

Holographic Charge Oscillations

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[arXiv:1412.2003](https://arxiv.org/abs/1412.2003) with Aristomenis Donos and
David Tong

Basic Questions

- Can we see finite density fermionic physics in holography?

Yes, by adding fermions...

Liu, McGreevy & Vegh,
Sachdev etc...

Yes, by working in $1+1$ dimensions...

Iqbal & Faulkner

- How about simply in the Reissner-Nordstrom black hole?

A Topical Example

- Power-law resistivities from Umklapp scattering require zero energy excitations at the lattice momentum
- Fermi surfaces provide a simple way to do this by scattering across a Fermi surface

$$\sigma^{-1} \sim T^2$$

- Reissner-Nordstrom produces similar physics through local criticality

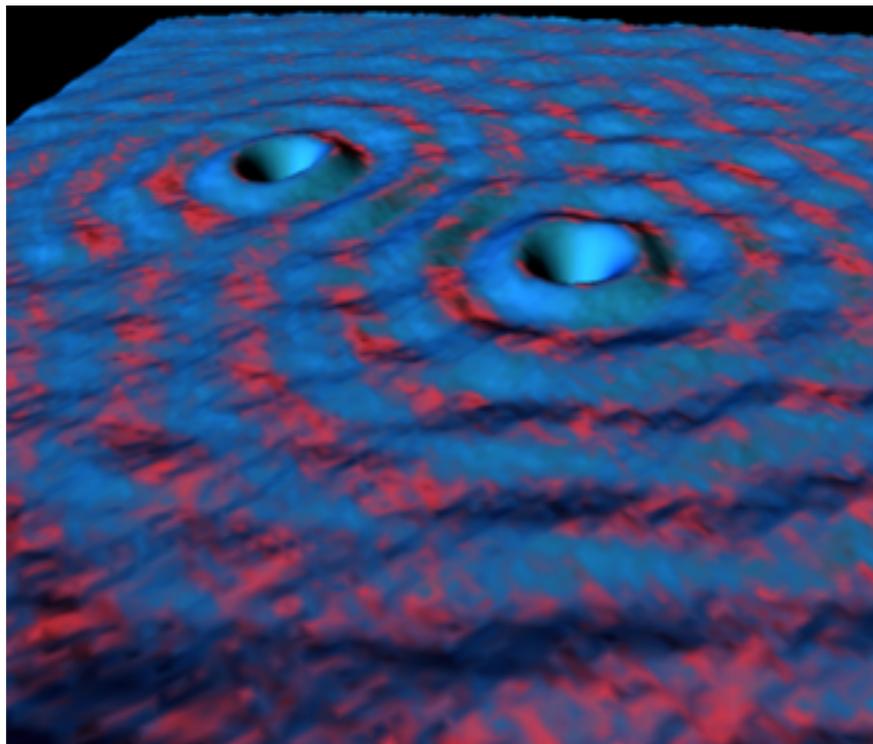
$$\omega \sim k^{1/z}$$

$$z \rightarrow \infty$$

$$\sigma^{-1} \sim T^{2\Delta_O(k_L)}$$

Charge Oscillations

- Adding a charged impurity to a metal gives rise to Friedel oscillations



$$\delta\rho \sim \frac{1}{r^3} \cos(2k_F r)$$

$(T = 0)$

M.F. Crommie, C.P. Lutz, D.M. Eigler, *Nature* 363, 524-527 (1993), IBM.

- Heuristically occurs because of finite size of low energy excitations

- Suppose we do the same in holography. We model an impurity via a chemical potential

