Flux & Hall states in Chern-Simons Matter theories with flavor

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1211.6045, 1311.6265 & 1411.3335

Summary

Motivation

Flavored ABJM with zero & finite temperature

Holographic dictionary - Running mass

ABJ & induced flux on probe D6-branes

Meson spectrum & breaking of the parity

Quantum Hall state

Motivation I

- Electrons at low T & strong magnetic field display a quantized Hall conductivity that develops a series of plateaus $\Rightarrow \frac{e^2}{h}\nu$

filling fraction

$$\nu = \frac{\text{charge density}}{\text{magnetic flux}}$$

- Aspects of the FQHE involve strongly coupled dynamics $\Rightarrow$ AdS/CFT

- Top down (& bottom up) models for (F)QHE exist: $D3/D7'$ & $D2/D8$

- Novel realization: Addition of flavor branes with an internal flux (parity break) in the backreacted ABJM $\Rightarrow$ Lots of advantages

- (Backreacted) ABJM is analytic background with good IR & UV

Motivation II

- **Internal flux** ➡ $\mathrm{WZ} \Rightarrow \int_{\Sigma_{D6}} \underbrace{\hat{\mathrm{C}}_1 \wedge \mathrm{F}}_{\text{internal}} \wedge \underbrace{\mathrm{F} \wedge \mathrm{F}}_{\text{external}} \sim \int_{\mathrm{AdS}_4} \mathrm{F} \wedge \mathrm{F}$

- Axionic term in AdS_4 $\Rightarrow$ Parity break $\Rightarrow$ CS term on the boundary

- ABJ insight $\Rightarrow$ specific choice of the internal flux

- Dependence of the filling fraction on the number of flavors

- (Further) expectations: striped phases, anyons...

ABJM Chern-Simons-matter theories

0806.1218

- describes the dynamics of multiple M2-branes at a $\mathbb{C}^4/\mathbb{Z}_k$ singularity

- Chern-Simons matter theory in 2+1 dim with gauge group $U(N)_k \times U(N)_{-k}$

- $\mathcal{N} = 6$ SUSY & $\lambda \sim \frac{N}{k}$ ⇒ rank of the gauge group; CS level

- When N & k are large $\overset{\text{type IIA}}{\Rightarrow}$ $AdS_4 \times \mathbb{CP}^3$ + fluxes with 24 SUSYs

- Massless quarks ⇒ D6-branes extending in the AdS_4 & wrapping an $\mathbb{RP}^3 \subset \mathbb{CP}^3$

Unquenching ABJM with smeared flavor

1105.6045 & 1211.0630

Write the $\mathbb{CP}^3$ & the RR 2-form as

$$ds^2_{\mathbb{CP}^3} = \frac{1}{4}\Big[ds^2_{\mathbb{S}^4} + \big(dx^i + \epsilon^{ijk} A^j x^k\big)^2\Big]$$

$\mathbb{S}^2$-bundle over $\mathbb{S}^4$ with $A^i \to SU(2)$ instanton on $\mathbb{S}^4$

$\mathcal{S}^i \to$ (rotated) basis of one-forms along $\mathbb{S}^4$

$E^i \to$ one-forms along the $\mathbb{S}^2$ fiber

$$F_2 = \frac{k}{2}\Big(E^1 \wedge E^2 - \big(\mathcal{S}^4 \wedge \mathcal{S}^3 + \mathcal{S}^1 \wedge \mathcal{S}^2\big)\Big)$$

Squash the metric & the RR 2-form as

$$T = \frac{3\,r_h}{4\pi}$$

$$ds^2 = L^2\, ds^2_{BH_4} + ds^2_6$$

$$ds^2_6 = \frac{L^2}{b^2}\Big[q\, ds^2_{\mathbb{S}^4} + (E^1)^2 + (E^2)^2\Big]$$

$q \to \mathbb{C}\mathbb{P}^3$ internal squashing

$b \to$ relative $AdS_4/\mathbb{C}\mathbb{P}^3$ squashing

Deformation parameter

$$\eta \equiv 1 + \frac{3N_f}{4k}$$

$$F_2 = \frac{k}{2}\Big[E^1 \wedge E^2 - \eta\,\big(\mathcal{S}^4 \wedge \mathcal{S}^3 + \mathcal{S}^1 \wedge \mathcal{S}^2\big)\Big]$$

