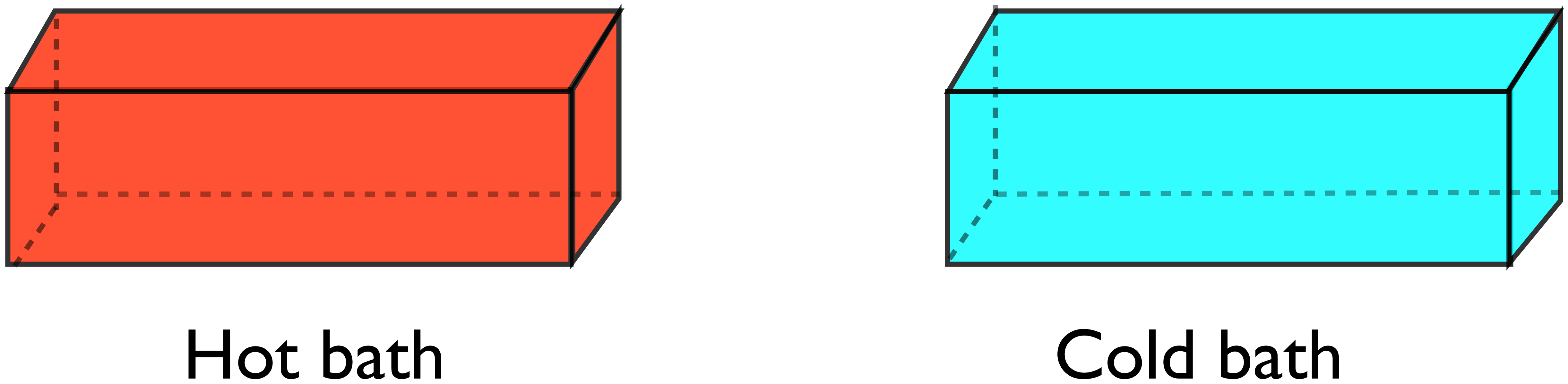
Black brane steady states

Based on work with I. Amado, H-C. Chang and A. Karch.

The problem I want to consider is as follows: at t=0 we prepare an initial state connected to two heat baths:



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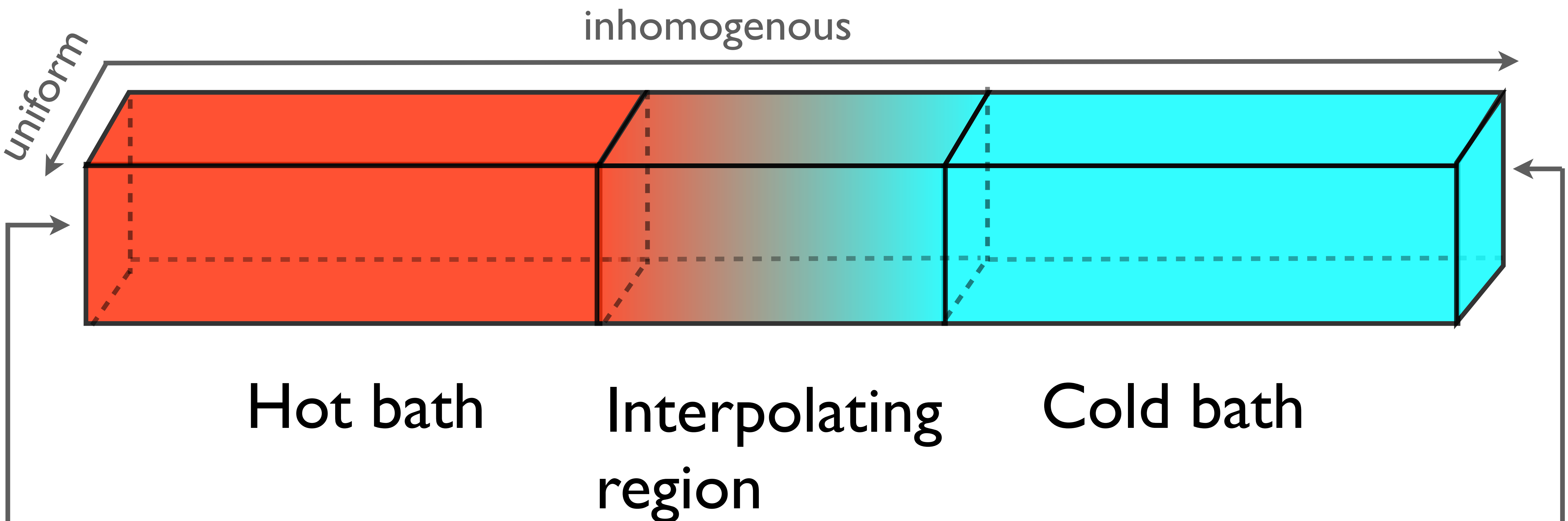

The boundaries are very far away and are held in thermodynamic equilibrium:

$$T^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

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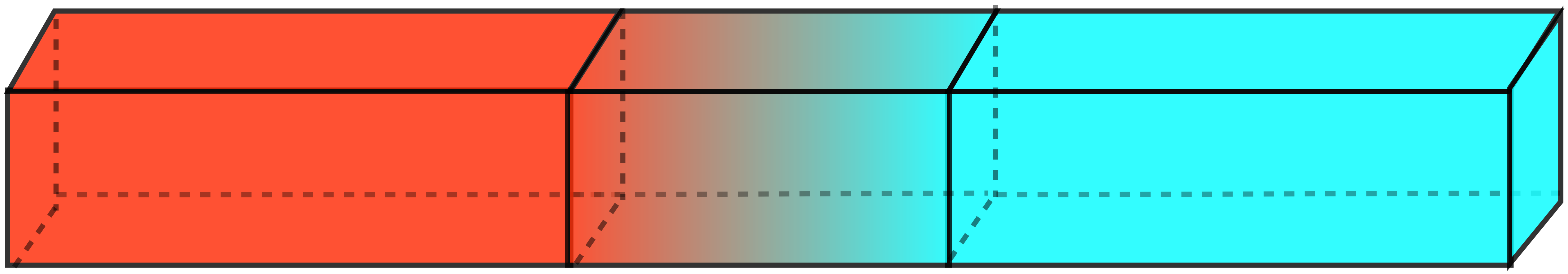
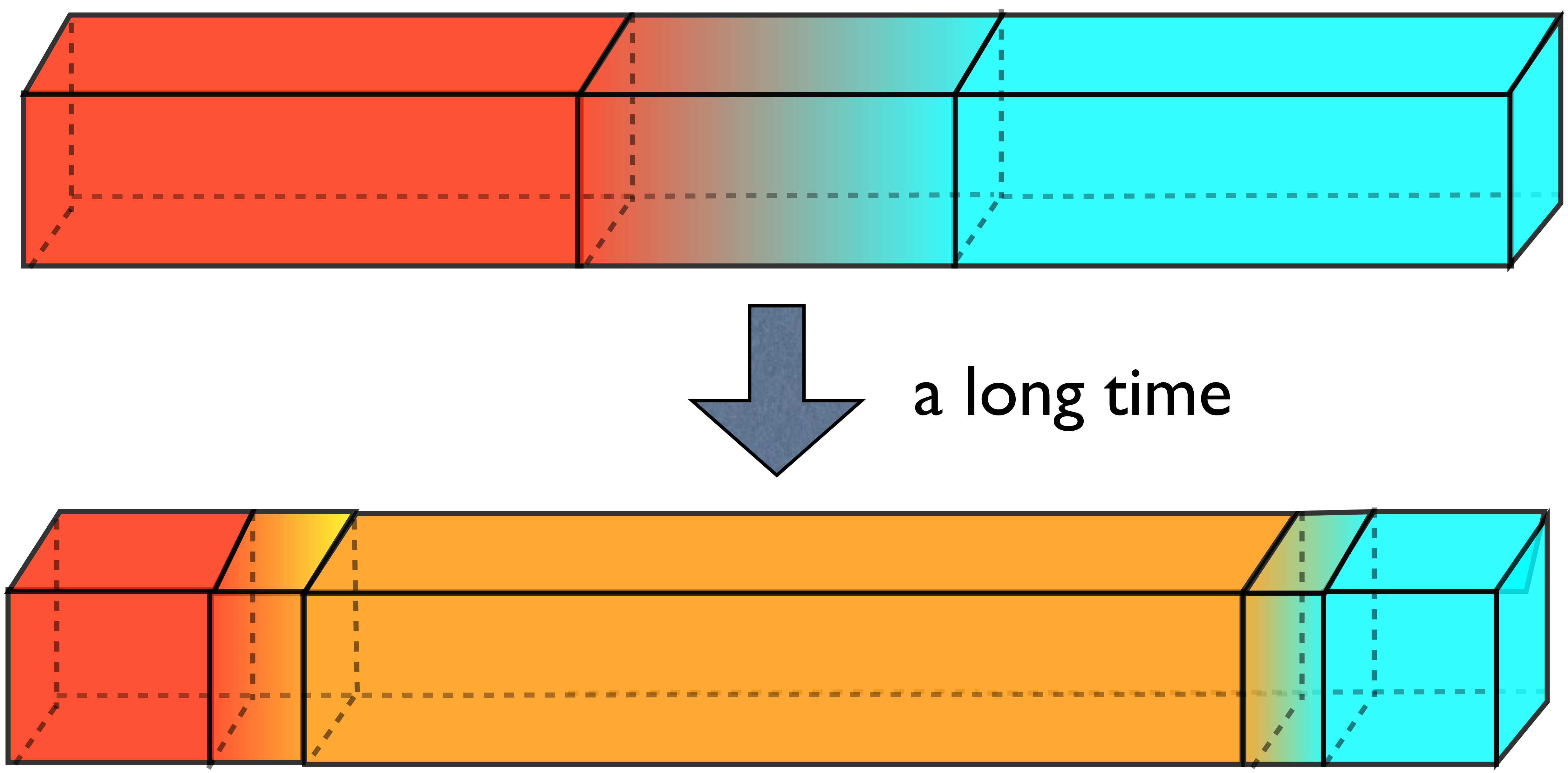


What can we say about the final state at late times?

Conjecture: If the conformal field theory thermalizes sufficiently fast* then the late time steady state is universal.



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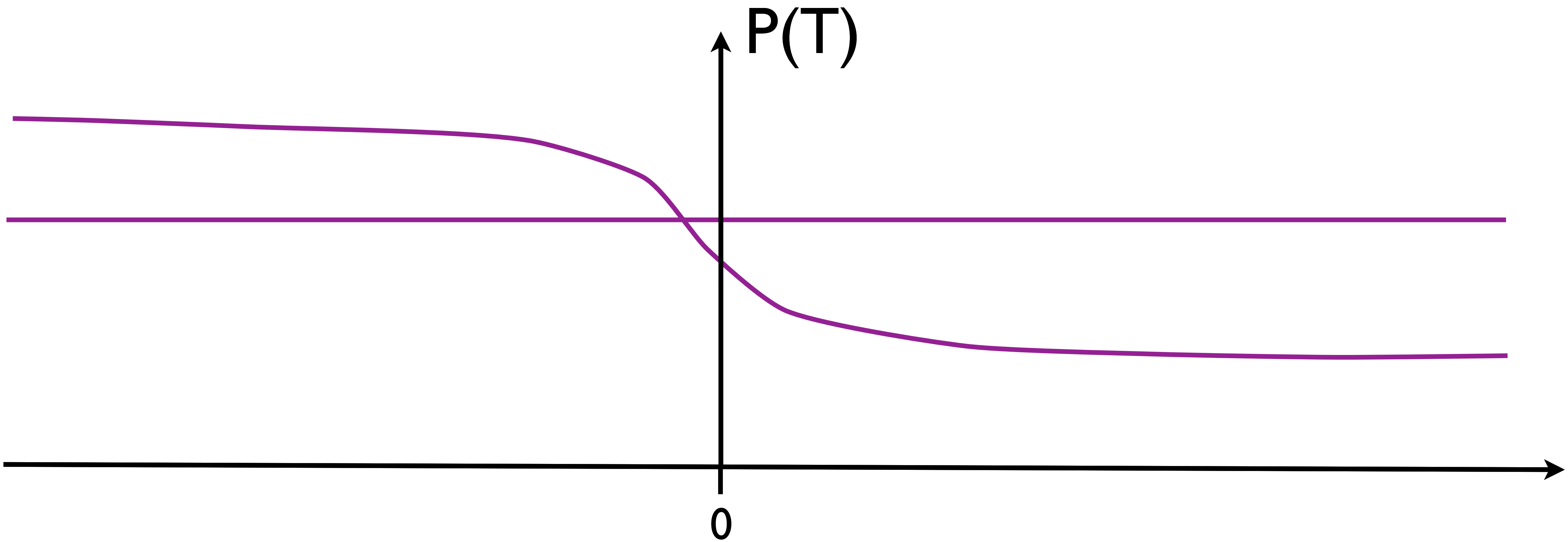