$$\delta\mu = C e^{-r^2/2R^2}$$

- Then within linear response the induced charge density is

$$\delta\rho(k) = \chi(k)\mu(k)$$

- Where $\chi(k)$ is the static susceptibility

$$\chi(k) = \langle J_t(k) J_t(-k) \rangle |_{\omega=0}$$

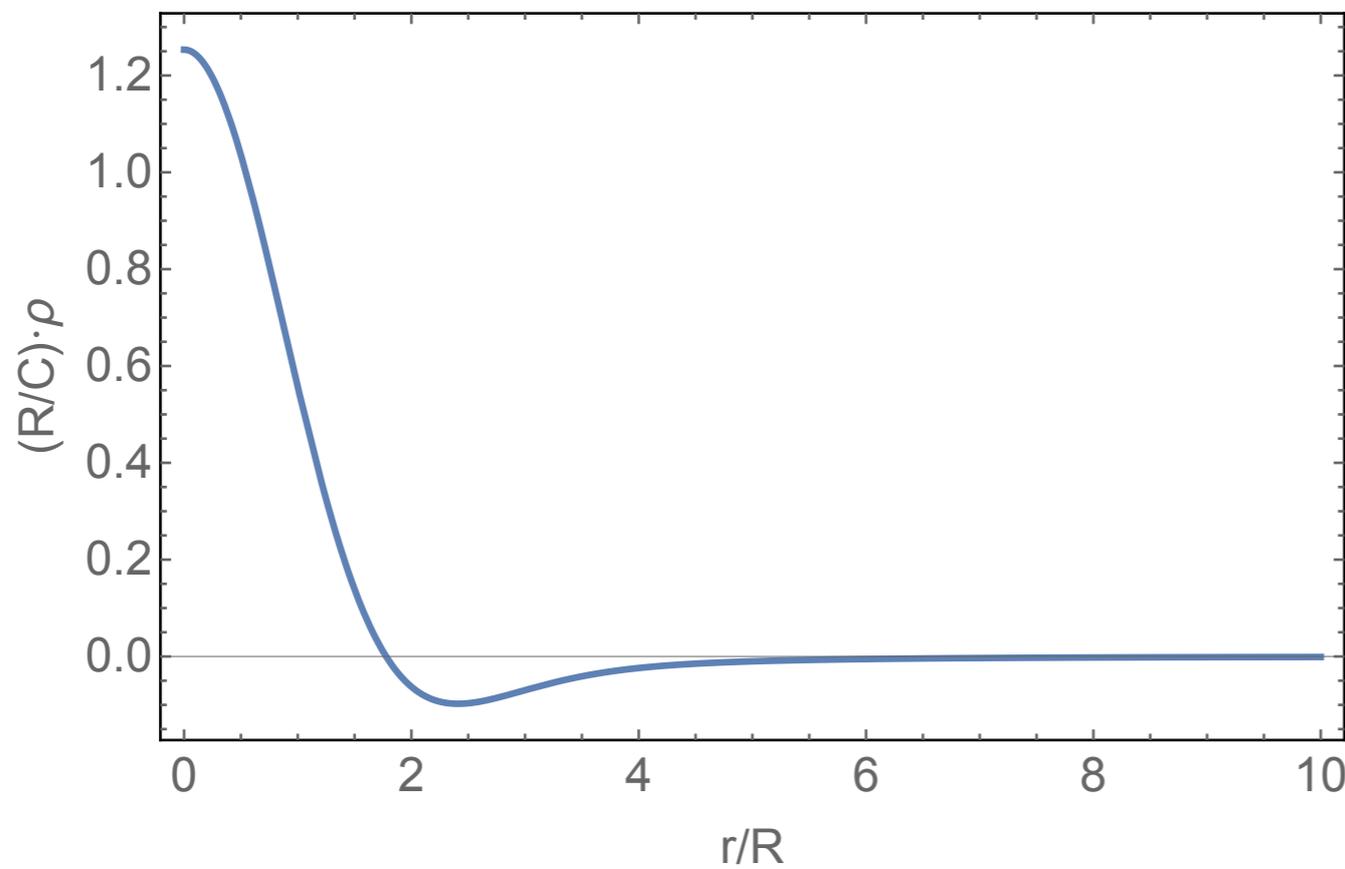
(See also Horowitz, Iqbal, Santos and Way)

