

CFT representation of (A)dS locality for fields with and without spin

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Mostly based on
PRD 86, 026004 (1204.0126) with Kabat, Lifschytz and Roy;
PRD 90 8, 086005 (1408.0415);
PRD 91, 086004 (1411.4657) with Xiao;
and 150X.XXXXX with Roy

- To understand how locality in a (particular) theory of Quantum Gravity breaks down. To what extent locality is satisfied. How does it change as we RG flow in the bulk.

Holographic theory like AdS/CFT helps us to fulfill that purpose in an elegant way.

The problem boils down to finding local bulk physics in terms of another local theory that we know and understand, namely boundary CFT.

- What does bulk locality have to say about black hole information problem.

- Review of the ‘bulk’ scalar field construction.

Use this framework to extend the prescription to various directions:

- Generalization to spin- 1, 2 and integer s .
- Local operators in terms of fields on a cut-off surface instead of at the conformal boundary. Connection with similar construction in dS (also for higher integer spins) and possibly to holographic RG framework.
- Order by order at $1/N$ expansions and finite N effects. Operators in the black hole background; especially inside the horizon.
- Black hole information problem and discussions.

Identification of normalizable and non-normalizable modes:

- GKPW prescription for Euclidean signature.

Gubser, Klebanov, Polyakov; Witten 1998

- Normalizable modes in Lorentzian spacetime. A 'Transfer function' was constructed which had support over the entire boundary. We will be working in this framework.

Balasubramanian et al.; Banks et al. 1998.; Bena 1999

Basic idea is to write local bulk fields (commuting at the spacelike separation) in terms of integrating boundary operators over some boundary region. So, may be we can write something like

$$\phi(x, z) = \int_{\text{boundary}} dx' K(x, z|x') \mathcal{O}(x')$$

with boundary integral over a tractable range?

Turns out that it is possible. For example, for a free massive scalar

$$\phi(t, x, z) \sim \int_{t'^2 + y'^2 < z^2} dt' d^{d-1} y' (\sigma z')^{\Delta-d} \mathcal{O}(t + t', x + iy')$$

For bulk-boundary AdS covariant distance

$$\sigma(z, x|z', x') = \frac{z^2 + z'^2 + (x - x')^2}{2zz'}$$

and boundary operator with dimension Δ . Here we will always work in Poincare coordinates for AdS_{d+1} with metric

$$ds^2 = G_{MN} dX^M dX^N = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2)$$

Hamilton, Kabat, Lifschytz, Lowe 2005.

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Smearing function is fixed and has the correct behavior under AdS isometry.

- For higher dimensional AdS spaces and for free massless or massive scalar fields, the integration is over finite boundary region (spacelike separated from the bulk field) provided we complexify boundary spacetime.

Hamilton et al. 2006, 2007.

- For timelike boundaries of AdS, solving bulk fields in terms of boundary is not a standard Cauchy problem and one can think of this construction as arising from starting with a retarded Green's function in dS.
- Other equivalent compromising alternatives are certainly possible. For example, momentum space representations.

Papadodimas and Raju 2013-2015

Review- Cont'd

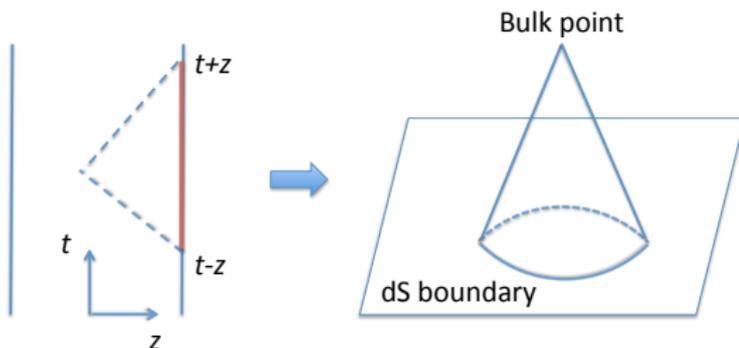


Figure : The smearing function support after translating to complexified coordinate (although here shown for 2-dimensional bulk). In terms of dS it could be thought of as the retarded Green's function.

