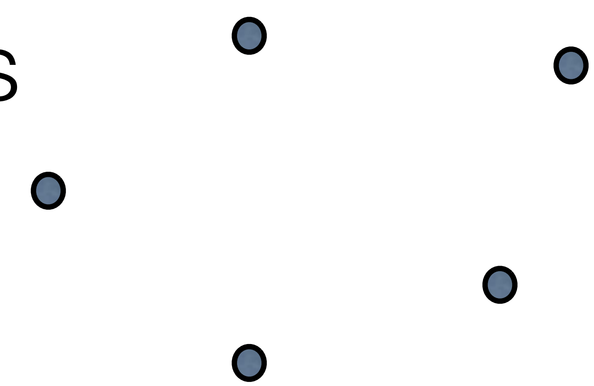
chaos (or ergodicity) + flat direction → evaporation

BH



entropy $\sim N^2$

gas



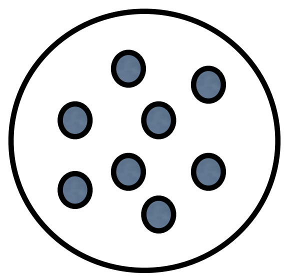
entropy $\sim N \times \infty$

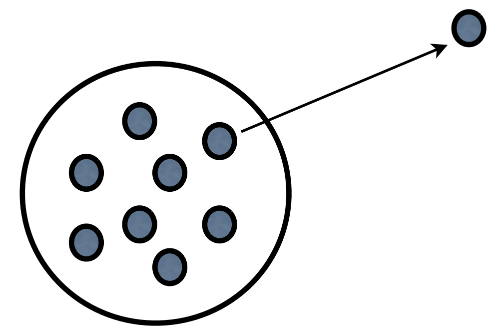
(space volume)

※ Flat direction must be sufficiently flat. Will be explained shortly. →

Once brane is emitted, it does not come back.

‘eigenvalues’ = position of D0-branes



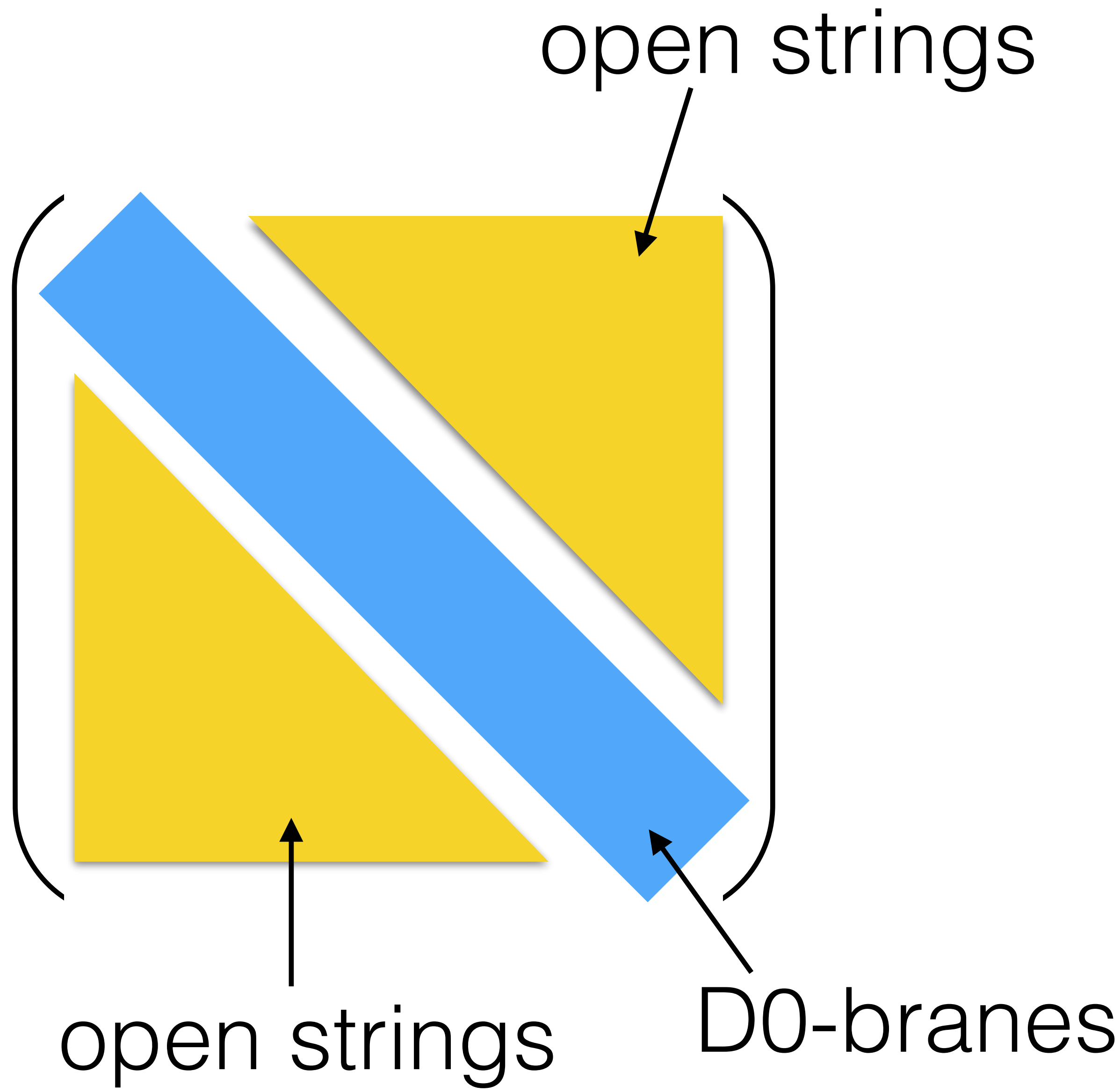


entropy $\sim N^2$ entropy $\sim (N-1)^2$

Emission rate $\sim \exp(-N)$

※ This is different from the emission of massless particles, in the sense D0-brane is heavy.

※ However the same mechanism would work for light particles at low-T, M-theory region, thanks to ‘quantum chaos’ + flat direction