Dilaton & RR 4-form

$$\boxed{e^{-\phi} = \frac{b}{4} \frac{\eta + q}{2 - q} \frac{k}{L}}$$

$$\boxed{F_4 = \frac{3kb}{4} \frac{\eta + q}{2 - q} L^2 \, \Omega_{AdS_4}}$$

The new AdS_4 radius is $\Rightarrow$ $$L^4 = 2\pi^2 \frac{N}{k} \frac{(2-q)\, b^4}{q(q + \eta q - \eta)}$$

Regime of validity $L >> 1 \,, \qquad e^{\phi} << 1$

$$\frac{N_f}{k} \sim 1 \quad \Rightarrow \quad \boxed{N^{\frac{1}{5}} << k << N}$$

(same as in the unflavored case)

$$N_f >> k \quad \Rightarrow \quad \boxed{N^{\frac{1}{5}} << N_f << N}$$

Flavor effects - Quark-antiquark potential

Computation of the wilson loop
dual to a hanging string
in $\mathcal{N}=4$ SYM

$$\boxed{V_{q\bar{q}} = -\frac{Q}{d}} \qquad Q = \frac{4\pi^2 L^2}{\left[\Gamma\left(\frac{1}{4}\right)\right]^4}$$

Maldacena & Rey

ABJM with flavor

$$\boxed{Q = \frac{4\pi^3\sqrt{2\lambda}}{\left[\Gamma\left(\frac{1}{4}\right)\right]^4}\,\sigma} \quad \Rightarrow \quad \sigma = \frac{1}{4}\,\frac{q^{\frac{3}{2}}\,(\eta+q)^2\,(2-q)^{\frac{1}{2}}}{(q+\eta q-\eta)^{\frac{5}{2}}}$$

Series expansion

$$\sigma = 1 - \frac{3}{8}\,\frac{N_f}{k} + \frac{9}{64}\left(\frac{N_k}{k}\right)^2 + \cdots$$

Dynamical quarks screen the Coulomb interaction

Embedding of a massive D6-brane probe

D6 extends in x^μ, r & $\mathbb{RP}^3$ $\begin{cases} \text{-2 directions inside } \mathbb{S}^4 \\ \text{-1 direction inside } \mathbb{S}^2 \end{cases}$

Write the $\mathbb{S}^2$ metric as $ds^2 = d\theta^2 + \sin^2\theta \, d\varphi^2$

D6 extends in φ with a profile in $\theta(r)$

Introduce new Cartesian-like coordinates

$$\rho = r^b \sin\theta$$

$$R = r^b \cos\theta$$

Generic embedding in the UV $\Rightarrow$ $R(\rho) \sim m + \dfrac{c}{\rho^{\frac{3}{b}-2}}$

SUSY embeddings $R = m$ & $c = 0 \Rightarrow \cos\theta(r) = \frac{m}{r^b}$

Holographic dictionary - running mass

bare quark mass → fundamental string ending on the D6-brane → Contribution from the anomalous dimension

Mass anomalous dimension in the unquenched ABJM $\Rightarrow \gamma_M = b - 1$

m_q will run with the scale due to the quark loops according to the Callan-Symanzik equation

Holographic realization

$$\frac{m_q\sqrt{\alpha'}}{\sqrt{\lambda}} \approx \frac{\sigma}{\sqrt{2}\,b}\, m_0\, \Lambda^{-\gamma_m} \quad \longrightarrow \quad \frac{\partial\, m_q}{\partial \log \Lambda} = -\gamma_m\, m_q$$

Callan-Symanzik equation

ABJ & induced flux on probe D6-branes

Axionic term in AdS_4 $\Rightarrow$ CS term on the bdy $\Rightarrow$ QHE $\Rightarrow$ Hall states

$\Downarrow$

Internal flux induced in ABJ $\Rightarrow$ $U(N+M)_k \times U(N)_{-k}$ with broken parity

- $N_f = 0$: the worldvolume gauge field can be understood as induced by the flat B_2 of the bulk

$$B_2 \sim \frac{M}{k} J \qquad \& \qquad \int_{\mathbb{CP}^1} B_2 = (2\pi)^2 \frac{M}{k}$$