Next step would naturally be generalization for free and interacting gauge fields.

Kabat, Lifschytz, Roy, DS 2012

Some Issues:

- Gauss constraint must somehow disturb the construction to some extent.

Heemskerk et al. 2012

Kabat, Lifschytz: 2012.

- Problem with lower dimension: $\text{AdS}_{d+1 \leq 3}$ as in lower dimensions, the normalizable modes themselves are fluctuating at the boundary.

Spins 1 and 2- Cont'd

We set the *holographic gauge* $A_z = 0$ which seems to be the easiest choice.

- For $\text{AdS}_{d+1>3}$, the field $\phi_\mu = zA_\mu$ satisfy the usual scalar wave equation with mass $m^2 R^2 = 1 - d$ which leads to its conformal dimension $\Delta = d - 1$ which still satisfies BF bound.

Breitenlohner, Freedman 1982

The smearing integral is given by

$$\begin{aligned} zA_\mu &\sim \int_{t'^2 + |y'|^2 = z^2} dt' d^{d-1} y' j_\mu(t + t', x + iy') \\ &= \frac{1}{\text{vol}(S^{d-1})} \int dt' d^{d-1} y' \delta(\sigma z') j_\mu(t + t', x + iy') \end{aligned}$$

Spins 1 and 2- Cont'd

In general dimensions, the bulk boundary correlator is

$$\langle zA_\mu(t, \mathbf{x}, z)j_\nu(0) \rangle = \frac{\Gamma(d/2)}{2\pi^{d/2}} \left(\frac{d-2}{d-1} \eta_{\mu\nu} l_1 - \frac{1}{2(d-1)(d-2)} \partial_\mu \partial_\nu l_2 \right)$$

where

$$l_n = \int_{t'^2 + |\mathbf{y}'|^2 = z^2} dt' d^{d-1} y' \frac{1}{(- (t + t')^2 + |\mathbf{x} + i\mathbf{y}'|^2)^{d-n}}$$

- Even though l_1 is only singular on the bulk lightcone, l_2 turns out to be singular in both bulk and boundary light cone.
- On the other hand, $\langle Fj \rangle$ correlators are only singular in bulk light cone.

Spins 1 and 2- Cont'd

Similar story for metric perturbation. Considering linearized perturbation of AdS metric

$$ds^2 = \frac{R^2}{z^2} (dz^2 + g_{\mu\nu} dx^\mu dx^\nu)$$

$$\text{with } g_{\mu\nu} = \eta_{\mu\nu} + \frac{z^2}{R^2} h_{\mu\nu}; \quad g_{zz} = g_{z\mu} = 0$$

In general $z^2 h_{\mu\nu}$ satisfy massless scalar wave equation. For $\text{AdS}_{d+1>3}$,

$$z^2 h_{\mu\nu} = \frac{1}{\text{vol}(B^d)} \int_{t'^2 + |y'|^2 < z^2} dt' dy' T_{\mu\nu}(t + t', x + iy')$$

Spins 1 and 2- Cont'd

Similar to the Maxwell case, using

$$\langle T_{\mu\nu}(x) T_{\alpha\beta}(0) \rangle = X_{\mu\nu\alpha\beta} \frac{1}{(x^2)^d} + Y_{\mu\nu\alpha\beta} \frac{1}{(x^2)^{d-1}} + Z_{\mu\nu\alpha\beta} \frac{1}{(x^2)^{d-2}}$$

we find

$$z^2 \langle h_{\mu\nu}(t, \mathbf{x}, z) T_{\alpha\beta}(0) \rangle = X_{\mu\nu\alpha\beta} J_0 + Y_{\mu\nu\alpha\beta} J_1 + Z_{\mu\nu\alpha\beta} J_2$$

where

$$J_n = \frac{1}{\text{vol}(B^d)} \int_{t'^2 + |\mathbf{y}'|^2 < z^2} dt' d^{d-1} y' \frac{1}{(- (t + t')^2 + |\mathbf{x} + i\mathbf{y}'|^2)^{d-n}}$$

- It turns out J_1 and J_2 is again singular on both bulk and boundary light cone.
- Upon calculating correlator with Weyl tensor we find, they only depend on l_0 and l_1 found previously and hence they lead to causal commutator.

