Anisotropic Holographic Insulators and Homes' Relation

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J. Erdmenger, B. Herwerth, S. Klug, RM, K. Schaalm, 1501.07615 + WIP

Introduction: High T_c and AdS/CFT

- Difficulty in understanding High T_c via AdS/CFT: Don't know the IR field theory (DOFs, operator content, action, scaling properties) - very different from AdS/QCD
- Need universal signatures to guide AdS/CFT model building
- Most prominent: Linear temperature resistivity

  • Many tunable bottom-up models , free parameters

  [HSV Lishitz: Charmousis etal, Sachdev+Huijse+Schwingle, Gouteraux+Kiritsis, Gouteraux, ...; Probe Branes: Kim+Kiritsis+Panagopoulos, Holographic Lattices: Horowitz+Santos, Gauntlett etal, Andrade+Withers, ...; Massive Gravity: Vegh etal; p-wave model Herzog etal. 1405.3714]

  • Different generic mechanisms , also with free parameters, assumptions

  [Hartnoll+Hofman 1201.3917, Davison+Schalm+Zaanen 1311.2451, Hartnoll 1405.3651]

- Momentum relaxation not completely universal, Drude works always, non-Drude non-universal

  [Modulated Lattices, linear axions, Q-lattices, Bianchi backgrounds, massive gravity]

- Need additional universal signatures , Homes' relation provides such a universal signature

Introduction: Homes' Relation

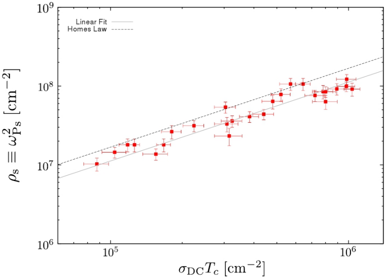
- Homes' Relation [Homes etal Nature 2004, PRB 2005]

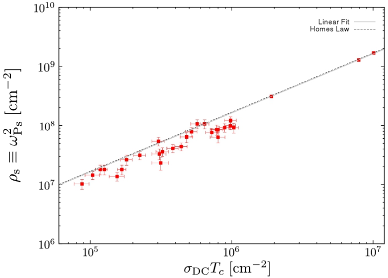
$$\rho_S|_{T=0} = C\sigma_{DC}(T_{c,+})T_c$$

C is universal for families of superconductors

In-Plane Cuprates, Elemental BCS: $C \approx 35$

C-Axis Cuprates, Dirty Limit BCS (Nb,Pb): $C \approx 65$





- Experimentally verified to good accuracy
- Derivation for clean/dirty BCS [Homes, C. C. et.al. Phys. Rev. B 72, 134517 (2005)]
- In Holographic Superconductors? Probe Limit [Erdmenger etal 1206.5305]

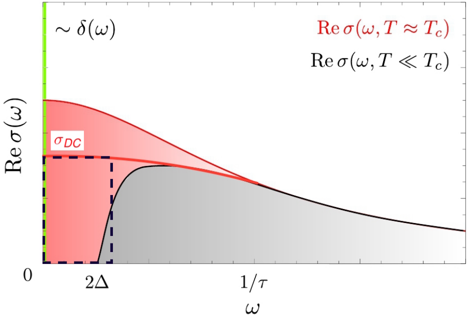
Homes' relation in the Dirty BCS

- Dirty BCS Very broad Drude peak ($\tau^{-1} > 2\Delta$)