Screening in AdS4

- In AdS4 conformal invariance fixes the scaling of the static susceptibility

$$\chi(k) \sim k$$

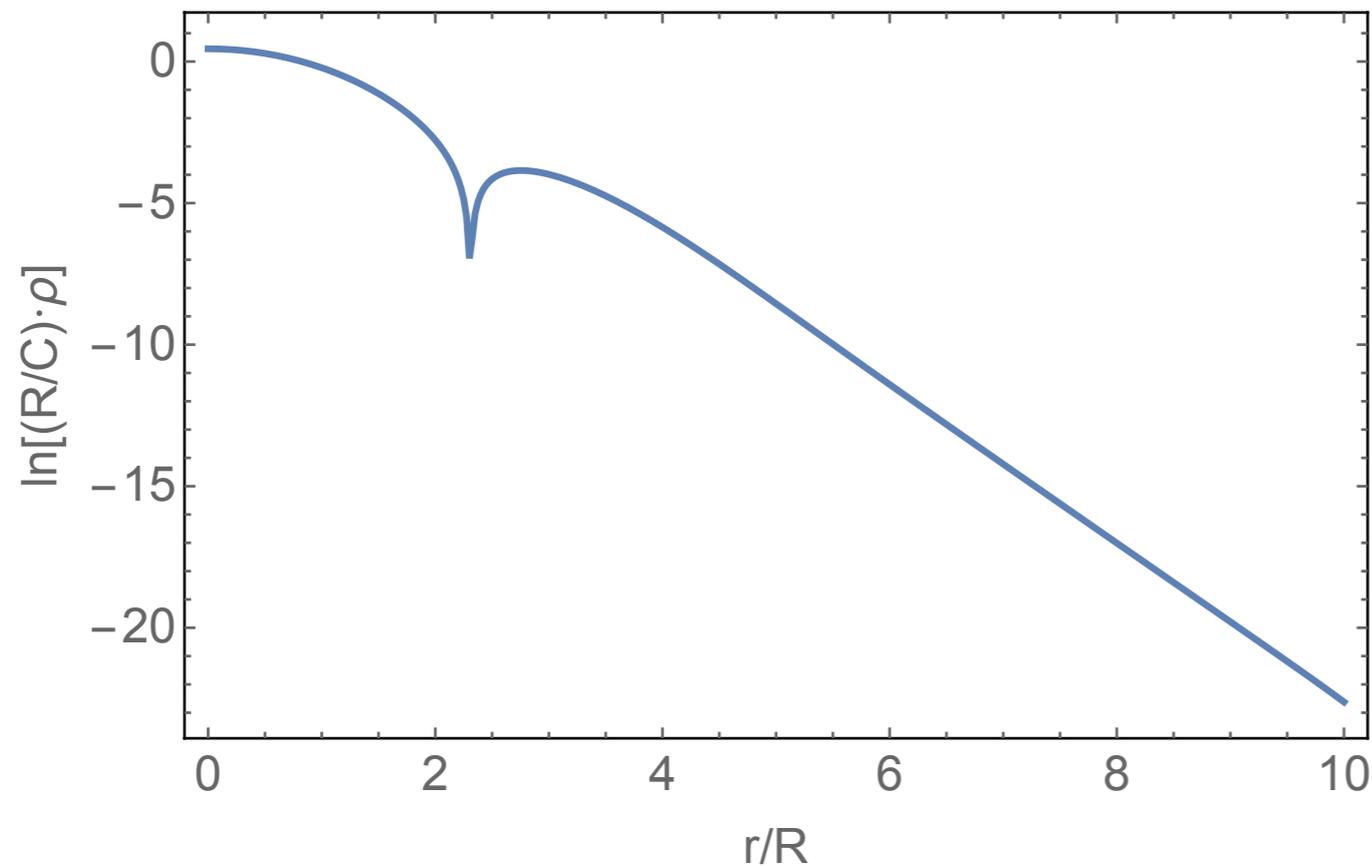
- For our Gaussian lump this gives the charge density