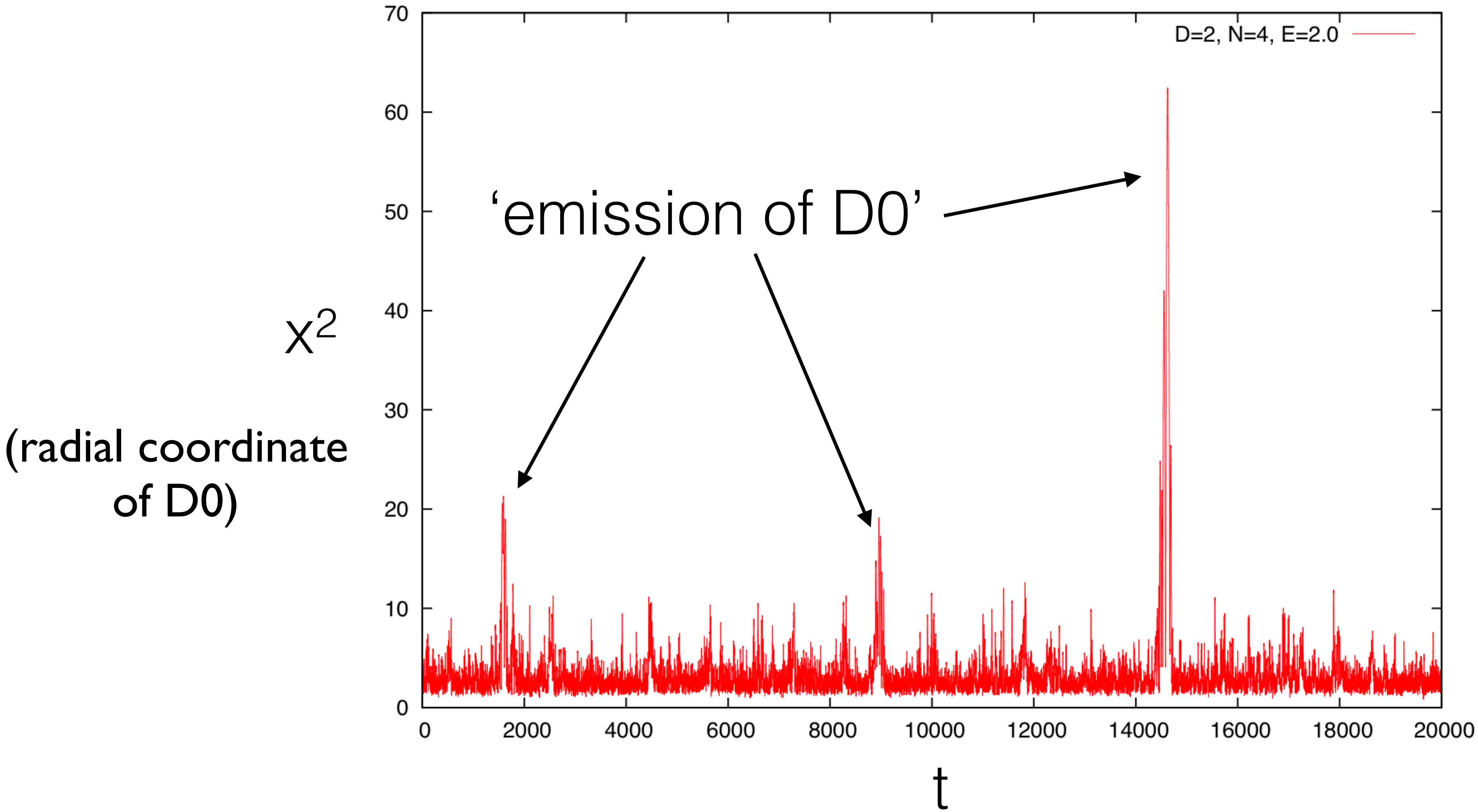
$X_M =$ 

eigenvalue of $(X_M{}^2)_{ij}$

$\doteqdot$ radial coordinate of D0

Distribution of the largest eigenvalue of $(X_M{}^2)_{ij}$

D=2 (2-matrix model), N=4

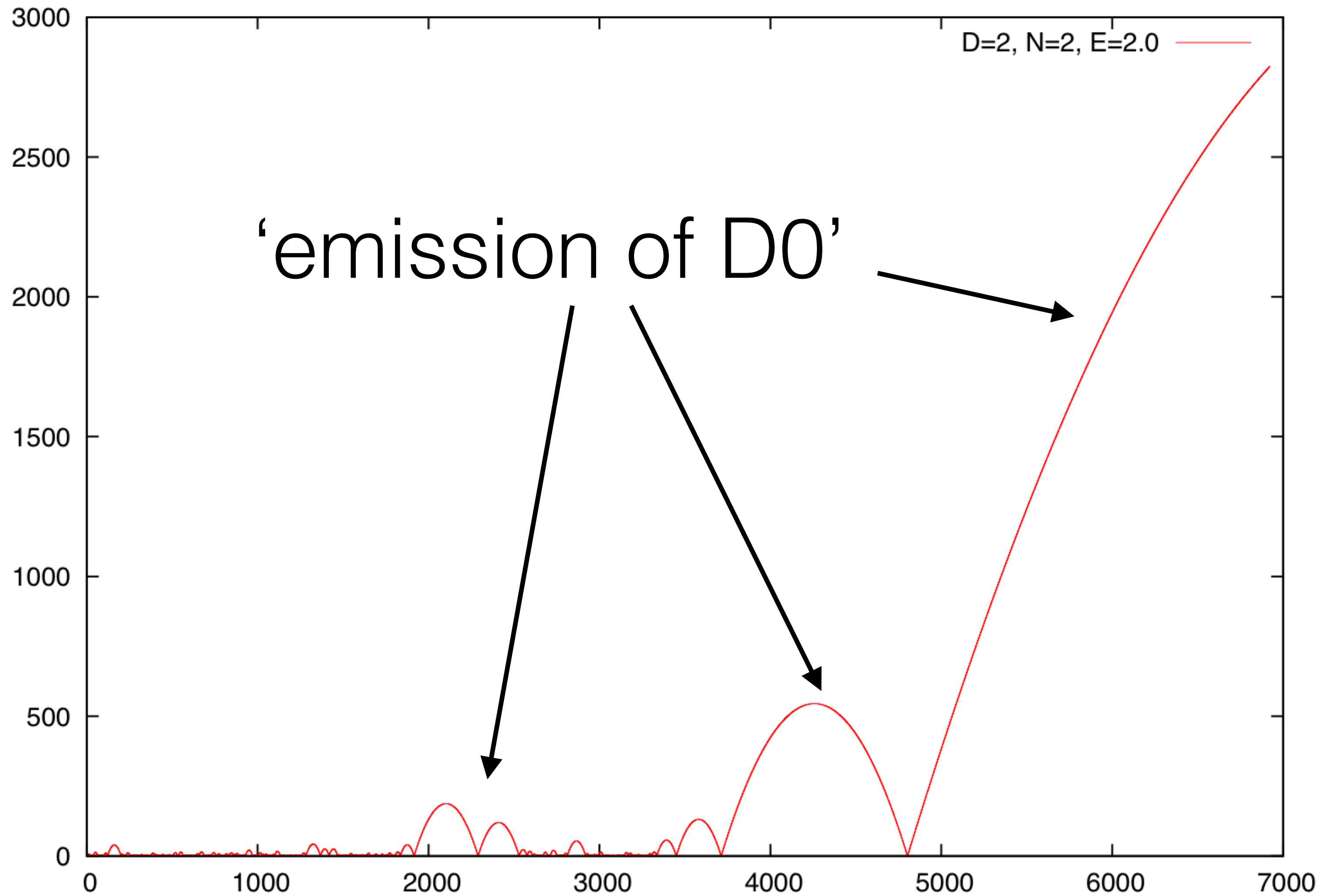


Rather unstable! BFSS (D=9) has the same instability.

