Hard-gapped holographic insulators and supersolids

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Based on
arXiv:1503.03481, with E. Kiritsis

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Outline

Introduction and the model

Motivation

The prototype of holographic models at finite density is the RN black hole. In its ground state, the conductivity for small ω is

$$\text{Re}(\sigma) \sim \delta(\omega) + \omega^n$$

[Hartnoll, Herzog & Horowitz 0810.1563]
[Horowitz & Roberts 0908.3677]

In the real world,

- There is a hard gap in superconductors and Mott insulators.
- The translational invariance is broken.

Introduction and the model

The action is

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{W(\phi)}{2}A^2 - \frac{Y(\phi)}{2}\sum_{i=1}^{2}(\partial\psi_i)^2\right]$$

where the leading IR behavior of V, Z, W, and Y are

$$V(\phi) = V_0 e^{-\delta\phi}, \qquad Z(\phi) = e^{\gamma\phi}, \qquad W(\phi) = W_0 e^{\eta\phi}, \qquad Y(\phi) = e^{\lambda\phi}.$$

Introduction and the model

- Conductivity with translational invariance

$$S=\int d^4x\sqrt{-g}\left[R-\frac{1}{2}(\partial\phi)^2-V(\phi)-\frac{Z(\phi)}{4}F^2\right]$$

[Charmousis, Goutéraux, Kim, Kiritsis & Meyer 1005.4690]

- Conductivity with momentum dissipation

$$S=\int d^4x\sqrt{-g}\left[R-\frac{1}{2}(\partial\phi)^2-V(\phi)-\frac{Z(\phi)}{4}F^2-\frac{Y(\phi)}{2}\sum_{i=1}^{2}(\partial\psi_i)^2\right]$$

The last term is responsible for the momentum dissipation. The massless scalars ψ_i take the form $\psi_i = kx_i$.

[Andrade & Withers 1311.5157; Donos & Gauntlett 1311.3292]

Introduction and the model

IR fixed points

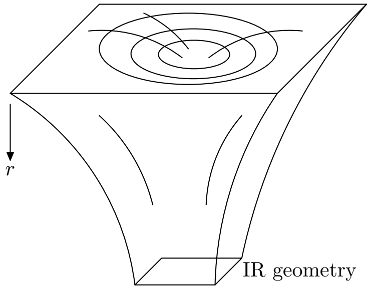
- IR charged
  - $\phi = \phi_*$: $AdS_2 \times \mathbb{R}^2$
  - $\phi \to \infty$: Hyperscaling-violating geometry ($z \neq 1$, θ)
- IR neutral
  - $\phi = \phi_*$: AdS_4
  - $\phi \to \infty$: Hyperscaling-violating geometry ($z = 1$, θ)

[Goutéraux & Kiritsis 1212.2625]

Introduction and the model

Hyperscaling-violating geometry (z, θ)

$$ds^2 = \frac{1}{r^2}\left(-\frac{g(r)}{h(r)}dt^2 + \frac{dr^2}{g(r)} + d\mathbf{x}^2\right)$$



$$ds^2 = \tilde{r}^{\frac{2\theta}{d}}\left(-\frac{dt^2}{\tilde{r}^{2z}} + \frac{d\tilde{r}^2 + d\mathbf{x}^2}{\tilde{r}^2}\right)$$

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + d\mathbf{x}^2\right)$$

Introduction and the model

Gubser criterion

$$Z(\phi) \sim e^{\gamma\phi}, \qquad V(\phi) \sim e^{-\delta\phi} \tag{1}$$

- IR charge solution:

$$z = \frac{\gamma^2 + 2\gamma\delta - 3\delta^2 + 4}{\gamma^2 - \delta^2}, \qquad \theta = \frac{4\delta}{\gamma + \delta}$$

$$\frac{z + 2 - \theta}{2z - 2 - \theta} > 0, \qquad \frac{z - \theta + 1}{2z - 2 - \theta} > 0, \qquad \frac{z - 1}{2z - 2 - \theta} > 0$$

