From strange metals to black holes and back: Effective theories of thermoelectric transport

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References and acknowledgments

Based on

- *Dissecting holographic conductivities*, to appear with R. Davison
- *Momentum dissipation and effective theories of coherent and incoherent transport*, JHEP **1501** (2015) 039 arxiv:1411.1062, with R. Davison
- *Holographic metals and insulators with helical symmetry*, JHEP **1409** (2014) 038 arxiv:1406.6351, with A. Donos and E. Kiritsis
- *Charge transport in holography with momentum dissipation*, JHEP **1404** (2014) 181, arxiv:1401.5436
- *Universal scaling properties of extremal holographic cohesive phases*, JHEP **1401** (2014) 080, arXiv:1308.2084

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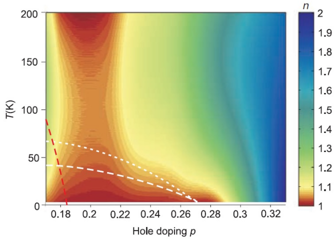
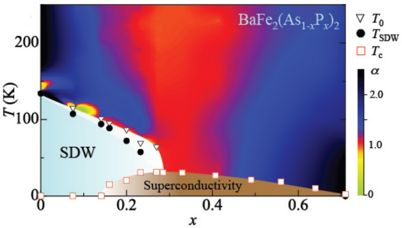
European Commission

Unconventional transport in strange metals

$La_{2-x}Sr_xCuO_4$



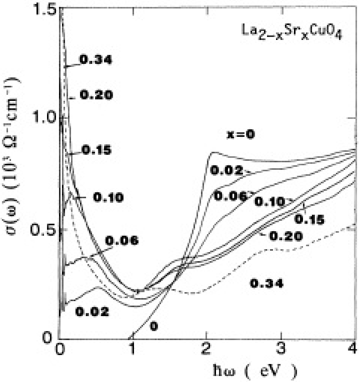
[Cooper et al, Science'09]

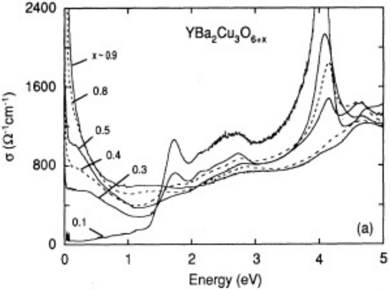


[Kasahara et al, PRB'10]

- Resistivity linear in T rather than quadratic (Fermi liquid).
- Short-lived quasiparticles.
- Strong coupling.

Frequency behaviour of the AC conductivity





[Cooper et al, PRB'91]

[Uchida et al, PRB'91]

- **Drude-like peak** at low frequencies for high doping.
- At intermediate doping, the peak broadens out.
- **Metal/insulator transition** at low doping.

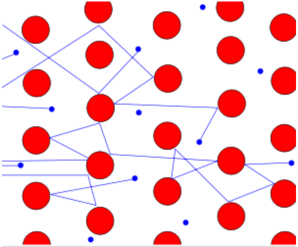
What is a Drude peak?

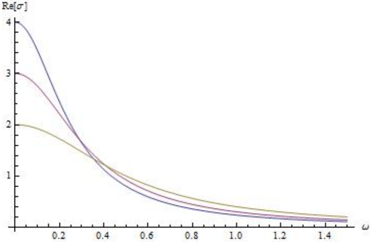
- Operationally: a purely imaginary pole close to the real axis

$$\boxed{\sigma(\omega, T) = \frac{n^2}{(\epsilon + p)(\Gamma - \imath\omega)}}$$

$$\Rightarrow \boxed{\sigma_{DC} = \sigma(\omega = 0, T) = n^2/\Gamma(\epsilon + p)}$$

- Needs a current relaxation rate Γ: may come from long-lived quasiparticles (e.g. Fermi liquid), almost conserved momentum ([Hartnoll & Hofman'12])...
- Observe that $\sigma_{DC} \to +\infty$ for $\Gamma \to 0$.
- The peak gets sharper as $\Gamma \to 0$.
- Its width $\mathcal{O}(\Gamma) \ll T$





Take home messages

- To explain the unconventional properties of strange metals, we need to understand the nature of thermoelectric transport in strongly-coupled systems without quasiparticles and/or with fast momentum relaxation.
- QFT approaches based on a quasiparticle picture of transport are typically not reliable.
- Gauge/gravity duality provides a new arena to describe strongly-coupled thermoelectric transport.