D=2 (2-matrix model), N=2

x^2

(radial coordinate of D0)



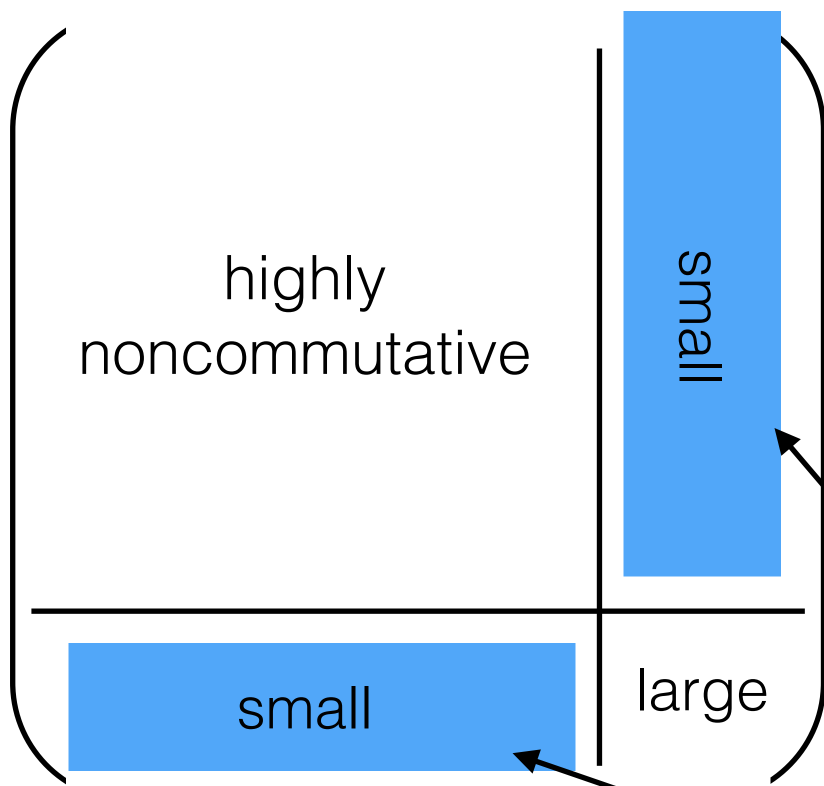
t

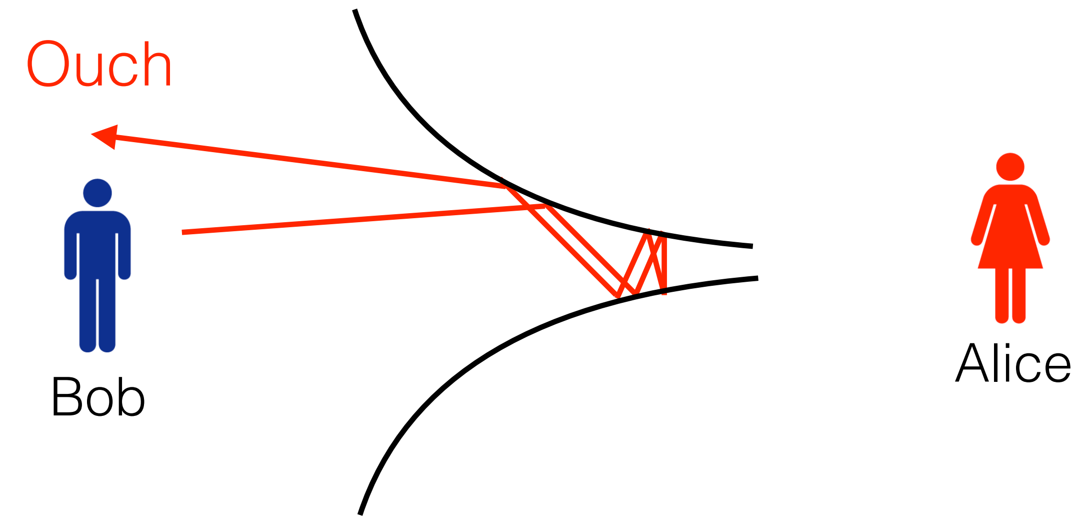
Even more unstable!

However, flat direction is too narrow!

(Very narrow)

$X_M =$



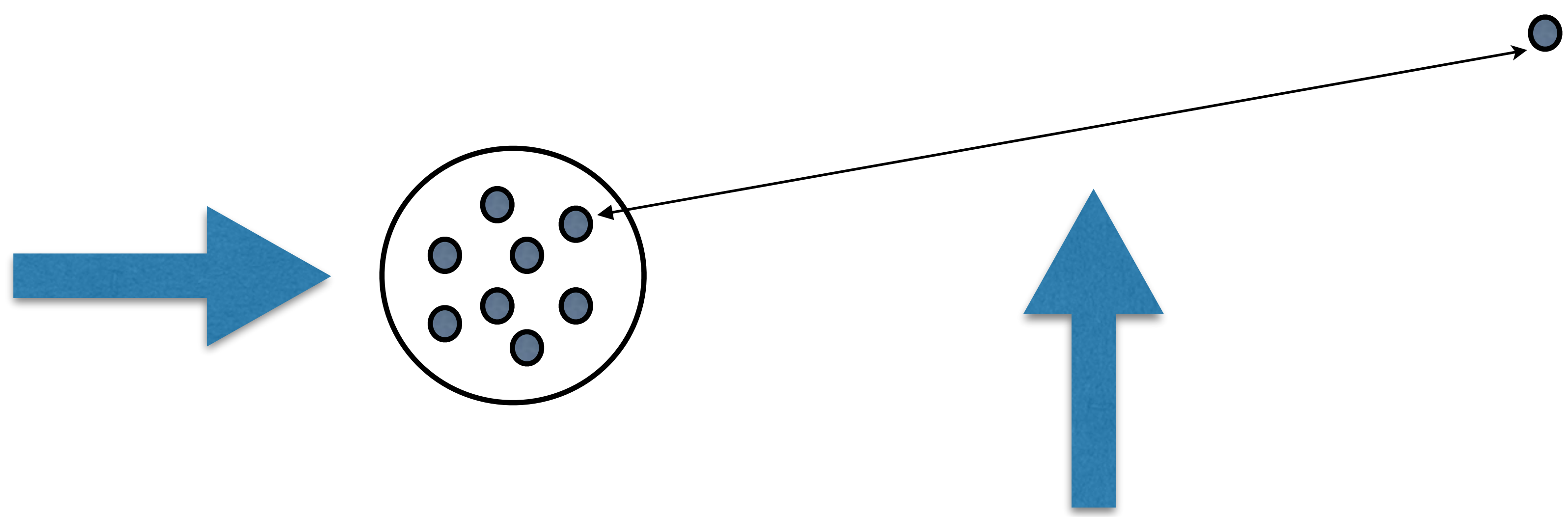


must be exactly zero
in order for D0 to
roll to infinity

This is just an artifact of the classical treatment.