Some Comments

- For both spin-1 and spin-2 case, from the form of smearing function it's clear that in the boundary limit we go to a localized operator at the boundary.
- For AdS_3 Maxwell, the bulk gauge field is proportional to a fluctuating gauge field at the boundary and the dictionary here is not well understood.
- The smearing distribution is pretty clever. For example, for AdS_3 , it already knows that the boundary conserved current is dual to the Chern-Simons gauge fields.

Jensen 2010

- The smearing distributions again have the correct AdS isometric properties.

Prescriptions for Higher Spin

- For higher integer spins (HS) similar gauge choices could be made, which can once again be used to write the HS equation of motion as scalar field equations.

DS and Xiao 2012

- The prescription turns out to be

$$\Phi_{\mu_1 \dots \mu_s} = \frac{\Gamma(s + \frac{d}{2} - 1)}{\pi^{\frac{d}{2}} \Gamma(s - 1)} \frac{1}{z^s} \int_{\mathcal{B}^d} dt' d^{d-1} \mathbf{y}' \left(\frac{z^2 - t'^2 - |\mathbf{y}'|^2}{z} \right)^{s-2} \mathcal{O}_{\mu_1 \dots \mu_s}(t + t', \mathbf{x} + i\mathbf{y}')$$

for field with integer spin $s > 1$. We see the field behaves like $z^{\Delta-s} = z^{d-2}$ near the boundary.

- This smearing function can also be shown to be AdS covariant. Also it could be seen that these higher spin operators are non-local.

- Similar construction can be carried out in de Sitter (dS) spacetime. In particular one way to construct local bulk operators in dS is to smear two sets of CFT operators.

Xiao 2014. For an alternative discussion see Lowe 2015

- In particular, for dS_{d+1} , one obtains (generalizable to higher spins)

$$\begin{aligned}\Phi(\eta, \mathbf{x}) = & A(\Delta, d) \int_{|\mathbf{x}'| < \eta} d^d \mathbf{x}' \tilde{K}_1(\eta, \mathbf{x}') \mathcal{O}_\Delta(\mathbf{x} + \mathbf{x}') \\ & + B(\Delta, d) \int_{|\mathbf{x}'| < \eta} d^d \mathbf{x}' \tilde{K}_2(\eta, \mathbf{x}') \mathcal{O}_{d-\Delta}(\mathbf{x} + \mathbf{x}')\end{aligned}$$

- Analytic continuation $z \rightarrow \eta$, $x^i \rightarrow ix^i$ and $R_{AdS} \rightarrow iR_{dS}$ takes AdS to flat patch dS with time η . Considering normalizable modes in AdS, leads to taking only +ve/-ve frequency modes in dS.
- Similar prescription needed to construct bulk in terms of “boundary operators” on a cutoff surface. Then analytic continuation between AdS and dS is visible (if normalizability condition is not imposed).

Hologram on a Cutoff Surface

- Construct massive bulk scalar field Φ in terms of operators on a cutoff surface at the large N limit. We again use Poincaré coordinates.
- Remember that near the AdS boundary ($z \rightarrow 0$) the field has two distinct behaviors ($\nu = \Delta - \frac{d}{2}$)

$$\Phi(z, x) = \frac{\phi_b(x)}{2\nu} z^\Delta + z^{d-\Delta} j(x)$$

where j and ϕ_b are defined via

$$j(x) = z^{-d+\Delta} \Phi(z, x)|_{z \rightarrow 0} \quad \text{and} \\ \phi_b(x) = z^{-2\nu} z \partial_z (z^{-d+\Delta} \Phi)|_{z \rightarrow 0} \leftrightarrow \mathcal{O}(x)$$

- We then similarly define

$$j_{cut}(x, z_0) = z^{-d+\Delta} \Phi(z, x)|_{z \rightarrow z_0} \quad \text{and} \\ \phi_{b,cut}(x, z_0) = z^{-2\nu} z \partial_z (z^{-d+\Delta} \Phi)|_{z \rightarrow z_0}$$