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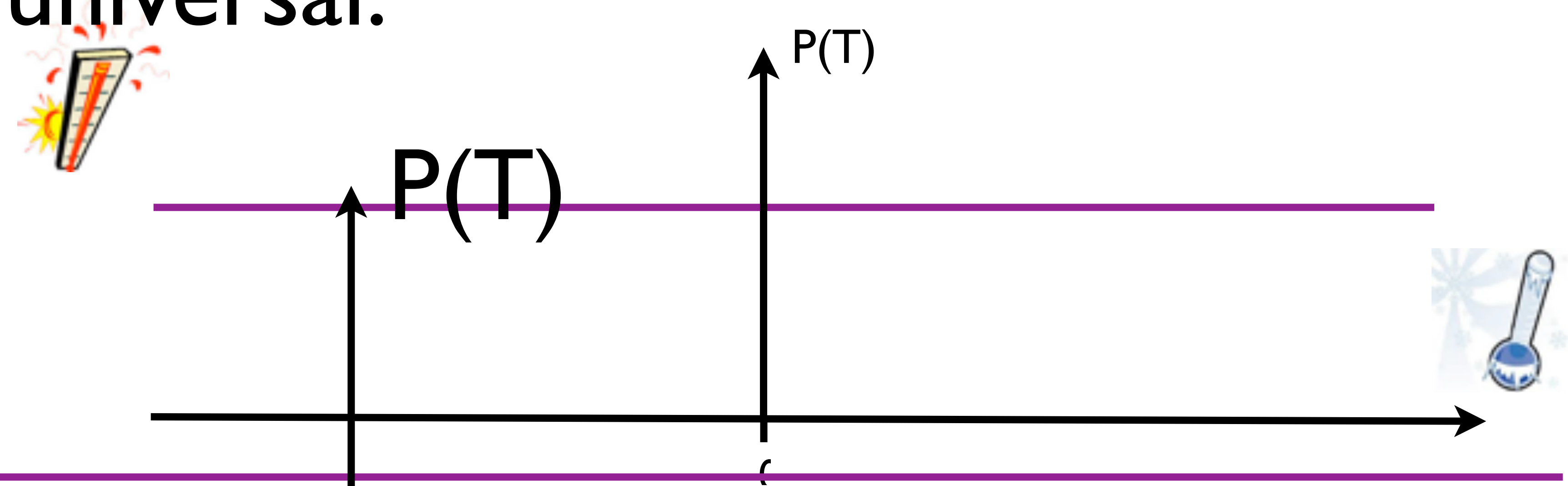

The pressure at late times will take one of 2 values:

(1) $$\frac{P}{P_0} = \frac{1}{d}\left(2(d-1) - (d-2)\sqrt{1-\delta p^2}\right)$$

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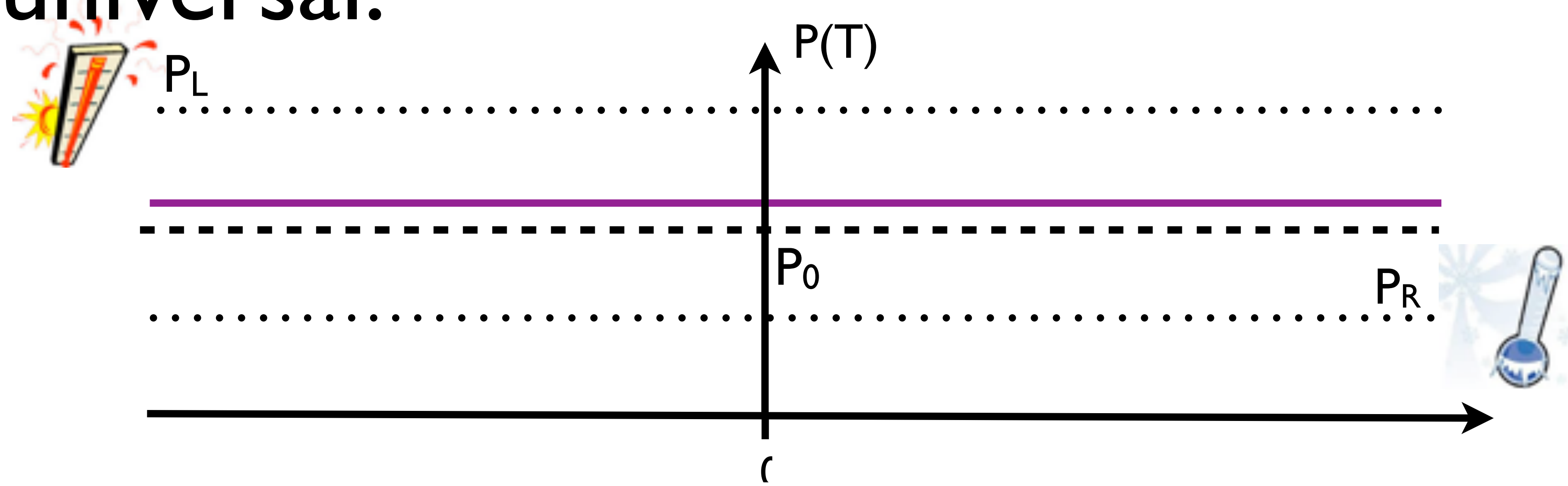

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$$(1) \quad \frac{P}{P_0} = \frac{1}{d}\left(2(d-1) - (d-2)\sqrt{1-\delta p^2}\right)$$

$$P_0 = \frac{P_L + P_R}{2} \qquad 0 < \delta p = \frac{P_L - P_R}{P_L + P_R} < 1 \qquad d = \text{dimension of spacetime}$$

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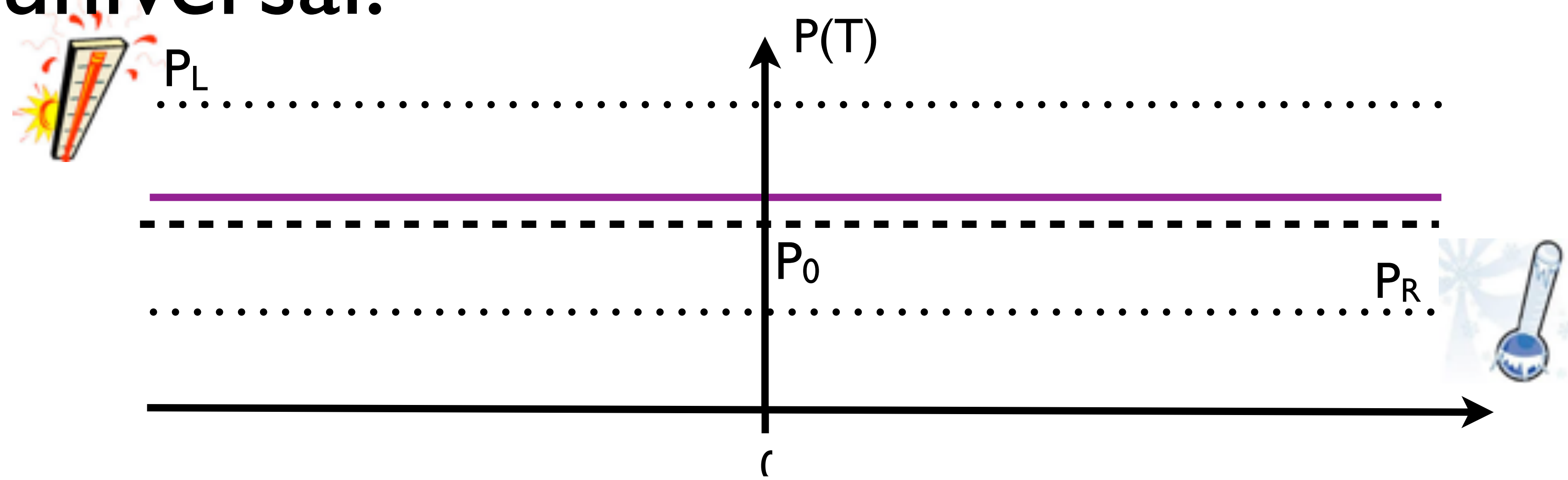

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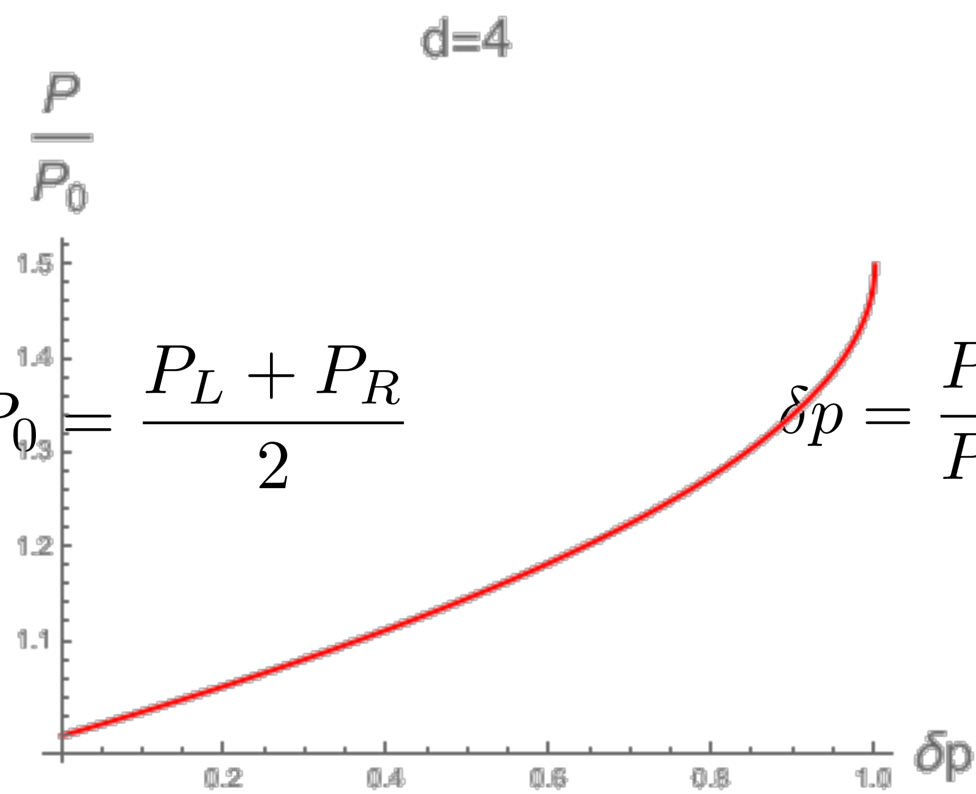

$$P_0 = \frac{P_L + P_R}{2} \qquad \delta p = \frac{P_L - P_R}{P_L + P_R}$$

$d =$ dimension of spacetime

See also Bhaseen et. al., 2013

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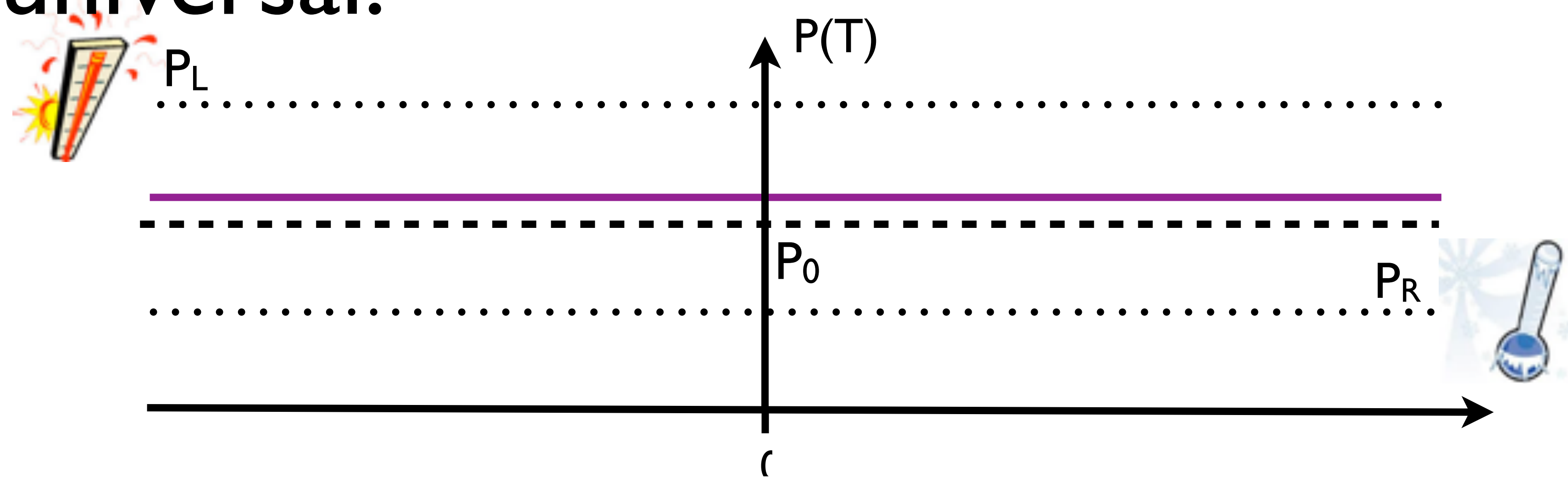