- IR neutral solution:

$$z = 1, \qquad \theta = \frac{2\delta^2}{\delta^2 - 1}$$

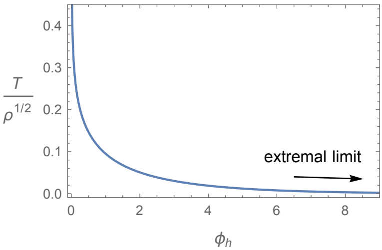
$$\delta^2 < 3$$

Introduction and the model

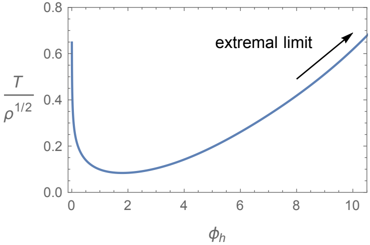
Finite temperature geometry and the extremal limit

$$S \sim T^{\frac{2-\theta}{z}}$$

$$\frac{2-\theta}{z} > 0$$



$$\frac{2-\theta}{z} < 0$$



Conductivity with translational invariance

To obtain the conductivity, we perturb the system by $\delta A_x = a_x(r)e^{-i\omega t}$ and $\delta g_{tx} = g_{tx}(r)e^{-i\omega t}$. Generically, for the metric

$$ds^2 = -D(r)dt^2 + B(r)dr^2 + C(r)(dx^2 + dy^2).$$

the equation for a_x after eliminating g_{tx} is

$$\left(Z\sqrt{\frac{D}{B}}a_x'\right)' + \left(Z\sqrt{\frac{B}{D}}\omega^2 - \frac{Z^2 A_t'^2}{\sqrt{BD}}\right) a_x = 0.$$

After we impose an appropriate boundary condition in the IR (to be discussed below), the asymptotic behavior in the UV is

$$a_x(r) = a_x^{(0)} + a_x^{(1)} r + \cdots\ ,$$

and the conductivity is calculated from

$$G = \frac{a_x^{(1)}}{a_x^{(0)}}, \qquad \sigma(\omega) = \frac{G}{i\omega}.$$

Conductivity with translational invariance

Schrödinger equation for the conductivity

After a change of variables by

$$\frac{d\xi}{dr} = \sqrt{\frac{B}{D}}, \qquad \tilde{a}_x = \sqrt{Z} a_x,$$

we can obtain a Schrödinger equation

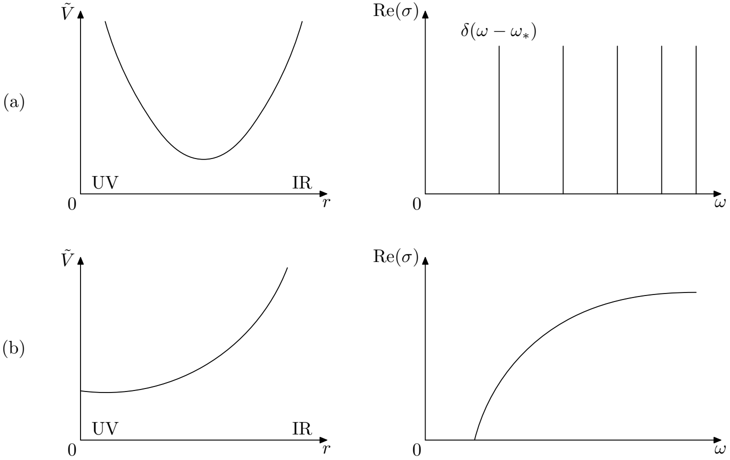
$$-\frac{d^2 \tilde{a}_x}{d\xi^2} + \tilde{V}(\xi)\tilde{a}_x = \omega^2 \tilde{a}_x.$$

The potential is given by

$$\tilde{V} = \frac{Z A_t'^2}{B} - \frac{D B' Z'}{4B^2 Z} + \frac{D' Z'}{4BZ} - \frac{D Z'^2}{4BZ^2} + \frac{D Z''}{2BZ}.$$

Conductivity with translational invariance

Schrödinger potential for the conductivity



Conductivity with translational invariance

Gapless case

$$\xi = \frac{\sqrt{h_0}}{g_0}\frac{2-\theta}{2z} r^{\frac{2z}{2-\theta}} \qquad S \sim T^{\frac{2-\theta}{z}}$$

$$\tilde{V}(\xi) = \frac{\nu^2 - 1/4}{\xi^2}, \qquad \nu = \frac{3z-\theta}{2z}$$