$$\rho_s \approx 2\sigma_{DC}(T_c)\Delta$$

- BCS relates the gap with critical temperature

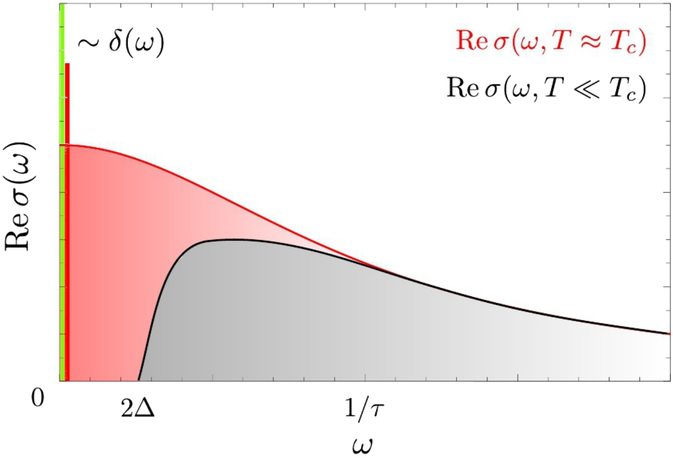
$$\Delta \sim T_c \quad \Rightarrow \quad \rho_s \sim \sigma_{DC} T_c$$



- Similar derivation in clean limit BCS

[Homes etal 2005]

Translation Invariance & Momentum Conservation



$$Re\sigma_n = K\delta(\omega) + \ldots\,, \quad Re\sigma_s = (K' + \rho_s)\delta(\omega) + \ldots$$

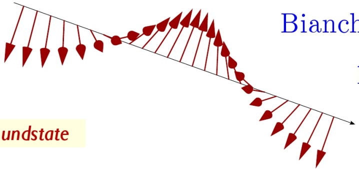
[Erdmenger metal 1206.5305] rewrote Homes' Law by sum rules, $\tau \propto D \propto 1/T$ from diffusion in the Probe Limit

Homes & Realistic Insulators/Superconductors

- A more realistic setup [Donos+Hartnoll 2012]
  - Momentum relaxation $\rightarrow$ Bianchi VII Spacetime
  - Uncondensed charged IR DOFs? $\rightarrow$ Cohesive SuCo Phase
  - Extra Lifshitz IR DOFs? $\rightarrow$ Holographic Cohesive Insulator

$$S = \int d^5x\sqrt{-g}\left(R + 12 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{m^2}{2}B_\mu B^\mu\right) - \frac{\kappa}{2}\int B\wedge F\wedge W$$

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + e^{2v_1(r)}\omega_1^2 + e^{2v_2(r)}\omega_2^2 + e^{2v_3(r)}\omega_3^2$$



Bianchi VII_0 helix geometry

period p; strength λ

has an "insulating" groundstate

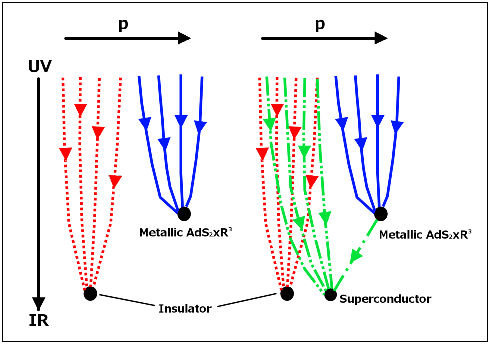
Homes & Holographic Insulator/Superconductor

- Add a charged scalar

$$S = \int d^5x\sqrt{-g}\left(R + 12 - \frac{1}{4}F^2 - \frac{1}{4}W^2 - \frac{m^2}{2}B^2\right)$$
$$- \int d^5x\sqrt{-g}\left(|(\partial_\mu - iqA_\mu)\rho|^2 + m_\rho^2|\rho|^2\right) - \frac{\kappa}{2}\int B\wedge F\wedge W$$

- Massless case ($m_\rho = m = 0$) and large $|\kappa| > 0.57$:

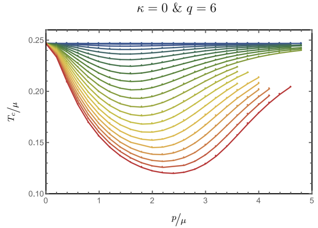
Cohesive **Superconducting ground state:** $*F|_{Horizon} \overset{T\to 0}{\to} 0$

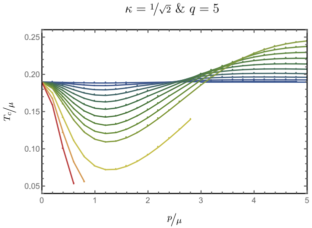


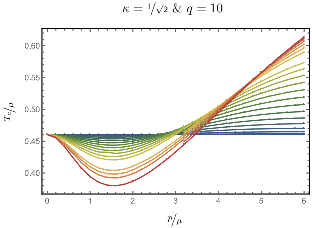
- All phases unstable towards superconductivity.