Hologram on a Cutoff Surface- Cont'd

- The correct prescription turns out to be

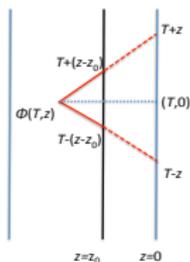
$$\begin{aligned}\Phi(z, x) = & \int d^d x' K_1(x'|x, z, z_0) \phi_{b,cut}(x', z_0) \\ & + \int d^d x' K_2(x'|x, z, z_0) j_{cut}(x', z_0)\end{aligned}$$

with some complicated K_1 and K_2 , which are of course also functions of Δ , d and the cutoff surface location z_0 .

- The expressions are simplest for massless scalars in AdS_2 where the final result for the bulk operator becomes ($z_0 = z/m$)

$$\begin{aligned}\phi(T, z) = & \frac{1}{2} [j_{cut}(T + (m-1)z_0, z_0) + j_{cut}(T - (m-1)z_0, z_0)] \\ & + \frac{1}{2} \int_{T-(z-z_0)}^{T+(z-z_0)} dT' \phi_{b,cut}(T', z_0)\end{aligned}$$

Hologram on a Cutoff Surface- Cont'd



This gives the correct $\text{AdS}_2/\text{CFT}_1$ prescription, as we take z_0 to zero. As we take $z_0 \rightarrow 0$ limit, the current $j_{cut} \rightarrow 0$.

Hologram on a Cutoff Surface- Cont'd

For integer m , it gives us the usual expected value of the bulk field in terms of cutoff surface boundary values once we impose normalizeability.

$$\Phi(T, z) = \sum_{i=1}^m \Phi(T_i, z_0) = \sum_{i=1}^m j_{cut}(T_i, z_0), \quad \text{with}$$

$$T_1 = T + (z - z_0), \quad T_{i+1} = T_i - 2z_0, \quad \dots \quad T_m = T - (z - z_0)$$

which is a special case for massless scalars in AdS_2 with $z_0 = z/m$.

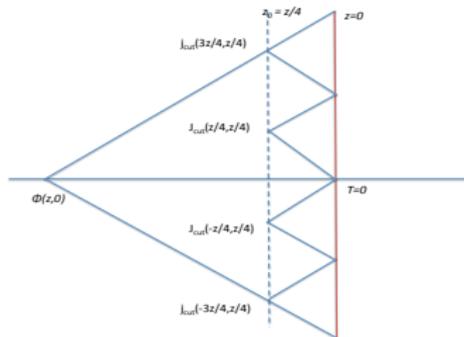


Figure : Smearing function for AdS_2/CFT_1 ($m = 4$) for cut-off surface $z_0 = z/4$ contains four delta functions as shown in the figure.

Connection to Holographic RG Framework

- In the standard holographic RG prescription,

$$Z = \int \mathcal{D}\tilde{\phi}(z_0, x) \Psi_{IR}(z_0, \tilde{\phi}) \Psi_{UV}(z_0, \tilde{\phi})$$

where Ψ_{IR} and Ψ_{UV} arises from integrating out relevant bulk fields against the exponential of the relevant part of the bulk action:

$$\Psi_{UV} = \int \mathcal{D}\phi \Big|_{\substack{z > z_0 \\ z < z_0}} e^{-\kappa^{-2} S \Big|_{\substack{z > z_0 \\ z < z_0}}}$$

Faulkner et al. 2010 and Heemskerck et al. 2010

- If one considers the UV factor is a local Gaussian for a single bulk scalar, i.e.

$$\Psi_{UV}(z_0, j_{cut}) = \exp \left\{ -\frac{1}{2h\kappa^2} \int d^d x (j_{cut}(x, z_0) + g(x))^2 \right\}$$

Then after the UV part is integrated out, one effectively induces double trace operators.