Take home messages

- We have learned a lot about gravity by confronting gravitational duals to Condensed Matter expectations.
- Intense effort has gone into deriving top-down models which could be dual to specific Condensed Matter systems.
- However, (bottom-up) holographic computations may pave the way for the construction of effective theories independent from holographic setups.

N.B.: We are interested in low energy phenomena. These depend mostly on the IR part of the spacetime where Lorentz symmetry is broken, so asymptotically AdS boundary conditions are not an obstacle. AdS is now a proxy to have a reliable holographic dictionary.

Outline

1. Holography with momentum relaxation and relation to hydrodynamics/memory matrices

2. Quantum critical transport from holography

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Holographic momentum relaxation

Momentum relaxation is needed for realistic transport properties.

We now know many ways to dissipate momentum in holography.

Simplest example to date: use **spatial scalar sources** to dissipate momentum [Andrade & Withers'13]:

Ward identities: $\nabla_\mu \langle T^{\mu\nu} \rangle = \nabla^\nu \psi_{I(0)} \langle O^I_\psi \rangle + F^{(0)\nu\mu}_{ext} \langle J_\mu \rangle \qquad \mu, \nu = t, x, y$

$$\nabla^\mu \langle J_\mu \rangle = 0$$

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(R + 6 - \frac{1}{4} F^2 + \partial\psi_x^2 + \partial\psi_y^2 \right)$$

Black brane solution with $\vec{\psi} = m\vec{x}$ and horizon radius r_0, [Bardoux, Caldarelli & Charmousis'12] parameterized by: T, μ, m (temperature, chemical potential and strength of momentum relaxation)

Holographic DC conductivities

DC thermoelectric conductivities can be calculated from horizon data [Blake & Tong'13, Andrade & Withers'13, Donos & Gauntlett'14] using radially conserved quantities [Iqbal & Liu'08]

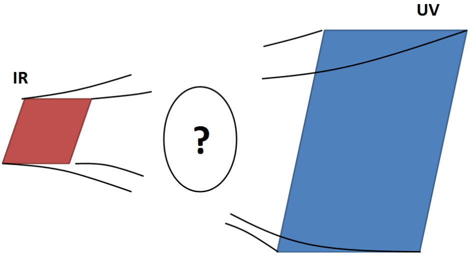
$$\sigma_{DC} = \boxed{1} + \boxed{\frac{\mu^2}{m^2}}, \qquad \bar{\kappa} = \boxed{\frac{4\pi s T}{m^2}}, \qquad \alpha = \boxed{\frac{4\pi\mu}{m^2 r_0}}$$

It is tempting to organize these results in terms of momentum relaxation and pair creation contributions.

The first would dominate the slow momentum relaxation regime, the second the fast momentum relaxation regime.

N.B.1:The pair creation interpretation is correct at zero density when particle-hole symmetry of the CFT is restored.
N.B.2: Equivalent formulæ exist for other systems (including a dilaton [**B.G.**'14, Donos & Gauntlett'14] or higher order kinetic terms [Taylor & Woodhead'14, Baggioli & Pujolas'14], massive gravity [Blake & Tong'13, Amoretti et al'14], Bianchi VII [Donos, Kiritsis & **B.G.**'14] and inhomogeneous lattices [Blake, Tong & Vegh'13, Donos & Gauntlett'14], random-field disorder [Davison, Schalm & Zaanen'13, Lucas, Schalm & Sachdev'14, O'Keefe & Peet'15])

But...



- Radially conserved quantities are deceptive: there is no simple map from horizon to boundary data in general.
- At zero density, the holographic DC heat conductivity $\bar{\kappa} = \frac{4\pi sT}{m^2}$ can only be related to a momentum relaxation rate for slow momentum relaxation [Davison & **B.G.**'14].
- What about non-zero charge density?

Linearized hydrodynamics

- To understand the origin of the various terms in the DC conductivities, we need to turn on the frequency dependence.

- We can do so in a small ω expansion in the context of linearized hydrodynamics [Policastro, Son & Starinets'02, Herzog'02]

- Focussing on the electric conductivity, we find that it receives two independent contributions

$$\sigma = \sigma_+ + \sigma_-$$

  which originate from two independent, decoupled gauge-invariant modes in the bulk.