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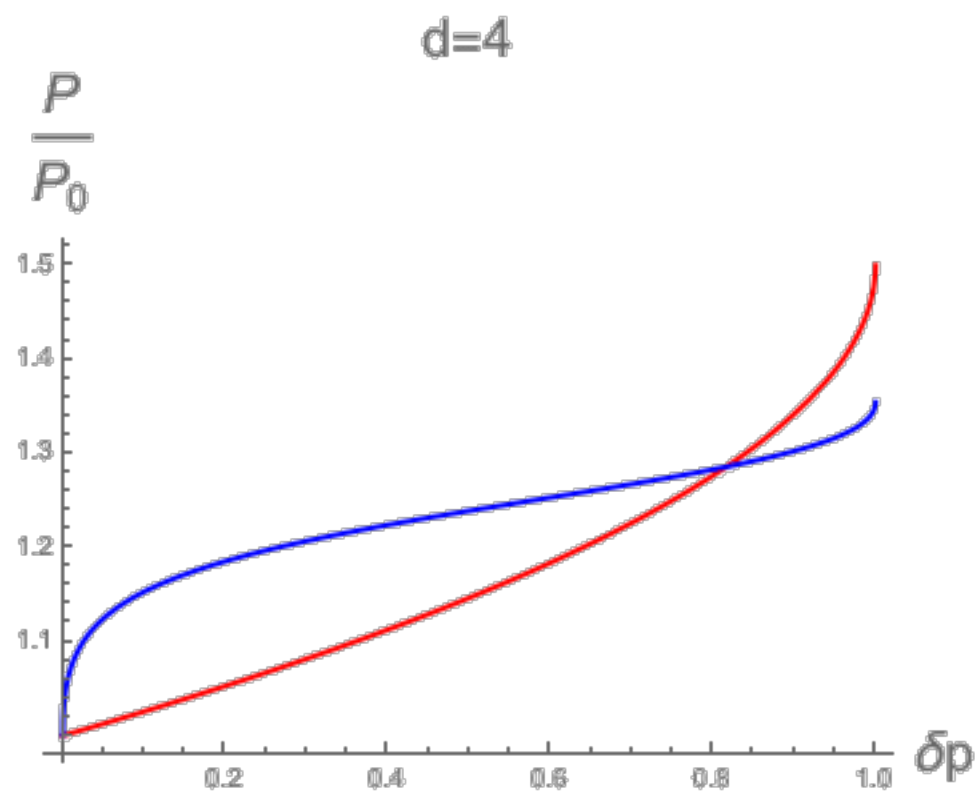
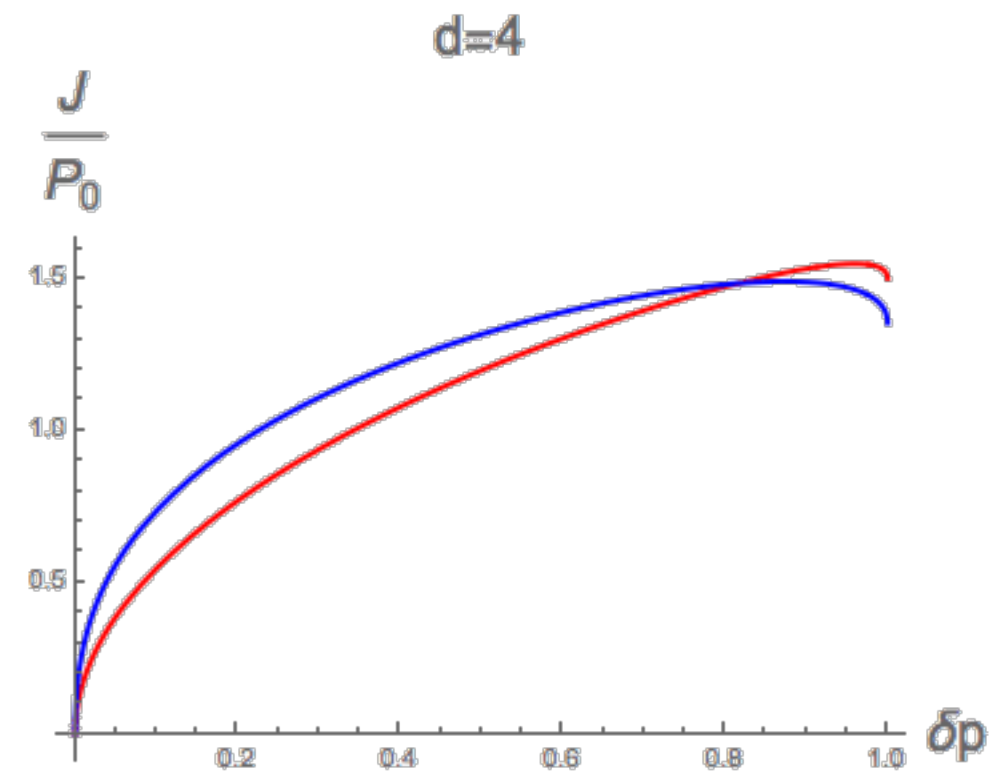


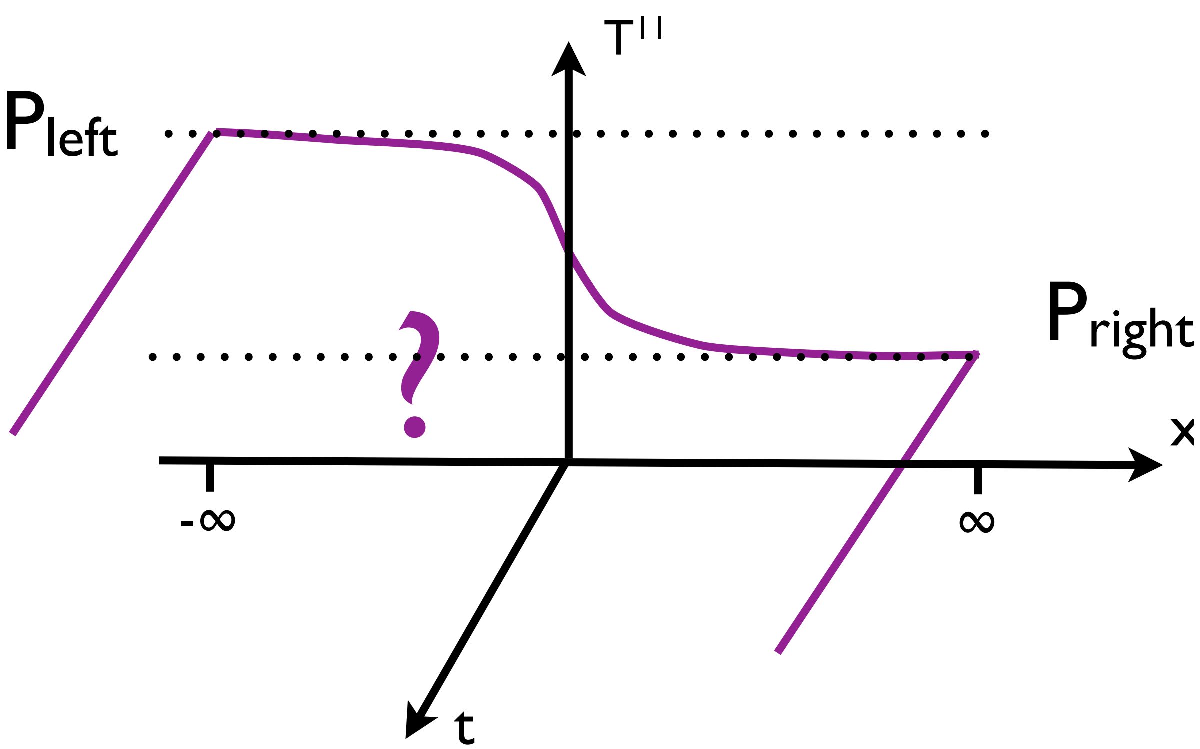


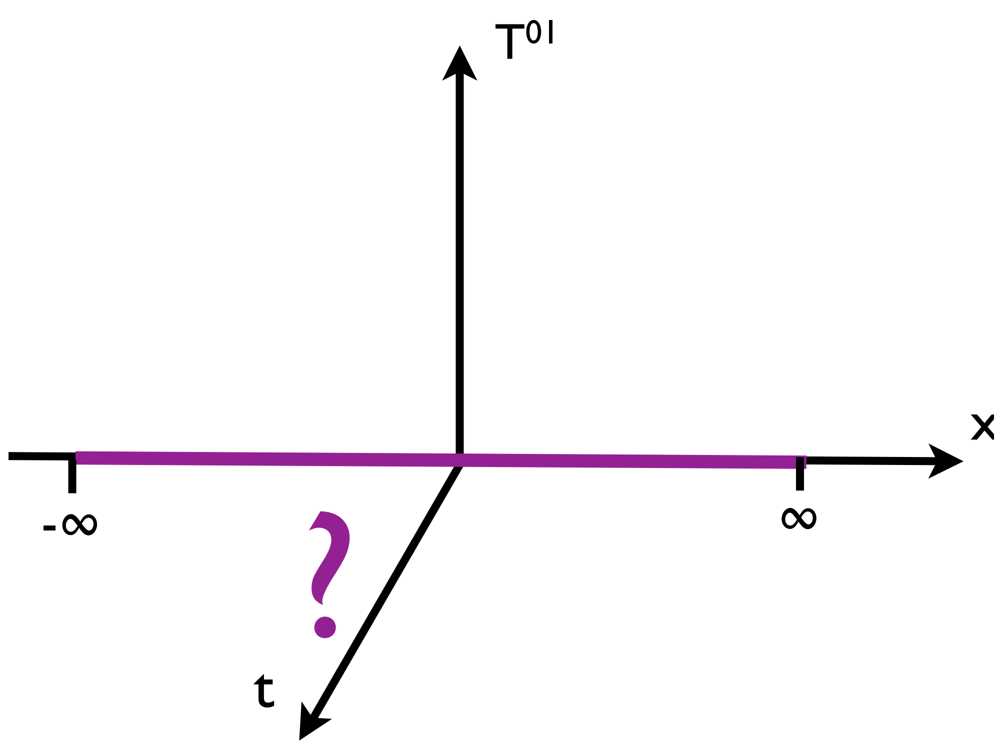
Plan:

- Prove the conjecture for 2d CFT’s
- Motivate the conjecture
- Provide evidence for the conjecture in non trivial configurations

Steady states in 2d CFT's

Formally, we are asking for the value of the energy momentum tensor at late times, given an initial condition and boundary condition.





In a conformal theory (using $ds^2 = -dt^2 + dx^2$)

$$T^{\mu\nu} = \begin{pmatrix} T_+(t+x) + T_-(-t+x) & T_-(-t+x) - T_+(t+x) \\ T_-(-t+x) - T_+(t+x) & T_+(t+x) + T_-(-t+x) \end{pmatrix}$$

Steady states in 2d CFT's

At x=∞ we have the right heat bath

$$T_+(\infty) + T_-(\infty) = P_{\text{right}}\,, \quad T_-(\infty) - T_+(\infty) = 0$$

At x=-∞ we have the left heat bath

$$T_+(-\infty) + T_-(-\infty) = P_{\text{left}}\,, \quad T_-(-\infty) - T_+(-\infty) = 0$$

Therefore, at t=∞ we have (See also, Bernard and Doyon, 2013; Bhaseen et. al., 2013)

In a conformal theory (using $ds^2 = -dt^2 + dx^2$)

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$$T^{11} = T_+(\infty) + T_-(-\infty) = \frac{1}{2}\left(P_{\text{left}} + P_{\text{right}}\right)\,,$$

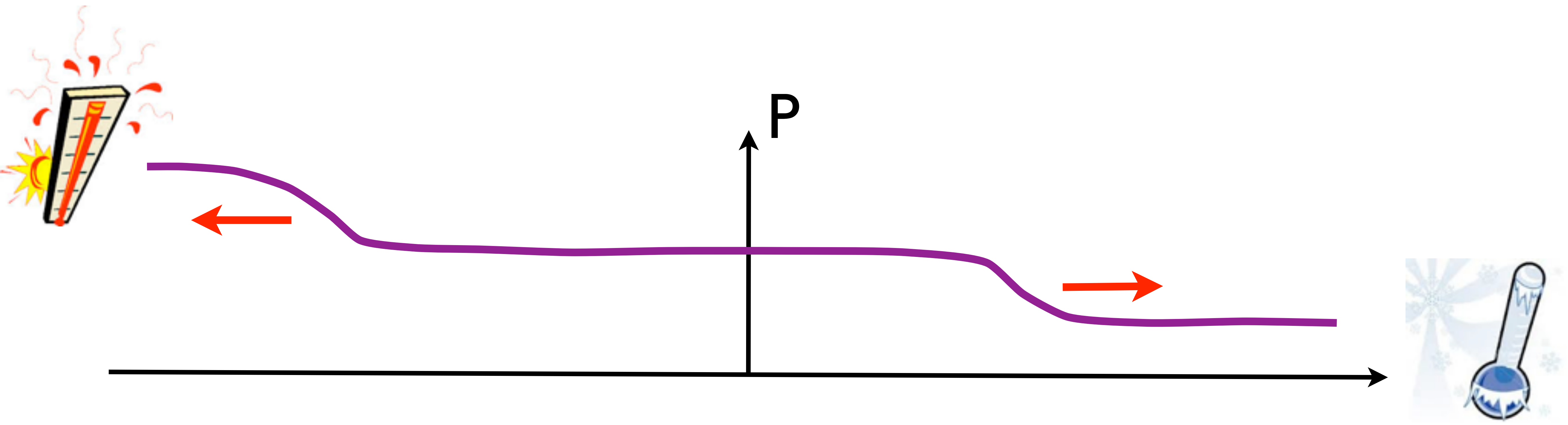
$$T^{01} = T_+(-\infty) - T_-(\infty) = \frac{1}{2}\left(P_{\text{left}} - P_{\text{right}}\right)$$

Steady states in 2d CFT's

Main ingredient:

$$T^{\mu\nu} = \begin{pmatrix} T_+(t+x) + T_-(-t+x) & T_-(-t+x) - T_+(t+x) \\ T_-(-t+x) - T_+(t+x) & T_+(t+x) + T_-(-t+x) \end{pmatrix}$$

The left and right moving modes push the disturbance to infinity at the speed of light, leaving a steady state region in between.