$$r \gg R$$
$$\delta\rho \sim CR^2/r^3$$

Finite Temperature

$$T \neq 0$$



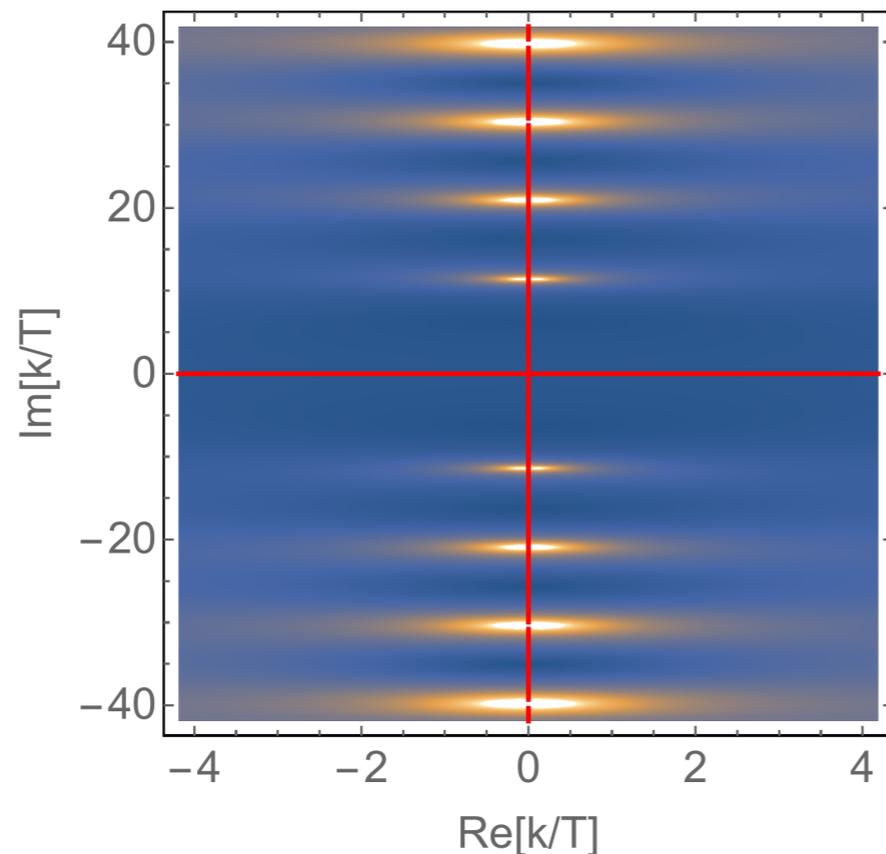
$$r \gg R, T^{-1}$$

$$\delta\rho \sim CR^2 \frac{e^{-r/\lambda}}{\sqrt{r\lambda^5}}$$

Here $\lambda \sim 1/T$ is the Debye screening length

Static Susceptibility

- The exponential fall-off can be understood by looking at $\chi(k)$ for complex k



$$\text{Im}k_* = 1/\lambda \sim T$$

- String of poles on the imaginary k axis.
Lowest pole dominates the large distance fall-off.

(c.f. quasinormal modes)