Contributions to the DC electric conductivity

$$\sigma_{DC} = \sigma_{+} + \sigma_{-}|_{\omega=0} = 1 + \frac{\mu^2}{m^2}$$

where

$$\sigma_{+}^{DC} = \sigma_Q + O(m^2)\,, \qquad \sigma_{-}^{DC} = \frac{\mu^2}{m^2} + 1 - \sigma_Q + O(m^2)\,,$$

and

$$\sigma_Q = \left(\frac{sT}{\epsilon + p}\right)^2\Bigg|_{m=0} = \frac{1}{9}\left(\frac{\mu^2 - 12r_0^2}{\mu^2 + 4r_0^2}\right)^2$$

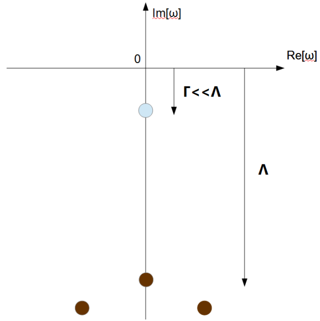
- The full DC conductivity is a non-trivial rearrangement of terms between σ_{+} and σ_{-}.
- Our DC results are actually non-perturbative in m.
- What is the low frequency dependence of σ_{+} and σ_{-}?

Coherent vs Incoherent transport [Hartnoll et al'13, Hartnoll'14]

Slow momentum relaxation $\Gamma \ll \Lambda$

Coherent transport dominated by momentum relaxation with rate Γ

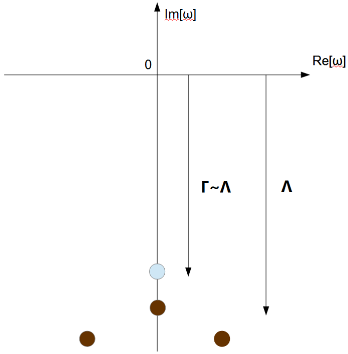
Purely imaginary pole $\omega \sim -\imath\Gamma$ close to the real axis $\Rightarrow$ sharp peak



Fast momentum relaxation $\Gamma \gtrsim \Lambda$

Incoherent transport dominated by energy/charge diffusion

No well-defined low energy excitation $\Rightarrow$ Constant optical conductivity



Our calculations yield at small values of m (slow momentum relaxation)

$$\sigma_+ = \sigma_Q + O(m^2)\,, \qquad \sigma_- = \frac{\frac{n^2}{\epsilon+P} + \# m^2}{\Gamma - i\omega} + O(m^2)\,,$$

where

$$\Gamma = \frac{4m^2 r_0}{12 r_0^2 + 3\mu^2} + O(m^4) = \frac{s m^2}{4\pi(\epsilon + P)} + O(m^4)$$

- We reproduce translationally invariant results obtained in [Hartnoll & Herzog'08, Hartnoll'09].
- σ_+ is a purely incoherent contribution without any (hydrodynamic) poles close to the real axis.
- σ_- is a purely coherent contribution which sets the height/width of the Drude peak. It includes higher-order corrections in m^2 ($\#$ is a complicated, uninformative number). Its $\omega = 0$ limit reproduces our non-perturbative DC result at the relevant order.

Other comments

- The story is similar for the other thermoelectric conductivities $\bar{\kappa}$ and α, both at zero and non-zero frequency.

- At zero density, we recover that the electric optical conductivity is totally incoherent (constant) for all frequencies [Herzog et al'07]: the current and the momentum operators decouple.

- At zero density and slow momentum relaxation, the heat conductivity takes the form (no incoherent contribution)

$$\bar{\kappa} = \frac{s}{\Gamma - i\omega} + O(\Gamma)\,, \qquad \Gamma = \frac{m^2}{4\pi T} + O(m^4)$$

in agreement with our previous results [**B.G.** & Davison'14].

Further comments

- We expect the same decomposition will hold in other setups.
- This provides an derivation of the thermoelectric conductivities of these states from the Kubo formula, which agrees with [Donos & Gauntlett'14].
- Can we obtain these results directly from a first-principles, hydrodynamic or memory matrix-type analysis?

Hydrodynamics with weak momentum relaxation

[Hartnoll et al'07] proposed:

$$\nabla_\mu T^{\mu\nu} = -\Gamma \delta^{\nu i} T^{0i} + F^{\nu\mu}_{ext} J_\mu \,, \qquad \nabla_\mu J^\mu = 0$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \eta \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \eta_{\alpha\beta}\partial_\lambda u^\lambda\right) + O(\partial^2)$$

$$J^\mu = n u^\mu - \sigma_Q \partial^\mu \left(\frac{\mu}{T}\right) + O(\partial^2)$$

By assumption $\Gamma \ll T$ in order to be able to neglect modifications to the constitutive relations.