Connection to Holographic RG Framework- Cont'd

It was postulated that

$$\Psi_{IR}(z_0, j_{cut}) = \int \mathcal{D}M|_{kz_0 < 1} \exp \left\{ -S_0 + \frac{1}{\kappa^2} \int d^d x j_{cut} \mathcal{O}_i \right\}$$

which is usually interpreted as the wavefunction for the QFT with a cut-off. From the bulk point of view, one can simply treat the j_{cut} 's appearing on the exponential of Ψ_{IR} to be the on-shell operators and the \mathcal{O}_i 's as their sources. Then after integrating the UV part,

$$\frac{\delta}{\delta \mathcal{O}} \frac{\delta}{\delta \mathcal{O}} Z|_{\mathcal{O} \rightarrow 0} = \int \mathcal{D}\tilde{\phi} \langle j_{cut} j_{cut} \rangle \Psi_{UV}[\tilde{\phi}] \propto \langle j_{cut} j_{cut} \rangle$$

(modulo some factors of \hbar and κ). Thus one can simply recover the cut-off bulk locality or relate the cut-off surface field theory correlators with the bulk partition function while flowing in holographic RG. UV-cutoff CFT will not give us local bulk fields.

1/N Smearing Function

- Add higher dimension operators: Bilocal operators or equivalently tower of higher dimensional operators to restore locality for scalar fields at the level of 3 point function.

$$\phi(z, x) = \int dx' K_{\Delta}(z, x|x') \mathcal{O}_{\Delta}(x') + \sum_I a_I \int dx' K_{\Delta_I}(z, x|x') \mathcal{O}_{\Delta_I}(x')$$

Kabat, Lifschytz, Lowe, 2011; Kabat, Lifschytz, 2012

- Clearly indicates how locality can break down as we go to finite N . Not enough higher dimensional operators to recover locality.

See also Papadodimas- Raju

- Similar story for gauge fields. Locality is satisfied to the extent of non-locality introduced by Gauss constraints.

Kabat, Lifschytz, Lowe, 2012, 2013

Operators behind black hole horizon

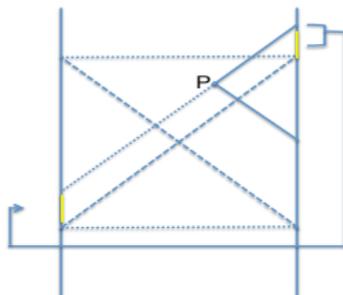


Figure : Subhorizon operator construction as thermofield double. The construction is similar in Rindler and BTZ.

This construction works for infinite N as the CFT correlators decay at large times, even though the smearing blows up.

$$\Phi(P) = \int_{\text{spacelike}} dy [z \sigma(P|y, z)]^{\Delta-2} \mathcal{O}_{\Delta}^R(y) + \int_{\text{timelike}} dy' [-z' \sigma(P|y', z')]^{\Delta-2} (-)^{\Delta} \mathcal{O}_{\Delta}^L(y')$$

Hamilton et al. 2007



Finite N Smearing Function

- The finite N effects can be incorporated in the smearing function only if they are cutoff at a large timescale t_{max} . This is because, at finite N the CFT correlation function starts to show small fluctuations at the time scale t_{max} , which when convolved with smearing function, blows up to give meaningless expressions.

Kabat and Lifschytz 2013

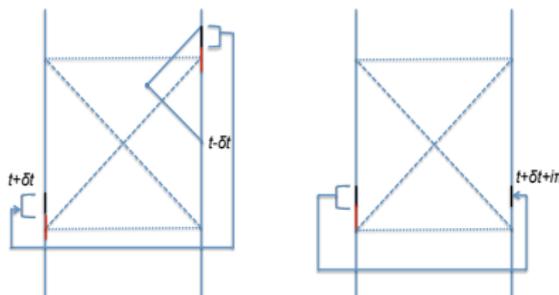


Figure : The second figure shows an alternate formulation using the mapping of left side smearing function to the right boundary. Thus in this case, smearing over only one boundary is sufficient.