Phase Diagram

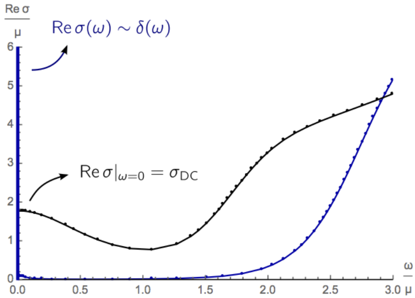
- T_c depending strongly on lattice data, Possible Quantum Phase Transition, Homogeneity/Overshooting







Optical Conductivity

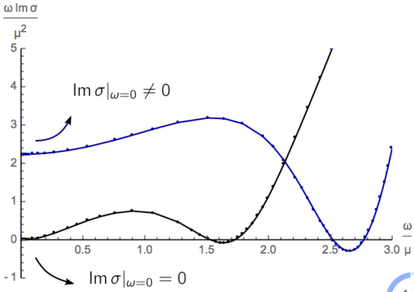


soft gap

Kramers-Kronig relations

$$\mathrm{Re}\, f(\omega) = \frac{2}{\pi}\mathcal{P}\int_0^\infty d\omega' \frac{\omega' \,\mathrm{Im}\, f(\omega')}{\omega'^2 - \omega^2}$$

$$\mathrm{Im}\, f(\omega) = -\frac{2\omega}{\pi}\mathcal{P}\int_0^\infty d\omega' \frac{\mathrm{Re}\, f(\omega')}{\omega'^2 - \omega^2}$$

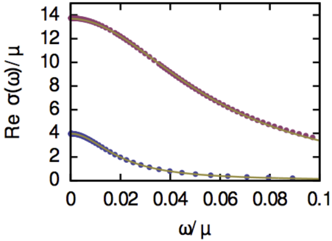


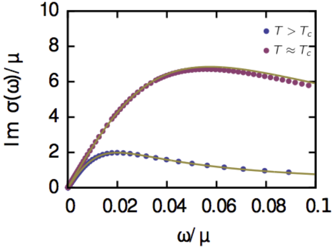
$$\mathrm{Re}\,\sigma(\omega) \sim \delta(\omega) \quad \Leftrightarrow \quad \mathrm{Im}\,\sigma(\omega) \sim \frac{i}{2}\frac{1}{\omega}$$

Drude Theory in Normal Phase

- **Drude model**

$$\sigma(\omega) = \frac{\sigma_{DC}}{1 - i\omega\tau} = \frac{n_n e^2 \tau}{m^*} \frac{1}{1 - i\omega\tau} = \frac{\omega_P^2}{4\pi} \frac{1}{1/\tau - i\omega}$$





$\hookrightarrow$ $\mathrm{Re}\,\sigma(\omega) = \frac{\sigma_{DC}}{1 + \omega^2\tau^2}$

$\hookrightarrow$ $\mathrm{Im}\,\sigma(\omega) = \frac{\omega\tau}{1 + \omega^2\tau^2}\sigma_{DC}$

N.B.: Not for medium/strong momentum relaxation

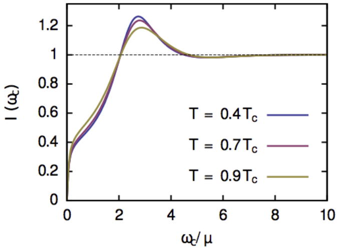
intermediate and IR DOFs mix $\rightarrow$ Homes' relation