Screening in RN

- Turn on a background chemical potential

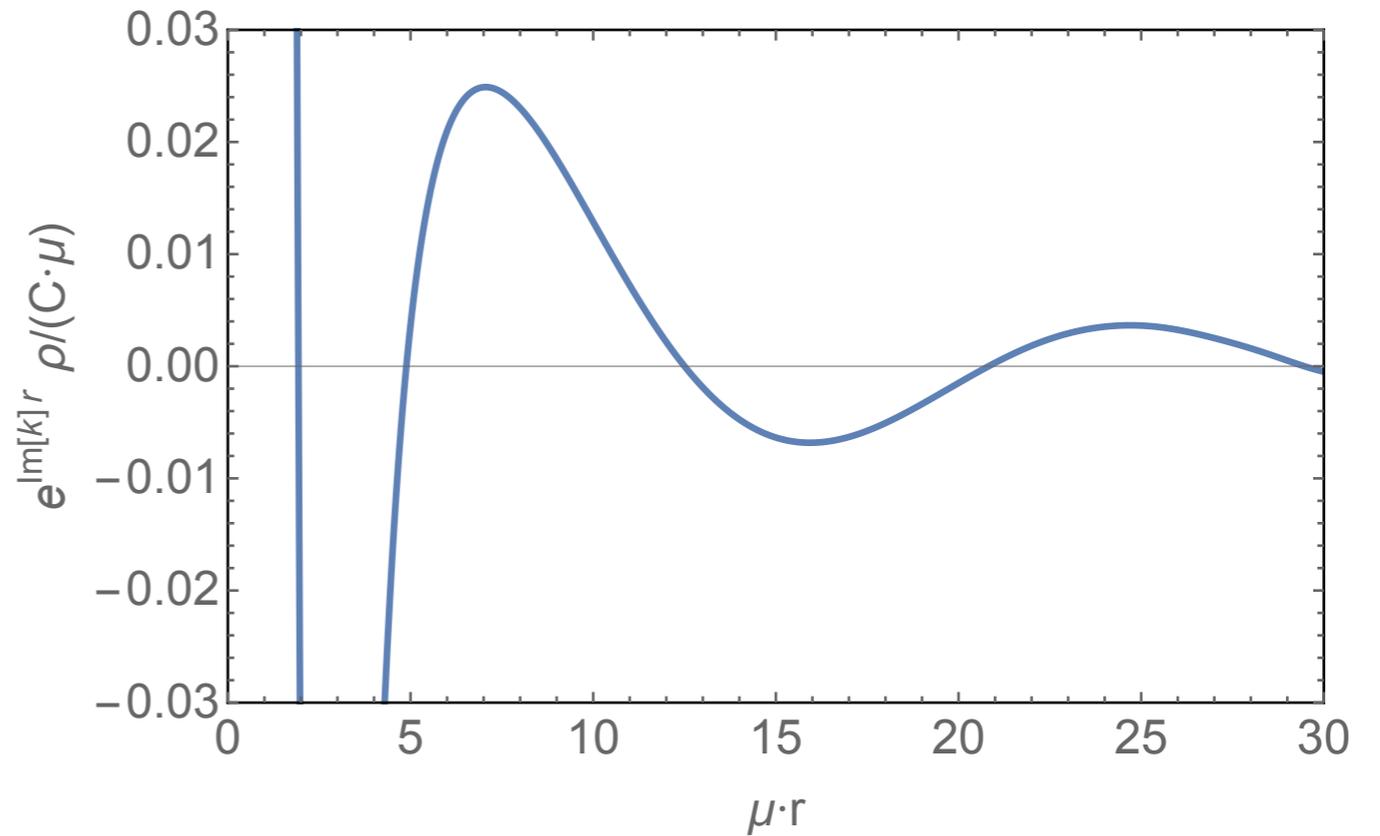
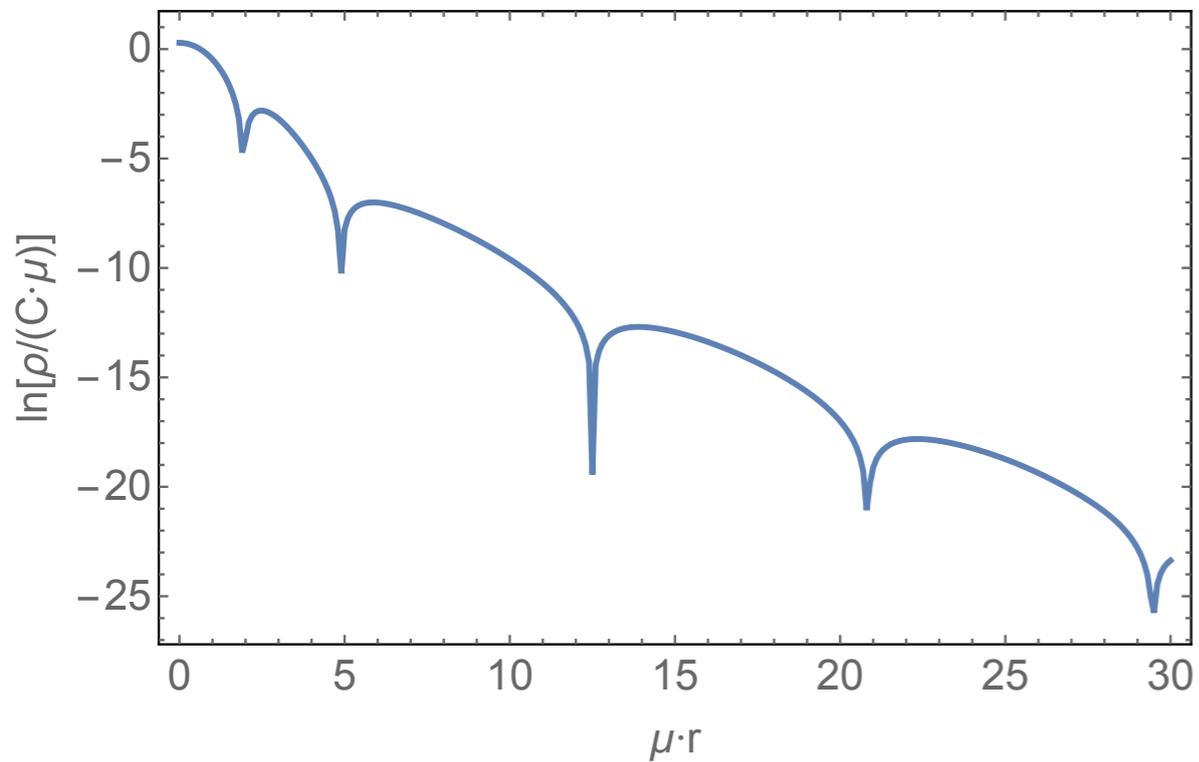
$$\mu = \mu_0 + C e^{-r^2/2R^2}$$