- $N_f \neq 0$: induced metric of the internal 3-cycle for a MN embedding

$$\frac{ds_7^3}{L^2} = \frac{1}{b^2} \Big[\underbrace{q\, d\alpha^2 + q \sin^2 \alpha\, d\beta^2}_{S^2_*} + \sin^2 \theta \, (d\psi + \cos\alpha\, d\beta)^2 \Big]$$

$\downarrow$ collapses at the tip for $\theta = 0$

Non collapsing S^2_* at the tip of the brane $r = r_*$

Turn on a flux through S^2_* with quantization condition

$$\frac{1}{2\pi\alpha'}\int_{\mathbb{S}^2_*} F = \frac{2\pi M}{k}$$

Ansatz $F_{int} = dA$ with $\mathrm{A} = \mathrm{L}^2\,\mathrm{a(r)}\,(\mathrm{d}\psi + \cos\alpha\,\mathrm{d}\beta)$

flux function determined through the quantization

$$a(r_*) = -\frac{\pi M}{kL^2} = -Q$$

BPS equations + kappa symmetry $\mathcal{N}$=1 SUSY solution

$$\cos\theta(r) = \left(\frac{r_*}{r}\right)^b$$

$$a(r) = -Q\left(\frac{r_*}{r}\right)^{\frac{b}{q}}$$

Meson spectrum & parity violation

$M \neq 0 \quad \Rightarrow \quad$ parity violation $\quad \Rightarrow \quad$ meson signature