F-Sum Rule

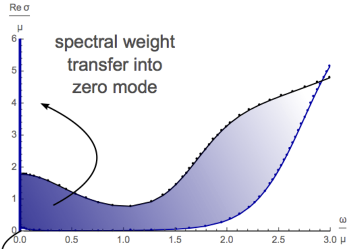
- Plasma frequency ~ Superfluid strength

$$\omega_{Ps}^2 = 8 \int_0^\infty d\omega \; \mathrm{Re}\, \sigma_s(\omega)$$

$$= 4\pi \frac{n_s e^2}{m} = \lambda_L^{-2} \equiv \rho_s$$



- Ferrell-Glover-Tinkham sum rule



$\sigma_{reg} \neq 0$ regular contribution due to soft gap c.f high Tc superconductors

$$\rho_s = \int_{0^+}^\infty d\omega \left[\mathrm{Re}\, \sigma_n(\omega) - \mathrm{Re}\, \sigma_s(\omega)\right]$$

assuming no residual regular or high frequency contribution

20

Holds irrespective of strength of momentum relaxation.

Two Fluid Model

- Two fluid model

$$\mathrm{Re}\,\sigma(\omega) = \frac{e}{m^*}\left(\chi_n(T)\frac{\tau}{1+\omega^2\tau^2} + \frac{\pi}{2}\chi_s(T)\delta(\omega)\right)$$

regular metallic part — superfluid part

- χ is controlling the strength

normal phase:

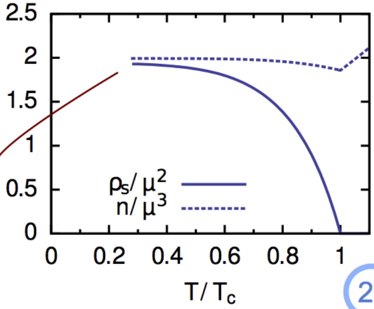
$$\chi_n(T > T_c) = n_n$$
$$\chi_s(T > T_c) = 0$$

superconducting phase:

$$\chi_n(T = 0) = 0$$
$$\chi_s(T = 0) = n_s$$

$$\rho_s(T \to 0) \approx n_s(T \to 0)$$

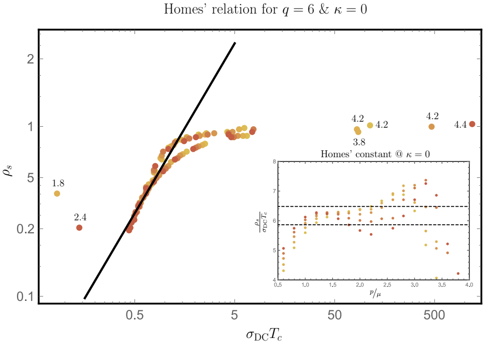
$$\chi_n(T \to 0) \approx 0 \qquad \chi_s(T \to 0) \approx n_s$$



N.B.: $\rho_s \neq n_s$ for stronger momentum relaxation

Homes' Relation for $\kappa = 0$

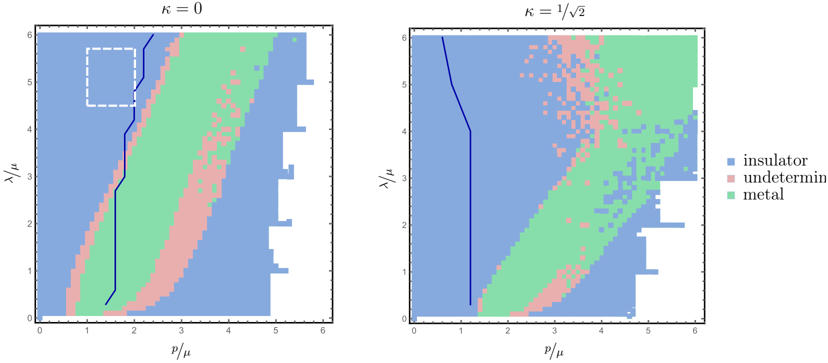
- Holds in intermediary regime: $\boxed{\frac{\lambda}{\mu} \approx 4.5 \ldots 6\,, \quad \frac{\rho}{\mu} \approx 1 \ldots 2}$