What was wrong?

short strings
mass <<T
$h\nu << k_B T$



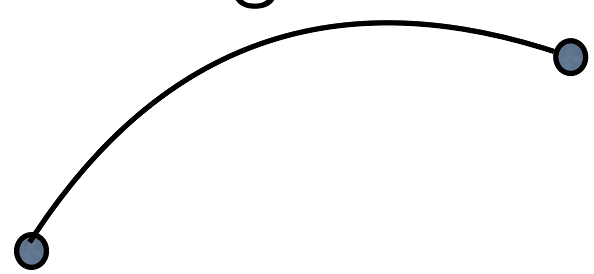
Long, heavy string
$h\nu$ is too big
classical approximation is NOT valid

classical approximation is valid when $h\nu << k_B T$

classical approximation is valid when hν << $k_B T$

open string mass = hν of harmonic oscillator

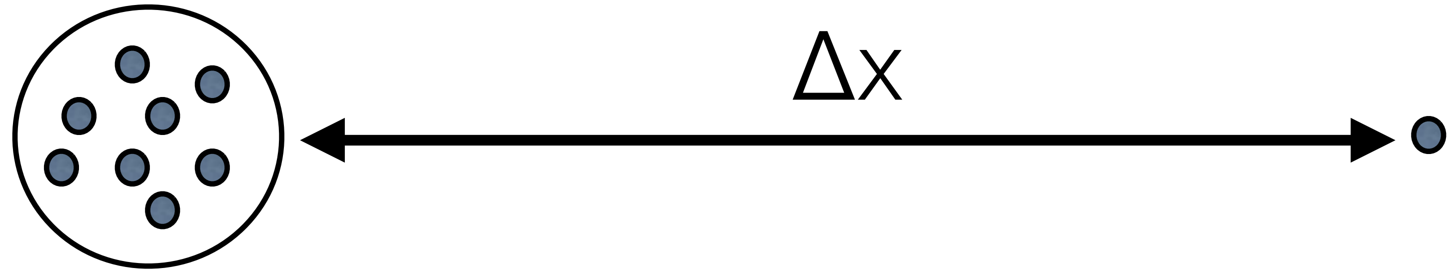
off diagonal element
= open string



diagonal element
= D0-brane

$$L = \frac{1}{2g_{YM}^2} \mathrm{Tr} \left(\sum_i (D_t X^i)^2 + \frac{1}{2} \sum_{i \neq j} [X_i, X_j]^2 \right)$$

SUSY makes flat direction flatter



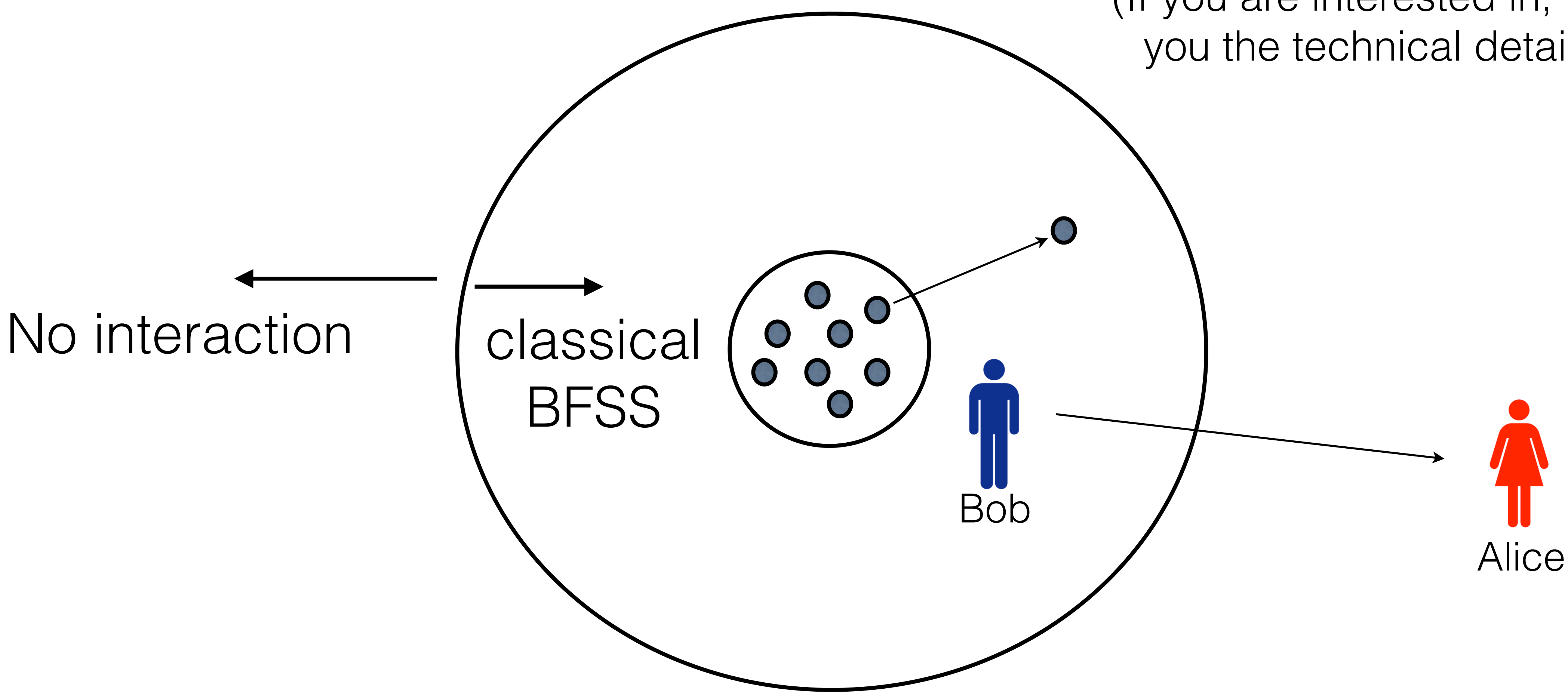
One-loop approximation
should be valid when Δx is large.

There, fermions are not negligible,
they cancel the attraction coming from bosons.

An "effective" model

- Turn-off the interaction (off-diagonal elements) once D0 goes beyond a threshold value.
- It can be done by keeping full SU(N) symmetry.

(If you are interested in, I can tell you the technical detail later.)



Classical time evolution mimics formation and evaporation of BH.

