2015.04.17

A Simple Holographic Superconductor with Momentum Relaxation

Keun-Young Kim
GIST, Korea

Gwangju Institute of Science and Technology

References

arXiv.org > hep-th > arXiv:1501.00446

High Energy Physics – Theory

A Simple Holographic Superconductor with Momentum Relaxation

Keun-Young Kim, Kyung Kiu Kim, Miok Park

arXiv.org > hep-th > arXiv:1409.8346

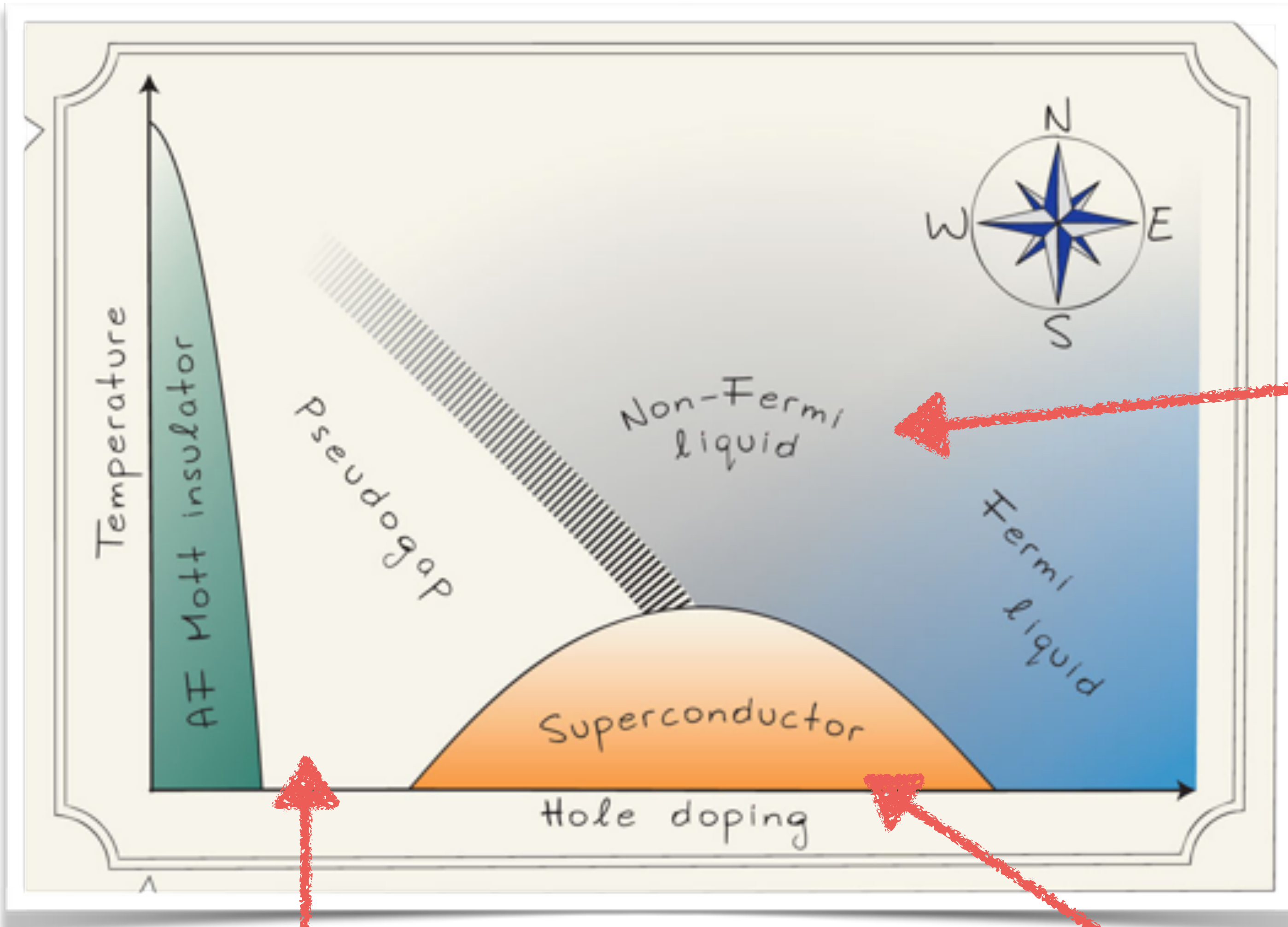
High Energy Physics – Theory

Coherent/incoherent metal transition in a holographic model

Keun-Young Kim, Kyung Kiu Kim, Yunseok Seo, Sang-Jin Sin

Motivation: Phenomenology

Cuprate phase diagram



Peter Wahl, 2012, Nature Physics

(11)
Andrea Amoretti
Matteo Baggioli
Mike Blake
Richard Davison
Blaise Gouteraux
Umut Gursoy
Niko Jokela
Elias Kiritsis
Daniele Musso
Jie Ren
Yunseok Seo

(4)
Daniel Arean
Johanna Erdmenger
Rene Meyer
Leopoldo Pando Zayas

Sang-Jin Sin

Introduction: Holographic model

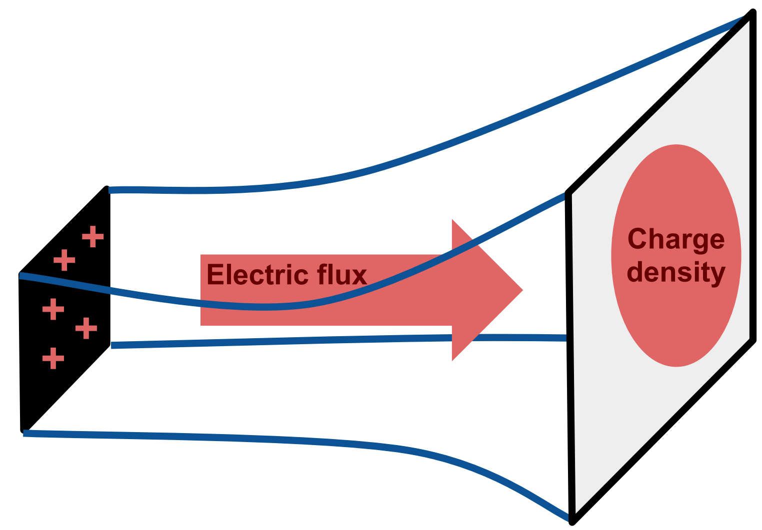
0803.3295: Hartnoll, Herzog, Horowitz

The first holographic superconductor

$$S_{\mathrm{HHH}} = \int_M \mathrm{d}^4x\sqrt{-g}\left[R + \frac{6}{L^2} - \frac{1}{4}F^2 - |D\Phi|^2 - m^2|\Phi|^2\right]$$

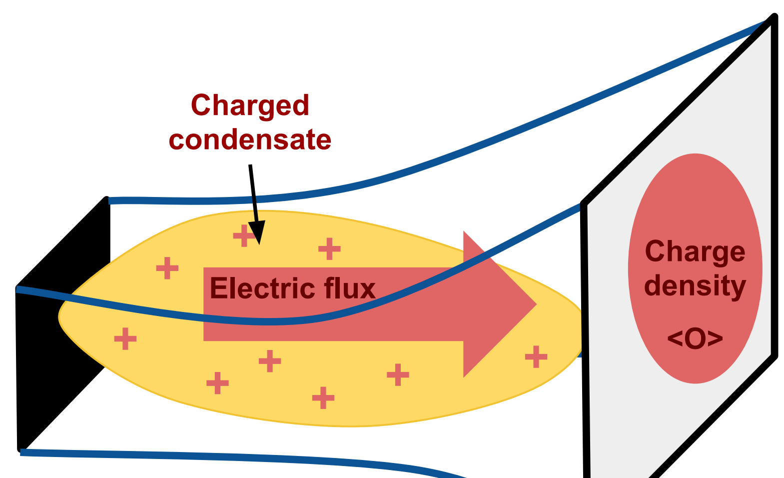
$\Phi = 0$

AdS-RN-black brane



$\Phi \neq 0$

Holographic superconductor

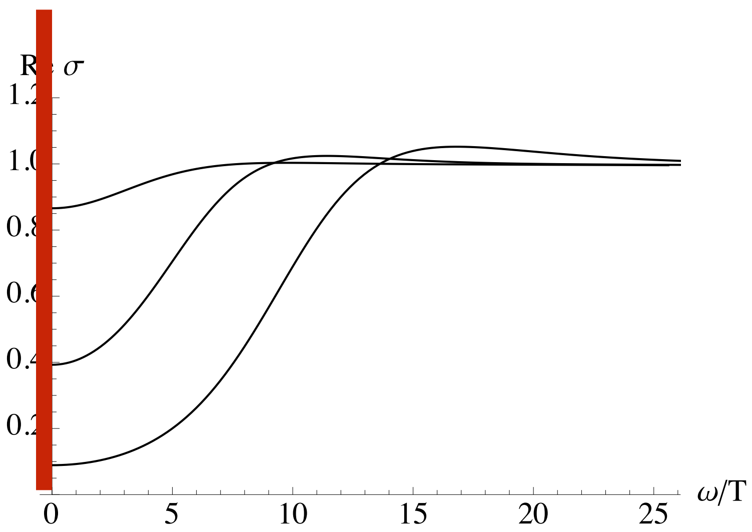


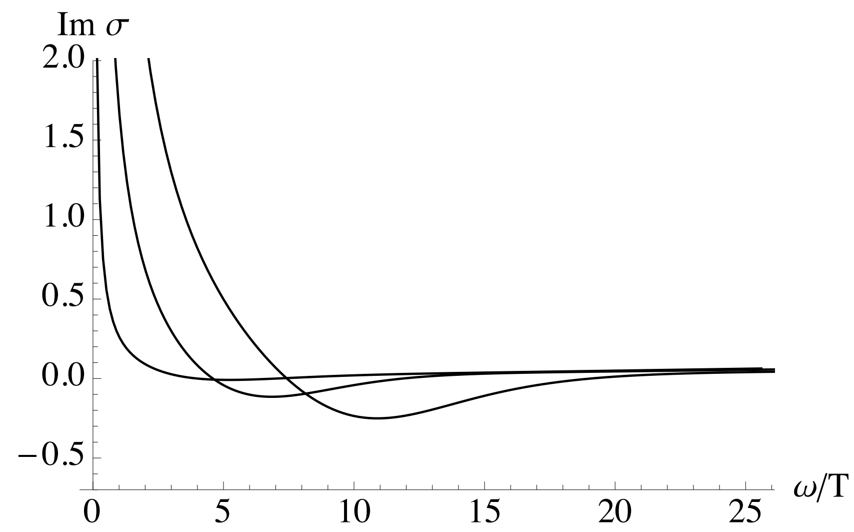
1106.4324: Hartnoll

Introduction: Holographic model

Conductivity: normal phase

0903.3234:Hartnoll





Two different delta functions

$$\mathrm{Im}\,\sigma \sim 1/\omega \qquad \Leftrightarrow \qquad \mathrm{Re}\ \sigma(\omega) \sim \delta(\omega)$$

Translation invariance + finite density

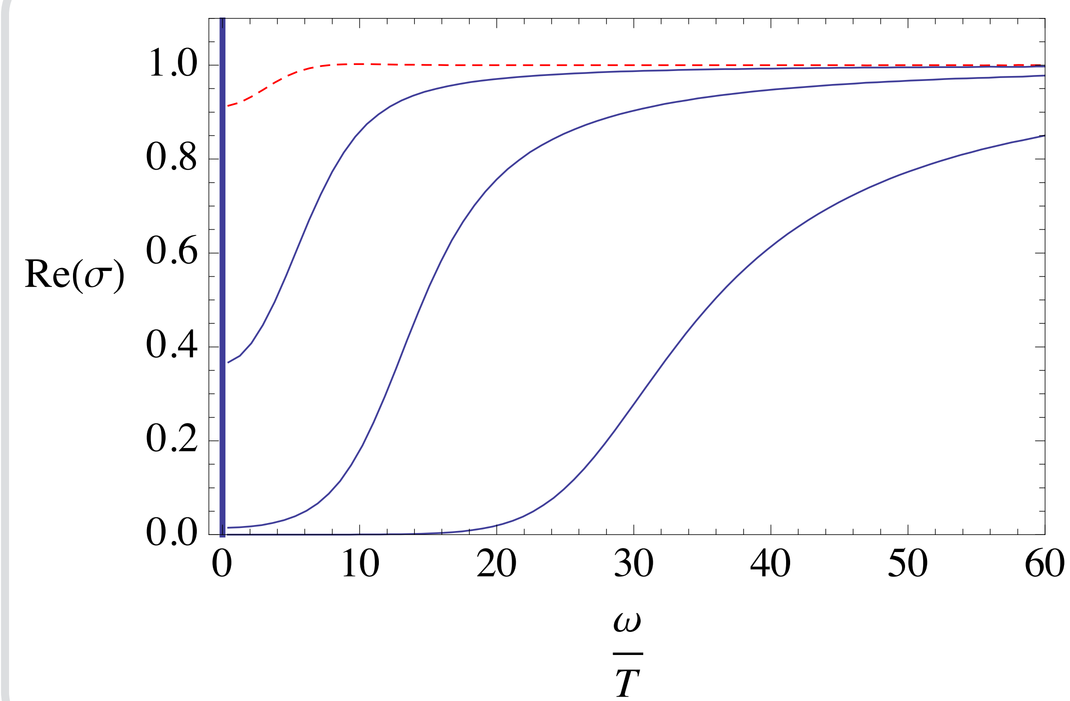
Kramers-Kronig relation

$$\chi(\omega) = \chi_R(\omega) + i\chi_I(\omega)$$

$$\chi_R(\omega) = \frac{1}{\pi}\mathcal{P}\int \frac{\chi_I(\omega')}{\omega' - \omega}d\omega' \,, \qquad \chi_I(\omega) = -\frac{1}{\pi}\mathcal{P}\int \frac{\chi_R(\omega')}{\omega' - \omega}d\omega'$$

0803.3295: Hartnoll, Herzog, Horowitz

Conductivity: superconducting phase



Introduction: Holographic model

1302.6586: Horowitz, Santos

The first holographic superconductor

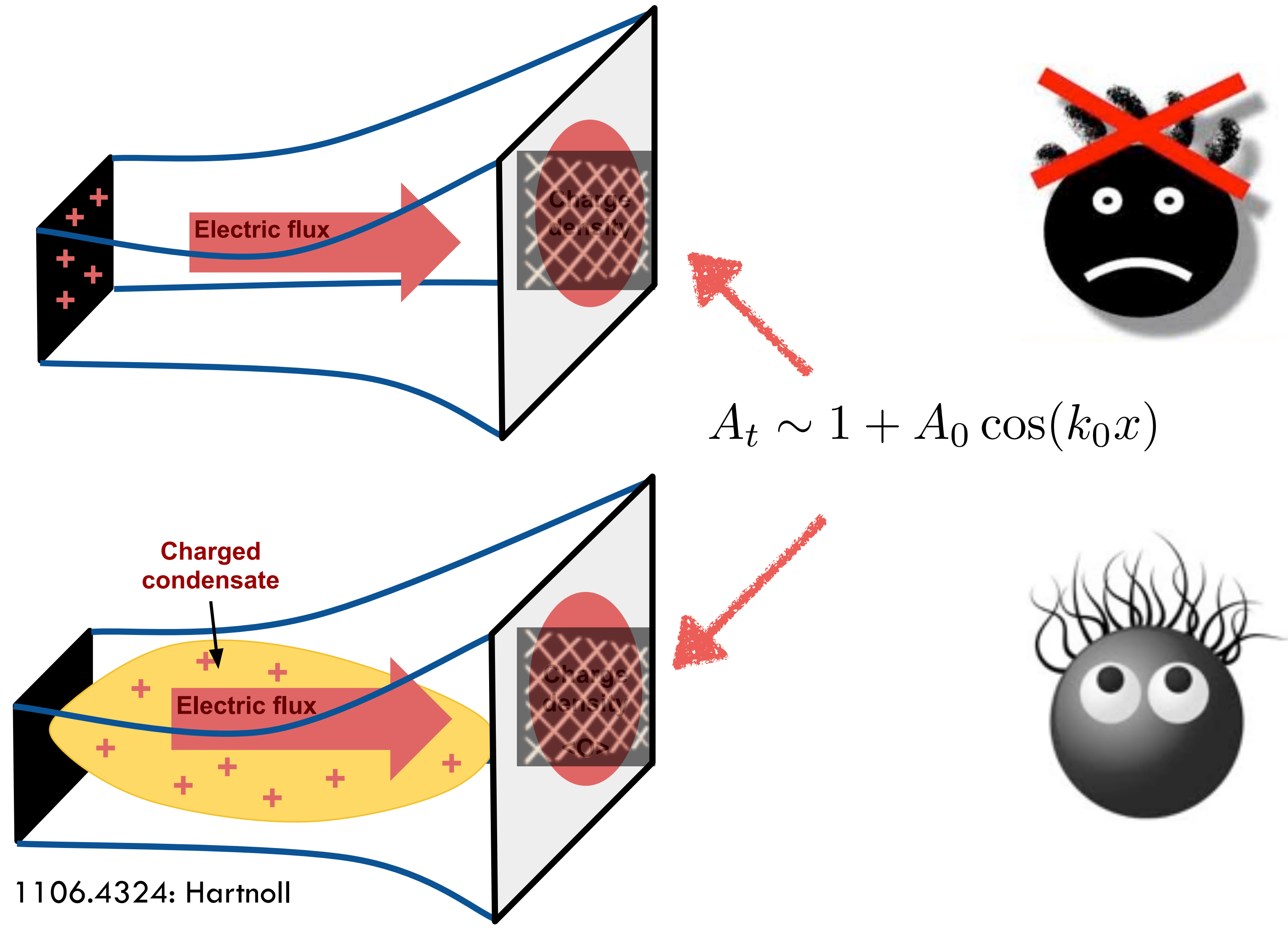
$$S_{\rm HHH} = \int_M \mathrm{d}^4x\sqrt{-g}\left[R + \frac{6}{L^2} - \frac{1}{4}F^2 - |D\Phi|^2 - m^2|\Phi|^2\right]$$