- At high $T/\mu_0 \gg 1$ the induced charge density falls off exponentially as in Schwarzschild.
- Remarkably there is a phase transition at

$$T_c \approx 0.33\mu_0$$

- For $T < T_c$ we begin to see oscillations in the charge density

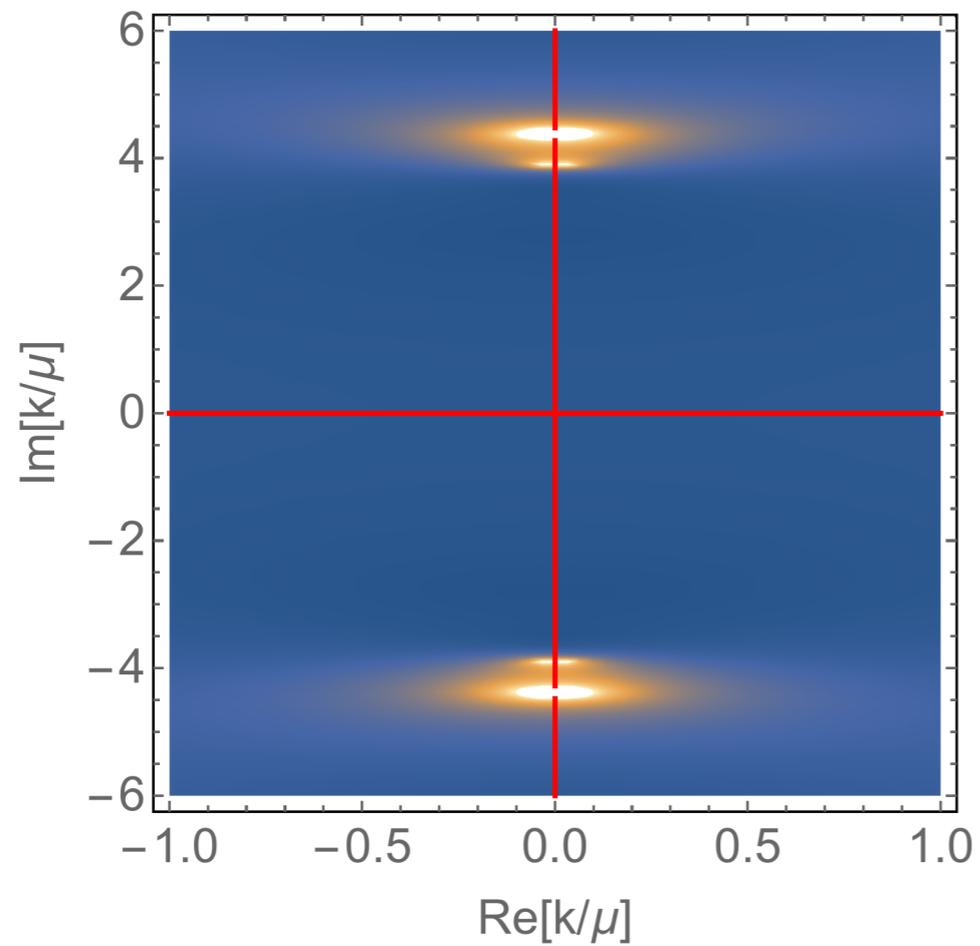
Holographic Charge Oscillations