More than 2 dimensions

Energy momentum conservation and conformal invariance imply:

$$\partial_\mu T^{\mu\nu} = 0\,, \quad T^\mu{}_\mu = 0$$

Within our ansatz

$$T^{\mu\nu}(t,x) = \begin{pmatrix} T^{00} & T^{01} & 0 \\ T^{01} & T^{11} & 0 \\ 0 & 0 & T_\perp \end{pmatrix}$$

Let us assume, in addition, that the system is described by a perfect inviscid fluid:

$$T^{\mu\nu} = \epsilon(P) u^\mu u^\nu + (\eta^{\mu\nu} + u^\mu u^\nu) P$$

energy density 4-velocity Pressure

More than 2 dimensions: ideal fluids

Energy momentum conservation and conformal invariance imply:

$$\partial_\mu T^{\mu\nu} = 0\,, \quad T^\mu{}_\mu = 0$$

If the pressure difference between the baths is small, then sound modes will dominate the dynamics

$$\epsilon = (d-1)P\,, \quad P = P_0 + \delta P(t,x)\,, \quad u^\mu = (1, \delta\beta(t,x), 0, \ldots, 0)$$

$$\delta P = P_-(x - c_s t) + P_+(x + c_s t)$$

$$\delta\beta(t,x) = \beta_0 + \frac{1}{dP_0 c_s}\left(P_+(x + c_s t) - P_-(x - c_s t)\right)\,,$$

speed of sound

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More than 2 dimensions: ideal fluids

The linearized equations for δP and $\delta\beta$ are wave equations. Their general solution is given by:

So we can use the same strategy as before to obtain the late time behavior of the pressure and velocity.

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So we can use the same strategy as before to obtain the late time behavior of the pressure and velocity:

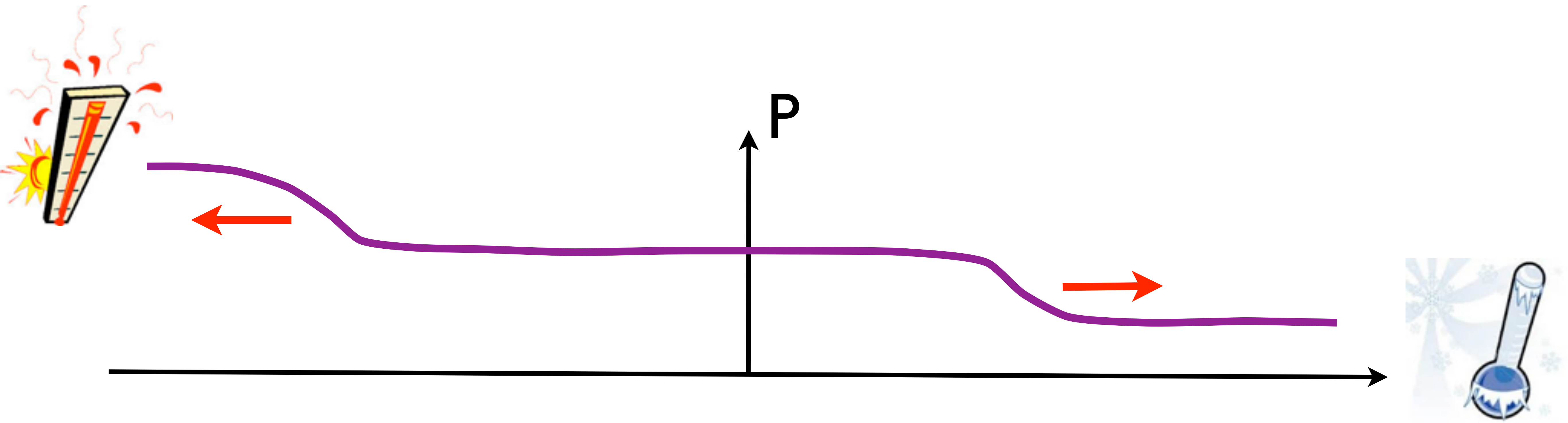
At $x \to \mp\infty$ we impose that the system is connected to a heat bath. This determines the $t \to \infty$ behavior

$$T^{00}(t\to\infty) = (d-1)P_0 \, , \quad T^{01}(t\to\infty) = \frac{\Delta P}{c_s} \, , \quad T^{11}(t\to\infty) = P_0$$

More than 2 dimensions: ideal fluids

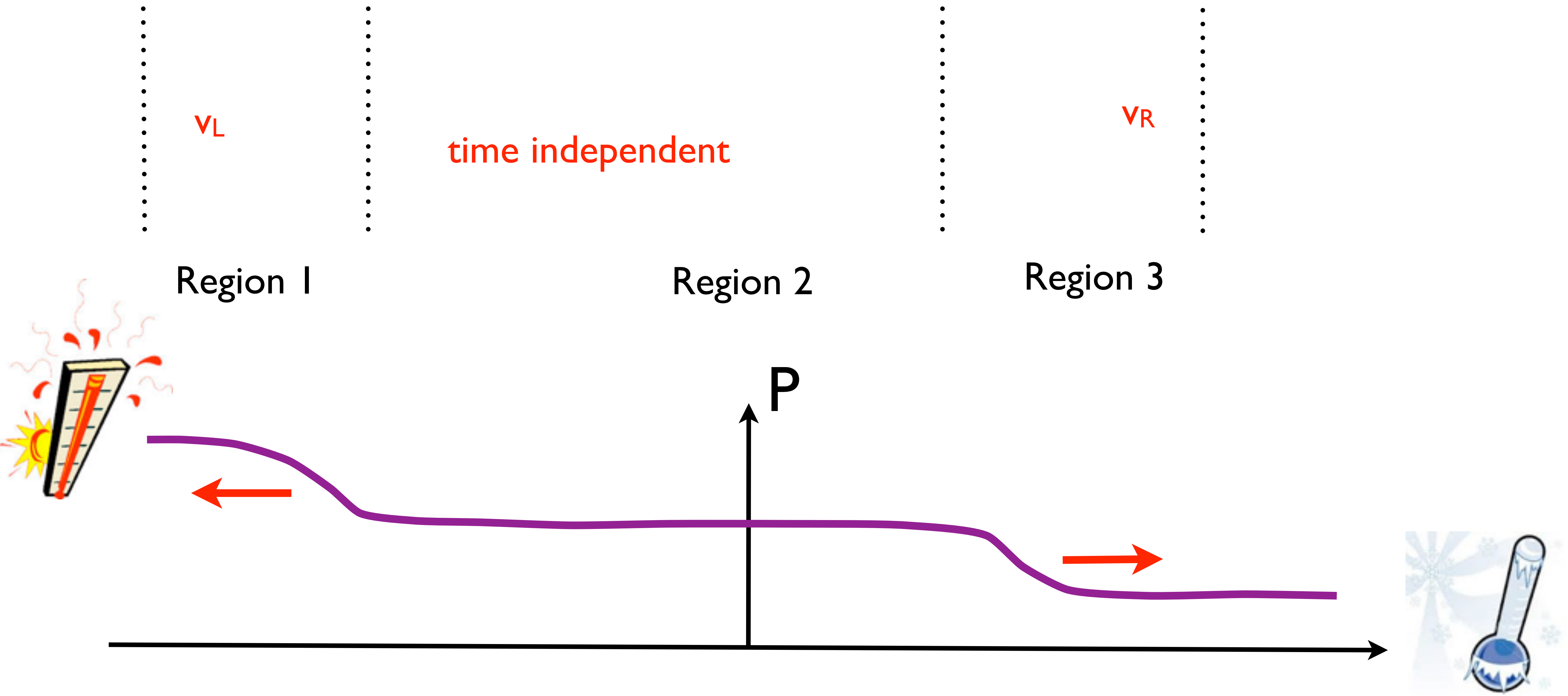
Once again, the left and right moving modes push the disturbance to infinity (at the speed of sound), leaving a steady state region in between.

If the initial disturbance is discontinuous then one can show that shock waves replace the role of sound waves. (Marti, Mueller, 1994)



A conjecture

At late times modes propagating towards the heat baths generate an intermediate steady state region.



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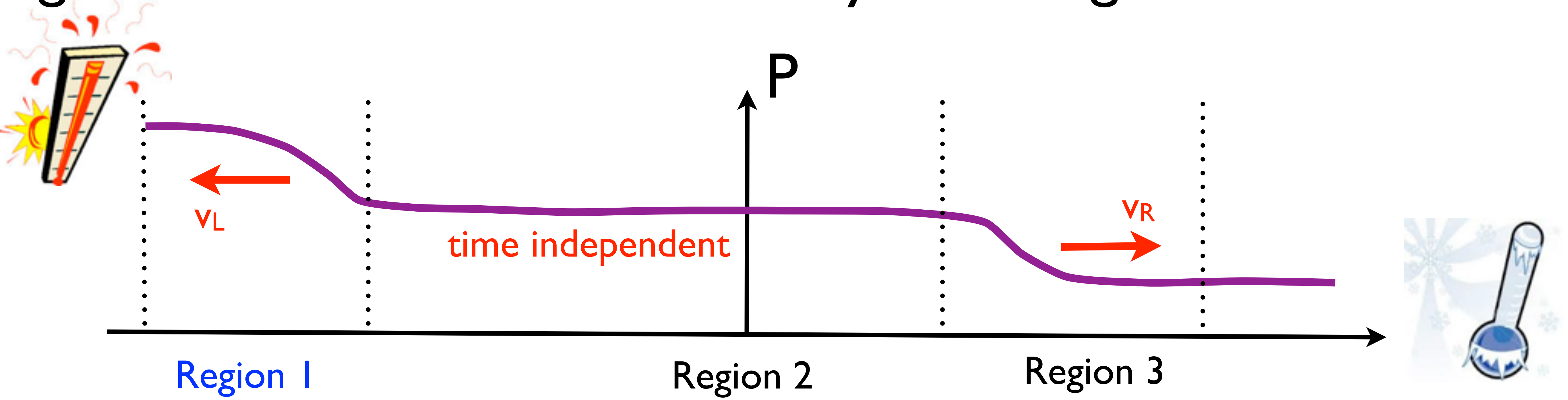

Region 1

$$T_1^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_L} W_L(x+v_L t) & W_L(x+v_L t) \\ W_L(x+v_L t) & -v_L W_L(x+v_L t) \end{pmatrix} + C_I^{\mu\nu}$$

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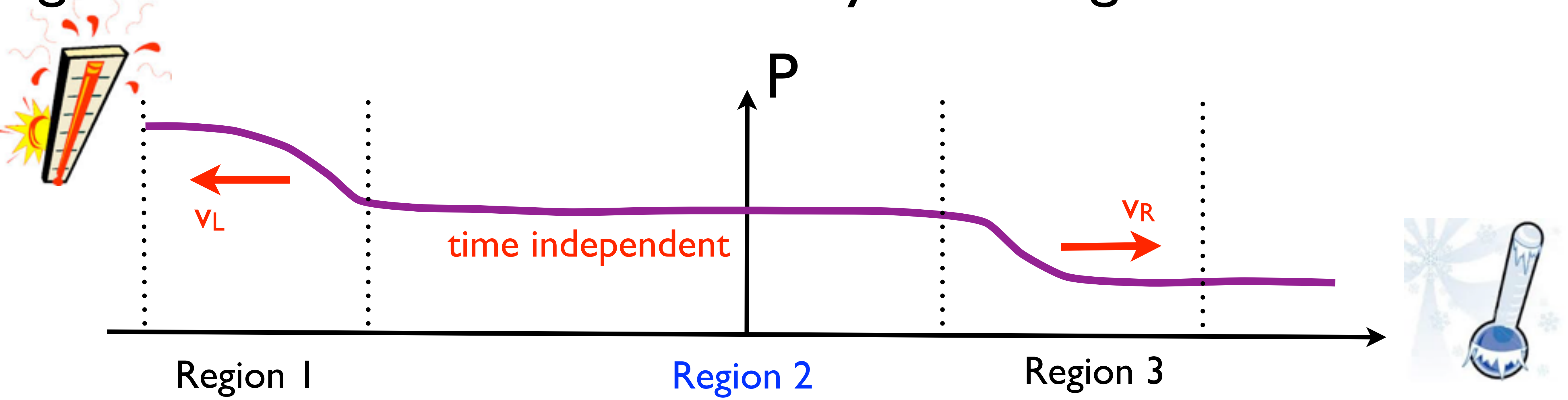