$\Phi = 0$

AdS-RN-black brane

$\Phi \neq 0$

Holographic superconductor



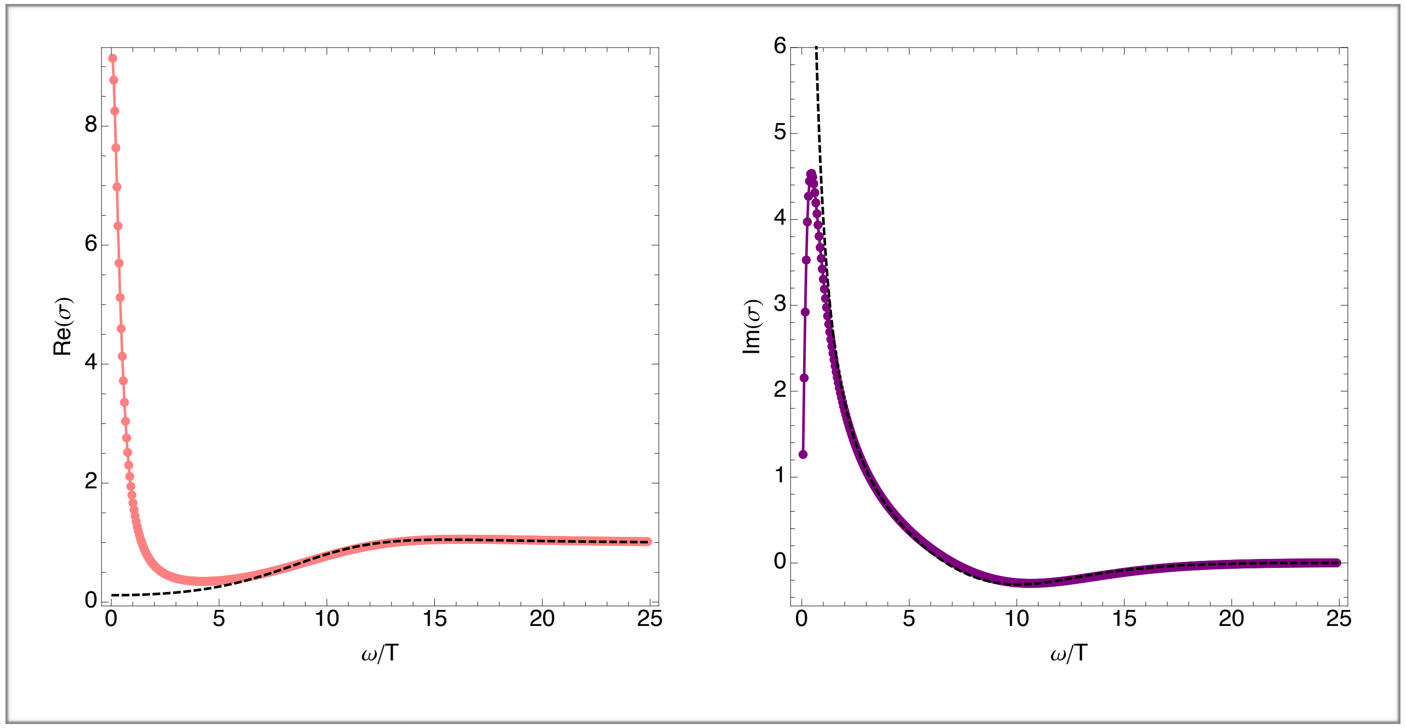
1106.4324: Hartnoll

Introduction: Holographic model

Momentum relaxation

$A_t \sim 1 + A_0 \cos(k_0 x)$

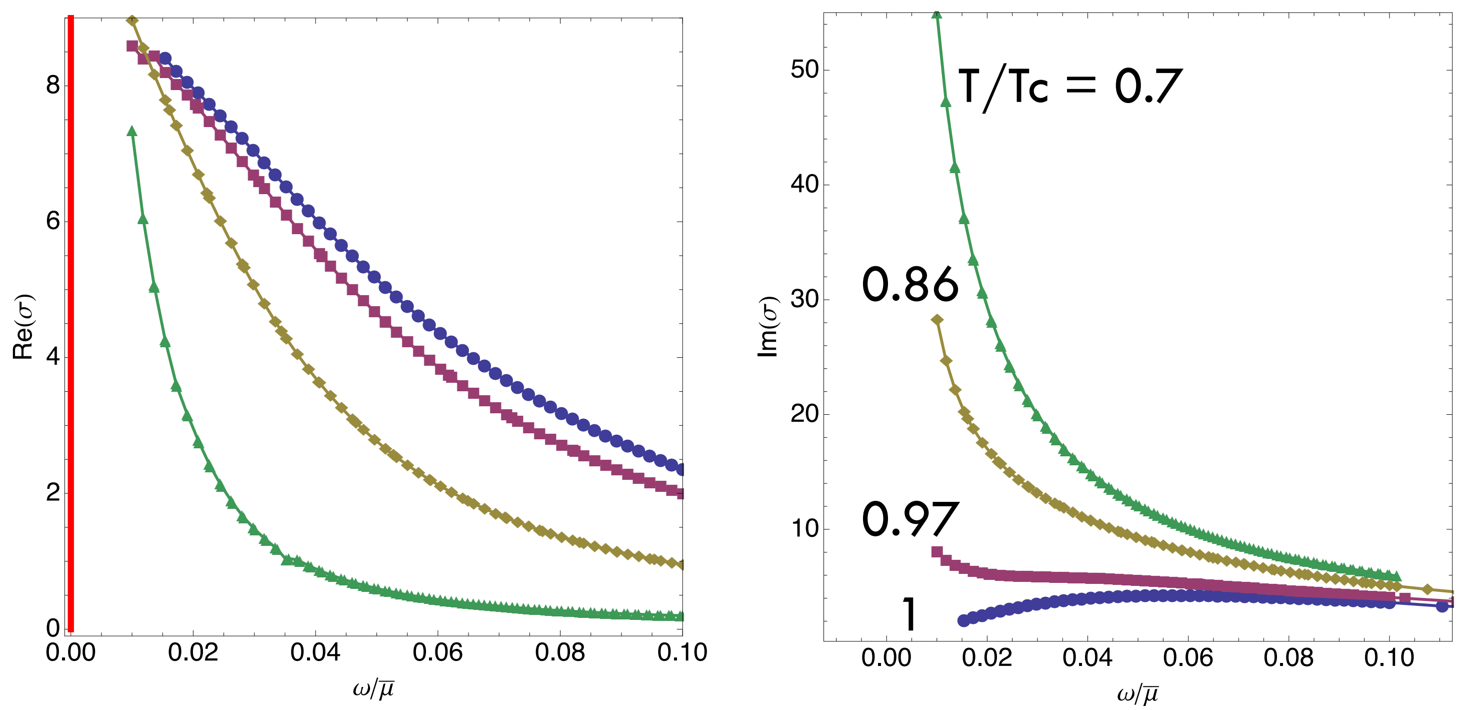
cf) $\phi \sim A_0 \cos(k_0 x)$



1204.0512, 1209.1098:
Horowitz, Santos, Tong

~~Two different delta functions~~

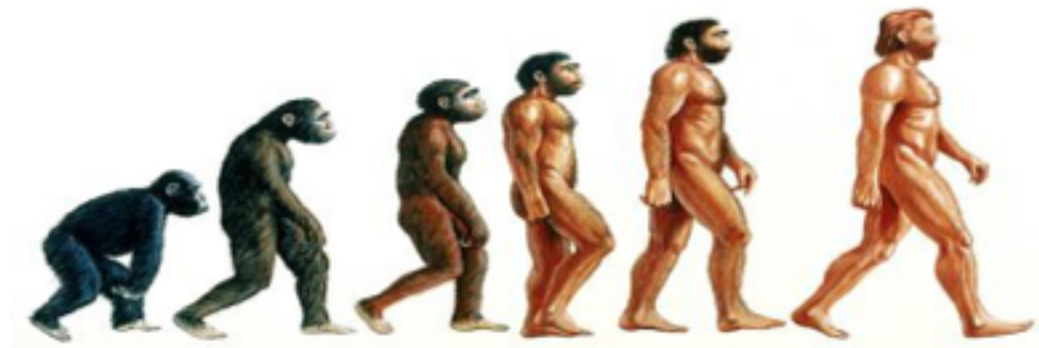
Only one delta function

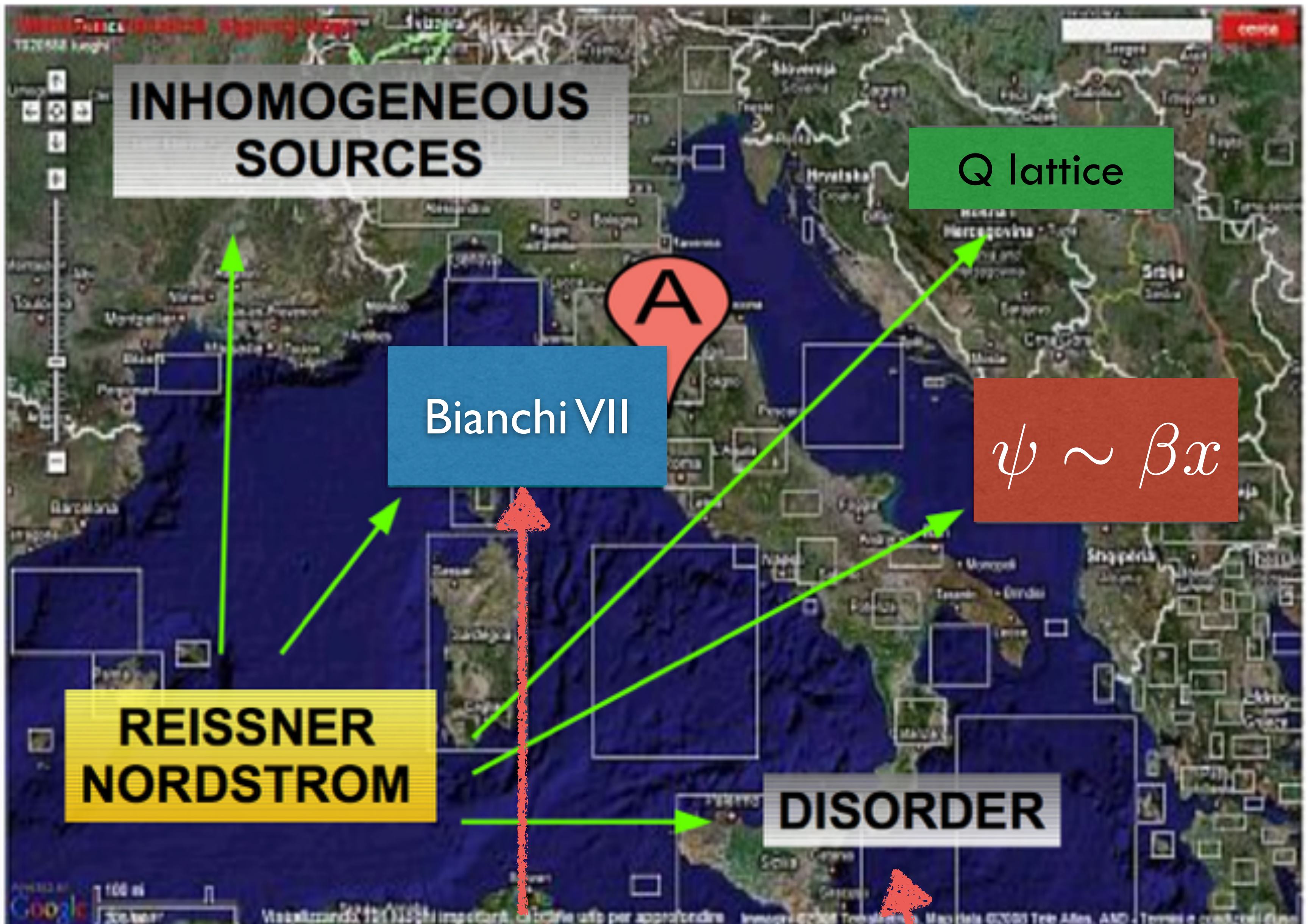


1302.6586: Horowitz, Santos

From Matteo Baggioli's slides

Holographic





Johanna Erdmenger
Rene Meyer

Daniel Arean
Leopoldo Pando Zayas

Momentum relaxation simplified (ODE)

$$\psi_I = \beta_{Ii} x^i = \beta \delta_{Ii} x^i$$

Andrade and Withers
1311.5157

Contents

Model

Action

$$S_{\rm HHH} = \int_M \mathrm{d}^{d+1}x\sqrt{-g}\left[R + \frac{d(d-1)}{L^2} - \frac{1}{4}F^2 - |D\Phi|^2 - m^2|\Phi|^2\right],$$

$$S_{\rm GH} = -2\int_{\partial M} \mathrm{d}^d x\sqrt{-\gamma}K \,,$$

$$D_M\Phi = (\nabla_M - iqA_M)\,\Phi$$

$$\boxed{S_\psi = \int_M \mathrm{d}^{d+1}x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1}^{d-1}(\partial\psi_I)^2\right]}$$

Equations of motion

$$R_{MN} - \frac{1}{2}g_{MN}\left(R + \frac{d(d-1)}{L^2} - \frac{1}{4}F^2 - |D\Phi|^2 - m^2|\Phi|^2 - \frac{1}{2}\sum_{I=1}^{d-1}(\partial\psi_I)^2\right)$$
$$= \frac{1}{2}\partial_M\psi_I\partial_N\psi_I + \frac{1}{2}F_{MQ}F_N{}^Q + \frac{1}{2}\left(D_M\Phi D_N\Phi^* + D_N\Phi D_M\Phi^*\right),$$

$$\nabla_M F^{MN} + iq(\Phi^* D^N\Phi - \Phi D^N\Phi^*) = 0\,,$$

$$\left(D^2 - m^2\right)\Phi = 0\,,$$

$$\nabla^2\psi_I = 0\,,$$

Ansatz

$$\mathrm{d}s^2 = -\mathcal{G}(r)e^{-\chi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\mathcal{G}(r)} + \frac{r^2}{L^2}(\mathrm{d}x^2 + \mathrm{d}y^2)\,,$$

$$A = A_t(r)\mathrm{d}t\,, \qquad \Phi = \Phi(r)\,, \qquad \psi_I = \beta_{Ii}x^i = \frac{\beta}{L^2}\delta_{Ii}x^i$$

Model

Action

$$S_{\mathrm{HHH}} = \int_M \mathrm{d}^{d+1}x\sqrt{-g}\left[R + \frac{d(d-1)}{L^2} - \frac{1}{4}F^2 - |D\Phi|^2 - m^2|\Phi|^2\right],$$

$$D_M\Phi = (\nabla_M - iqA_M)\,\Phi$$

$$S_{\mathrm{GH}} = -2\int_{\partial M} \mathrm{d}^d x\sqrt{-\gamma}K\,,$$

$$\boxed{S_\psi = \int_M \mathrm{d}^{d+1}x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1}^{d-1}(\partial\psi_I)^2\right]}$$

