LOOKING FOR AN ON-SHELL REGULATOR

FRoGS
FRont of pro-Galician Scientists

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Holograv 2015 - Firenze

(Based on work with P. Benincasa and D. Gordo)

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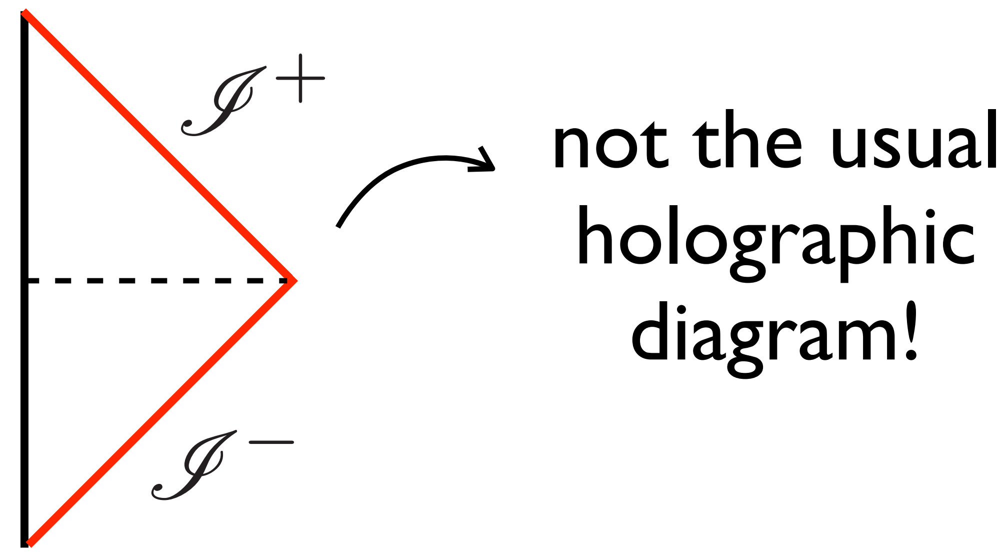
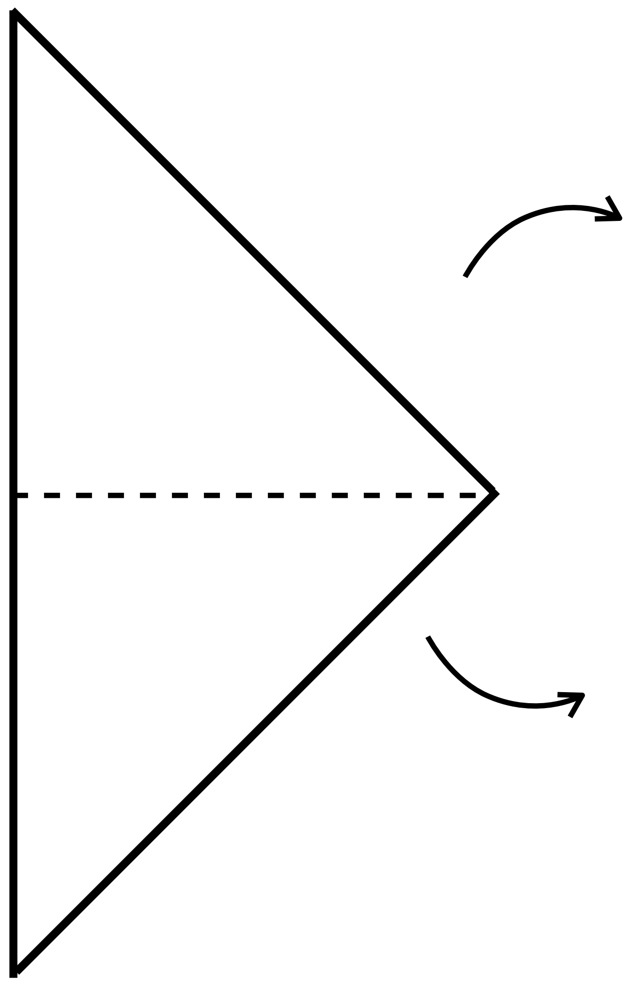
"Holography" without gravity

- Take a QFT in flat Minkowski spacetime



- What information can we gain on the QFT from asymptotic structure?
- Soft theorems, scattering amplitudes, ...