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Region 2

$$T^{\mu\nu}(x) = \begin{pmatrix} \epsilon(x) & J(x) \\ J(x) & P(x) \end{pmatrix}$$

Conservation:

$$J'(x) = 0\,, \quad P'(x) = 0$$

A conjecture

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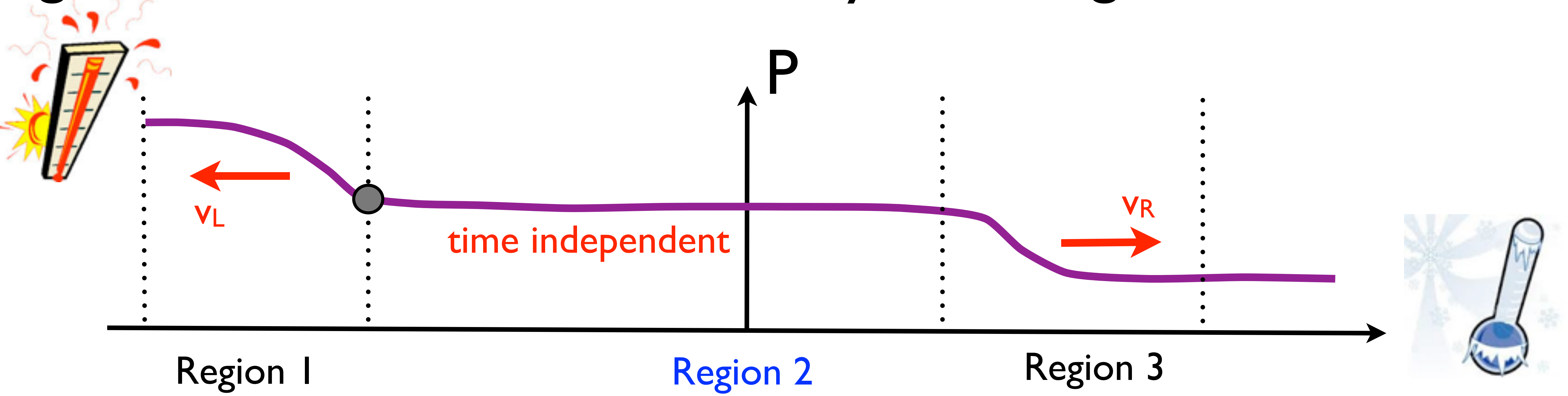

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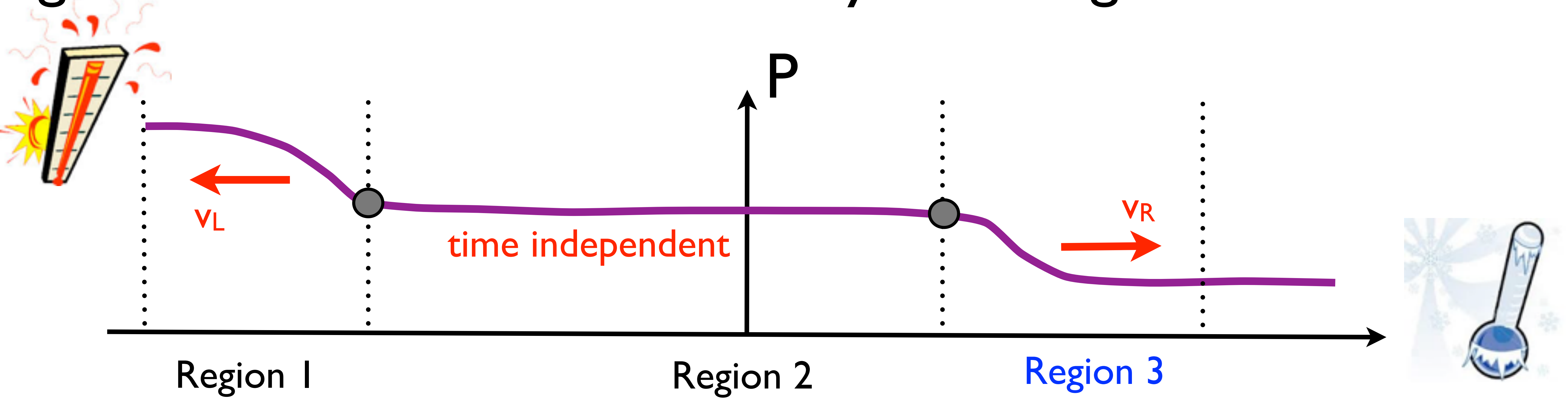

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Region 2

$$T^{\mu\nu} = \begin{pmatrix} \epsilon(x) & J \\ J & P \end{pmatrix}$$

Region 3

$$T_3^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_R} W_R(x - v_R t) & W_R(x - v_R t) \\ W_R(x - v_R t) & -v_R W_L(x - v_R t) \end{pmatrix} + C_{III}^{\mu\nu}$$

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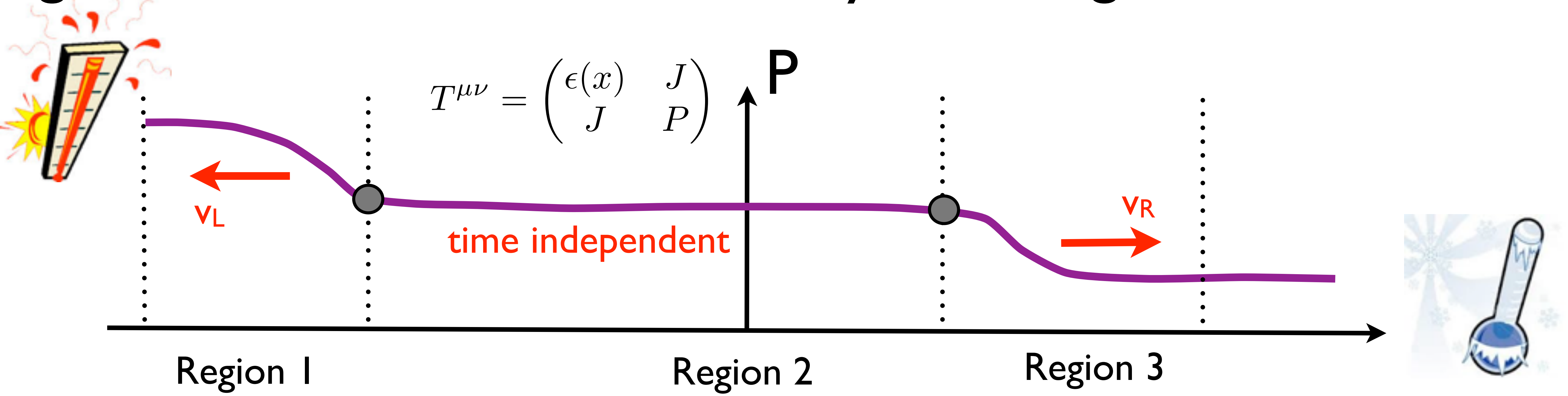
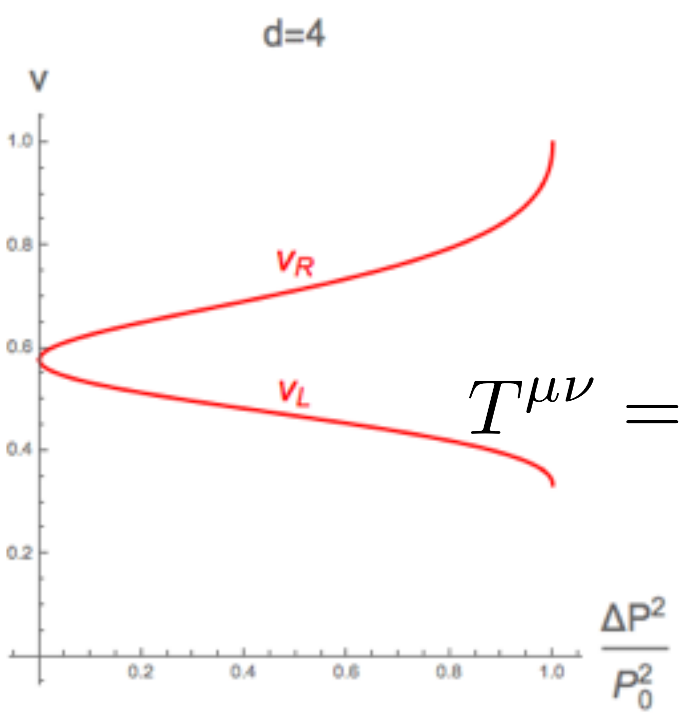
At late times modes propagating towards the heat baths generate an intermediate steady state region.



We find:



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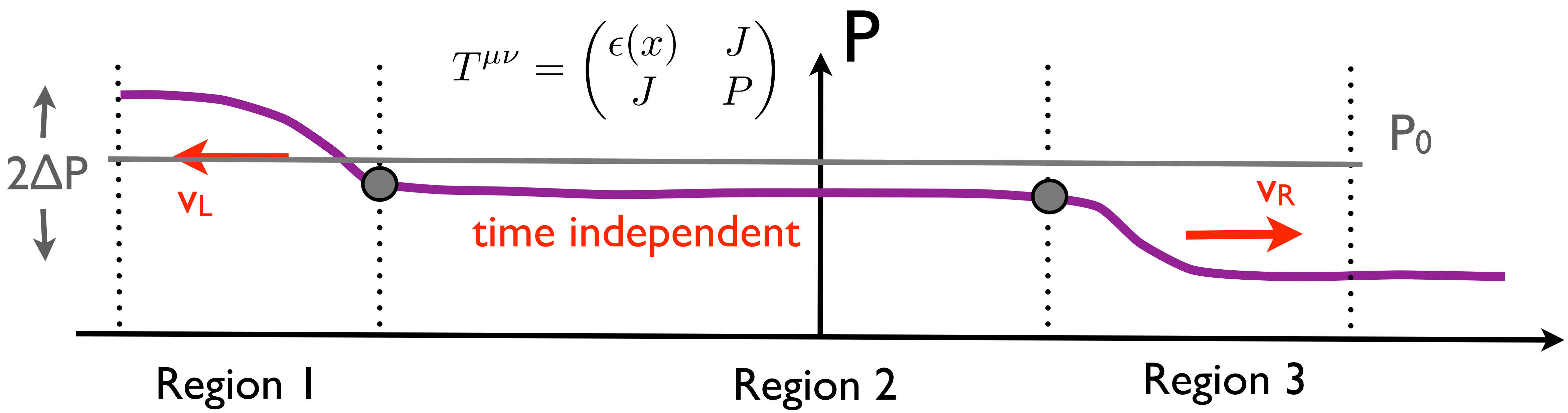
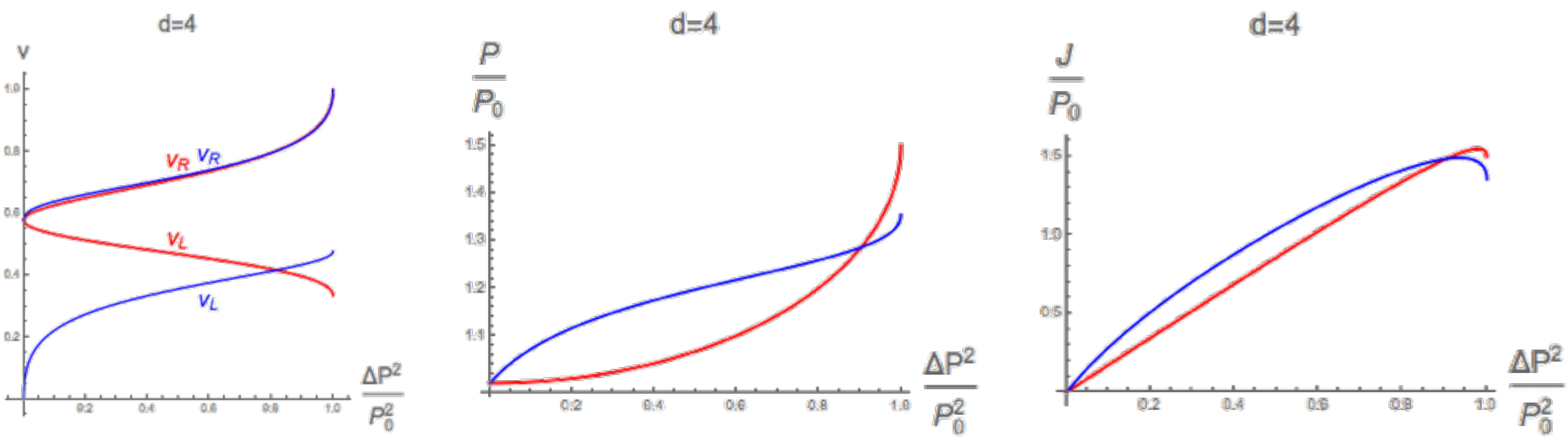


We find:

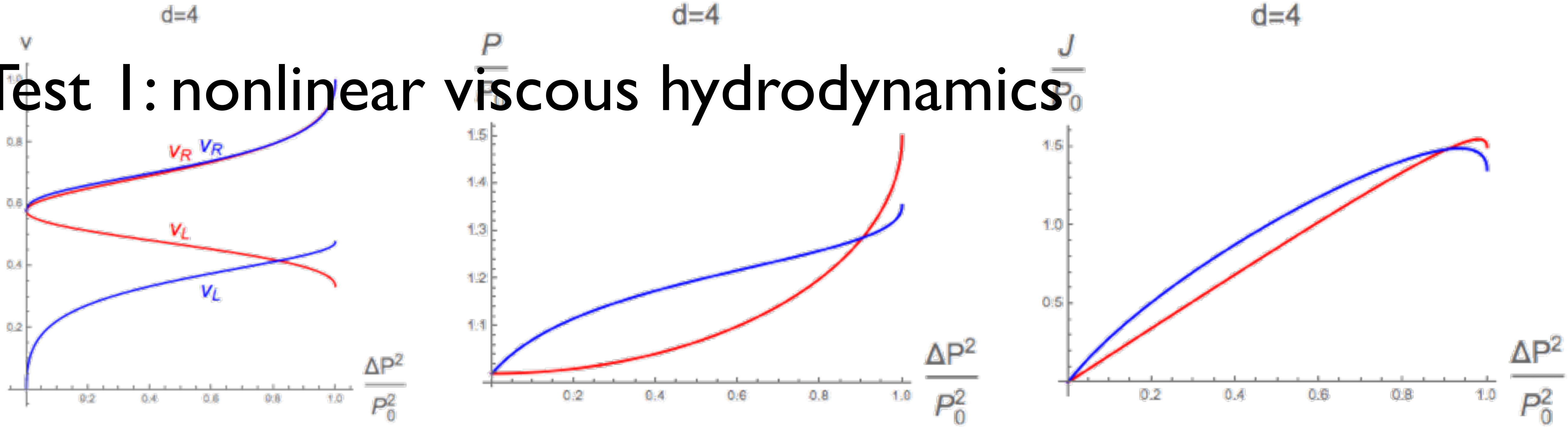
(See also Bhaseen et. al., 2013)



A conjecture

We find:

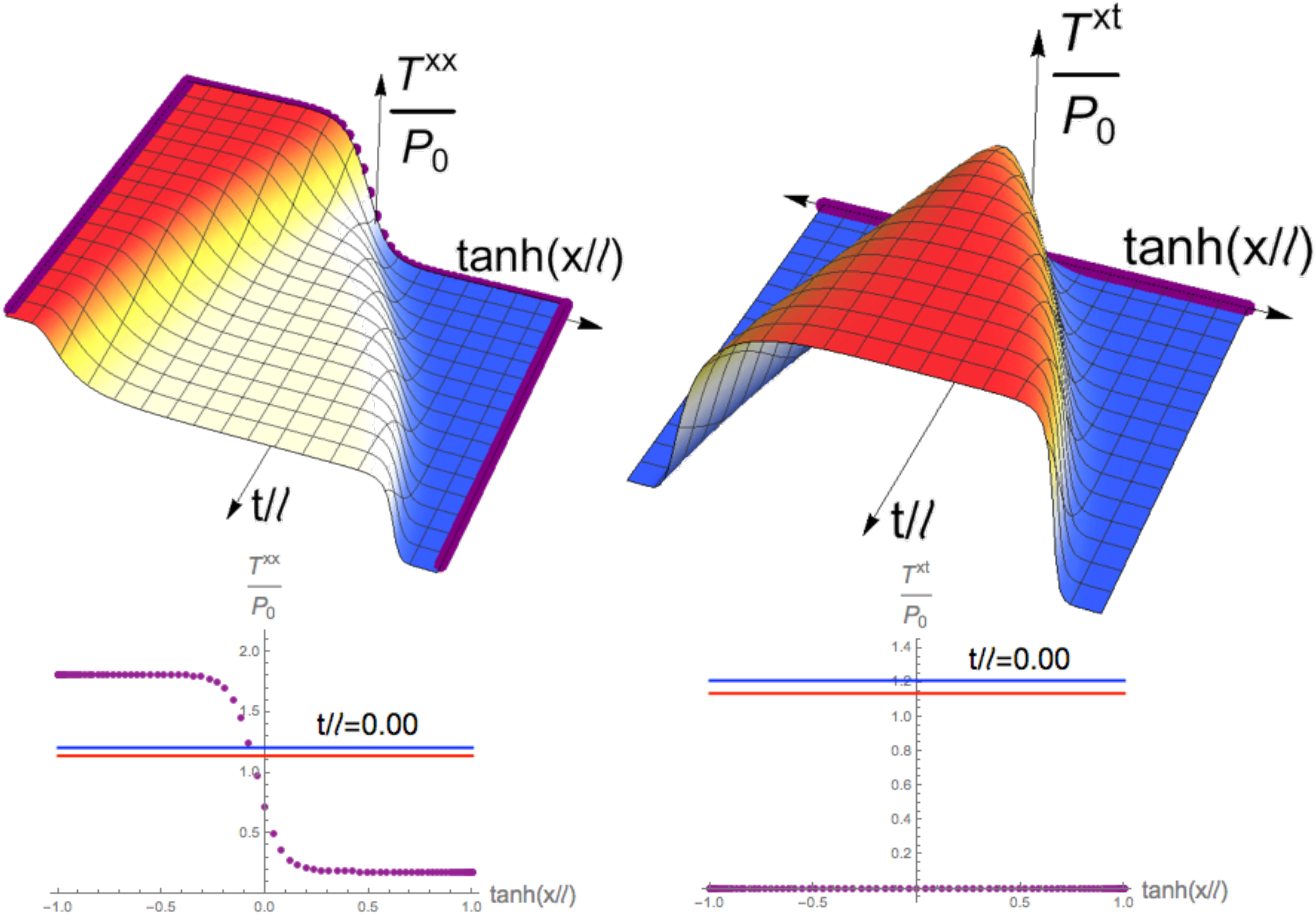
Test 1: nonlinear viscous hydrodynamics



Testing the conjecture: viscous hydro

(Baier, Romatschke, Son, Starinets, Stephanov, 2007)

We find ($d=3$, $\Delta P/P_0=0.8$)



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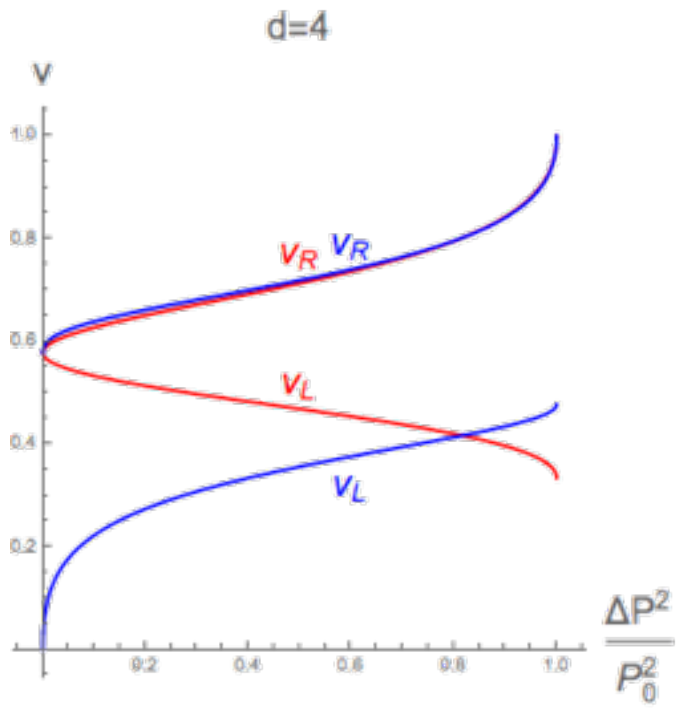
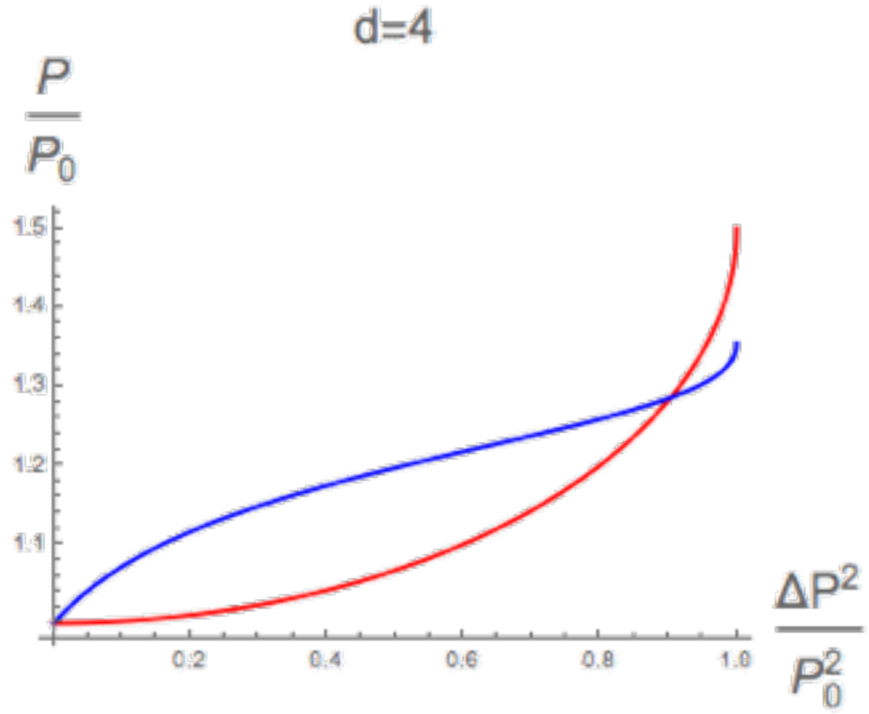
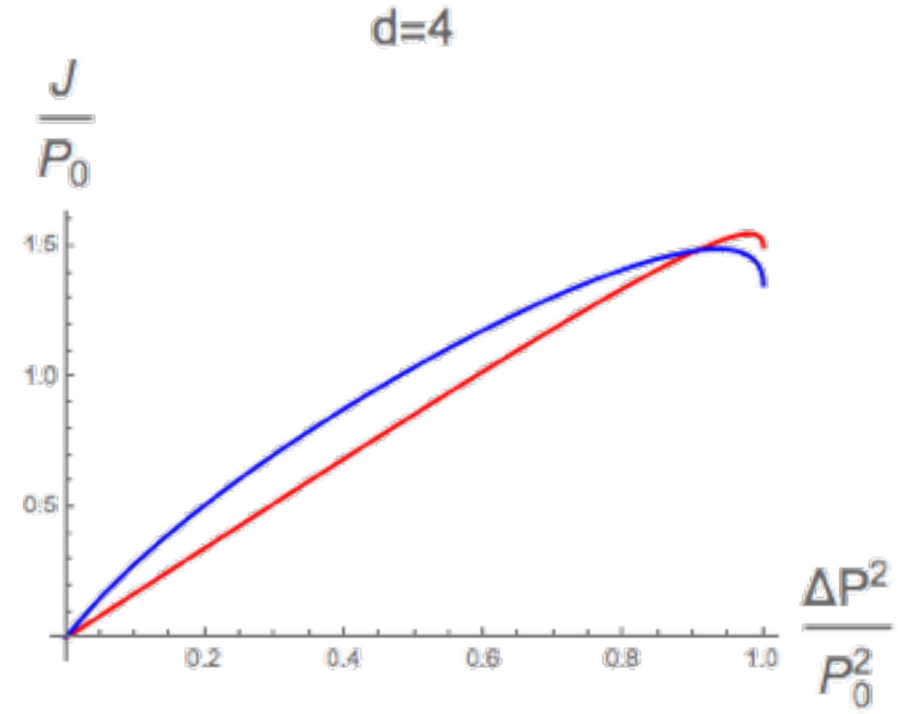




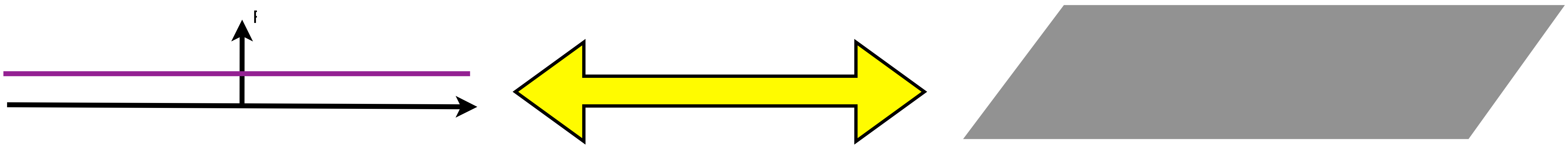


Test 1: nonlinear viscous hydrodynamics.

Test 2: Holography.