- Strong lattice important: $\ln \rho_s = \ln C + \ln \sigma_{DC}(T_c)T_c$
- Holographic Homes' constant: $\boxed{C = 6.2 \pm 0.3}$
- In-plane/BCS $(35/8 \approx 4.4)$ and Out-plane/dirty $(65/8 \approx 8.1)$
- On the border of applicability of Drude/Two Fluid model

Zero Temperature Phase Diagram (preliminary)

- Zero Temperature Numerics difficult
- Phases distinguished by behavior of DC conductivity (Insulating/Conducting)



- Homes' relation holds close to the insulating/conducting phase transition, as well as the minimum of the critical temperature $\rightarrow$ WIP

Conclusions & Outlook

- Homes' relation a candidate for a universal relation in high temperature superconductivity
- For holographic superconductors: probe limit [Erdmenger et al 2012]
- Problematic: Translation invariance & IR D.O.Fs
- Break translation invariance via Bianchi VII_0
  Use Insulating Geometry of [Donos+Hartnoll '12]
  Use Cohesive Superconducting Phase
- Sum rules hold, Homes relation holds in a regime of intermediary momentum dissipation
- To Do List:
  - Homes' law for other values of κ
  - Understand the QPTs with and without superconducting order parameter
  - Understand Relation with QC Physics & (Non-)Planckian dissipation
  - Fluctuations at $T = 0$, ρ_s vs. n_s, other instabilities?

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Matt Visser

Ben Craps
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Towards Homes' Relation in AdS/CFT [1206.5305]

- Three Assumptions
  - 1. FGT sum rule: $\rho_s + \int_{0_+}^{\infty} Re\sigma_{xx,s}(\omega)d\omega = \underbrace{\int_0^{\infty} Re\sigma_{xx,n}(\omega)d\omega}_{=\omega_{Pn}^2/8}$
  - 2. Tanner's relation: $n_s \sim n_n \quad \Rightarrow \quad \omega_{Pn}^2 \sim \omega_{Ps}^2$
  - 3. Drude model: Two scales ω_{Pn}^2 and $\tau \Rightarrow \sigma_{DC} \sim \omega_{Pn}^2 \tau(T_c)$
- S & P-wave superconductor in probe limit: $D_{M/R} \sim \frac{1}{T}$

$$\rho_s = \omega_{Ps}^2 \overset{(2)}{\sim} \omega_{Pn}^2 \overset{(3)}{\sim} \sigma_{DC} T_c \quad \Leftrightarrow \quad T_c \tau(T_c) \sim T_c D_{M/R}|_{T_c} = const.$$

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- Finite backreaction changes T_c and $D_{M/R}$

  $\rightarrow$ Changing the charge of the order parameter
- What could have gone wrong?
  - Need to introduce momentum relaxation beyond probe limit.
  - Sum rules may not hold (at $T = 0$?)
  - Is the Drude model valid? Is it useful?
  - Uncondensed charged IR DOFs? Extra Lifshitz IR DOFs?

Zero Temperature Solutions

- Break translations by Bianchi VII_0 Helix [1212.2998]

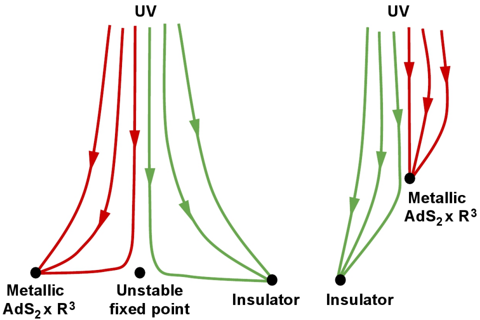
$$\omega_1 = dx\,, \quad \omega_2 + i\omega_3 = e^{ipx}(dy + idz)$$

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + \sum_{i=1}^{3} e^{2v_i(r)}\omega_i^2$$

- Charge density and Helix Field, Order Parameter

$$A = a(r)dt\,, \quad B = w(r)\omega_2\,, \quad \rho = \rho(r) \in \mathbb{R}$$

- QPT by μ or Helix source/pitch: $|\kappa_c| \approx 0.57$ [1212.2998]