The IR limit is at $\xi \to \infty$, which happens when $\frac{z}{2-\theta} > 0$. In this case, the extremal limit of the small black hole branch is at $T \to 0$. Moreover, the Gubser criterion implies that we have $\nu > \frac{1}{2}$.

$$\tilde{a}_x \sim \sqrt{\xi} H_\nu^{(1)}(\omega\xi) \sim e^{i\omega\xi}.$$

The current-current correlator is gapless in this case.

$$\mathrm{Re}(\omega) \sim \delta(\omega) + \omega^{2\nu-1}$$

Conductivity with translational invariance

Gapped case

$$\xi = \frac{\sqrt{h_0}}{g_0}\frac{2-\theta}{2z}r^{\frac{2z}{2-\theta}} \qquad\qquad S \sim T^{\frac{2-\theta}{z}}$$

$$\tilde{V}(\xi) = \frac{\nu^2 - 1/4}{\xi^2}, \qquad \nu = \frac{3z-\theta}{2z}$$

The IR limit is at $\xi \to 0$, which happens when $\frac{z}{2-\theta} < 0$. In this case, the extremal limit of the small black hole branch (that is now thermodynamically unstable) is at $T \to \infty$.

$$\tilde{a}_x \;=\; C_1\sqrt{\xi}J_{-\nu}(\omega\xi) + C_2\sqrt{\xi}J_{\nu}(\omega\xi) \;\sim\; C_1\,\xi^{1/2-\nu} + C_2\,\xi^{1/2+\nu}.$$

The second linearly independent solution is normalizable when $|\nu| < 1$, and is non-normalizable when $|\nu| > 1$. If there are two normalizable solutions, the boundary condition in the IR depends on how the singularity is resolved, and thus the calculation of the correlator is unreliable and can only be fixed when the singularity is properly resolved.

Conductivity with translational invariance

Gapped case

$$\xi = \frac{\sqrt{h_0}}{g_0}\frac{2-\theta}{2z} r^{\frac{2z}{2-\theta}} \qquad\qquad S \sim T^{\frac{2-\theta}{z}}$$

$$\tilde{V}(\xi) = \frac{\nu^2 - 1/4}{\xi^2}, \qquad \nu = \frac{3z-\theta}{2z}$$

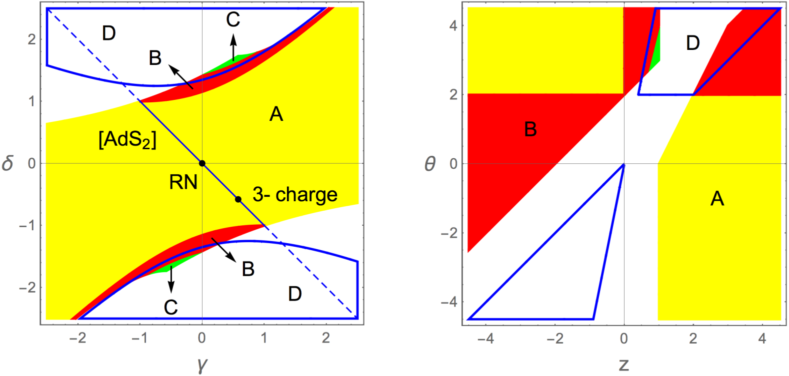
The IR limit is at a constant non-zero ξ, which happens when $\frac{z}{2-\theta} \to 0$. In this case, the extremal limit is at a constant T, and the Schrödinger potential is a constant V_0. The general solution for a_x is

$$a_x = C_1 e^{\sqrt{V_0-\omega^2}\,\xi} + C_2 e^{-\sqrt{V_0-\omega^2}\,\xi}.$$

When $\omega^2 > V_0$, the first solution describes the in-falling wave with the $\omega \to \omega + i\epsilon$ prescription. If we analytically continue the $\omega^2 > V_0$ solution to $\omega^2 < V_0$, the solution for a_x is unambiguous for all ω.