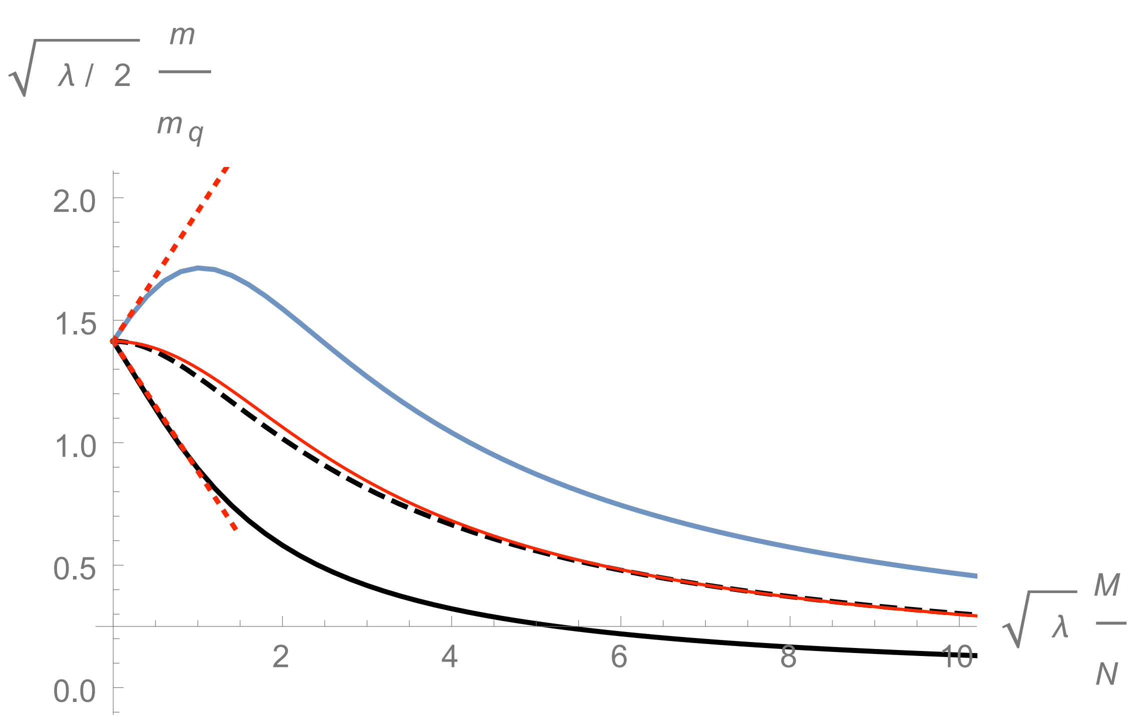
Focus on the vector mesons

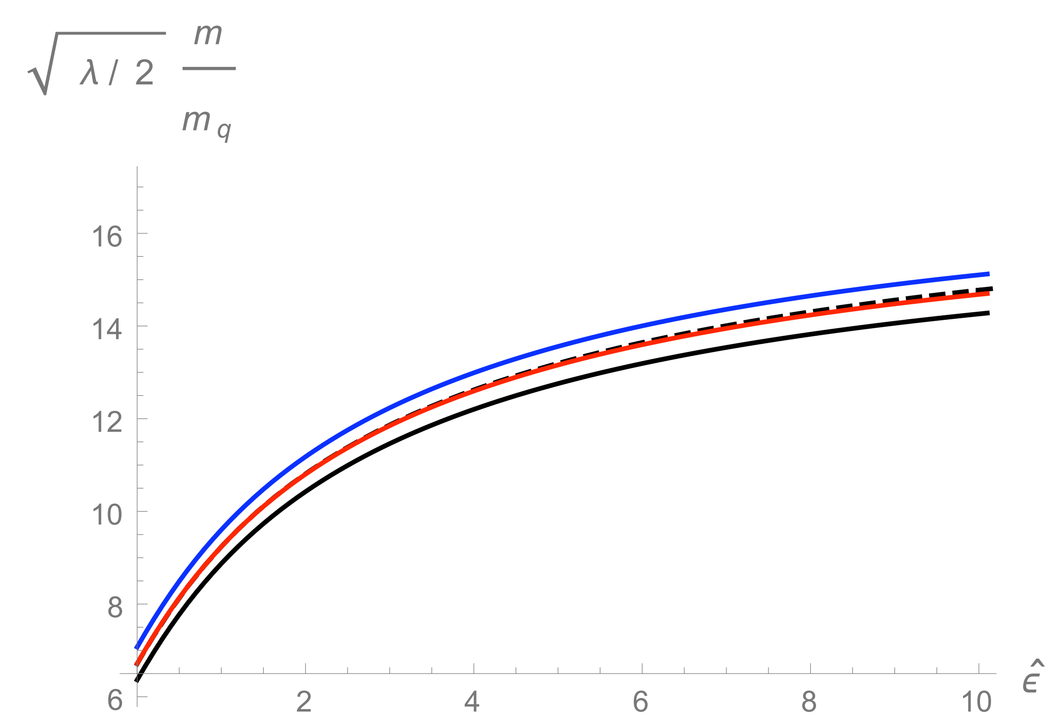
$$\delta A_\mu = \xi_\mu \, e^{ik_\nu x^\nu} R(r) \qquad \text{for} \qquad \mu = 0, 1, 2$$

ξ_μ: polarization vector

$$\chi_\pm = \sqrt{1 - \frac{k^2}{\omega^2}}\,\xi_1 \pm i\xi_2$$

$$\chi_+ = 0 \quad \& \quad \chi_- \neq 0$$
$$\chi_- = 0 \quad \& \quad \chi_+ \neq 0$$





E, B, charge density & currents

Motivation: contact with CMT and particularly QHE

$$A = L^2 \Big[a_0(r)\, dt + (Et + a_x(r))\, dx + (B\, x + a_y(r))\, dy + a(r)\, (d\psi + \cos\alpha\, d\beta) \Big]$$

There is a SUSY analytical solution with 1 supercharge!!!

$$E = B, \quad a_x(r) = 0, \quad \cos\theta(r) = \left(\frac{r_*}{r}\right)^b, \quad a(r) = -Q \left(\frac{r_*}{r}\right)^{\frac{b}{q}}$$

$$a_0'(r) = -\, a_y'(r) = (4-3b)\, \frac{b^2}{q}\, Q\, B\, r^{2(b-2)}\, \frac{r^2 - r_*^2}{r^{2b} - r_*^{2b}}$$