Finite N Smearing Function (Cont'd)

- This cutoff prescription induces exponentially small non-locality which might indicate a firewall for large BHs in AdS, after the Page time.

$$\phi_{modified} = \phi_{semiclassical} - e^{-dS/2\Delta} \mathcal{O}$$

- This is one way of showing that these large time scales can play an important role in the bulk non-locality, which is otherwise invisible in general.
- However, this firewall could still be an artifact of working with the semiclassical saddle points. It could be that in a full quantum gravity path integrals, late time integrals are dominated by geometries without a horizon and hence circumventing the necessity of firewalls.

Soloduhkin 2005, Germani and DS 2015

Finite N Smearing Function (Cont'd)

This construction can also be extended to BHs formed by collapse processes.

Work in progress. Roy and DS

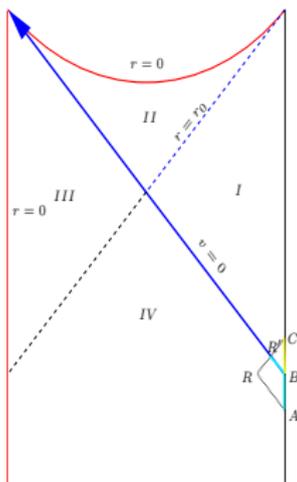


Figure : AdS_{d+1} -Vaidya geometry for a null-shell collapse.

Similar construction for AdS_2 -Vaidya by Lowe and Roy 2008

Finite N Smearing Function (Cont'd)

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Work in progress. Roy and DS

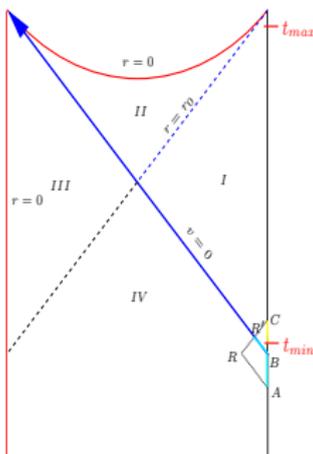


Figure : AdS_{d+1} - Vaidya construction with cut-offs.

Finite N Smearing Function (Cont'd)

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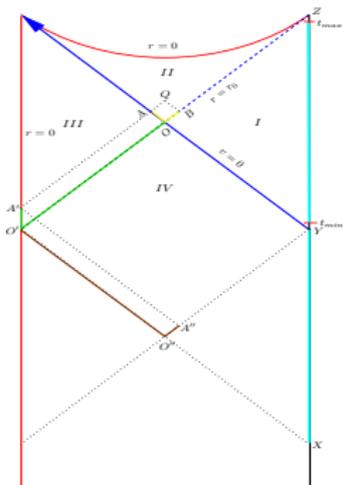


Figure : Smearing of points inside horizon for AdS_{d+1} -Vaidya geometry with cut-offs.

- Extension of HS construction to three-point level? Any possible Weyl like “gauge invariant” HS fields?
- Interactions and finite N effects for dS construction.
- Exploring in further details the relation between cut-off smearing construction and holographic RG?
- General Backgrounds? Interesting from the point of view of the state dependent construction of Papadodimas-Raju (PR) and 150X.XXXXX and hence the firewall paradox.

Some Comments: Kabat, Lifschytz, Roy, DS 2012

- Sharpen the connection between state dependence of PR and smearing construction. Important to avoid the objections of AMPSS and MP.
- Contact with other approaches?

Heemskerk et al. 2009, Penedones 2010, Bekaert et al. 2014

THANK YOU

$$ds^2 = -(r^2 - 1)dt^2 + (r^2 - 1)^{-1}dr^2 + r^2 dx^2$$

$$\phi(t, x, r) = \int d\omega dk e^{-(i\omega t - ikx)} a_{\omega k} f_{\omega k}(r)$$

$$\phi \sim r^{-\Delta} \mathcal{O}, \quad f_{\omega k}(r) \sim r^{-\Delta}$$

So,

$$a_{\omega k} = \int e^{(i\omega t - kx)} \mathcal{O}$$

Then $K = F.T.\{f_{\omega k}\}$ as $\phi = \int K \mathcal{O}$. But turns out $f_{\omega k}$ blows up exponentially as $k \rightarrow \pm\infty$.