Future directions

- More detail about the thermalization & scrambling processes.
- What can we learn from 1/N corrections?
- Can we make a better effective theory? Learn from QGP industry? Determine the potential by Euclidean simulation?
- Can we somehow mimic emission of massless particles?
- (1+1)-, (1+2)- and (1+3)-d YM; BH/BS topology change.
- Full quantum simulation?
- Firewall? No Firewall?

backup

‘instant Lyapunov exponent’

$$\begin{pmatrix} \delta\dot{V}_M \\ \delta\dot{X}_M \end{pmatrix} = \begin{pmatrix} 0 & \mathcal{M}_{MN} \\ \delta_{MN} & 0 \end{pmatrix} \begin{pmatrix} \delta V_N \\ \delta X_N \end{pmatrix}$$

$$\mathcal{M}_{MN}\delta X_N = \delta[X_N, [X_M, X_N]]$$

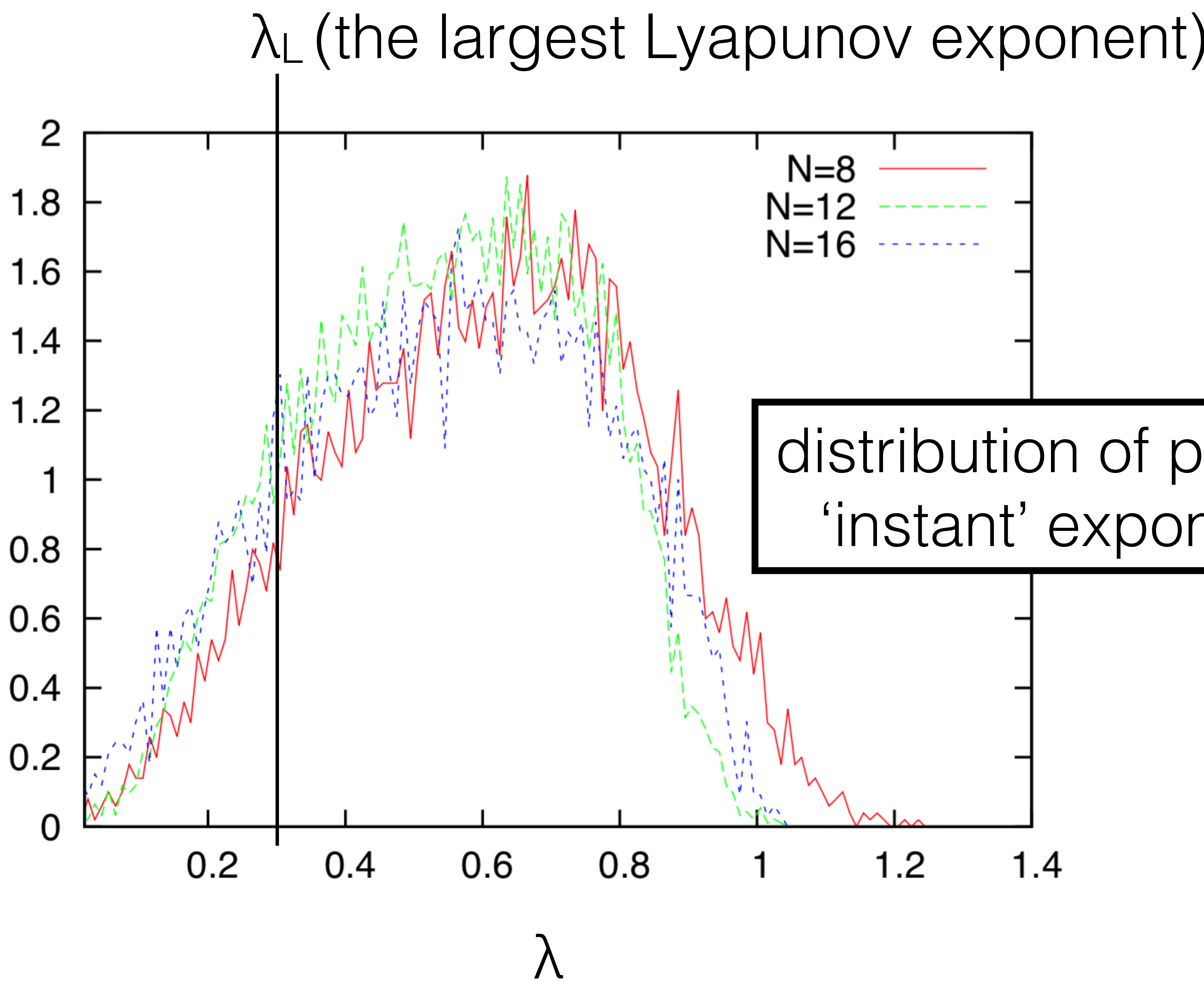
$$\mathcal{M}\vec{v} = \lambda^2\vec{v} \quad \Rightarrow \quad \begin{pmatrix} 0 & \mathcal{M} \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \pm\lambda\vec{v} \\ \vec{v} \end{pmatrix} = \pm\lambda \begin{pmatrix} \pm\lambda\vec{v} \\ \vec{v} \end{pmatrix}$$

λ (in λ^2): real

$\pm\lambda$: real or pure imaginary

our first guess: this gives λ_L at large-N.

surprise....



distribution of positive ‘instant’ exponents

why?

$$\mathcal{M}_{MN}\delta X_N = \delta[X_N,[X_M,X_N]]$$

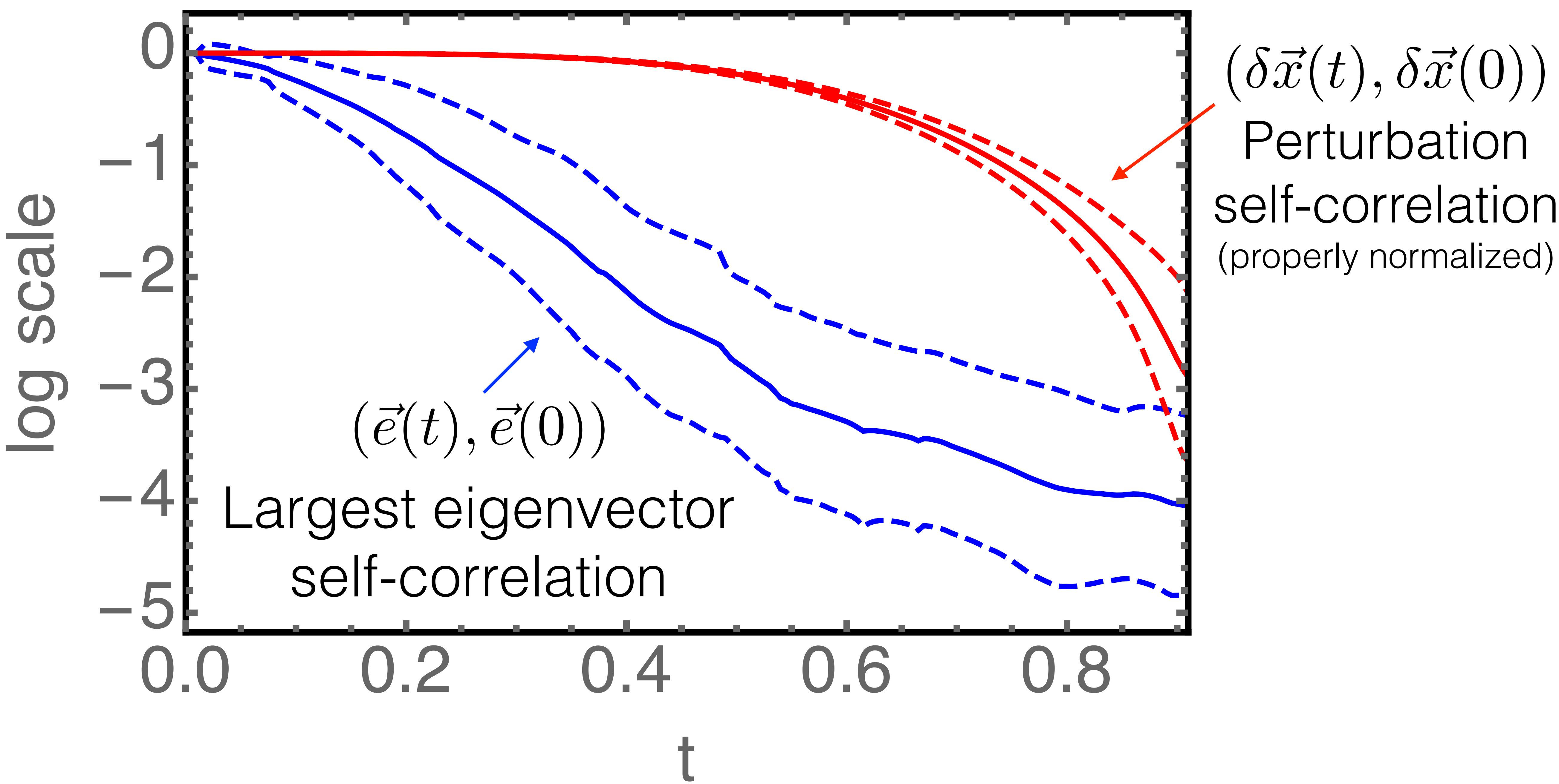
$$\mathcal{M}\vec{v} = \lambda^2\vec{v}$$