Equations of motion

$$\chi' + r\Phi'^2 + \frac{rq^2A_t^2\Phi^2e^{\chi}}{\mathcal{G}^2} = 0\,,$$

$$\Phi'^2 + \frac{e^{\chi}A_t'^2}{2\mathcal{G}} + \frac{2\mathcal{G}'}{\mathcal{G}r} + \frac{2}{r^2} - \frac{6}{\mathcal{G}L^2} + \frac{m^2\Phi^2}{\mathcal{G}} + \frac{q^2A_t^2\Phi^2e^{\chi}}{\mathcal{G}^2} = \frac{-\beta^2}{r^2\mathcal{G}L^2}$$

$$\Phi'' + \left(\frac{\mathcal{G}'}{\mathcal{G}} - \frac{\chi'}{2} + \frac{2}{r}\right)\Phi' + \left(\frac{q^2e^{\chi}A_t^2}{\mathcal{G}^2} - \frac{m^2}{\mathcal{G}}\right)\Phi = 0$$

$$A_t'' + \left(\frac{\chi'}{2} + \frac{2}{r}\right)A_t' - \frac{2q^2\Phi^2}{\mathcal{G}}A_t = 0$$

Ansatz

$$\mathrm{d}s^2 = -\underline{\mathcal{G}(r)}e^{\underline{-\chi(r)}}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\mathcal{G}(r)} + \frac{r^2}{L^2}(\mathrm{d}x^2 + \mathrm{d}y^2)\,,$$

$$A = \underline{A_t(r)}\mathrm{d}t\,, \qquad \Phi = \underline{\Phi(r)}\,, \qquad \psi_I = \underline{\beta_{Ii}x^i} = \frac{\beta}{L^2}\delta_{Ii}x^i$$

Background solution

KK, Kim, Park(2015)

Ansatz

$$ds^2 = -\mathcal{G}(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{\mathcal{G}(r)} + \frac{r^2}{L^2}(dx^2 + dy^2)\,,$$

$$A = A_t(r)dt\,, \qquad \Phi = \Phi(r)\,, \qquad \psi_I = \beta_{Ii}x^i = \frac{\beta}{L^2}\delta_{Ii}x^i$$

Equations of motion

$$\chi' + r\Phi'^2 + \frac{rq^2A_t^2\Phi^2e^\chi}{\mathcal{G}^2} = 0\,,$$

$$\Phi'^2 + \frac{e^\chi A_t'^2}{2\mathcal{G}} + \frac{2\mathcal{G}'}{\mathcal{G}r} + \frac{2}{r^2} - \frac{6}{\mathcal{G}L^2} + \frac{m^2\Phi^2}{\mathcal{G}} + \frac{q^2A_t^2\Phi^2e^\chi}{\mathcal{G}^2} = \frac{-\beta^2}{r^2\mathcal{G}L^2}$$

$$\Phi'' + \left(\frac{\mathcal{G}'}{\mathcal{G}} - \frac{\chi'}{2} + \frac{2}{r}\right)\Phi' + \left(\frac{q^2e^\chi A_t^2}{\mathcal{G}^2} - \frac{m^2}{\mathcal{G}}\right)\Phi = 0$$

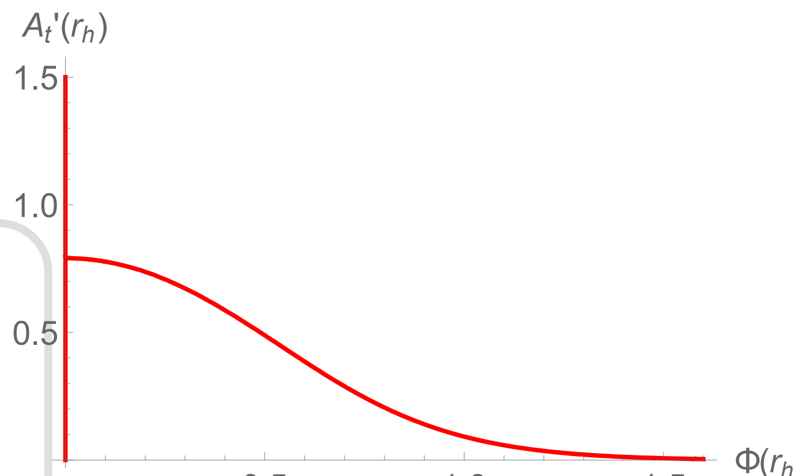
$$A_t'' + \left(\frac{\chi'}{2} + \frac{2}{r}\right)A_t' - \frac{2q^2\Phi^2}{\mathcal{G}}A_t = 0$$

Horizon

$$\chi(r_h)$$

$$\mathcal{G}(r_h) = 0$$

$$A_t(r_h) = 0 \qquad \boxed{\begin{matrix} A_t'(r_h) \\ \Phi(r_h) \end{matrix}}$$



Boundary

$$\chi(r) \sim \chi^{(0)} + \frac{(\Phi^{(1)})^2}{2r^2} + \frac{4\Phi^{(1)}\Phi^{(2)}}{3r^3}\cdots\,,$$

$$\mathcal{G}(r) \sim \frac{r^2}{L^2} + \frac{(\Phi^{(1)})^2}{2L^2} + \frac{\beta^2}{2L^2} + \frac{\mathcal{G}^{(1)}}{rL^2} + \cdots\,,$$

$$A_t(r) \sim A_t^{(0)} - \frac{A_t^{(1)}}{r} + \cdots\,,$$

$$\Phi(r) \sim \boxed{\frac{\Phi^{(1)}}{r}} + \frac{\Phi^{(2)}}{r^2} + \cdots\,,$$

Normal state solution

No condensate $\Phi = 0$

$$ds^2 = -\mathcal{G}(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\mathcal{G}(r)} + \frac{r^2}{L^2}(\mathrm{d}x^2 + \mathrm{d}y^2)\,, \quad \chi(r) = 0\,,$$

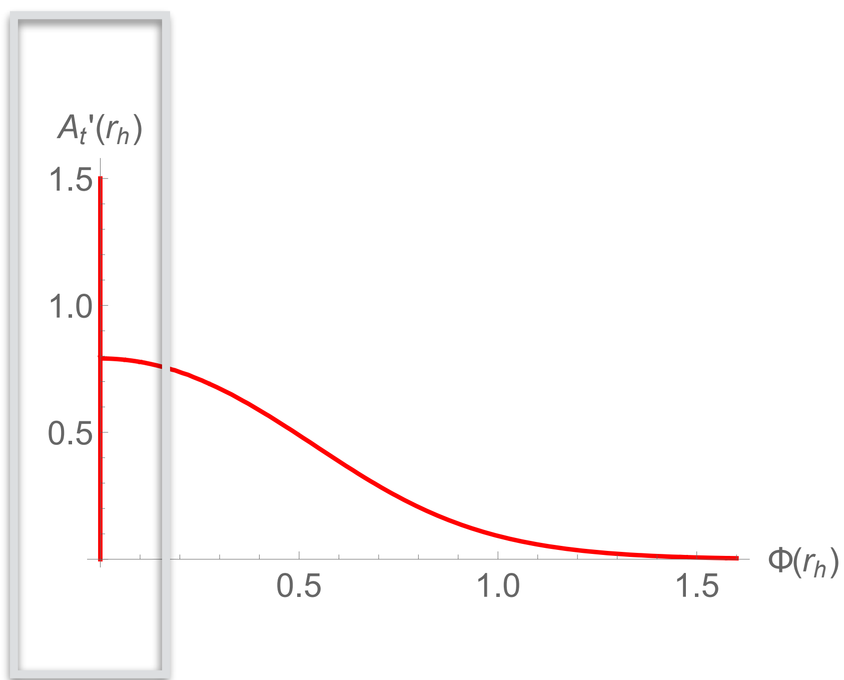
$$\mathcal{G}(r) = \frac{1}{L^2}\left(r^2 \boxed{-\frac{\beta^2}{2}} - \frac{m_0}{r} + \frac{\mu^2}{4}\frac{r_h^2}{r^2}\right)\,,$$

$$A = \frac{\mu}{L}\left(1 - \frac{r_h}{r}\right)\mathrm{d}t\,,$$

$$\psi_I = \beta_{Ii}x^i = \frac{\beta}{L^2}\delta_{Ii}x^i\,,$$

$$m_0 = r_H^3\left(1 + \frac{\mu^2}{4r_h^2} \boxed{-\frac{\beta^2}{2r_h^2}}\right)$$

$$T_H = \frac{\mathcal{G}'(r_h)}{4\pi} = \frac{1}{4\pi L^2}\left(3r_h - \frac{\mu^2 + \boxed{2\beta^2}}{4r_h}\right)$$



Contents

Normal state solution

Bardoux, Caldarelli, Charmousis (2012)
Andrade, Withers(2013)
KK, Kim, Park(2015)

No condensate $\Phi = 0$

$$ds^2 = -\mathcal{G}(r)dt^2 + \frac{dr^2}{\mathcal{G}(r)} + \frac{r^2}{L^2}(dx^2 + dy^2)\,, \quad \chi(r) = 0\,,$$

$$\mathcal{G}(r) = \frac{1}{L^2}\left(r^2 \boxed{-\frac{\beta^2}{2}} - \frac{m_0}{r} + \frac{\mu^2}{4}\frac{r_h^2}{r^2}\right)\,,$$

$$A = \frac{\mu}{L}\left(1 - \frac{r_h}{r}\right)dt\,,$$

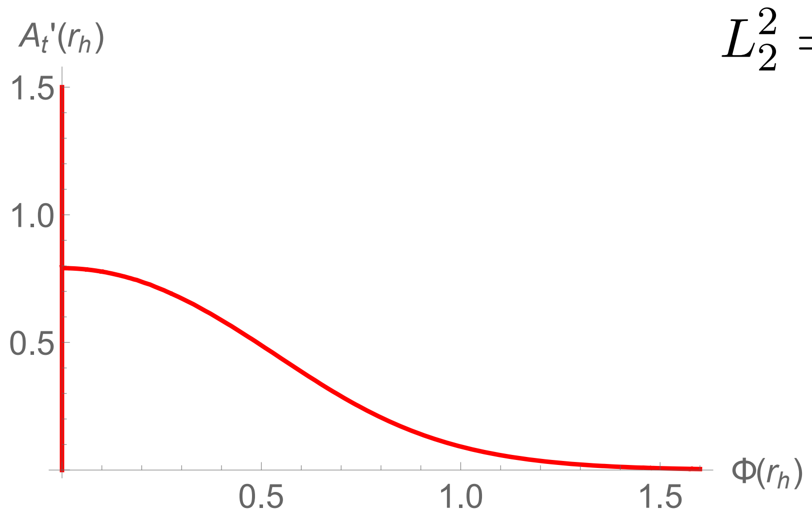
$$\psi_I = \beta_{Ii}x^i = \frac{\beta}{L^2}\delta_{Ii}x^i\,,$$

$$m_0 = r_H^3\left(1 + \frac{\mu^2}{4r_h^2} \boxed{-\frac{\beta^2}{2r_h^2}}\right)$$

$$T_H = \frac{\mathcal{G}'(r_h)}{4\pi} = \frac{1}{4\pi L^2}\left(3r_h - \frac{\mu^2 + \boxed{2\beta^2}}{4r_h}\right)$$

Near horizon geometry of the extremal black brane Andrade, Withers(2013)

$\mathrm{AdS}_2 \times \mathbb{R}^{d-1}$ with the effective radius of AdS_2



$$L_2^2 = \frac{L_{d+1}^2}{d(d-1)}\frac{(d-1)\beta^2 + (d-2)^2\mu^2}{\beta^2 + (d-2)^2\mu^2}$$

d=3

$$L_2^2 = \frac{L^2}{6}\left(1 + \frac{\frac{\beta^2}{\mu^2}}{1 + \frac{\beta^2}{\mu^2}}\right)$$

Instability of normal state at zero T

BF bound of scalar field with mass M in AdS_{d+1} with the radius L_{d+1}

$$M^2 L_{d+1}^2 = -\frac{d^2}{4} \qquad \boxed{M^2 L_2^2 = -\frac{1}{4} \quad (d=1)}$$

- Effective AdS_2 radius at zero T

$$L_2^2 = \frac{L^2}{6}\left(1 + \frac{\frac{\beta^2}{\mu^2}}{1+\frac{\beta^2}{\mu^2}}\right)$$

- Effective mass near horizon at zero T

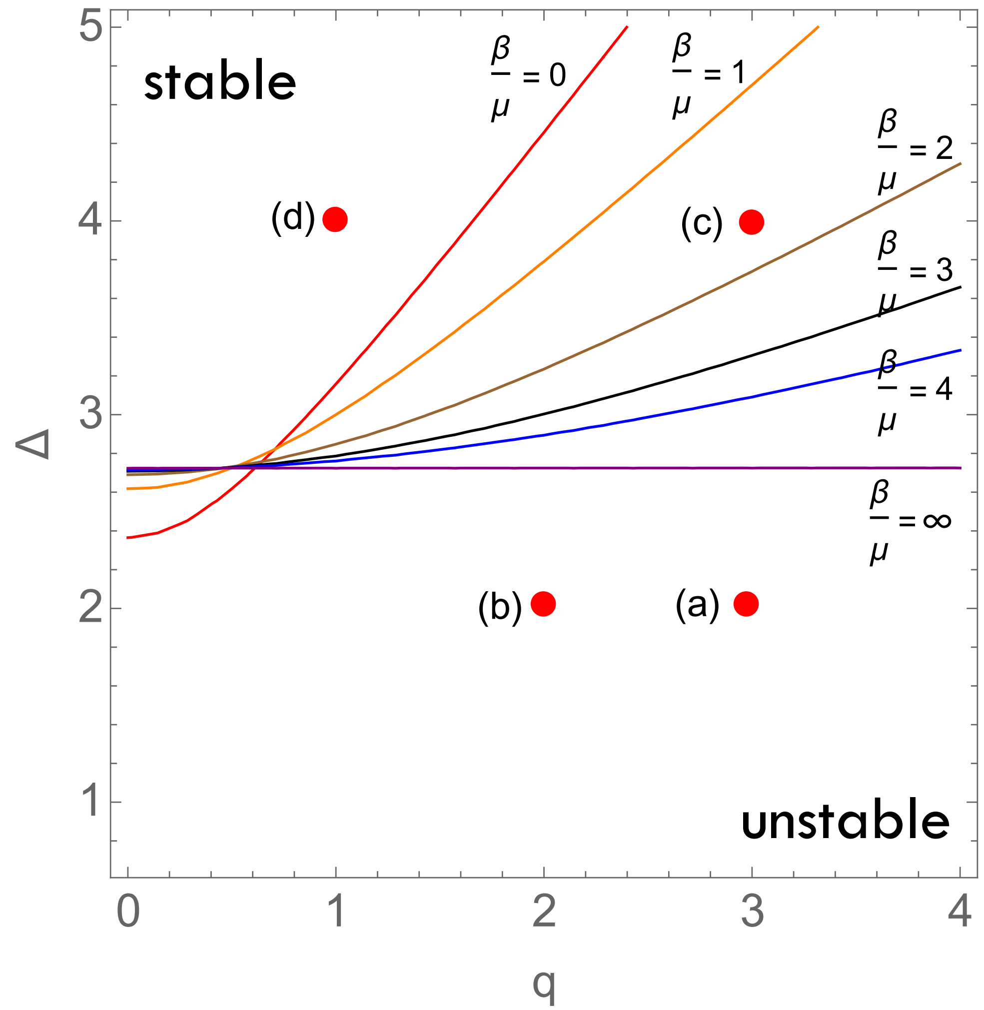
$$\left(\nabla^2 - \boxed{m^2 - q^2 g^{tt} A_t^2}\right)\Phi = 0$$

$$m_{\text{eff}}^2 = \lim_{r\to r_h}\lim_{T\to 0}\left(m^2 + q^2 g^{tt} A_t^2\right) = m^2 - \frac{2q^2/L^2}{1+\frac{\beta^2}{\mu^2}}$$