Two Mechanisms of Instability

- Usual superconducting instability: $AdS_2 \times \mathbb{R}^3$ with the charged scalar mode $\delta\rho \sim r^\alpha$ unstable if

$$3 + m_\rho^2 - 2q^2 < 0$$

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- Insulating geometry with charged hair: $(\kappa = 1/\sqrt{2})$

$$a = a_0 r^{5/3} + \ldots, \quad w = w_0 + w_1 r^{4/3} + \ldots, \quad U = U_0 r^2 + \ldots$$

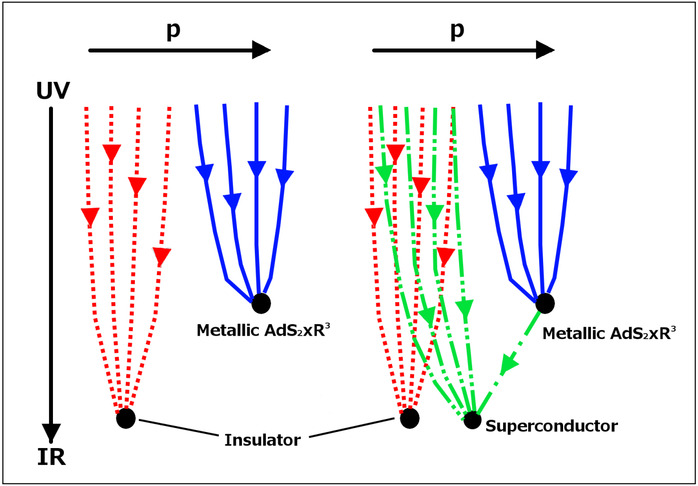
$$e^{v_1} = e^{v_{10}} r^{-1/3} + \ldots, \quad e^{v_2} = e^{v_{20}} r^{2/3} + \ldots, \quad e^{v_3} = e^{v_{30}} r^{1/3} + \ldots,$$

$$\rho = \rho_0 + \rho_1 r^{4/3} + \ldots$$

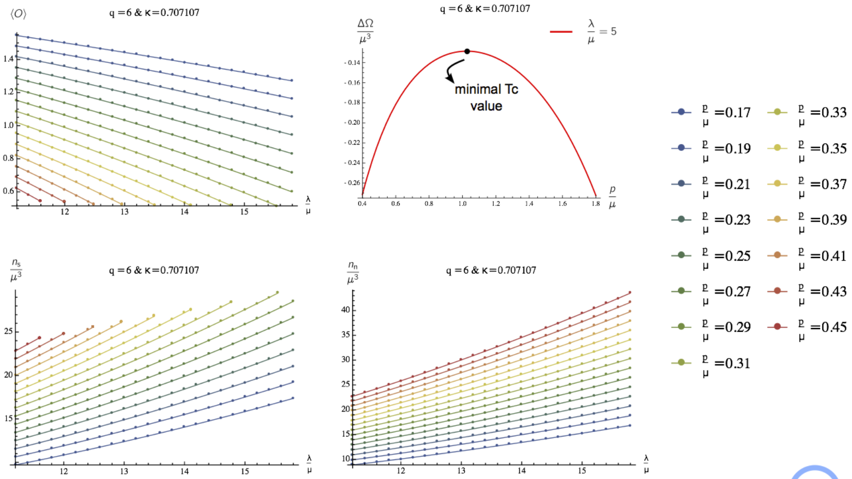
- Cohesive Phase (no flux): $\int_{\mathbb{R}^3} *F \Big|_{horizon} = 0$
- Flows to both insulating and superconducting fixed points
- Superconducting ground state expected to have lower free energy

Two Mechanisms of Instability

- Usual superconducting instability: $AdS_2 \times \mathbb{R}^3$
- Free energy competition Insulator/Superconductor