Asymptotic structure



$$\mathcal{H}_+ = \{|\mathrm{in}\rangle\}$$

$$\mathcal{H}_- = \{|\mathrm{out}\rangle\}$$

$$|\mathrm{out}\rangle = S|\mathrm{in}\rangle$$

S-matrix is the matrix of change of basis

Asymptotic structure

- Asymptotic symmetry group;
  it defines one-particle states, and it imposes constraints on the form of S

- Unitarity: $S^\dagger S = \mathbb{1}$

- Singularity structure of the S-matrix
  (e.g. via locality/cluster decomposition pple)

Some restrictions

- Perturbative approach $$M_n^{\text{pert}} = \underset{\substack{\wr \\ g^{n-2}}}{M_n^{\text{tree}}} + \sum_{L=1}^{\infty} \underset{\substack{\wr \\ g^{n+2L-2}}}{M_n^{\text{L}}} \qquad \left(\begin{array}{c} M_n \\ \wr \\ < \text{in}|\text{S}|\emptyset > \end{array} \right)$$

- Four-dimensional QFTs with only massless particles.

- There is a three-point interaction per coupling constant

Some restrictions

- Perturbative approach $$M_n^{\text{pert}} = \underset{\substack{\propto \\ g^{n-2}}}{M_n^{\text{tree}}} + \sum_{L=1}^{\infty} \underset{\substack{\propto \\ g^{n+2L-2}}}{M_n^{\text{L}}} \quad \left(\begin{array}{c} M_n \\ \wr \\ < \text{in}|\text{S}|\emptyset > \end{array} \right)$$
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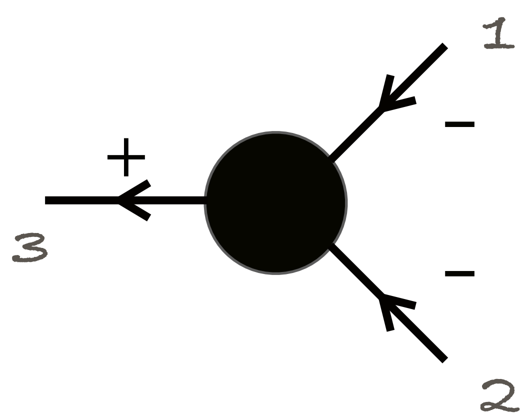
Goal! Compute all the perturbative series for the scattering amplitude from asymptotic info

Basic building blocks

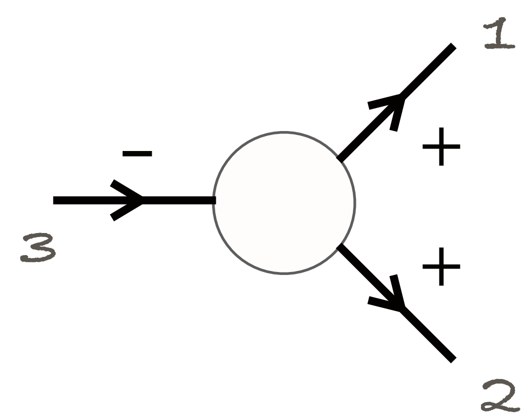
- M_3 is fixed by Poincaré invariance (plus allowed singularity structure)