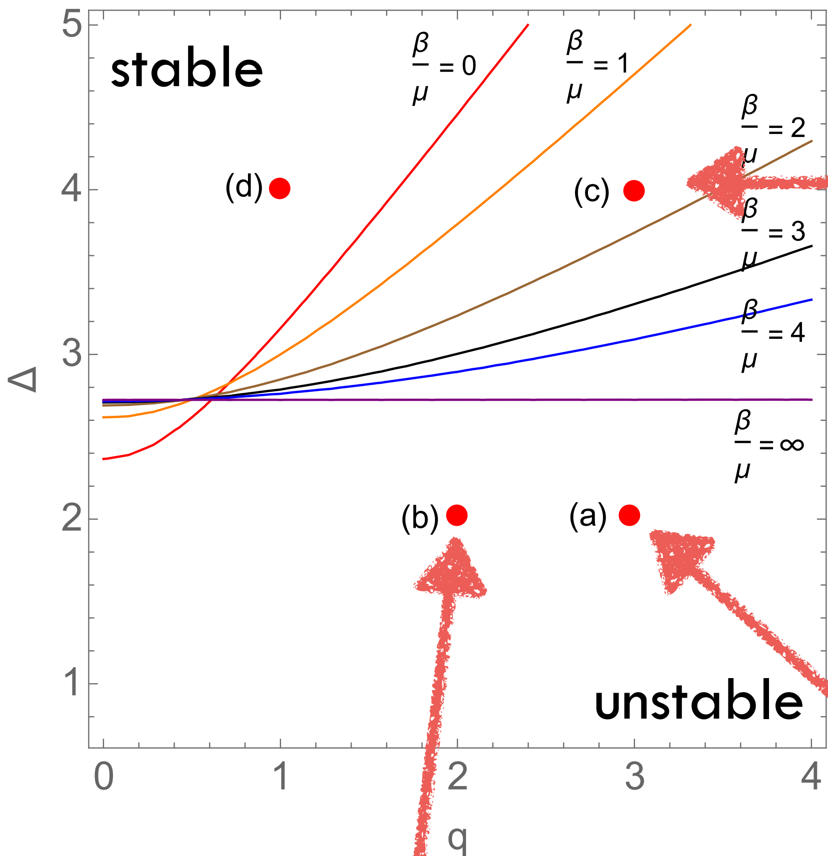
Instability condition

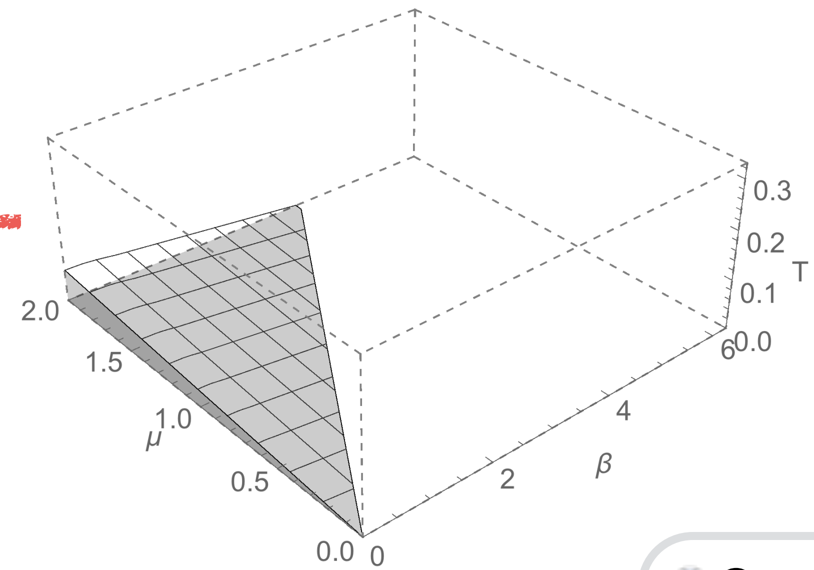
$$m_{\text{eff}}^2 L_2^2 = \left[m^2 L^2 - \frac{2q^2}{1+\frac{\beta^2}{\mu^2}}\right]\left[\frac{1}{6}\left(1+\frac{\frac{\beta^2}{\mu^2}}{1+\frac{\beta^2}{\mu^2}}\right)\right] < -\frac{1}{4}$$

$$m^2 L^2 = \Delta(\Delta - 3)$$



Phase diagram



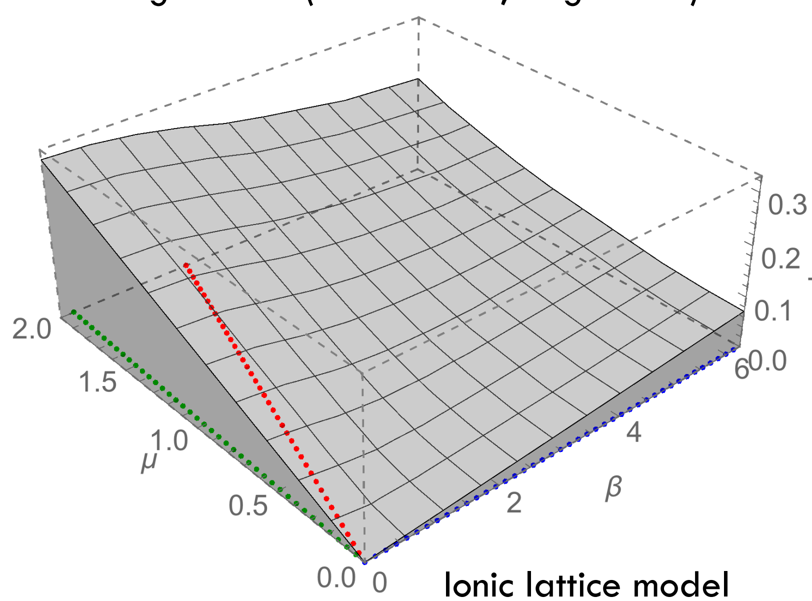


(c) $\Delta = 4, q = 3$

- Quantum phase transition near $\beta/\mu = 1.5$
- Zero T case (Ongoing)

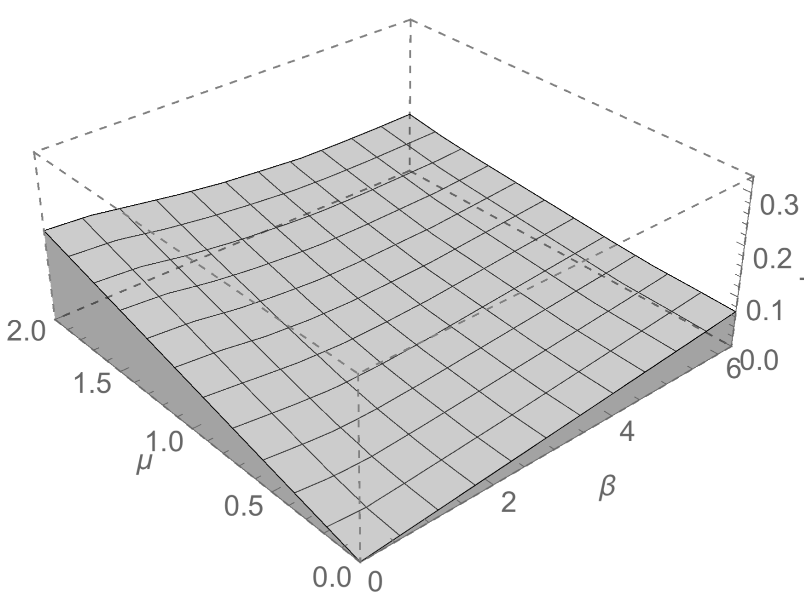
Q-lattice(1410.1761, Ling et al),
single scalar(1401.6501, Koga et al)

- non-monotonic beta-dependence of transition temperature
- beta-induced superconductor (Ongoing, other momentum relaxation models?)

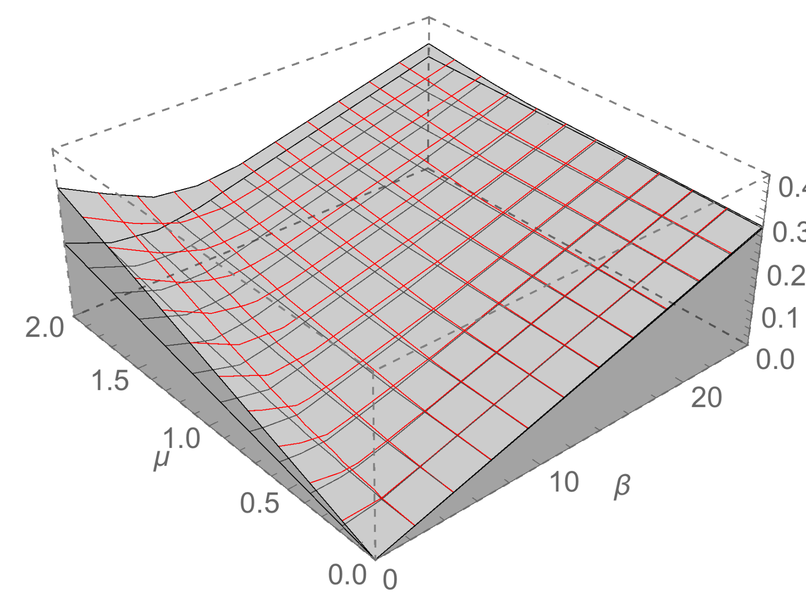


Ionic lattice model
(1302.6586: Horowitz, Santos)

(a) $\Delta = 2, q = 3$



(b) $\Delta = 2, q = 2$



Contents

Action for fluctuations

KK, Kim, Park(2015)

Action

$$S_{\mathrm{HHH}} = \int_M \mathrm{d}^4x\sqrt{-g}\left[R + \frac{6}{L^2} - \frac{1}{4}F^2 - |D\Phi|^2 - m^2|\Phi|^2\right],$$

$$S_{\mathrm{GH}} = -2\int_{\partial M} \mathrm{d}^3x\sqrt{-\gamma}K\,, \quad S_\psi = \int_M \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right]$$

Equations of motion

$$R_{MN} - \frac{1}{2}g_{MN}\left(R + \frac{d(d-1)}{L^2} - \frac{1}{4}F^2 - |D\Phi|^2 - m^2|\Phi|^2 - \frac{1}{2}\sum_{I=1}^{d-1}(\partial\psi_I)^2\right)$$
$$= \frac{1}{2}\partial_M\psi_I\partial_N\psi_I + \frac{1}{2}F_{MQ}F_N{}^Q + \frac{1}{2}\left(D_M\Phi D_N\Phi^* + D_N\Phi D_M\Phi^*\right),$$

$$\nabla_M F^{MN} + iq(\Phi^* D^N\Phi - \Phi D^N\Phi^*) = 0\,,$$
$$\left(D^2 - m^2\right)\Phi = 0\,,$$
$$\nabla^2\psi_I = 0\,,$$

Ansatz

$$\mathrm{d}s^2 = -\mathcal{G}(r)e^{-\chi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\mathcal{G}(r)} + \frac{r^2}{L^2}(\mathrm{d}x^2 + \mathrm{d}y^2)\,,$$

$$A = A_t(r)\mathrm{d}t\,, \qquad \Phi = \Phi(r)\,, \qquad \psi_I = \beta_{Ii}x^i = \frac{\beta}{L^2}\delta_{Ii}x^i$$

Fluctuations

$$\delta A_x(t,r) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}a_x(\omega,r)$$

$$\delta g_{tx}(t,r) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}r^2 h_{tx}(\omega,r)$$

$$\delta\psi_1(t,r) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}\xi(\omega,r)$$

Action for fluctuations

KK, Kim, Park(2015)

Action

$$S^{(2)}_{\rm ren} = \frac{V_2}{2}\int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\left(-\rho\bar{a}^{(0)}_x h^{(0)}_{tx} - \epsilon\bar{h}^{(0)}_{tx}h^{(0)}_{tx} + \bar{a}^{(0)}_x a^{(1)}_x - 3\bar{h}^{(0)}_{tx}h^{(3)}_{tx} + 3\bar{\xi}^{(0)}\xi^{(3)}\right)$$

Equations of motion

$$a_x'' + \left(\frac{\mathcal{G}'}{\mathcal{G}} - \frac{\chi'}{2}\right)a_x' + \left(\frac{\omega^2}{\mathcal{G}^2}e^\chi - \frac{2q^2\Phi^2}{\mathcal{G}}\right)a_x + \frac{r^2e^\chi A_t'}{\mathcal{G}}h_{tx}' = 0\,,$$

$$h_{tx}' + \frac{A_t'}{r^2}a_x + \frac{i\beta\mathcal{G}e^{-\chi}}{r^2\omega}\xi' = 0\,,$$

$$\xi'' + \left(\frac{\mathcal{G}'}{\mathcal{G}} - \frac{\chi'}{2} + \frac{2}{r}\right)\xi' - \frac{i\beta\omega e^\chi}{\mathcal{G}^2}h_{tx} + \frac{\omega^2e^\chi}{\mathcal{G}^2}\xi = 0\,,$$

$$h_{tx} = h^{(0)}_{tx} + \frac{1}{r^2}h^{(2)}_{tx} + \frac{1}{r^3}h^{(3)}_{tx} + \cdots\,,$$

$$a_x = a^{(0)}_x + \frac{1}{r}a^{(1)}_x + \cdots\,,$$

$$\xi = \xi^{(0)} + \frac{1}{r^2}\xi^{(2)} + \frac{1}{r^3}\xi^{(3)} + \cdots\,,$$

Ansatz

$$\mathrm{d}s^2 = -\mathcal{G}(r)e^{-\chi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\mathcal{G}(r)} + \frac{r^2}{L^2}(\mathrm{d}x^2 + \mathrm{d}y^2)\,,$$

$$A = A_t(r)\mathrm{d}t\,,\qquad \Phi = \Phi(r)\,,\qquad \psi_I = \beta_{Ii}x^i = \frac{\beta}{L^2}\delta_{Ii}x^i$$

Fluctuations

$$\delta A_x(t,r) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}a_x(\omega,r)$$

$$\delta g_{tx}(t,r) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}r^2h_{tx}(\omega,r)$$

$$\delta\psi_1(t,r) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}\xi(\omega,r)$$

AC electric conductivity

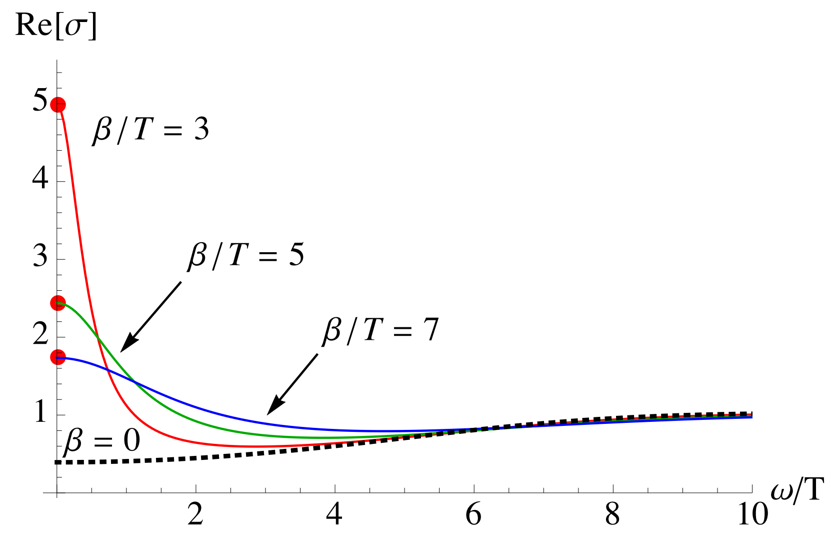
KK, Kim, Sin, Seo(2014)

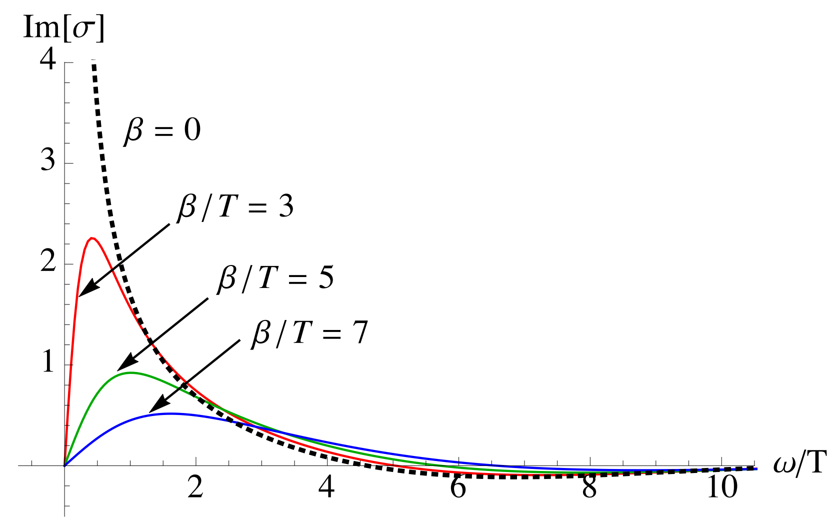
DC limit

$$\boxed{\sigma = 1 + \frac{\mu^2}{\beta^2}}$$

Andrade, Withers 1311.5157

$$\mu/T = 6$$

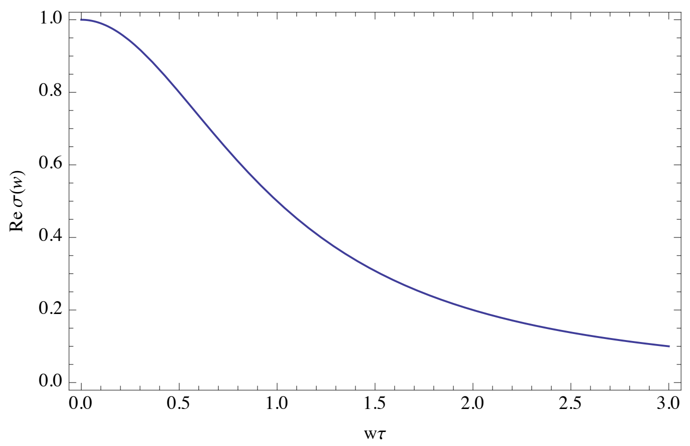


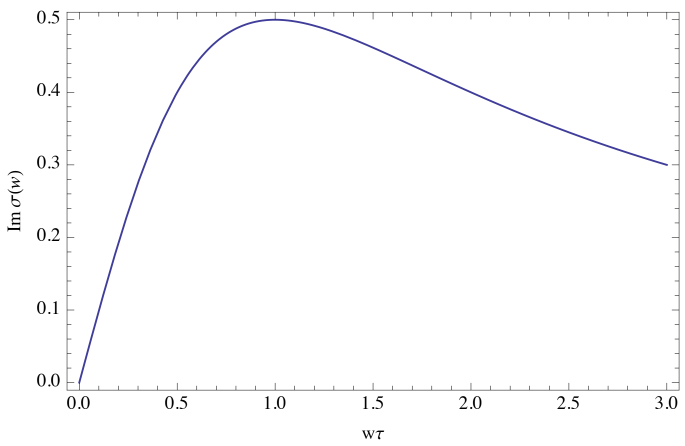


Drude model

$$\frac{dp}{dt} = -\frac{1}{\tau}p + qE \qquad p = mv, \quad J = nqv$$

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau} \qquad \sigma_0 = \frac{nq^2\tau}{m}$$