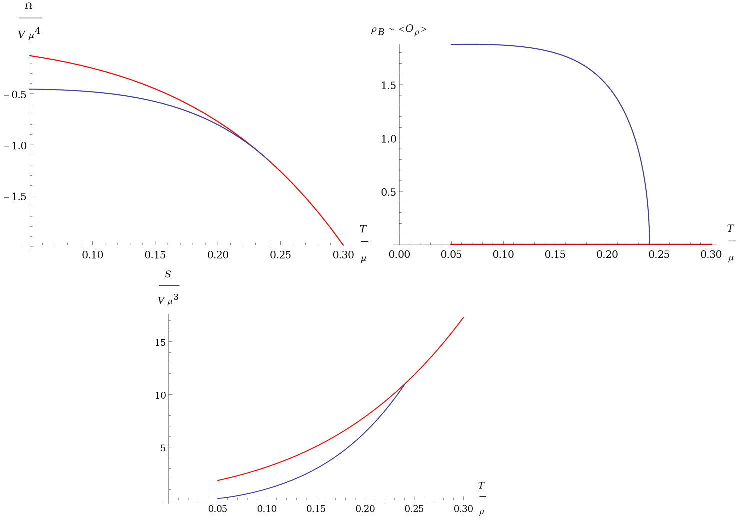


Zero Temperature Thermodynamics



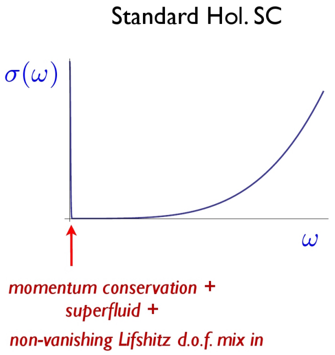
Thermodynamics

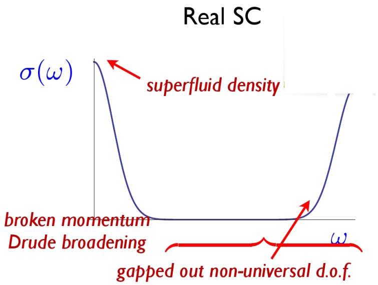
- Second order phase transition of mean field type:



Holographic vs. Real Superconductor

- Ways to improve the situation:
  - Momentum relaxation $\rightarrow$ Bianchi VII Spacetime
  - Uncondensed charged IR DOFs? $\rightarrow$ Cohesive Phase
  - Extra Lifshitz IR DOFs? $\rightarrow$ Holographic Insulator





- Probes: (semi-holography)

$$S = \int d^d x \left[-\frac{1}{2}(\partial\phi)^2 - \phi \mathcal{O}_{\mathcal{CFT}} \right] + S_{CFT}$$

$$G(\omega, k) = \frac{1}{\omega - vk + \Sigma(\omega)}$$

$$\Sigma(\omega) \sim e^{-(k^z/\omega)^{1/(z-1)}}$$

Interaction with geometric d.o.f. in Lifshitz background (z dynamical critical exponent)

$$\Sigma(\omega) \sim \frac{1}{N}\omega^c$$

Standard loop correction

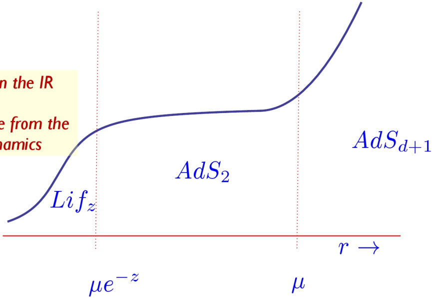
Faulkner, Liu, McGreevy, Vegh; Faulkner, Polchinski; Sachdev; Hartnoll, Polchinski, Silverstein, Tong,....