Notice
- Non-perturbative result
- $M_3 \neq 0$ only for complex momenta

- Massless particle in four dimensions $|\mathrm{in}\rangle = |p, h\rangle$



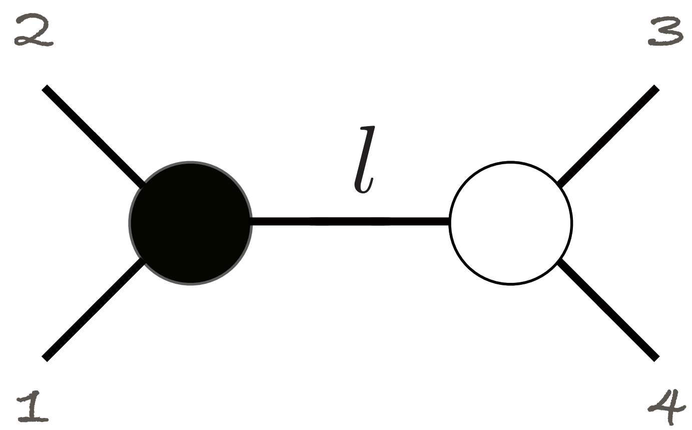
$h_1 + h_2 + h_3 < 0$



$h_1 + h_2 + h_3 > 0$

$$\sim g\,\delta(p_1 + p_2 + p_3)\,(p_1 \cdot p_2)^{\pm(h_3 - h_2 - h_1)}\,(p_2 \cdot p_3)^{\pm(h_1 - h_2 - h_3)}\,(p_3 \cdot p_1)^{\pm(h_2 - h_3 - h_1)}$$

Gluing three-point amplitudes



$$= \int_{\mathbb{R}^4 \subset \mathbb{C}^4} \mathrm{d}^4 \mathrm{l}\, \delta^+(\mathrm{l}^2)\, \mathrm{M}_3(1,2,\mathrm{l}) \mathrm{M}_3(-\mathrm{l},3,4)$$

- Any desired diagram can be built in this way. They are called on-shell diagrams.

- What is it computing? (What contour?)

Gluing three-point amplitudes

- Using unitarity (via BCFW techniques), [ARKANI-HAMED, BOURJAILY, CACHAZO, GONCHAROV, POSTNIKOV, TRNKA '12] showed how to represent any amplitude of planar N=4 SYM with on-shell diagrams

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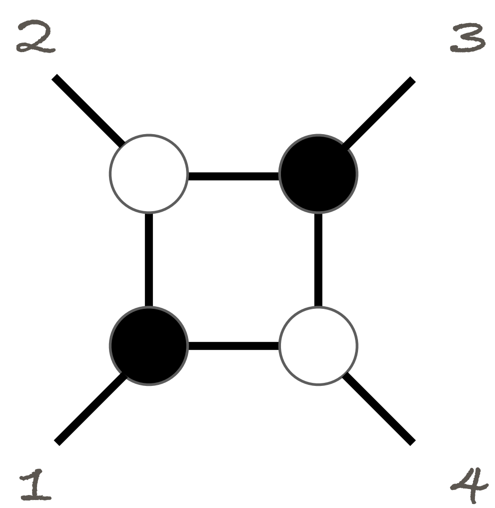
$$M_4^{\text{tree}} =$$



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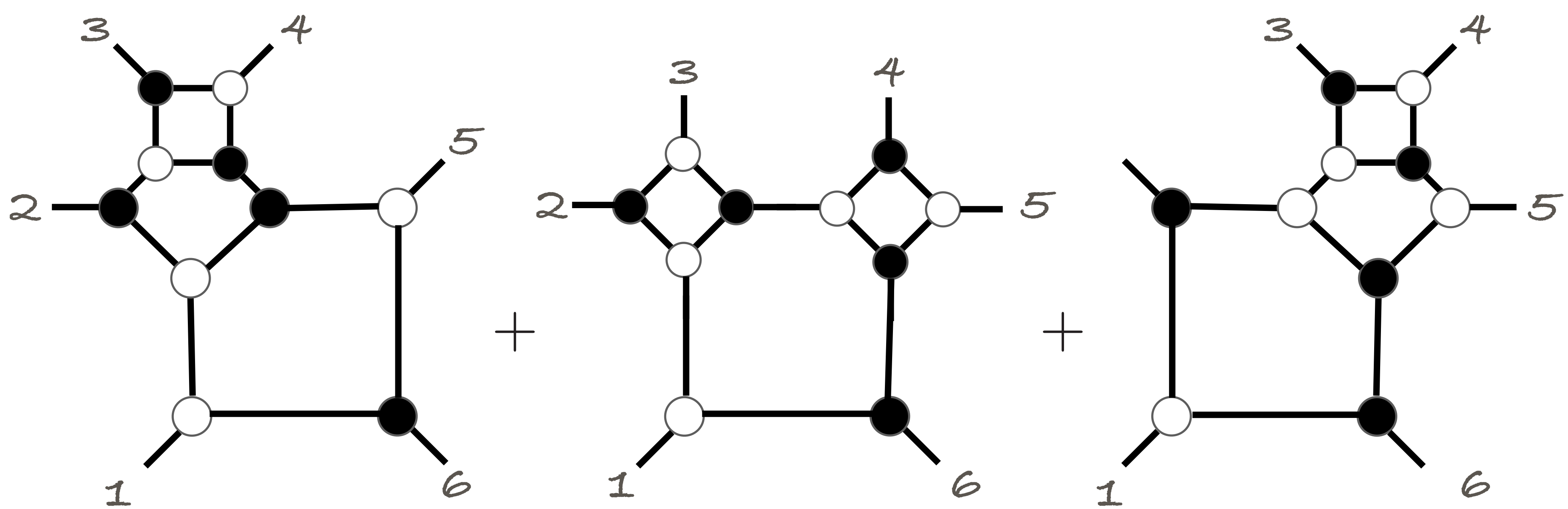
$$M^{\text{tree}}_{6,\text{NMHV}} =$$