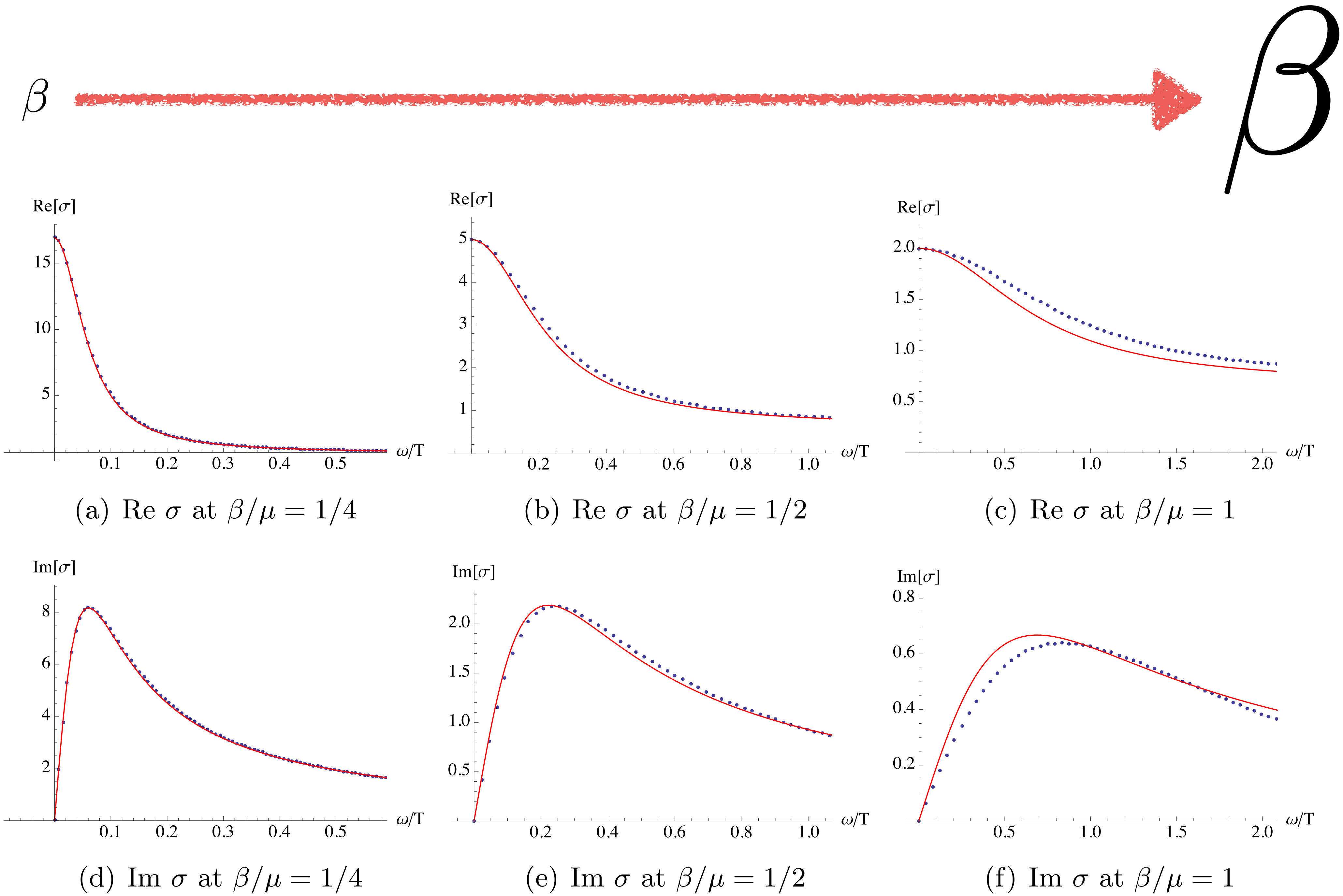
Ward identity

Andrade, Withers 1311.5157

$$\nabla^\nu \langle T_{\nu\mu} \rangle = \partial_\mu \phi \langle \mathcal{O} \rangle + F_{\mu\nu} \langle J^\nu \rangle$$

$$\partial_t \langle \delta p_x \rangle = \beta \langle \delta \mathcal{O} \rangle + \langle J^t \rangle \delta E_x$$

Drude/non-Drude



(a) Re σ at $\beta/\mu = 1/4$ (b) Re σ at $\beta/\mu = 1/2$ (c) Re σ at $\beta/\mu = 1$

(d) Im σ at $\beta/\mu = 1/4$ (e) Im σ at $\beta/\mu = 1/2$ (f) Im σ at $\beta/\mu = 1$

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau} + \sigma_Q$$

1405.3651: Hartnoll

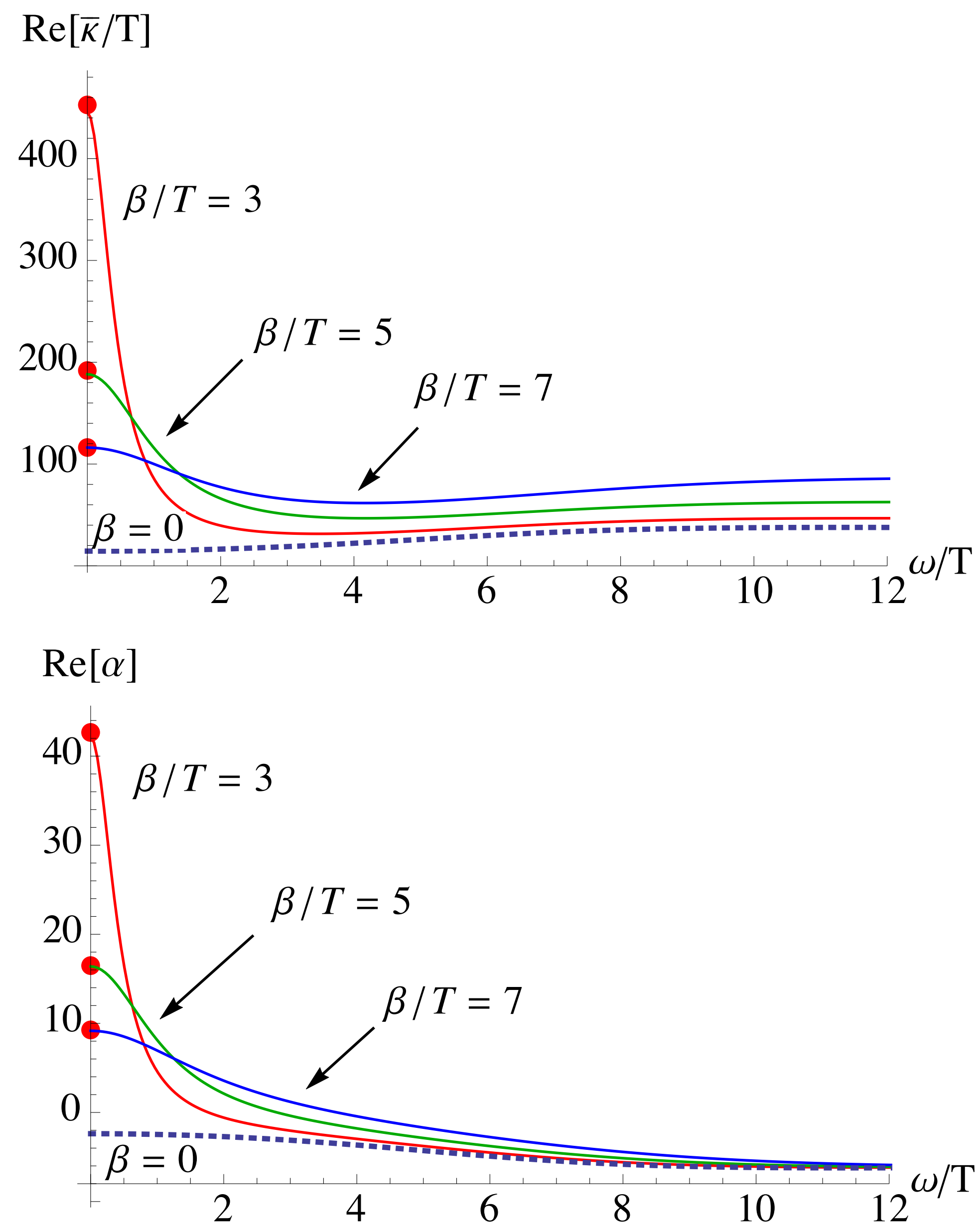
Thermal and thermoelectric conductivity

$\mu/T = 6$

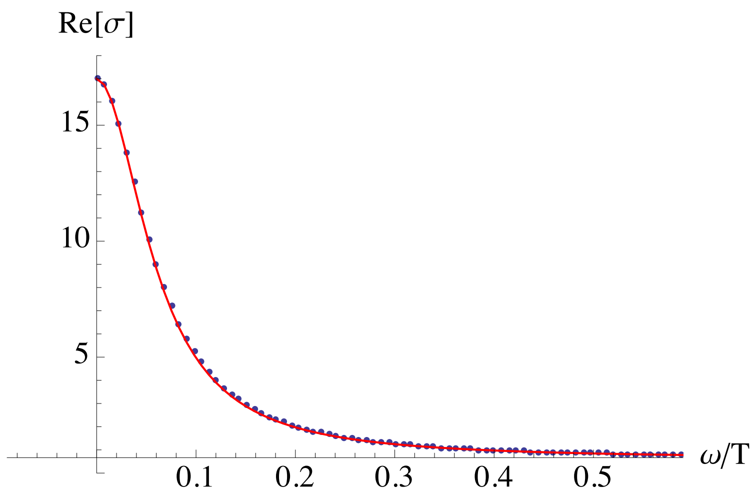
$$\frac{\bar{\kappa}}{T} = \frac{(4\pi)^2}{\beta^2} r_0^2$$

DC results:
Donos and Gauntlett
1406.4742

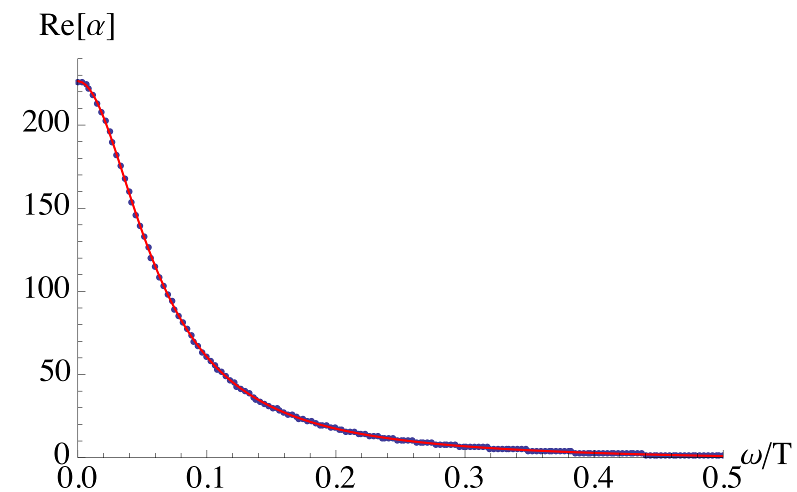
$$\alpha = \frac{4\pi\mu}{\beta^2} r_0$$