One can then linearize these equations around thermodynamic equilibrium to describe the relaxation of the (almost) conserved charges $\delta\epsilon$, π_x, δn and compute the thermoelectric conductivities from linear response.

Thermoelectric conductivities from hydrodynamics

Hydrodynamics predict [Hartnoll et al'07]

$$\sigma = \sigma_Q + \frac{n^2}{(\epsilon + p)(\Gamma - i\omega)}\,, \qquad \alpha = -\frac{\mu}{T}\sigma_Q + \frac{ns}{(\epsilon + p)(\Gamma - i\omega)}$$

$$\bar{\kappa} = \frac{\mu^2}{T}\sigma_Q + \frac{Ts^2}{(\epsilon + p)(\Gamma - i\omega)}$$

- At zero density, these formulæ agree with our holographic results at $O(\Gamma^0)$.
- At non-zero density, these formulæ are compatible with the holographic DC results, but only at $O(\Gamma^{-1})$: they do not capture $O(\Gamma)$ terms correcting the weight of the Drude peak, which are the same $O(\Gamma)$ as σ_Q.
- There is a similar discrepancy with existing memory-matrix calculations [Lucas & Sachdev'15].

1. Holography with momentum relaxation and relation to hydrodynamics/memory matrices

2. Quantum critical transport from holography

Anomalous dimensions for charge and energy

- If transport is dominated by quantum criticality (fast momentum relaxation), then what matters most are the scaling properties of our system.

- At fast momentum relaxation, charge and energy density are the operators of interest [Hartnoll'14].

- We can imagine giving them anomalous dimensions

$$[\epsilon] = d + z - \theta\,, \qquad [n] = \frac{1}{2}(\zeta + d - \theta)$$

$\theta = 0$ and $\zeta = d - \theta$ recover the usual dimensions.

- These translate as anomalous scalings for observables of experimental interest [Huijse et al.'11, **B.G.**'13,'14, Karch'14]

$$[s] = d - \theta\,, \qquad [\sigma] = \zeta - 2$$

Holographic quantum critical points with anomalous dimensions

$$[\epsilon] = d + z - \theta \,, \qquad [n] = \frac{1}{2}(\zeta + d - \theta)$$

- Examples of non-trivial θ in CMT include Fermi surfaces [Huijse et al'11] or systems with dimensional crossovers. Recently proposed examples of CMT systems with non-trivial ζ [Karch'15].

- Non-trivial values of θ and ζ abound in holographic setups, like EMD [Charmousis et al'10, **B.G.** & Kiritsis'11, Huijse et al'11, **B.G.** & Kiritsis'12, Gath et al'12, **B.G.**'13,'14] or probe branes [Ammon, Kaminski & Karch'12, Karch'14].

Scaling theory of the cuprate strange metals

- It was proposed by [Hartnoll & Karch'15] that cuprates strange metals have ($d = 2$)

$$z = \frac{4}{3}\,, \qquad \theta = 0 \qquad \zeta = \frac{2}{3}$$

  assuming that transport is dominated by the quantum critical sector, time-reversal invariant and not particle-hole symmetric.

- This way, [Hartnoll & Karch'15] can match the scaling of the resistivity, the Hall Lorenz ratio, the Hall angle, the magnetoresistance and the thermopower.
  Not thermodynamic quantities like the specific heat or the magnetic susceptibility though.
  See also [Khveshchenko'15]

- Effective theory of thermoelectric transport inspired by holography, but the validity of which does not necessarily rely on the existence of a holographic model.

Summary

- Do not trust radially conserved quantities!

$$\sigma_{DC} = 1 + \frac{\mu^2}{m^2} = \boxed{\sigma_Q} + \boxed{\frac{\mu^2}{m^2} + 1 - \sigma_Q} + O(m^2)$$

- Holography can suggest effective models of thermo-electric transport without need for quasiparticles.

- Hydrodynamic/Memory matrix formulation of transport with momentum relaxation.

- Scaling theory of quantum critical transport.

- These effective models can be generalized beyond their holographic origin and be contrasted to more realistic systems.