Conductivity with translational invariance

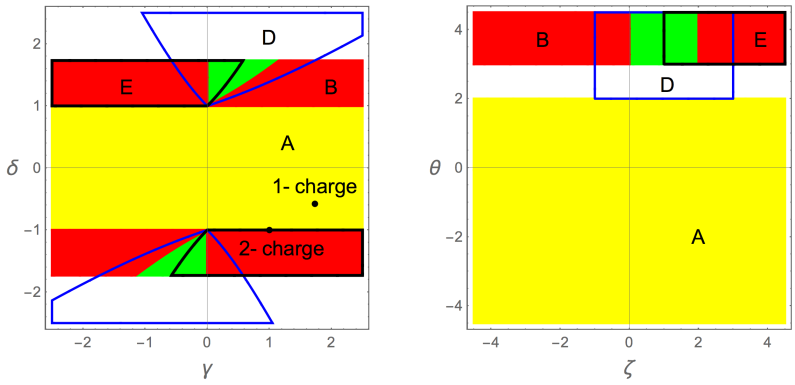
IR charged solution



In region A (yellow), the extremal limit is at $T \to 0$; $\sigma(\omega)$ gapless.
In region B (red), the extremal limit is at $T \to \infty$; $\sigma(\omega)$ gapped.
In region C (green), the extremal limit is at $T \to \infty$; $\sigma(\omega)$ gapless.

Conductivity with translational invariance

IR neutral solution



In region A (yellow), the extremal limit is at $T \to 0$; $\sigma(\omega)$ gapless.
In region B (red), the extremal limit is at $T \to \infty$; $\sigma(\omega)$ gapped.
In region C (green), the extremal limit is at $T \to \infty$; $\sigma(\omega)$ gapless.

Conductivity with momentum dissipation

$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{Y(\phi)}{2}\sum_{i=1}^{2}(\partial\psi_i)^2 \right]$$

To calculate the conductivity, we perturb the system by

$$\delta A_x = a_x(r)e^{-i\omega t}, \qquad \delta g_{tx} = g_{tx}(r)e^{-i\omega t}, \qquad \delta\psi_1 = \chi(r)e^{-i\omega t}$$

After eliminating g_{tx} and defining

$$b_x \equiv \frac{ik}{\omega}\sqrt{\frac{D}{B}}CY\chi',$$

we obtain the two coupled equations for a_x and b_x.

[Andrade & Withers 1311.5157; Goutéraux 1401.5436]

Conductivity with momentum dissipation

After inserting the hyperscaling-violating geometry to the equations, we obtain

$$-\frac{d^2\tilde{a}_x}{d\xi^2}+\frac{c_1}{\xi^2}\,\tilde{a}_x=\omega^2\tilde{a}_x+\frac{d_1}{\xi^\alpha}\tilde{b}_x,$$
$$-\frac{d^2\tilde{b}_x}{d\xi^2}+\left(\frac{c_2}{\xi^2}+\frac{c_3}{\xi^{2\alpha-2}}\right)\tilde{b}_x=\omega^2\tilde{b}_x+\frac{d_2}{\xi^\alpha}\tilde{a}_x,$$

where

$$\alpha=2-\frac{\kappa\lambda-2}{2z},$$

and c_1, c_2, c_3, d_1, and d_2 are constants. The coefficients c_1 and c_2 are given by $c_1=\nu_1^2-1/4$ and $c_2=\nu_2^2-1/4$, where

$$\nu_1=\frac{3z-\theta}{2z},\qquad \nu_2=\frac{2-z-\theta-\kappa\lambda}{2z}.$$

The terms involving the power α are subleading corrections. The gap in the conducttivity remains.

Conductivity with momentum dissipation

DC conductivity at finite temperature

There is a radially conserved quantity Π at $\omega = 0$. By taking advantage of this quantity, a formula for the DC conductivity is obtained

$$\sigma_{\mathrm{DC}} = \frac{\Pi}{i\omega\lambda_1}\bigg|_{r=r_h} = Z_h + \frac{q^2}{k^2 C_h Y_h} = e^{\gamma\phi_h} + \frac{q^2}{k^2 C_h e^{\lambda\phi_h}}.$$

[Blake & Tong 1308.4970; Goutéraux 1401.5436; Donos & Gauntlett 1401.5077]

Conductivity with momentum dissipation

DC conductivity for near-extremal geometries

The near-extremal black hole has an analytic solution in the IR [Charmousis, Goutéraux, Kim, Kiritsis & Meyer 1005.4690].