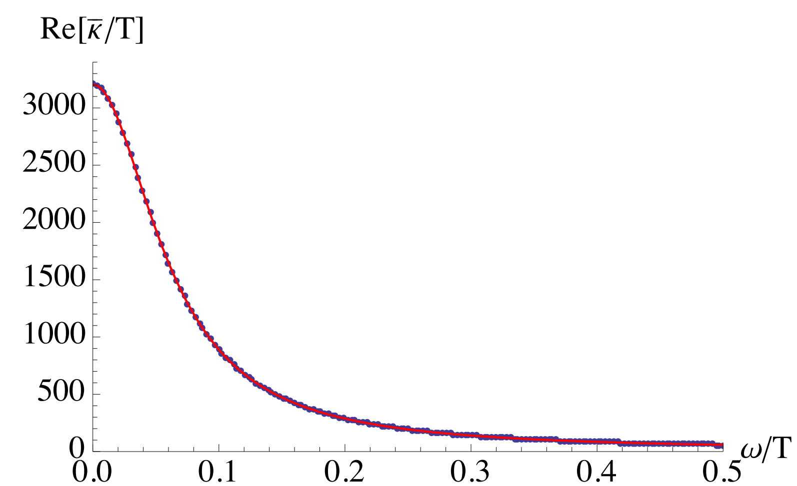
Relaxation time



(a) Re σ at $\beta/\mu = 1/4$



(a) Re α

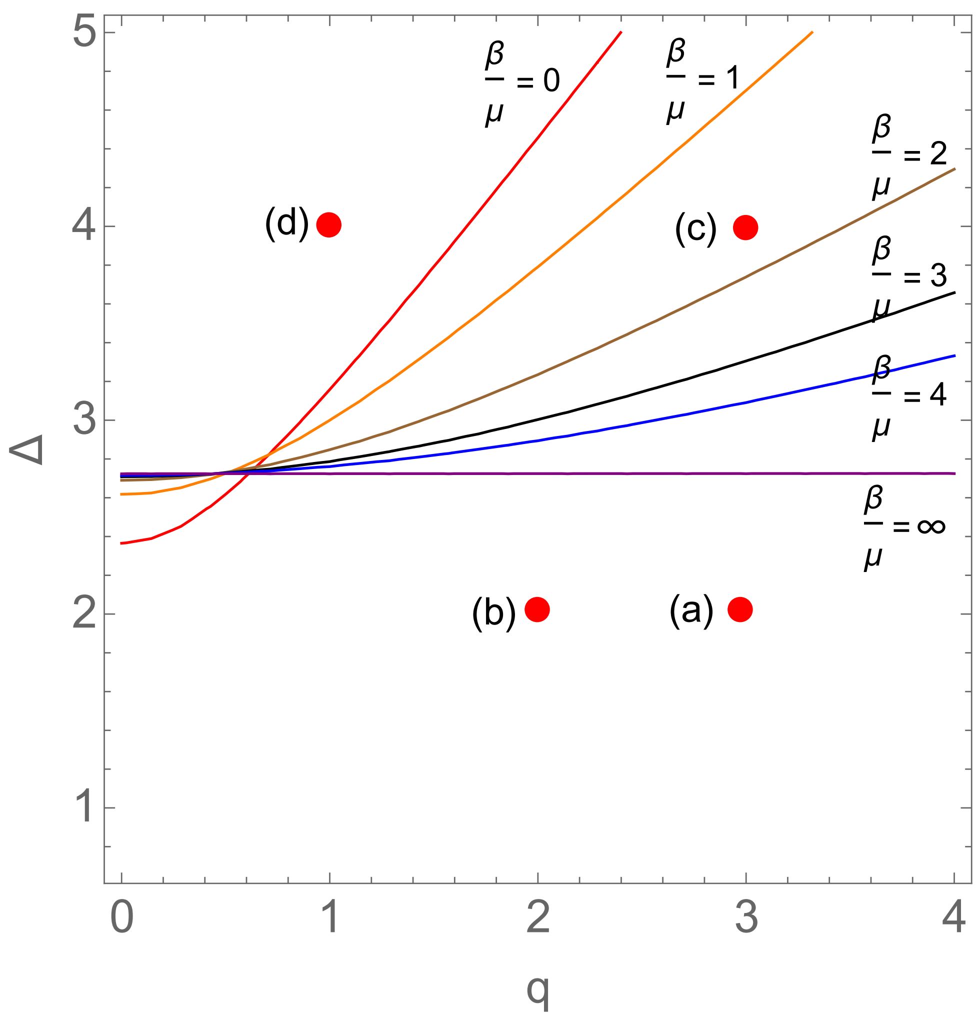


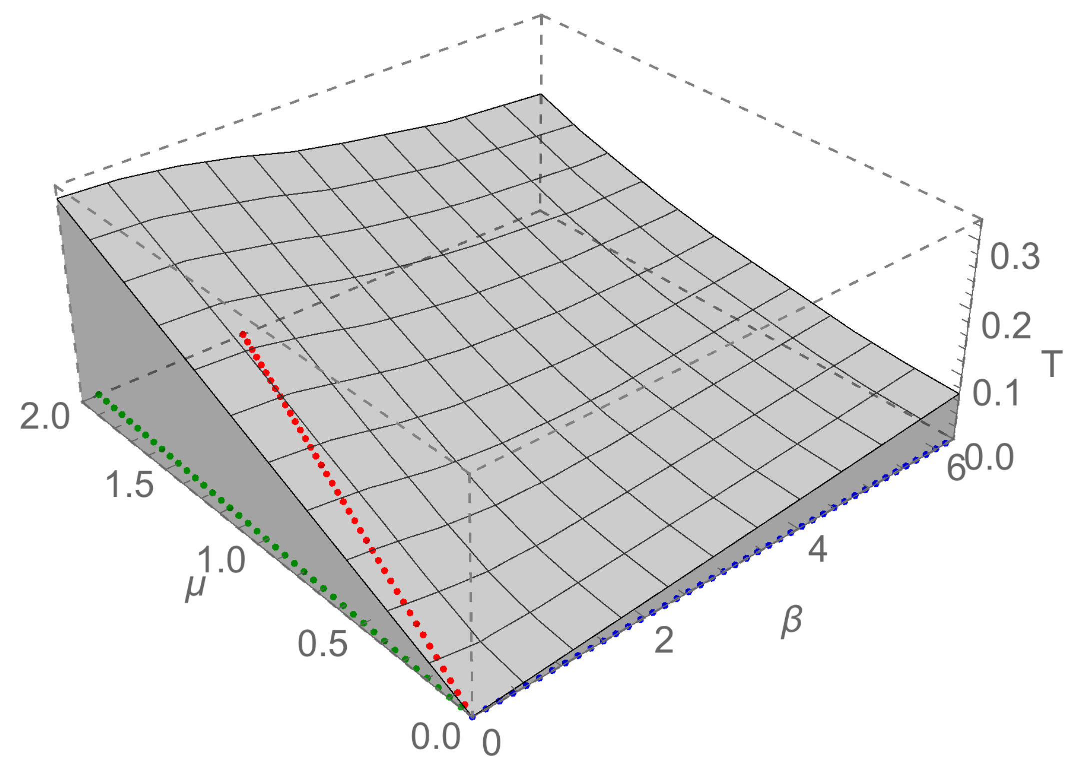
(c) Re $\bar{\kappa}/T$

Same relaxation time

Contents

Parameter set

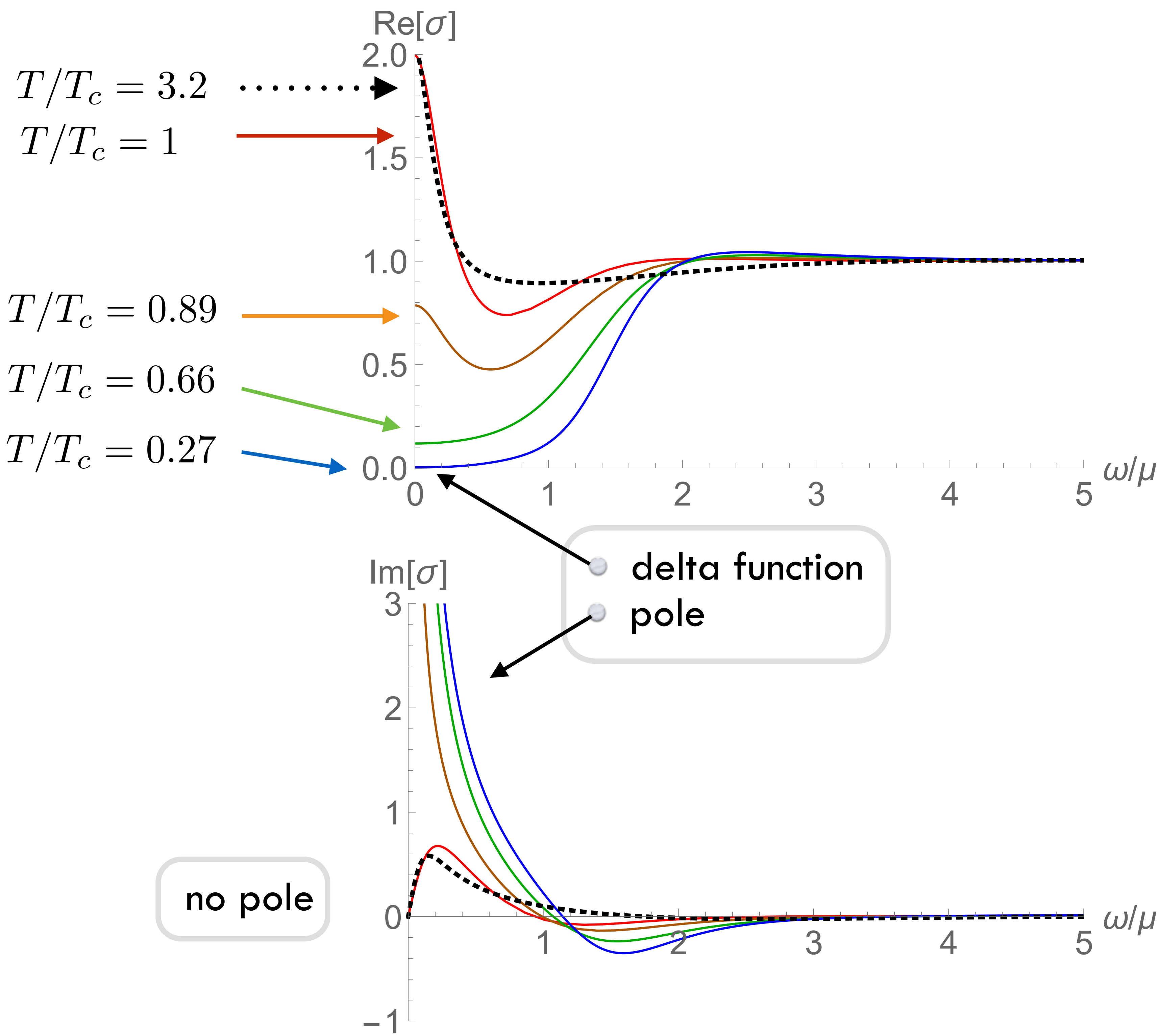




(a) $\Delta = 2, q = 3$

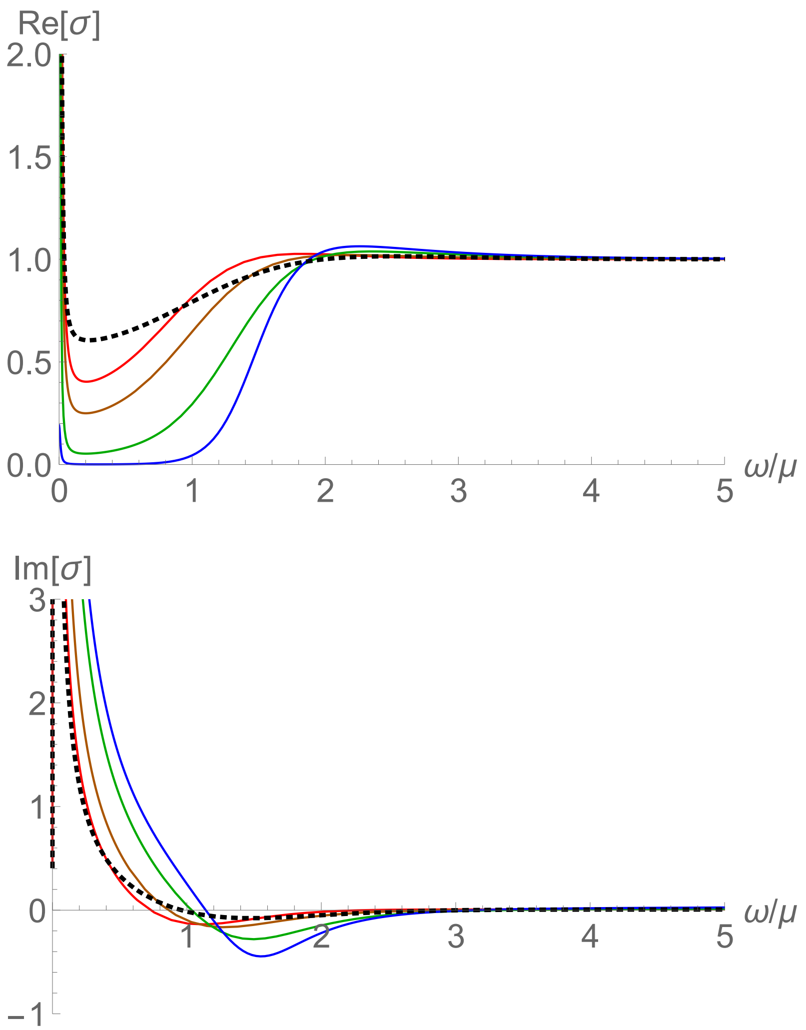
Electric AC Conductivity

$\beta/\mu = 1$

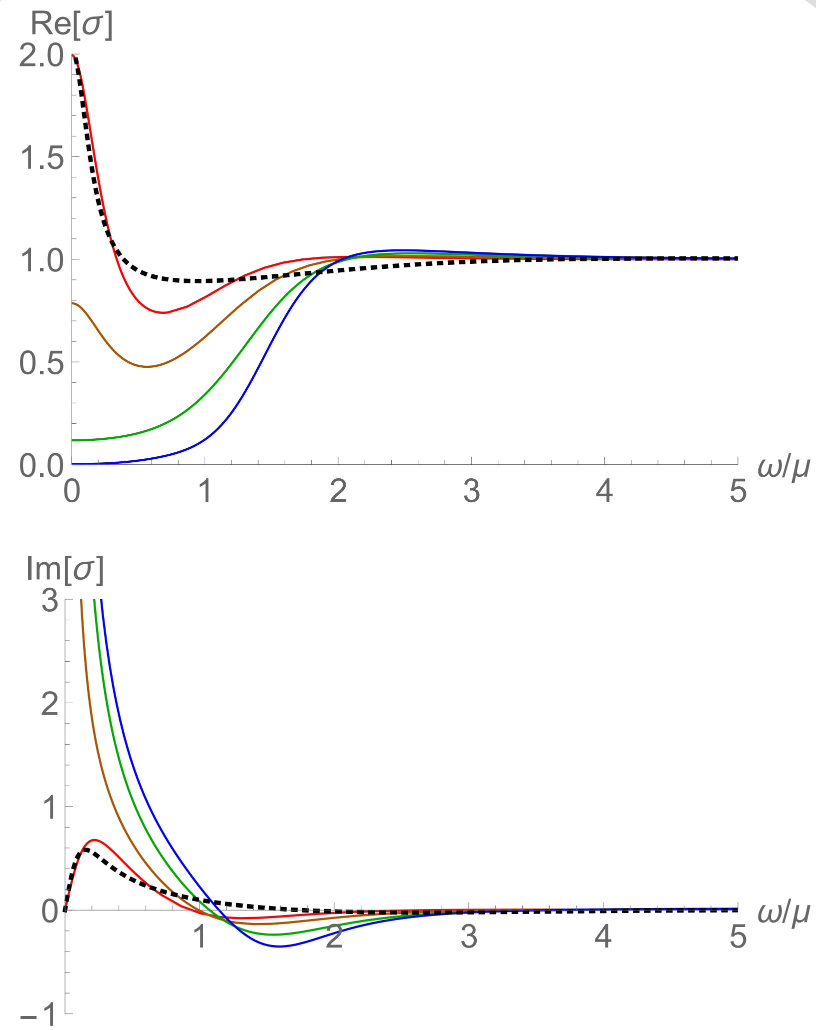


β-Effect

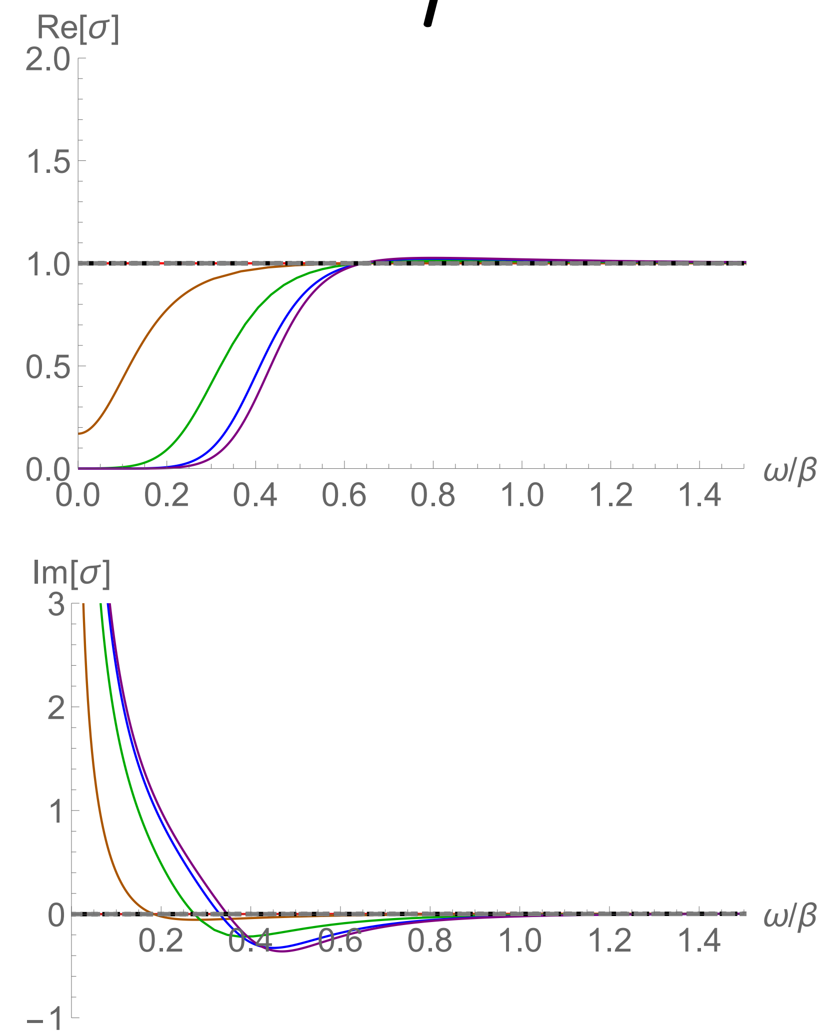
β → β



(a) $\beta/\mu = 0.1$, $T/T_c = 1.52, 1, 0.94, 0.76, 0.37$ (dotted, red, orange, green, blue)