Testing the conjecture: Holography

Let us start by considering an equilibrated configuration



A planar event horizon:

$$ds^2 = 2dt\,(dr - A(r)dt) + r^2 d\vec{x}^2$$

$$A(r) = r^2 \left(1 - \left(\frac{4\pi T}{3r}\right)^3\right)$$

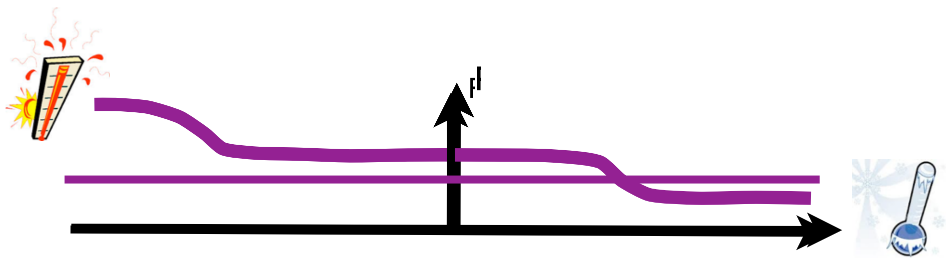
$$P(T) = p_0 \left(\frac{4\pi T}{3}\right)^3$$

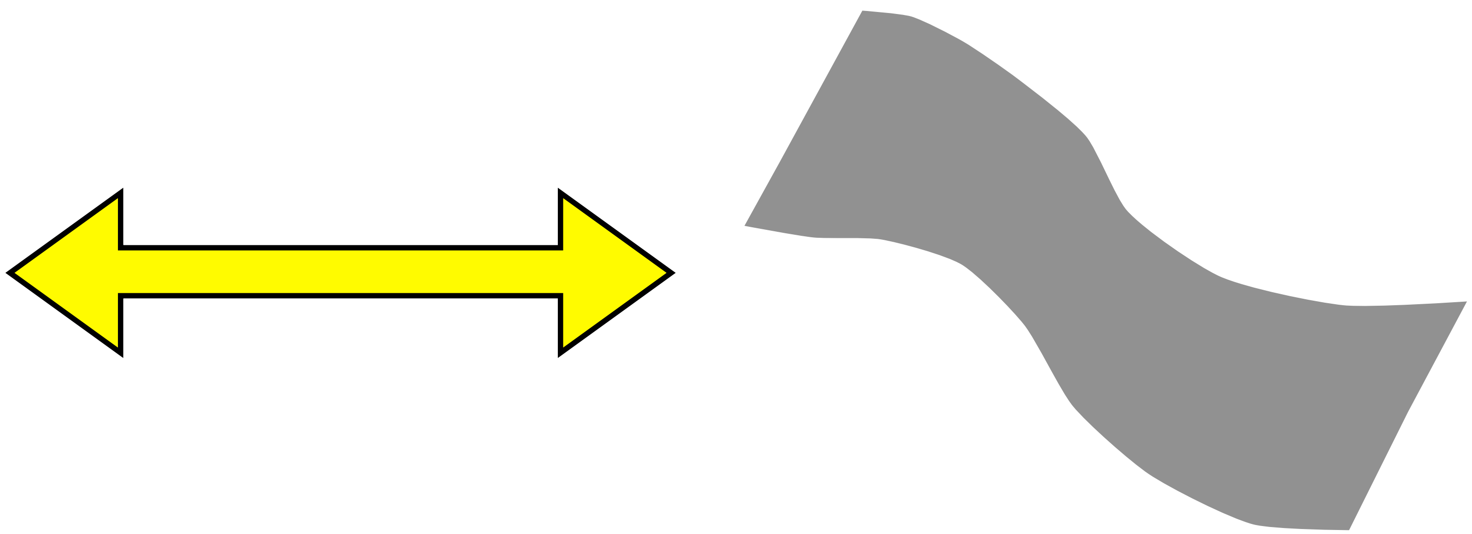
e.g., in ABJM

$$p_0 = \frac{2N^2}{9\sqrt{2\lambda}} \qquad \lambda = \frac{N}{k}$$

Testing the conjecture: Holography

Out of equilibrium we want to start with:





$$P(T_L) = p_0 \left(\frac{4\pi T_L}{3} \right)^3$$

$$P(T_R) = p_0 \left(\frac{4\pi T_R}{3} \right)^3$$

A planar event horizon:

$$ds^2 = 2dt\,(dr - A(r,z)dt) + r^2 d\vec{x}^2$$

$$A(r,z) = r^2 \left(1 - \left(\frac{a_1(z)}{3r} \right)^3 \right)$$

$$a_1(-\infty) = \frac{4\pi T_L}{3}$$

$$a_1(\infty) = \frac{4\pi T_R}{3}$$

Testing the conjecture: Holography

Out of equilibrium we want to start with:

and evolve it forward in time

$$ds^2 = 2dt\,(dr - A(r,z)dt) + r^2 d\vec{x}^2$$

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Testing the conjecture: Holography

Out of equilibrium we want to start with:

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$$A(r,z) = r^2 \left(1 - \left(\frac{a_1(z)}{3r}\right)^3\right)$$

and evolve it forward in time. Using

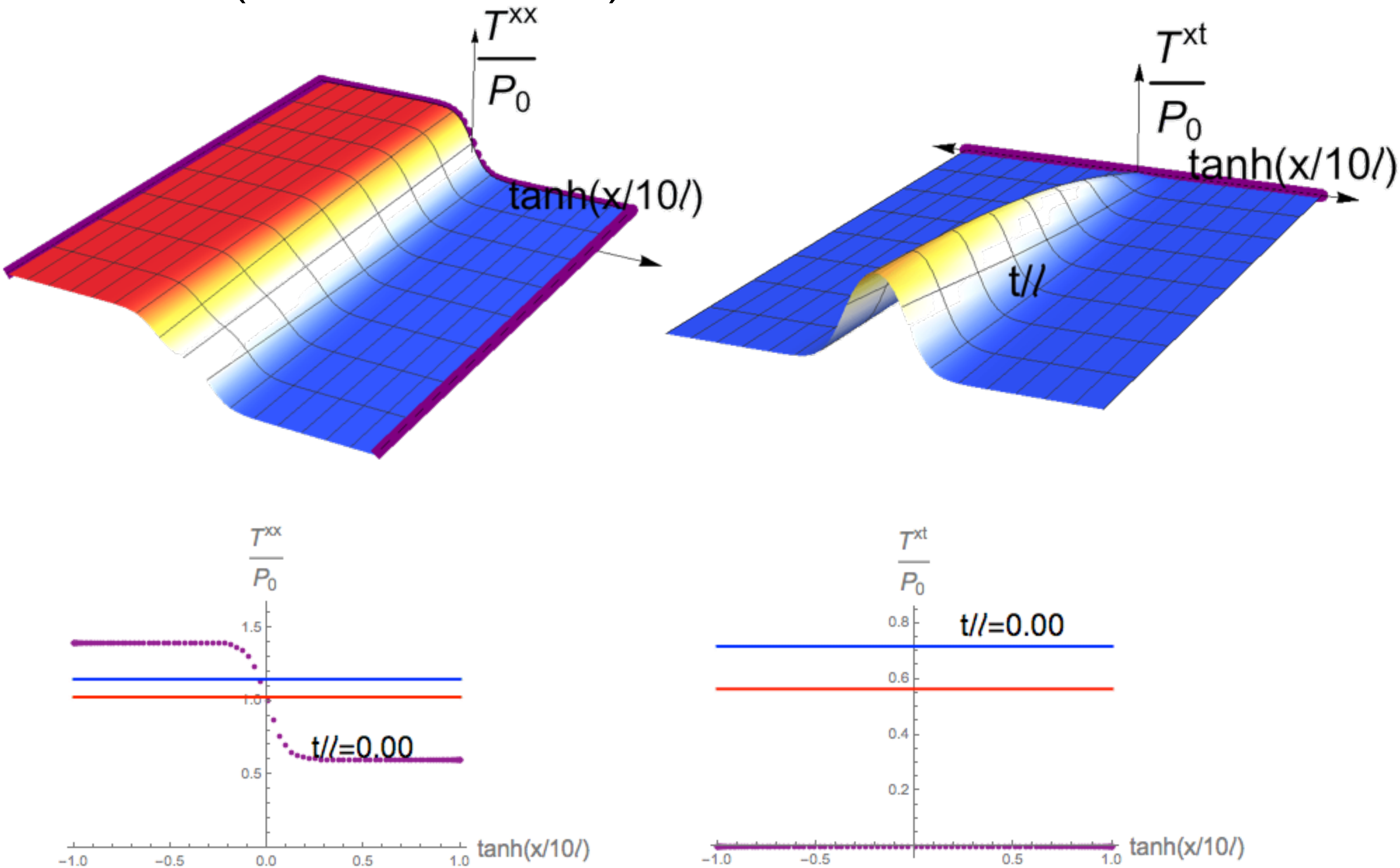
$$ds^2 = 2dt(dr - A(t,z,r)dt - F(t,z,r)dz) + \Sigma^2(t,r,z)\left(e^{B(t,z,r)}dx_\perp^2 + e^{-B(t,z,r)}dz^2\right)$$

the Einstein equations reduce to a set of nested linear differential equations in the radial coordinate 'r'. We have solved these equations numerically.

(Chesler, Yaffe, 2012)

Testing the conjecture: Holography

We find ($d=3$, $\Delta P/P_0=0.4$)



Testing the conjecture: Holography

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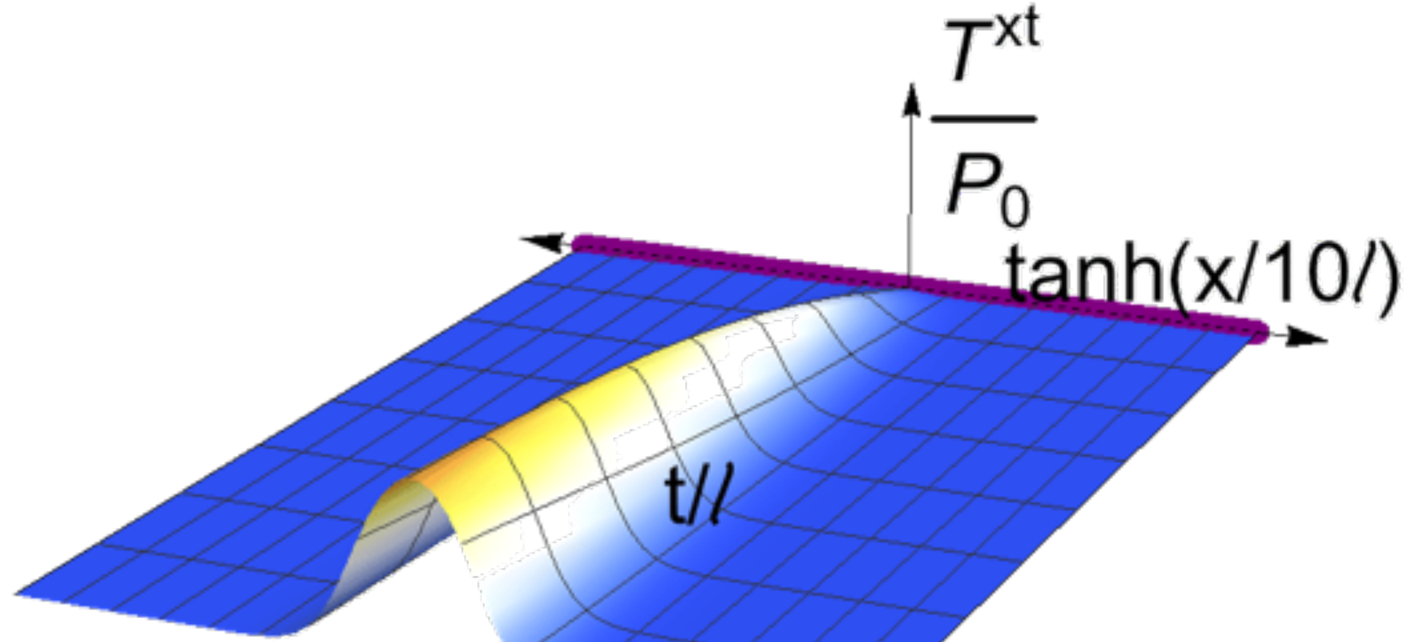
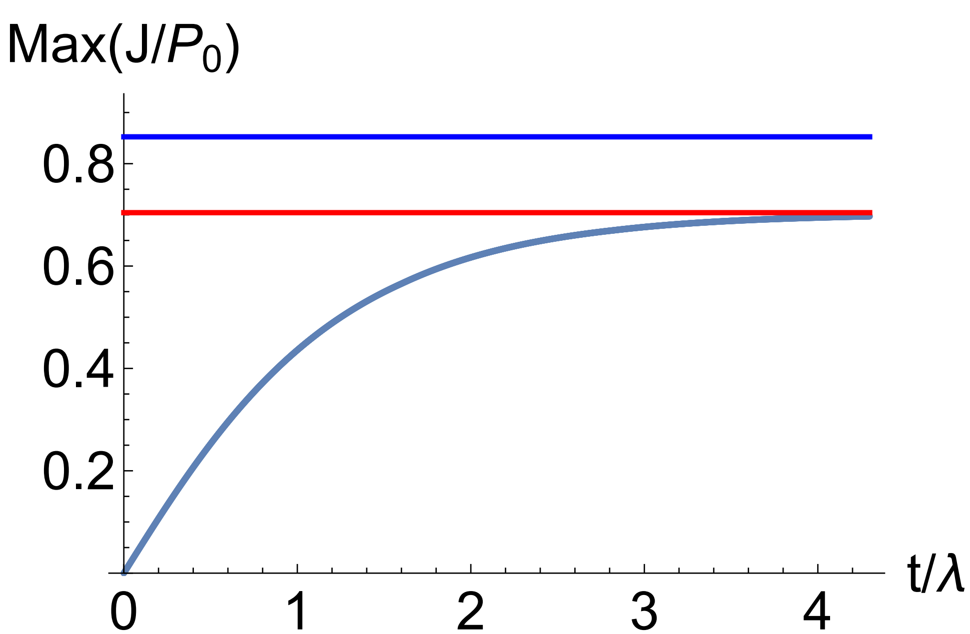
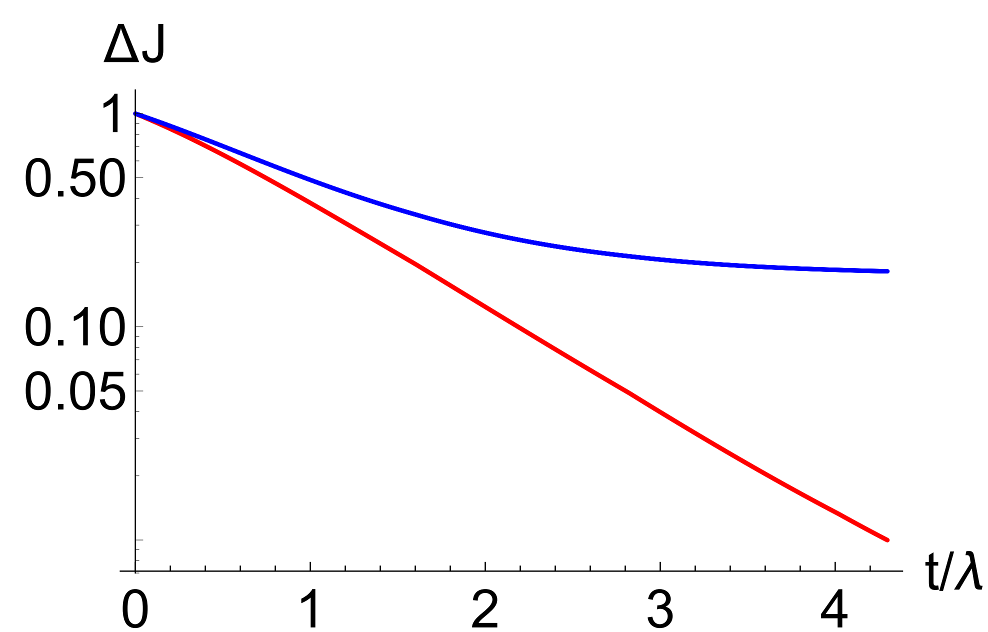