The DC conductivity for the near-extremal black hole is

$$\sigma_{\mathrm{DC}} \sim \tilde{r}_h^{\alpha_1} + \frac{q^2}{k^2} \tilde{r}_h^{\alpha_2}.$$

In terms of z and θ,

$$\alpha_1 = \frac{\theta - 4}{2 + z - \theta}, \qquad \alpha_2 = \frac{\theta - 2 + \kappa\lambda}{2 + z - \theta}.$$

Conductivity with momentum dissipation

DC conductivity for extremal geometries

$$\sigma_{\rm DC} = \frac{\Pi}{i\omega\lambda_1^{(0)}}$$

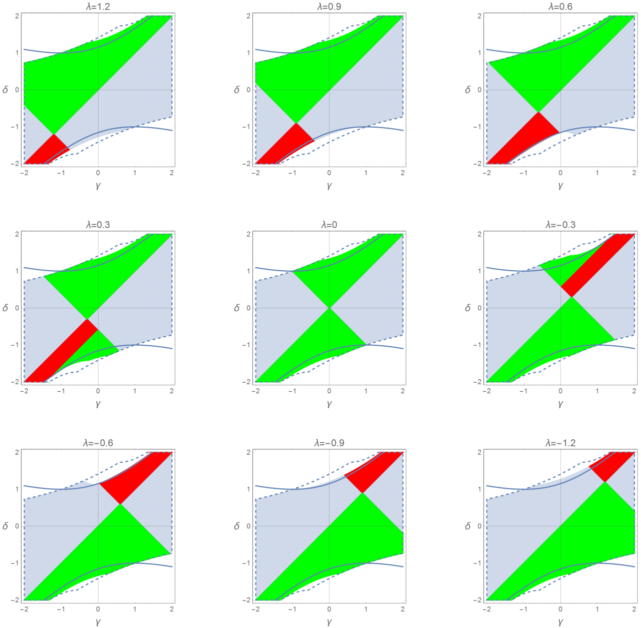
- Case 1: $\nu_2 > 1$. The solution for b_x is $b_x \sim \sqrt{Z_2}\xi^{1/2+\nu_2}$ The radially conserved quantity Π evaluated at the IR is

$$\Pi \sim r^{\frac{(\gamma-\delta)(\gamma-5\delta-4\lambda)-4}{(\gamma-\delta)^2}} \to 0.$$

- Case 2: $\nu_2 < -1$. The solution for b_x is $b_x \sim \sqrt{Z_2}\xi^{1/2-\nu_2}$. The radially conserved quantity evaluated in the IR is

$$\Pi \to \text{constant}$$

Conductivity with momentum dissipation



Holographic supersolids

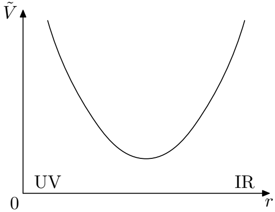
The action is

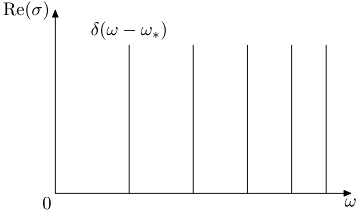
$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{W(\phi)}{2}A^2 - \frac{Y(\phi)}{2}\sum_{i=1}^{2}(\partial\psi_i)^2\right]$$

where the leading IR behavior of V, Z, W, and Y are

$$V(\phi) = V_0 e^{-\delta\phi}, \qquad Z(\phi) = e^{\gamma\phi}, \qquad W(\phi) = W_0 e^{\eta\phi}, \qquad Y(\phi) = e^{\lambda\phi}.$$

(a)





Summary

- From an Einstein-Maxwell-Dilaton system, the AC conductivity can have a hard gap and a discrete spectrum, with a $\delta(\omega)$ due to the translational invariance.
- In the presence of IR irrelevant momentum dissipation, the correlator still has a discrete spectrum, with the only difference that the zero frequency δ-function has now disappeared.
- The gapped geometries with momentum dissipation can describe a metal or an insulator, depending on the parameters.
- We also find holographic supersolids, which are hard-gapped superconductors with translational symmetry breaking.

Thank you!

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European Union
European Social Fund

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