it depends on $X_M(t)$, and hence on t.

‘growing direction’ changes very rapidly.
Perturbation grows to some directions
and shrinks along other directions.

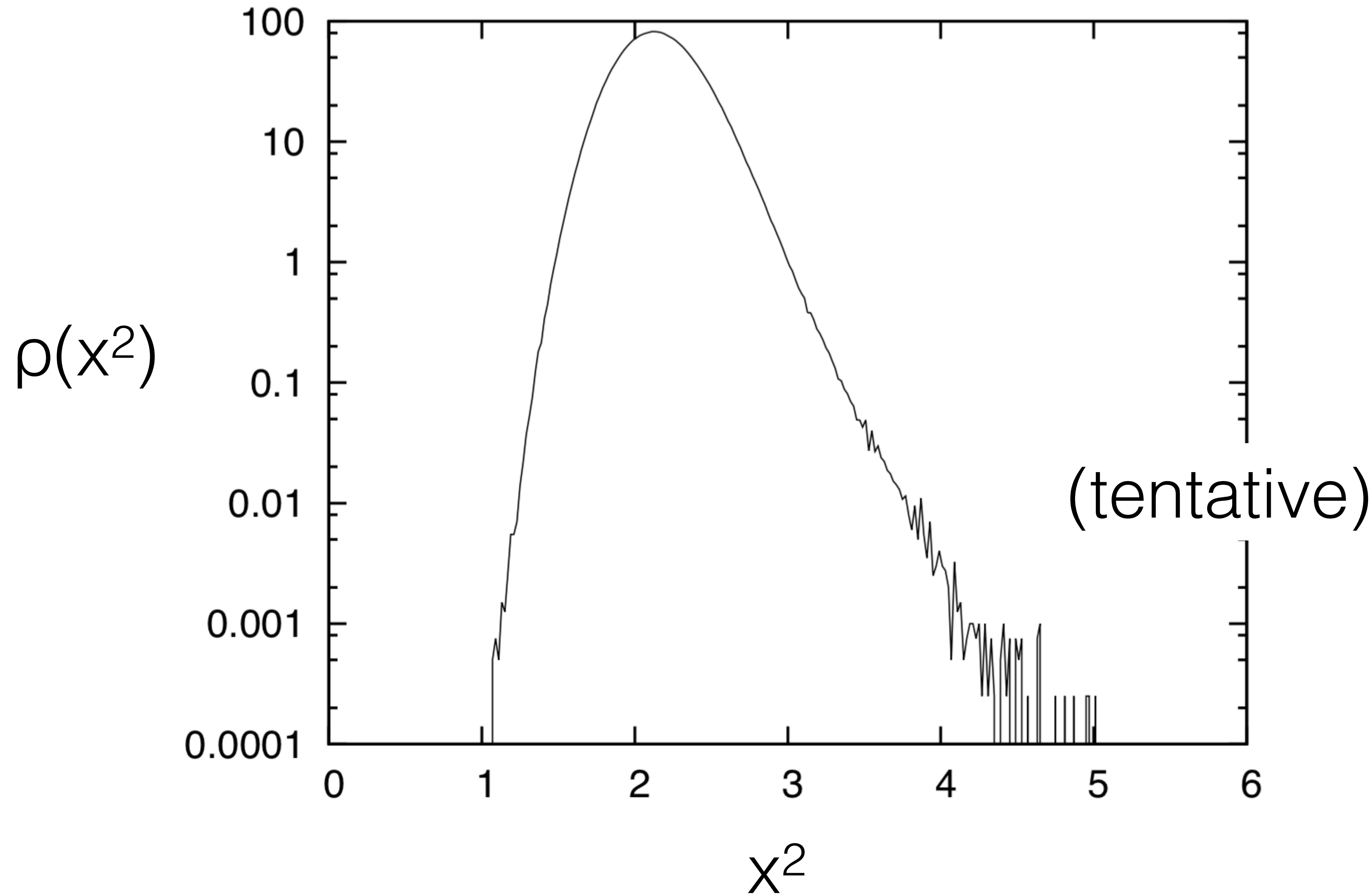
A nontrivial cancellation makes
the Lyapunov exponent rather ‘small’.

If X_M moved even faster, the exponent could be zero.