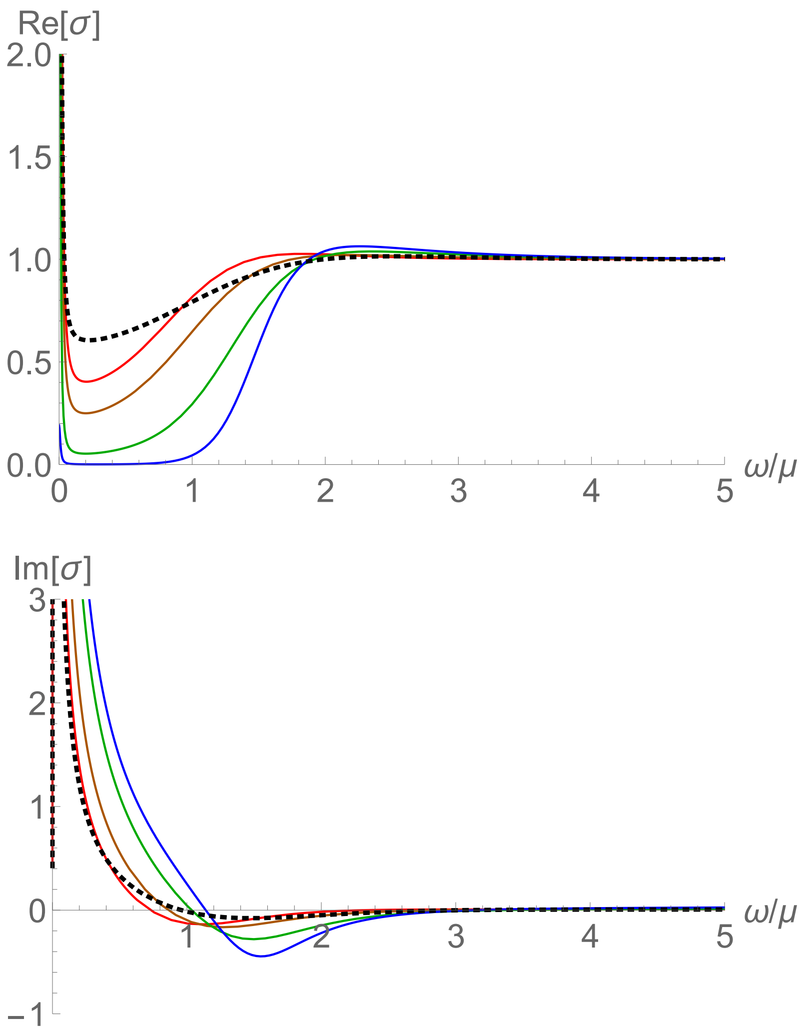


(b) $\beta/\mu = 1$, $T/T_c = 3.2, 1, 0.89, 0.66, 0.27$ (dotted, red, orange, green, blue)

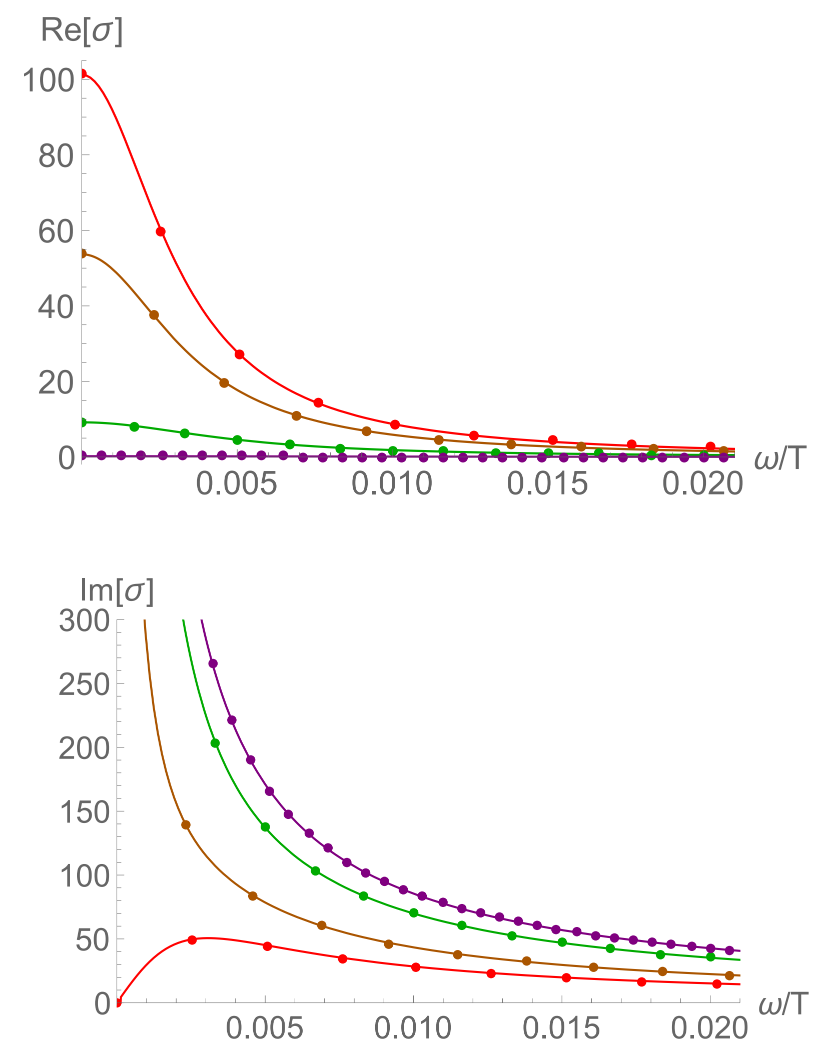


(c) $\beta/\mu \to \infty (\mu = 0)$, $T/T_c = 13.2, 3.5, 1, 0.95, 0.7, 0.4, 0.25$ (dashed, dotted, red, orange, green, blue, purple)

Two fluid model and Drude peak



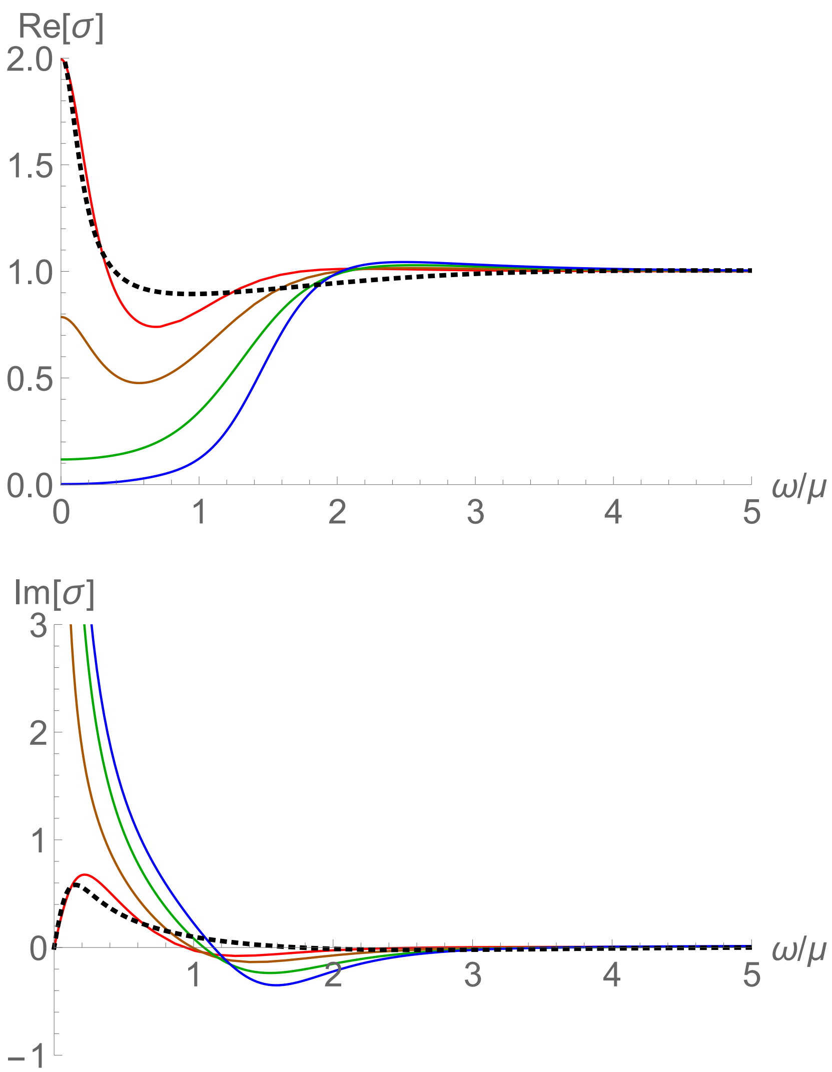
(a) $\beta/\mu = 0.1$, $T/T_c = 1.52, 1, 0.94, 0.76, 0.37$ (dotted, red, orange, green, blue)



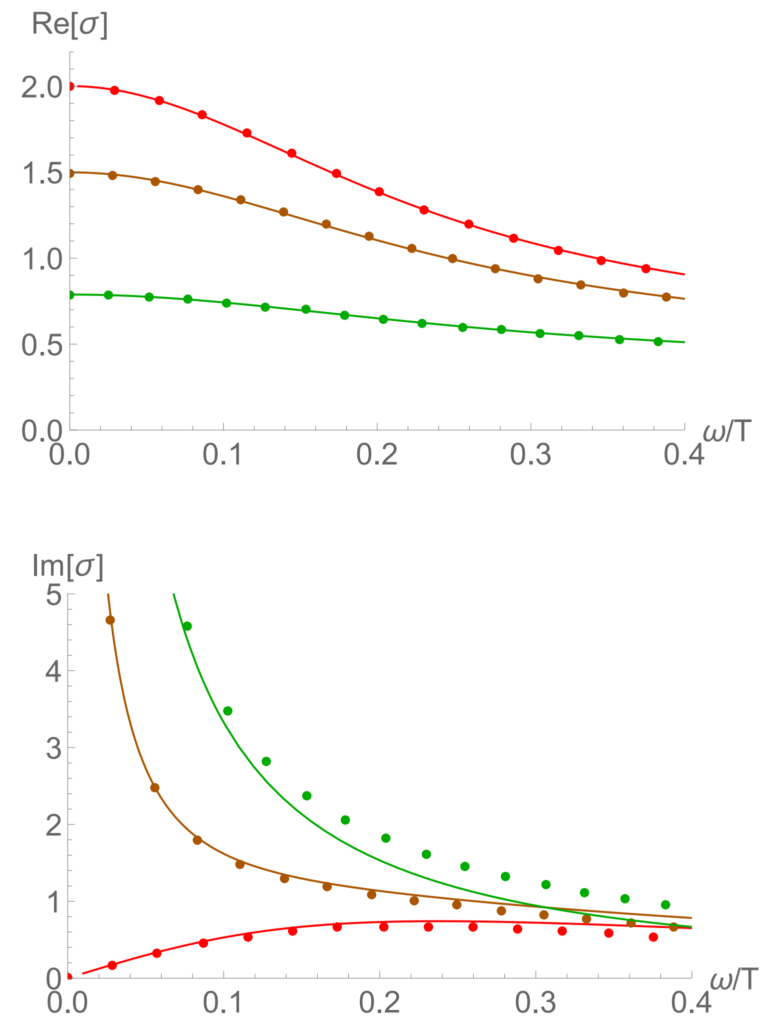
Two fluid model

$$\sigma(\bar{\omega}) = i\frac{K_s}{\bar{\omega}} + \frac{K_n \tau}{1 - i\bar{\omega}\tau}$$

Two fluid model and Drude peak



(b) $\beta/\mu = 1$, $T/T_c = 3.2, 1, 0.89, 0.66, 0.27$ (dotted, red, orange, green, blue)

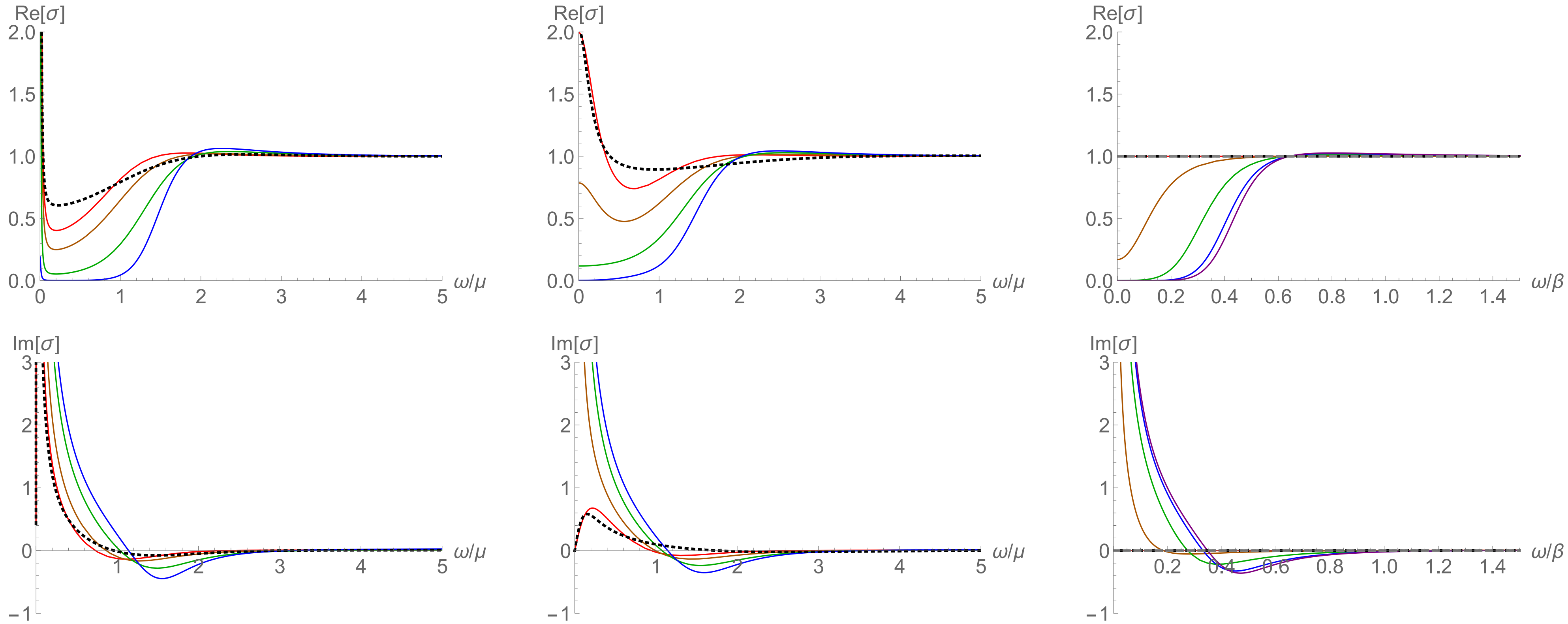


Two fluid model

$$\sigma(\bar{\omega}) = i\frac{K_s}{\bar{\omega}} + \frac{K_n \tau}{1 - i\bar{\omega}\tau} + K_0$$