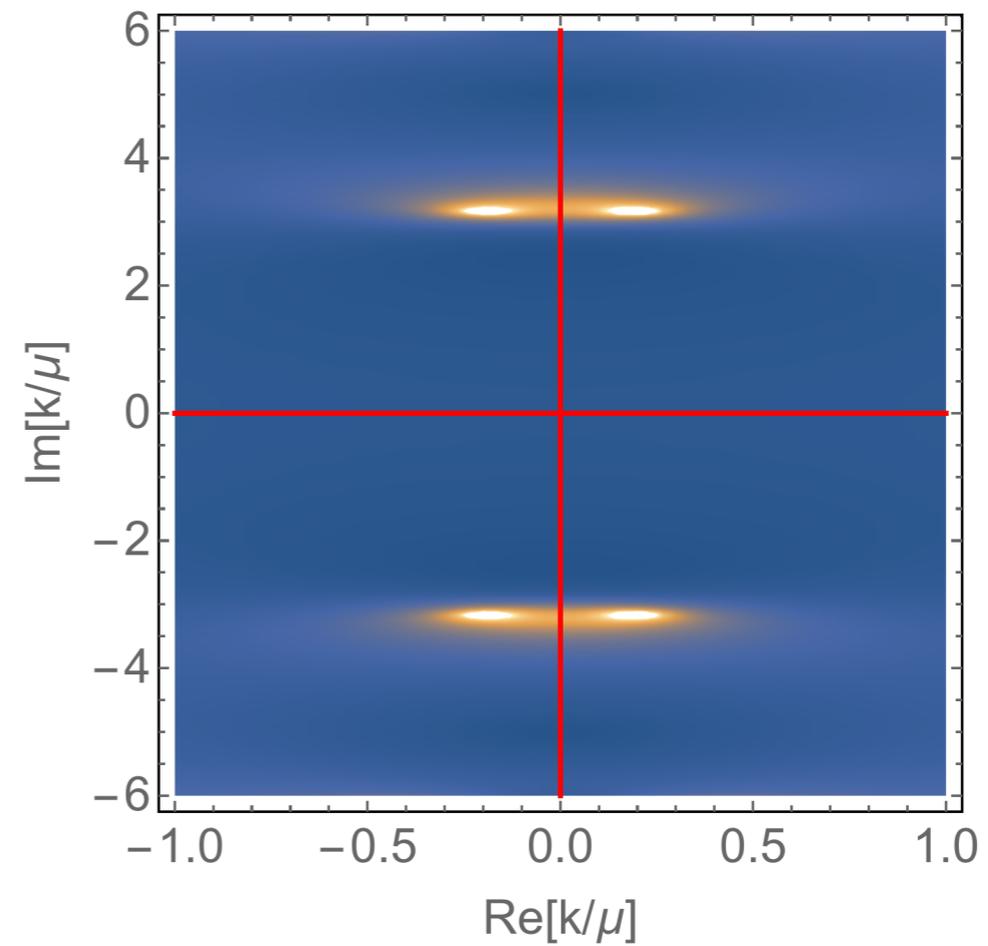
$$\delta\rho \sim \frac{e^{-r/\lambda}}{\sqrt{r}} \cos(r/\xi)$$

$$r \gg R, T^{-1}, \mu^{-1}$$

RN Static Susceptibility

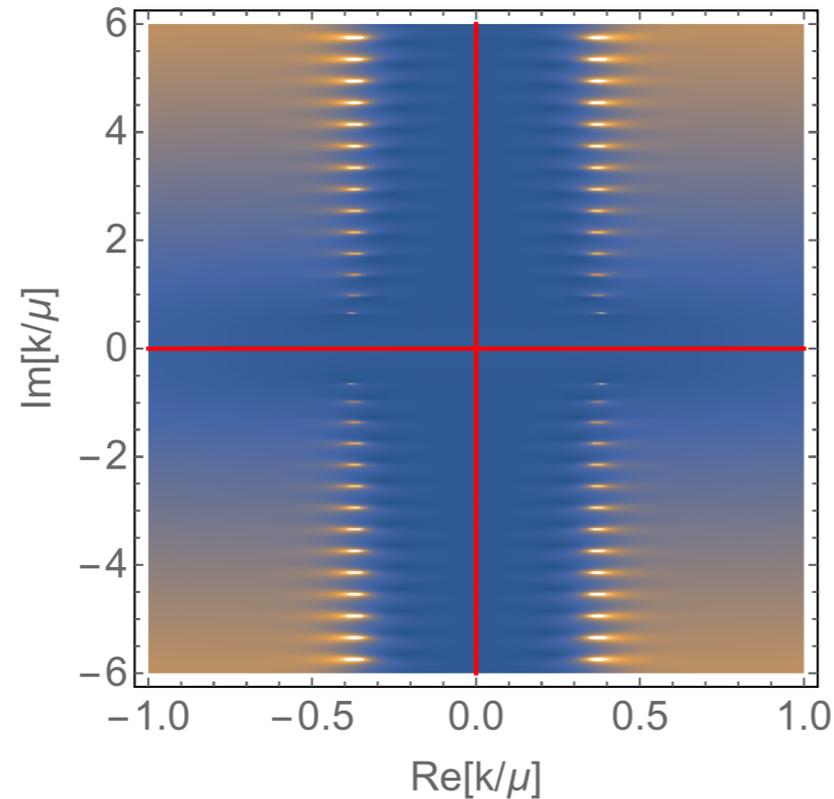


$$T > T_c$$



$$T < T_c$$

- At $T=0$, the poles coalesce into a branch cut.