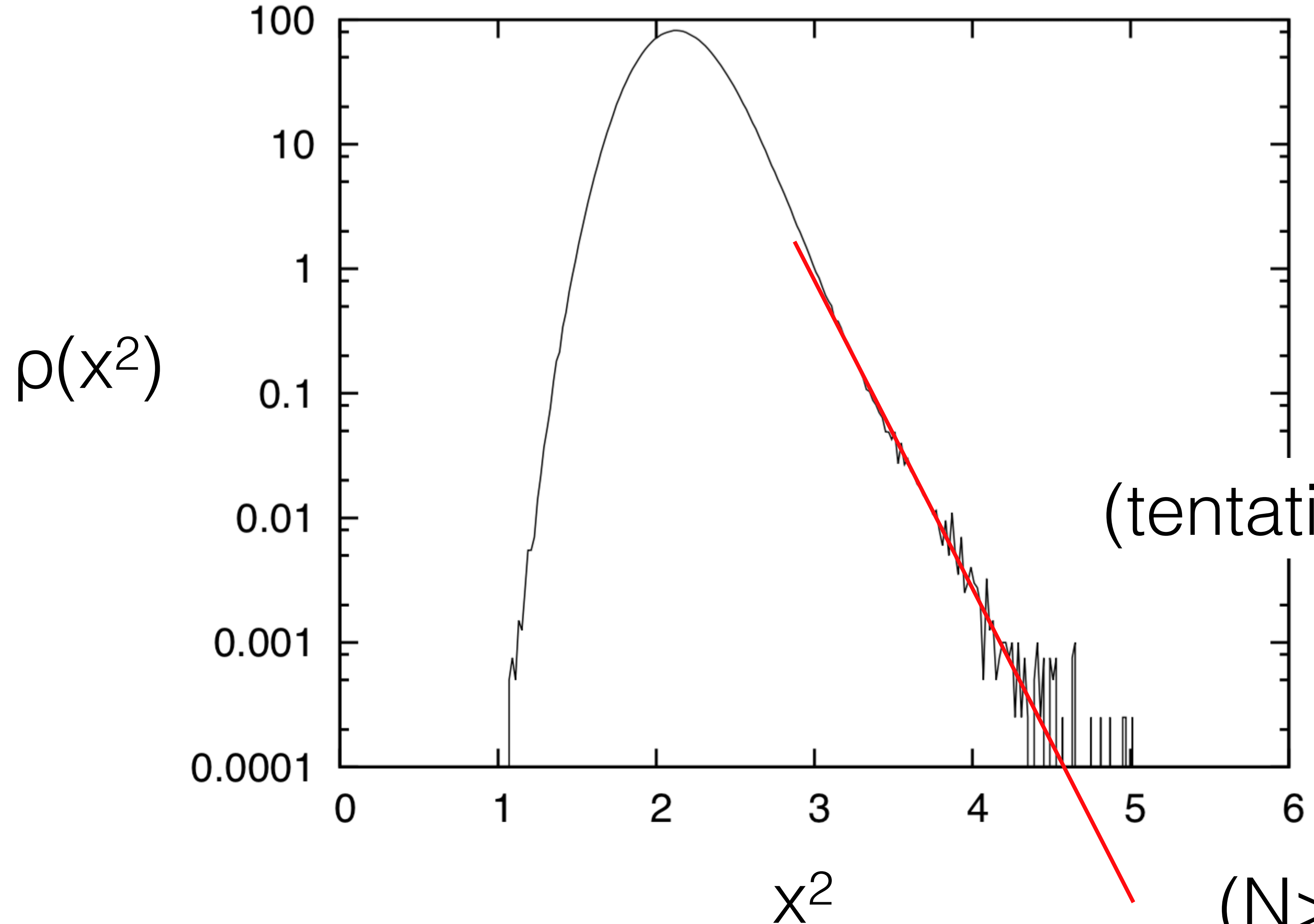
Perturbation cannot catch up with evolving eigenvector!

Distribution of the largest eigenvalue of $(X_M{}^2)_{ij}$



N=4
E/N^2=3.0

Distribution of the largest eigenvalue of $(X_M{}^2)_{ij}$



N=4
$E/N^2=3.0$

(tentative)

'Emission rate' is consistent with $\exp(-cNx^2)$

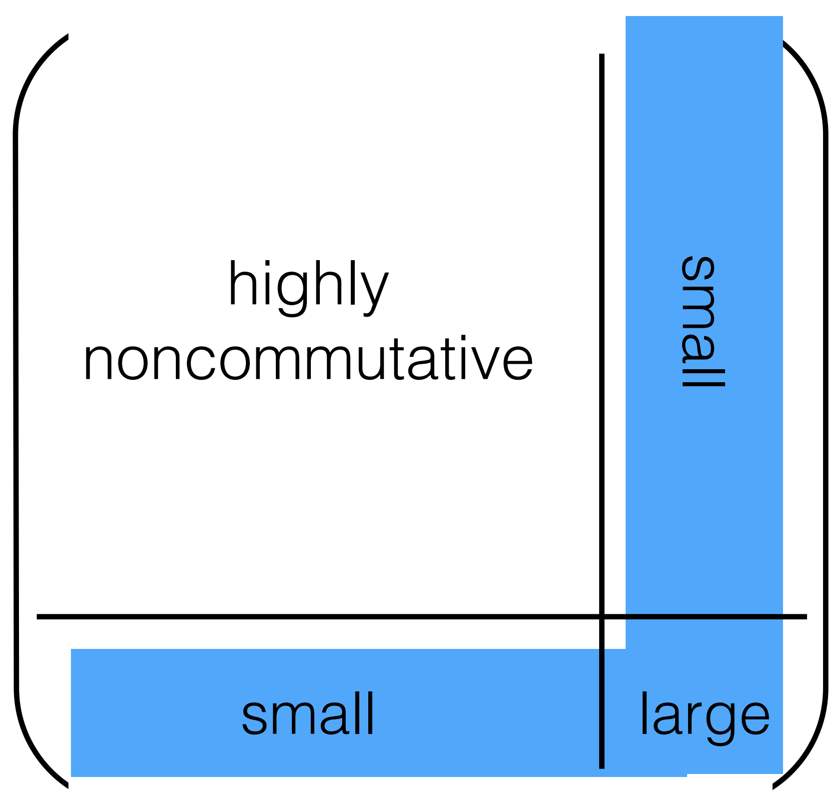
(N>3; c is a constant)

(systematic simulations are going on)

Let's diagonalize $(X_M^2)_{ij}$. suppose $x_1 < x_2 < \ldots << x_N$.

$$(X_M^2)_{ij} = \left(\begin{array}{cccc|c} x_1 & & & & \\ & x_2 & & & \\ & & \ddots & & \\ & & & & \\ \hline & & & & x_N \end{array}\right)$$