General EOM are invariant under $SO_+(1,1)$ symmetry transformations

In terms of boundary variables

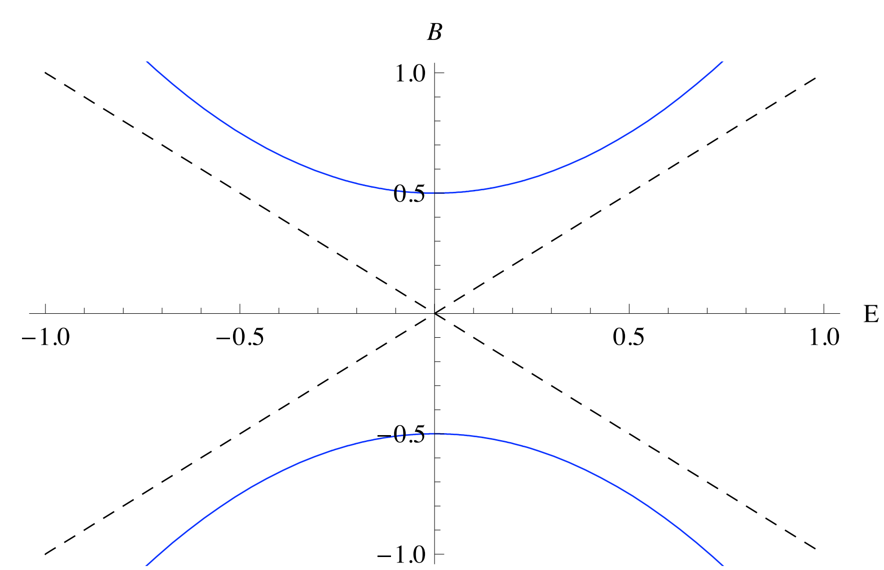
$$\begin{pmatrix} E \\ B \end{pmatrix} \to \mathcal{M}_\gamma \begin{pmatrix} E \\ B \end{pmatrix} \quad \& \quad \begin{pmatrix} d \\ j_y \end{pmatrix} \to \mathcal{M}_\gamma \begin{pmatrix} d \\ j_y \end{pmatrix}$$

$$\text{with} \quad \mathcal{M}_\gamma \equiv \begin{pmatrix} \cosh\gamma & \sinh\gamma \\ \sinh\gamma & \cosh\gamma \end{pmatrix}$$

$$\mathcal{Q}_1 \equiv E^2 - B^2\,, \qquad \mathcal{Q}_2 \equiv d^{\,2} - j_y^{\,2} \qquad \& \qquad \mathcal{Q}_3 \equiv B\,d - E\,j_y$$

Regularity at the tip: $\frac{d}{B} = \frac{j_y}{E} = I(r_*)$

$$\mathcal{Q}_2 = \mathcal{Q}_3 I(r_*) = -\mathcal{Q}_1 I(r_*)^2$$



Quantum Hall State

Internal flux + EM fields ⇨ Non-vanishing $\int \hat{C}_1 \wedge F \wedge F \wedge F$

Regularity at the tip: $\frac{d}{B} = \frac{j_y}{E} = I(r_*)$

Calculate the conductivities ⇨ $\sigma_{xx} = 0 \quad \& \quad \sigma_{xy} = \frac{N\sigma}{2\pi\sqrt{2\lambda}} \frac{j_y}{E} = \frac{\nu}{2\pi}$

Filling fraction ⇨ $$\nu = \frac{M}{2}\left[1 + \frac{3N_f}{4k}\int_{r_*}^{\infty} \cos\theta(r)\,\frac{a'(r)}{Q}\,dr\right]$$

SUSY

$$\nu = \frac{M}{2}\left[1 + \frac{3N_f}{8k}(1-\gamma_m)\right]$$

Flavor correction: can be thought as intrinsic disorder due to the dynamical flavors

Future directions

- Massive backreacted ABJM

ABJM in the IR $\Rightarrow$ Massless backreacted ABJM in the UV

$\Downarrow$ flow of the filling fraction

$$\nu_{IR} = \frac{M}{2} \quad \Rightarrow \quad \nu_{UV} = \frac{M}{2}\left[1 + \frac{3N_f}{4k}\int_{r_*}^{\infty} \cos\theta(r)\,\frac{a'(r)}{Q}\,dr\right]$$

- Backreacted ABJM at finite T - Thermodynamics

1st order PT $\Rightarrow$ MN embedding = incompressible Hall phase

BH embedding = metallic compressible phase

- Alternative quantization: Hall fluid to anyon superfluid