- Probes: (semi-holography)

$$S = \int d^dx \left[-\frac{1}{2}(\partial\phi)^2 - \phi\mathcal{O}_{\mathcal{CFT}} \right] + S_{CFT}$$

still $\mathcal{O}(N^2)$ d.o.f. in the IR

do not cleanly separate from the U(1) goldstone dynamics



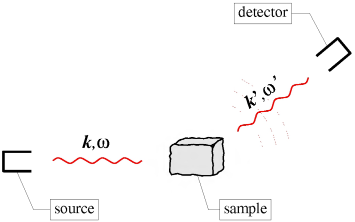
Definitions & Measurement

- (Super)fluid density from plasma frequency

$$\rho_n = \omega_{Pn}^2 = \frac{4\pi n_n e^2}{m_*}$$

$$\rho_s = \omega_{Ps}^2 = \frac{4\pi n_s e^2}{m_*} = \lambda_L^{-2}$$

- Reflectivity measurements



$$R(\omega) = \left|\frac{1-n(\omega)}{1+n(\omega)}\right|^2$$

Kramers-Kronig $\Rightarrow$ $\arg(R)$

$$\epsilon(\omega) = (n(\omega) + i\kappa(\omega))^2$$

$$\boxed{\sigma(\omega) = \frac{\omega}{4\pi i}\left(\epsilon(\omega) - \epsilon_\infty\right) \rightarrow \sigma_{DC}}$$

$$\boxed{\omega_P^2 = \lim_{\omega\rightarrow 0}(-\omega^2) Re(\epsilon(\omega))}$$

Superfluid Density and Sum Rules

- Oscillator Strength sum rule (Thomas-Reiche-Kuhn sum rule)

$$\frac{\omega_P^2}{8} = \int_0^\infty d\omega \underbrace{Re(\sigma(\omega))}_{\sim\delta(\omega_f-\omega_i)|<\omega_f|H_{int}|\omega_i>|^2}$$

- Spectral Weights

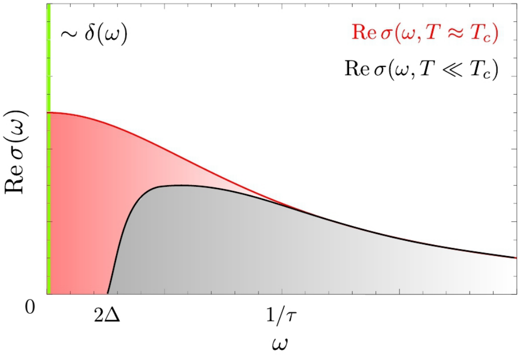
$$N_n = \frac{\omega_{Pn}^2}{8} = \int_0^\infty Re\sigma(\omega)|_{T>T_c} \ , \quad N_s = \int_{0_+}^\infty Re\sigma(\omega)|_{T<T_c}$$

- Ferrell-Glover-Tinkham sum rule (no missing spectral weight)

$$\boxed{\rho_s = 8(N_n - N_s)}$$

Superfluid Density and Sum Rules

- Ferrell-Glover-Tinkham sum rule (no missing spectral weight) $\boxed{\rho_s = 8(N_n - N_s)}$



$$Re\sigma(\omega) = \rho_s \delta(\omega) \qquad (\omega < 2\Delta)$$

- Holds in Holographic Superconductors with similarities to high Tc cuprates (after introducing a lattice) [1302.6586]

Homes' Law & Planckian Dissipation

[Zaanen 2004]

- Normal State: Two Time Scales
  - Plasma frequency ω_{Pn} (density of charge carriers)
  - Scattering rate $\tau^{-1}(T)$ (relaxation timescale τ)

$$\Rightarrow \sigma_{DC}(T_c) = \frac{\omega_{Pn}^2 \tau(T_c)}{4\pi} = \frac{n_n e^2 \tau(T_c)}{m_*}$$

- A fraction of the charge carriers condenses: Tanner's law

$$\boxed{n_s \approx \frac{n_n}{4}}$$

- Planckian dissipation: (Quantum Criticality) [van der Marel etal 2003]

$$\boxed{\tau(T_c) \sim \frac{\hbar}{k_B T_c}}$$

- From these assumptions Homes' law follows:

$$\sigma_{DC}(T_c) T_c \sim \underbrace{\frac{n_s e^2}{m_*}}_{=\omega_{Ps}^2 = \rho_s} \tau(T_c) T_c \sim \rho_s \frac{\hbar}{k_B}$$