FGT sum rule

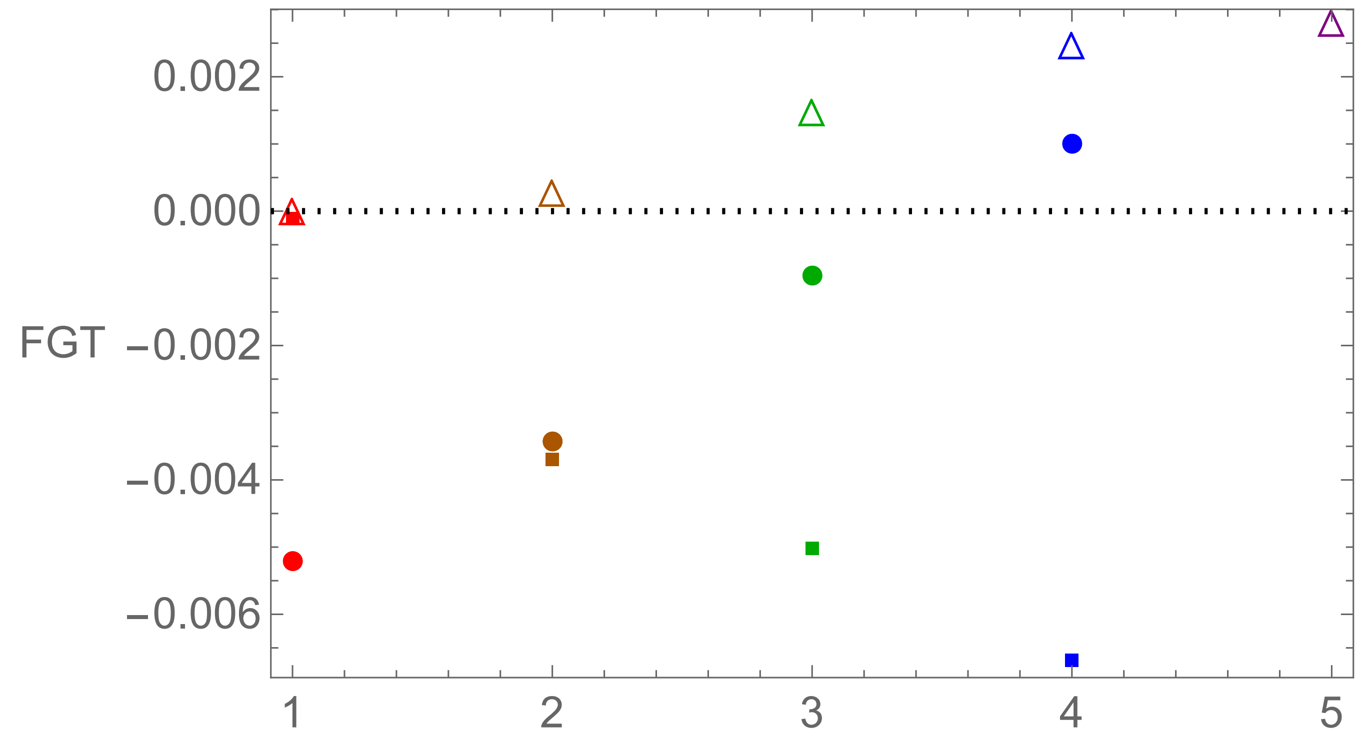
β



$$\beta$$

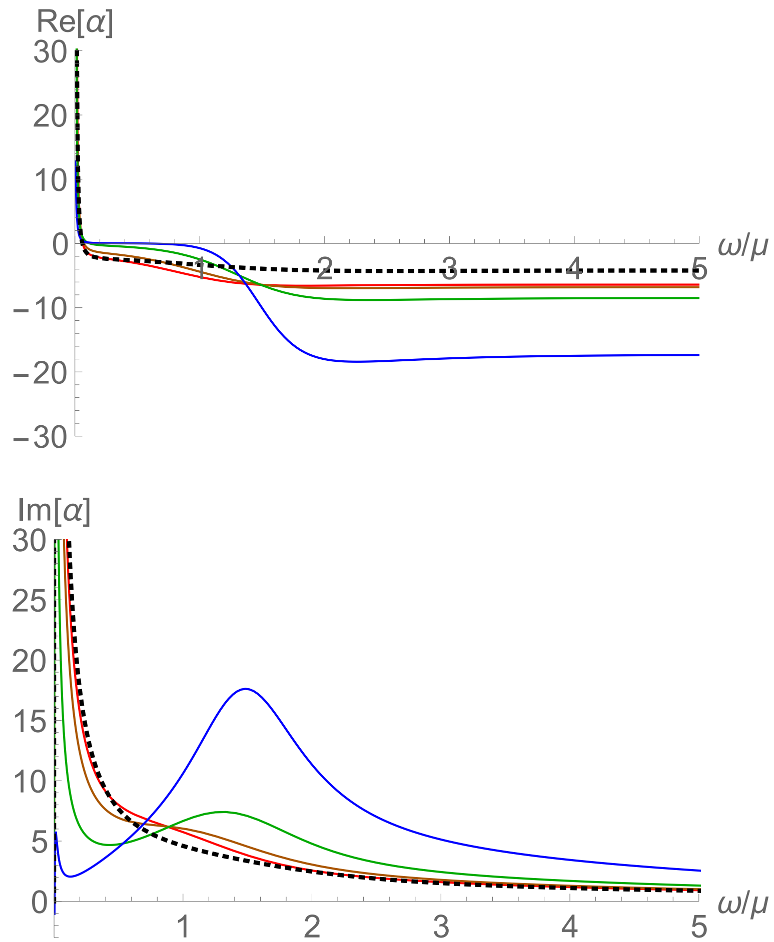
Sum rule works!

$$\mathrm{FGT} \equiv \int_{0^+}^{\infty} \mathrm{d}\omega \mathrm{Re}[\sigma_n(\omega) - \sigma_s(\omega)] - \frac{\pi}{2} K_s = 0$$

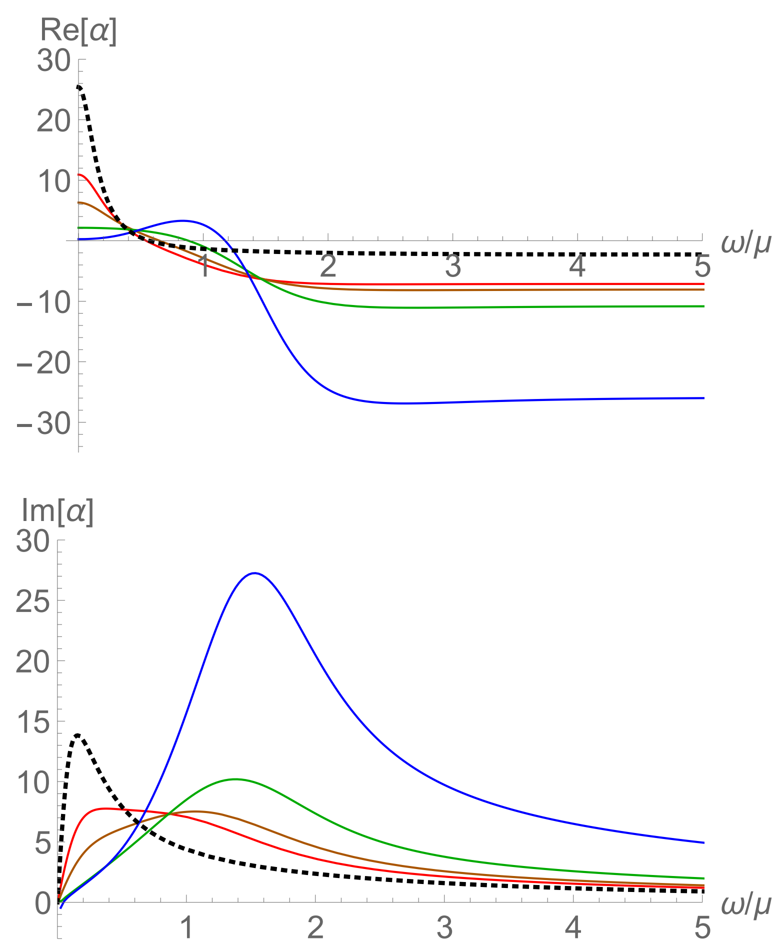


Thermoelectric AC Conductivity

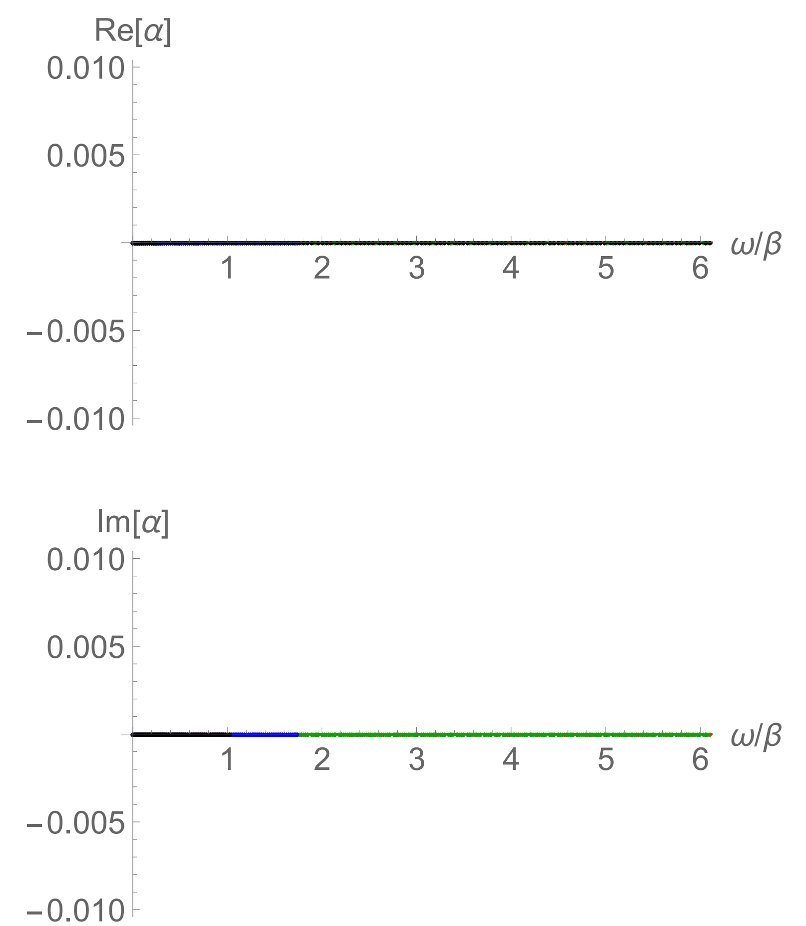
β $\longrightarrow$ β



(a) $\beta/\mu = 0.1$, $T/T_c = 1.52, 1, 0.94, 0.76, 0.37$ (dotted, red, orange, green, blue)



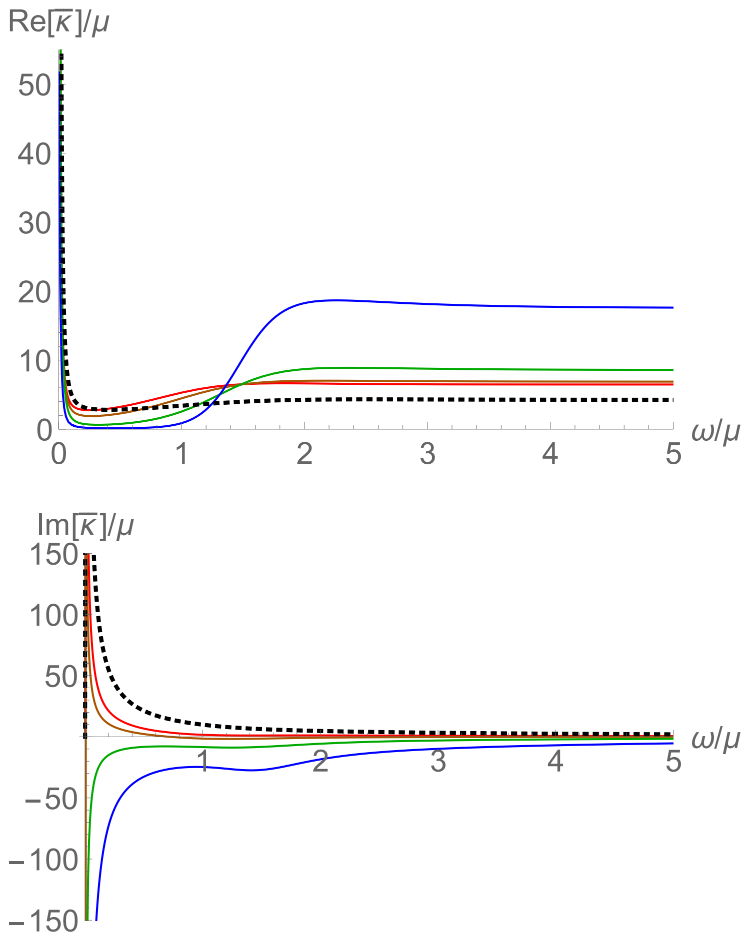
(b) $\beta/\mu = 1$, $T/T_c = 3.2, 1, 0.89, 0.66, 0.27$ (dotted, red, orange, green, blue)



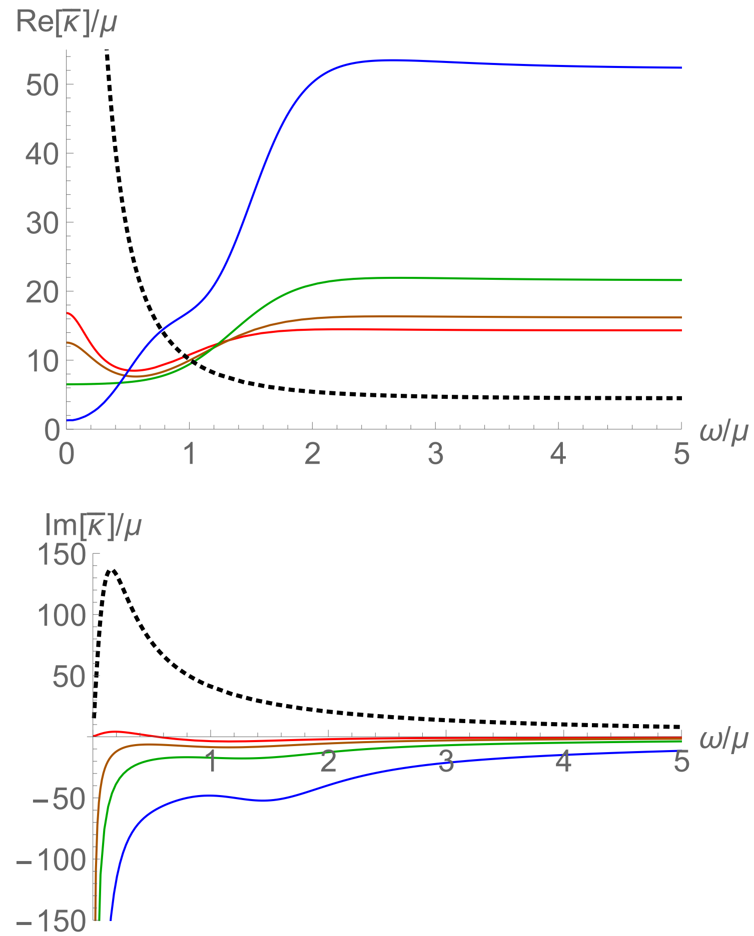
(c) $\beta/\mu \to \infty (\mu = 0)$, $T/T_c = 13.2, 3.5, 1, 0.95, 0.7, 0.4, 0.25$ (dashed, dotted, red, orange, green, blue, purple)

Thermal AC Conductivity

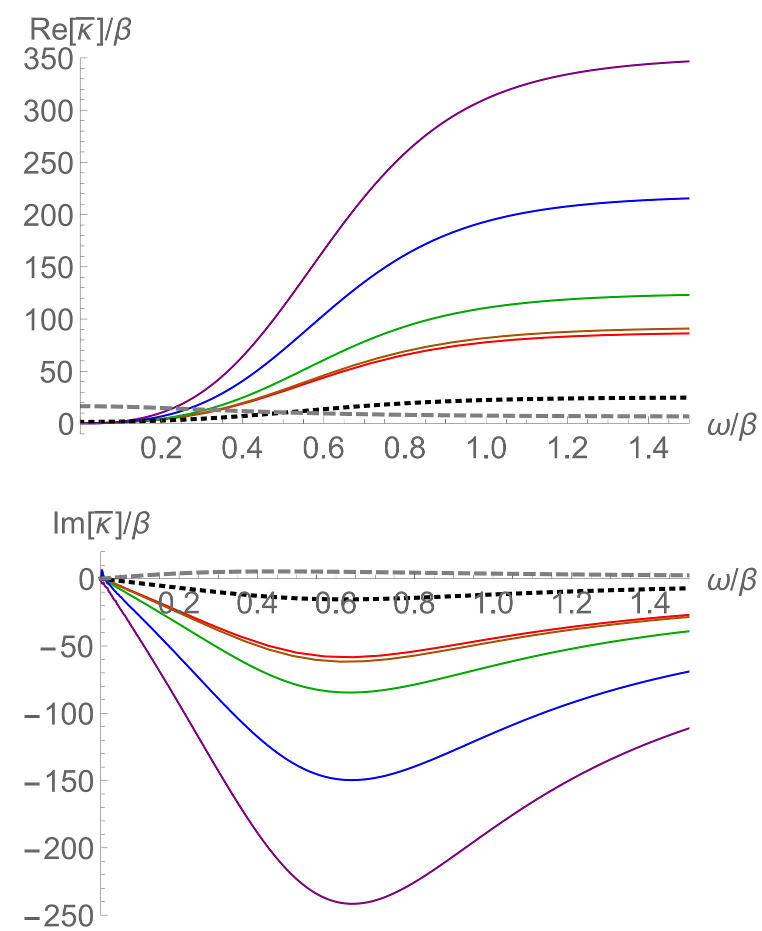
β $\longrightarrow$ β



(a) $\beta/\mu = 0.1$, $T/T_c = 1.52, 1, 0.94, 0.76, 0.37$ (dotted, red, orange, green, blue)



(b) $\beta/\mu = 1$, $T/T_c = 3.2, 1, 0.89, 0.66, 0.27$ (dotted, red, orange, green, blue)



(c) $\beta/\mu \to \infty (\mu = 0)$, $T/T_c = 13.2, 3.5, 1, 0.95, 0.7, 0.4, 0.25$ (dashed, dotted, red, orange, green, blue, purple)

Summary and outlook

- superconductor + momentum relaxation effect in a simple set-up

$$S_{\mathrm{HHH}} = \int_M \mathrm{d}^{d+1}x\sqrt{-g}\left[R + \frac{d(d-1)}{L^2} - \frac{1}{4}F^2 - |D\Phi|^2 - m^2|\Phi|^2\right] ,$$
$$S_{\mathrm{GH}} = -2\int_{\partial M} \mathrm{d}^d x\sqrt{-\gamma}K \ , \quad S_\psi = \int_M \mathrm{d}^{d+1}x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1}^{d-1}(\partial\psi_I)^2\right]$$

- New holographic superconductor? (β -induced)
  - Need to check other holographic superconductor models with momentum relaxation
- three AC conductivities: electric, thermoelectric, and thermal
  - consistency check: DC limit, FGT sum rule
- Homes law $\rho_{\mathrm{s}} = C\sigma_{\mathrm{DC}}(T_c)T_c$
- More curate like holographic model

Thank you!