- The branch cut terminates at a complex momentum

$$k_*/\mu_0 = 1/2\sqrt{2} + i/2$$

- In contrast to Friedel oscillations, ours remain exponentially damped.

Local Criticality

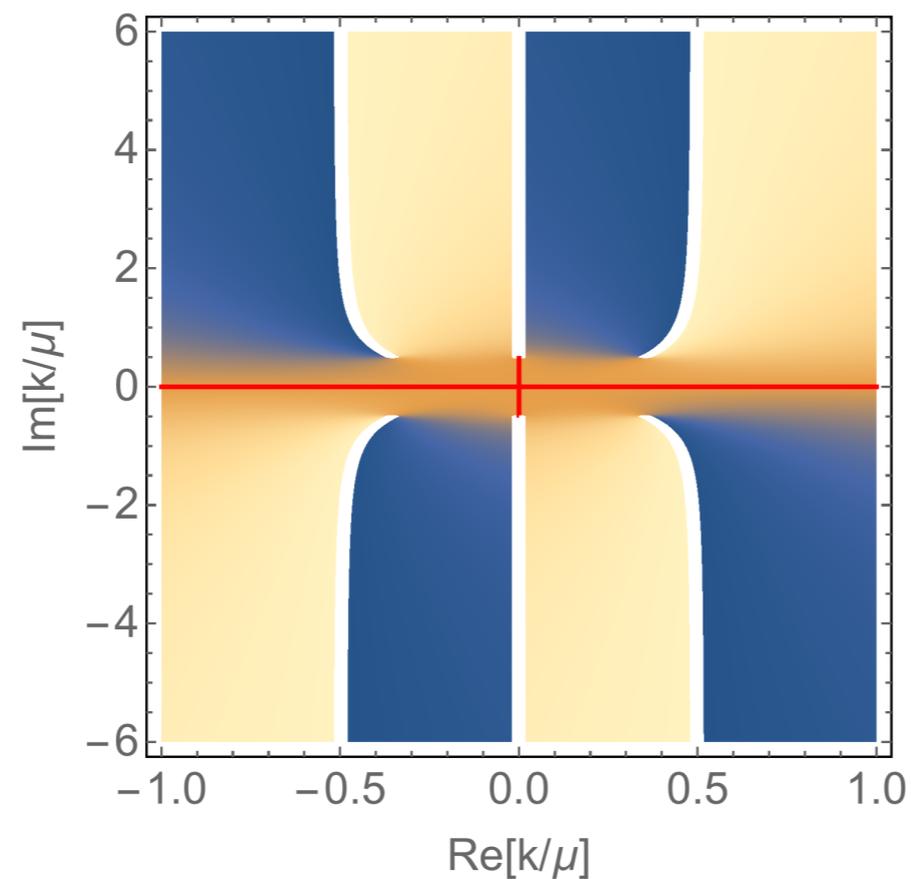
- Remarkably, this branch cut seems to directly arise from the local criticality.
- In the IR $J_t(k)$ couples to operators of dimension

$$\delta_{\pm}(k) = \frac{1}{2} + \nu_{\pm}(k)$$

$$\nu_{\pm} = \frac{1}{2} \sqrt{5 + 8(k/\mu_0)^2 \pm 4\sqrt{1 + 4(k/\mu_0)^2}}$$

- Because of the square roots ν_{\pm} have branch cuts in the complex k plane

- The branch cut in $\chi(k)$ is numerically found to arise from a branch cut in $\nu_-(k)$



- The branch cut terminates at the point

$$\nu_-(k_*) = 0$$

Comments

- We studied the effect of a charged impurity on the RN geometry.
- We found that below a critical temperature, the induced charged density exhibited oscillations.
- This is another example of the connection between Fermi-surface physics and local criticality.

Open Questions

- Can we get a power law fall-off to these oscillations?

c.f. charge density wave
instabilities

- Is holography suggesting the existence of a complex Fermi momentum at strong coupling?

Can we make sense of this in
field theory? In $\mathcal{N} = 4$ Super-
Yang-Mills?

Thank you!