$X_M =$



should be projected out from the interaction

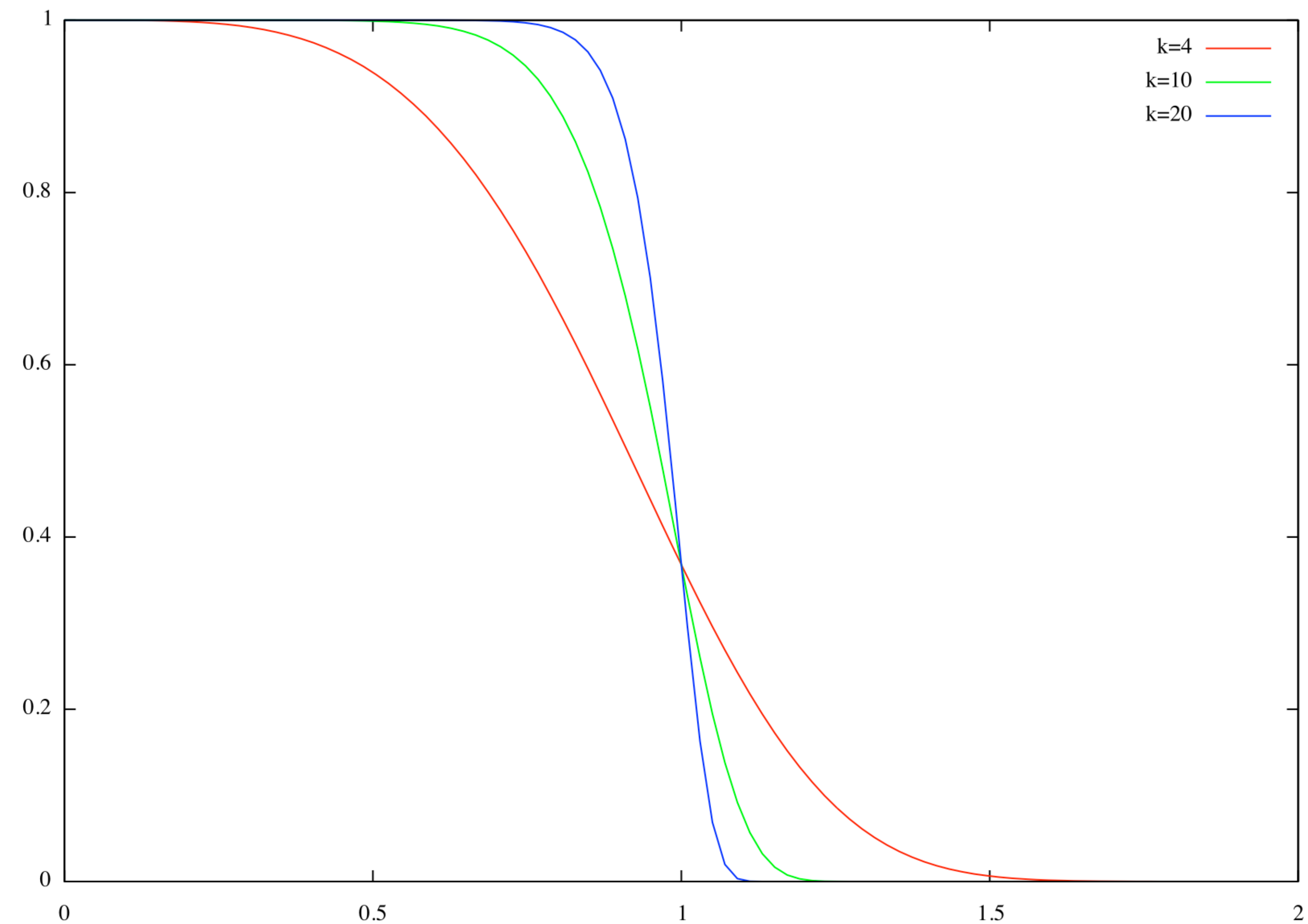
- There are many ways to construct a gauge-covariant projector. For example,

$f(X_M^2/R^2)$

with

$f(x) = \exp(-x^k)$

$$\left(\begin{array}{ccc|c} 1 & & & \\ & \ddots & & \\ & & 1 & \\ \hline & & & 0 \end{array}\right)$$



- Interaction term can be modified as

$$\mathrm{Tr}\,[X_I,X_J]^2 \to \mathrm{Tr}(f(X_M{}^2/R^2)\,[X_I,X_J]^2)$$

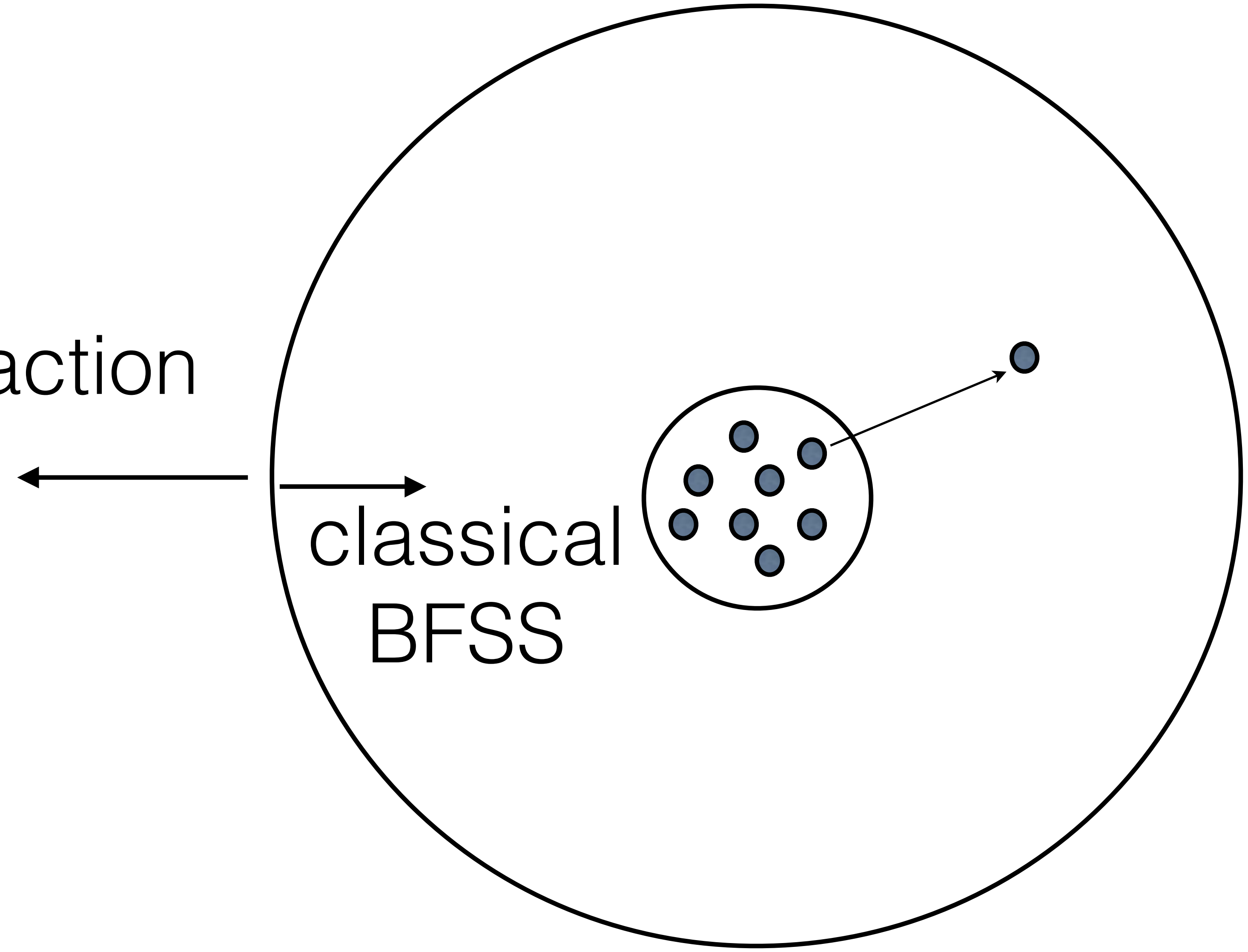
or

$$\widetilde{X} = f(X_M{}^2/R^2) \cdot X \cdot f(X_M{}^2/R^2)$$

$$\mathrm{Tr}\,[X_I,X_J]^2 \to \mathrm{Tr}([\widetilde{X}_I,\widetilde{X}_J]^2)$$

Once an eigenvalue of $X_M{}^2$ becomes larger than R^2, it propagates freely.

No interaction



Add a small 'mass term' $m^2 \, \mathrm{Tr} \, \tilde{X}_I^2$

which suppresses the random walk of BH.

(If m is small enough, it does not affect the emission)

Classical time evolution mimics formation and evaporation of BH.