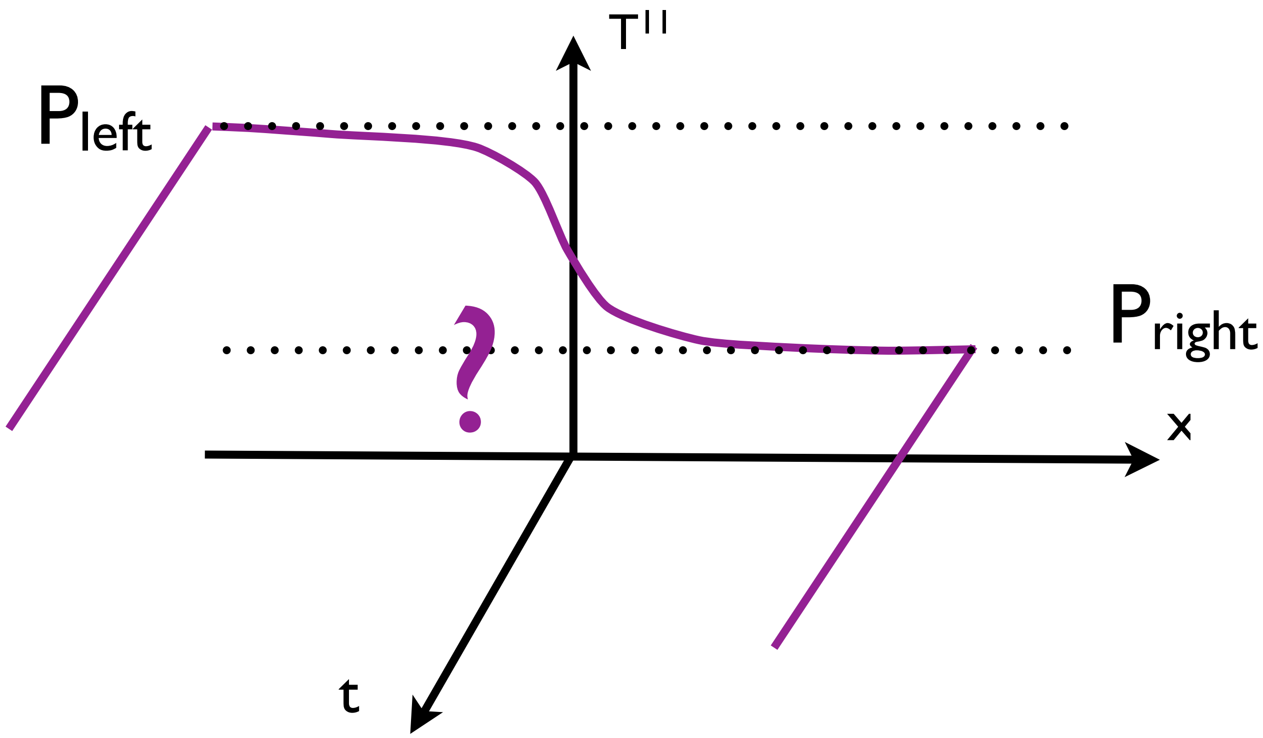
For each 't' we plot the maximal value of J/P_0

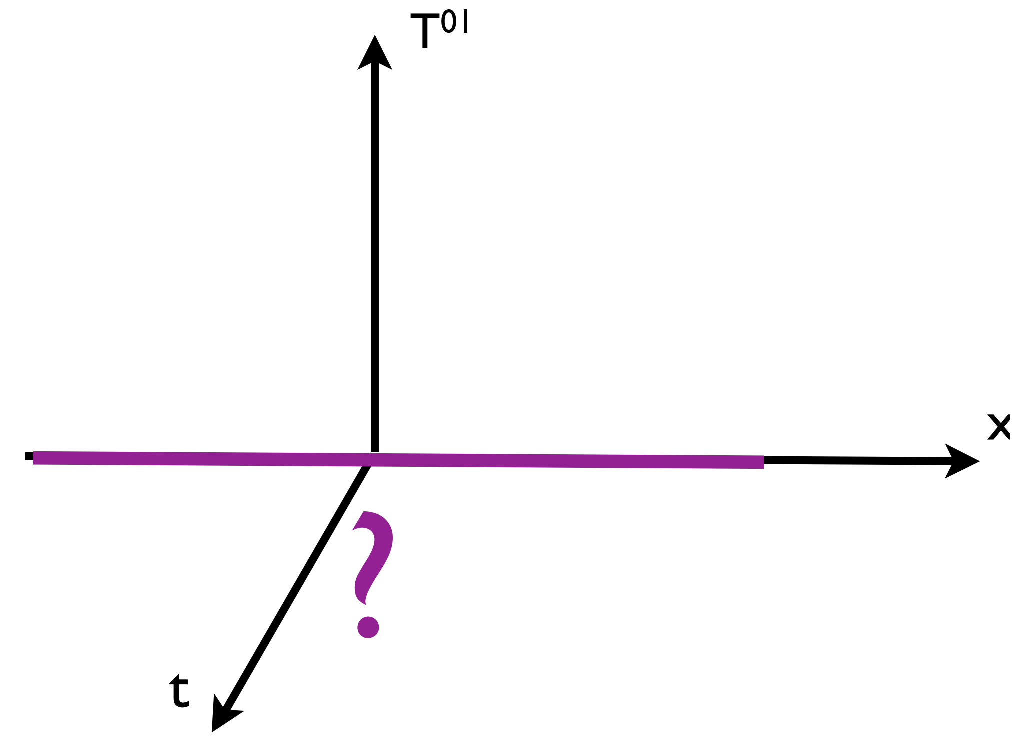


For each 't' we plot the difference: (max(J)-J)/J



Summary





In a 2d CFT we find

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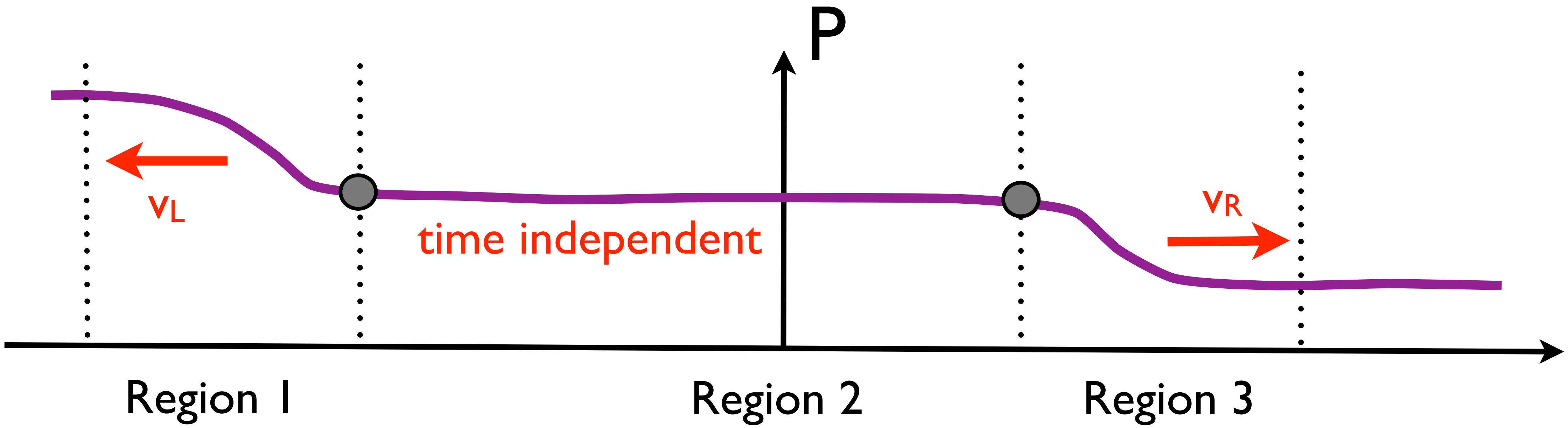
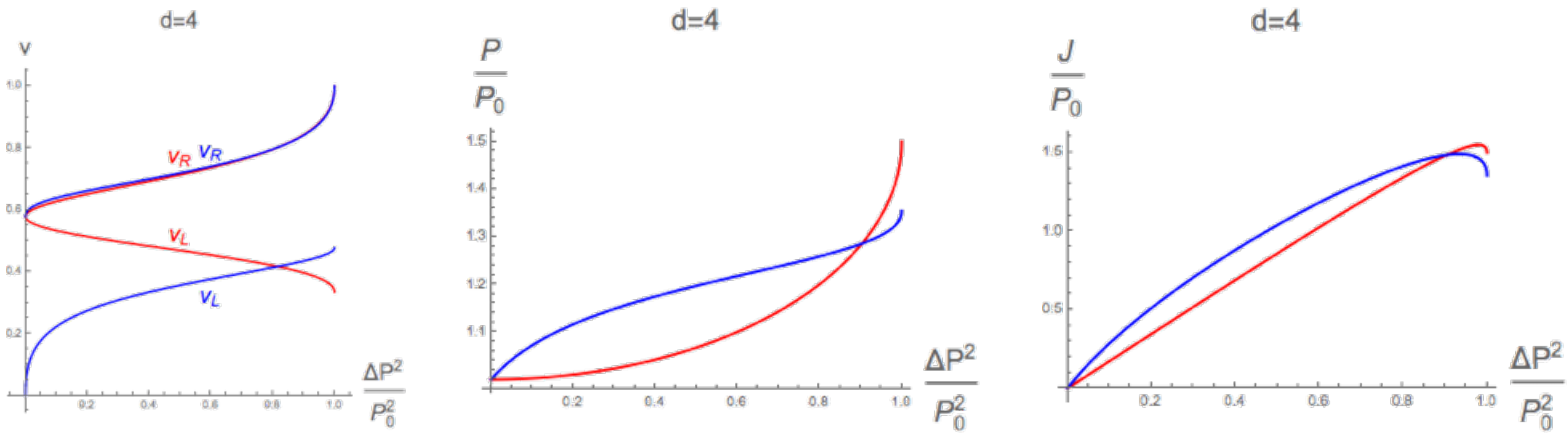
$$T^{01} = T_-(-\infty) - T_+(\infty) = \frac{1}{2}\Big(P_{\text{left}} - P_{\text{right}}\Big)$$

Summary

Otherwise, using the conjecture:



We find:



What about the blue branch?

Thank you