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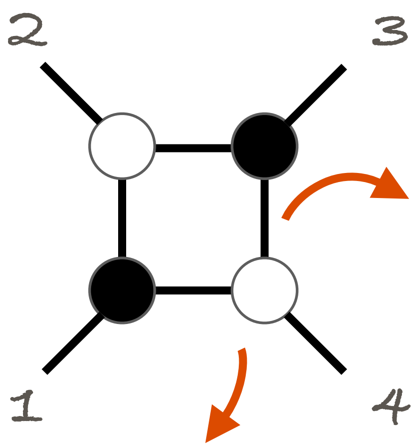

$\int \mathrm{d}^4\mathrm{l}\,\delta^+(\mathrm{l}^2)$

$\sim \int \mathrm{d}\zeta_{\mathrm{I}}$

$\delta^4(\sum p_i)$

$$\#\zeta_I\,|_{\text{free}} = 3I - 4(V-1)$$

$$I : \#\left(\begin{smallmatrix}\text{internal}\\ \text{lines}\end{smallmatrix}\right)$$

$$V : \#\,(\text{3-pt vertices})$$

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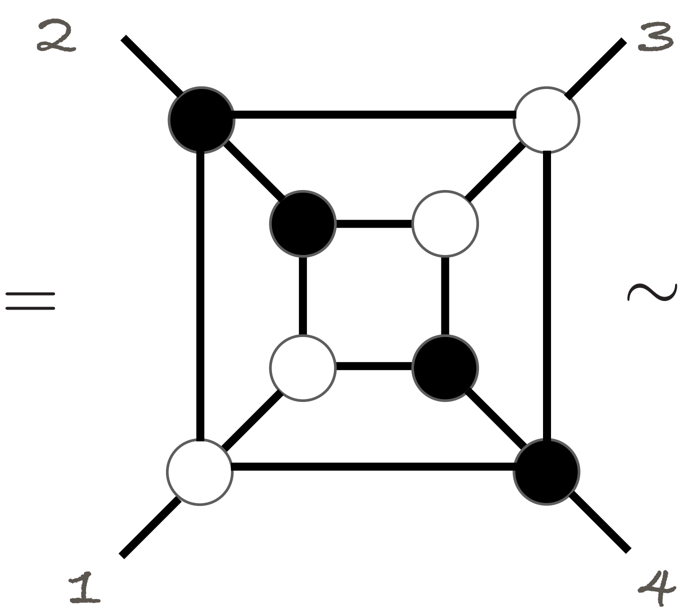
$$M_4^{\mathrm{L}=1} =$$



$$\sim \int \mathrm{d}^4 \zeta_{\mathrm{I}}$$

What are we computing

$$M_n^L = \int \prod_{k=1}^{L} \mathrm{d}^4 \mathrm{l_k}\, \mathrm{F}(\mathrm{p_i}, \mathrm{l_k}) = \int (\mathrm{integrand})$$

- To integrate or not to integrate
- Contour of integration?
- Need to regulate! What can the regulator be?
  First answer given by [FERRO, LUKOWSKI, MENEGHELLI, PLEFKA, STAUDACHER '12] , inspired by integrability techniques

An on-shell regulator?

[BENINCASA, E.C., GORDO '13]

- Deform helicities of three-point amplitudes

$$h_i \rightarrow h_i + \varepsilon_i$$

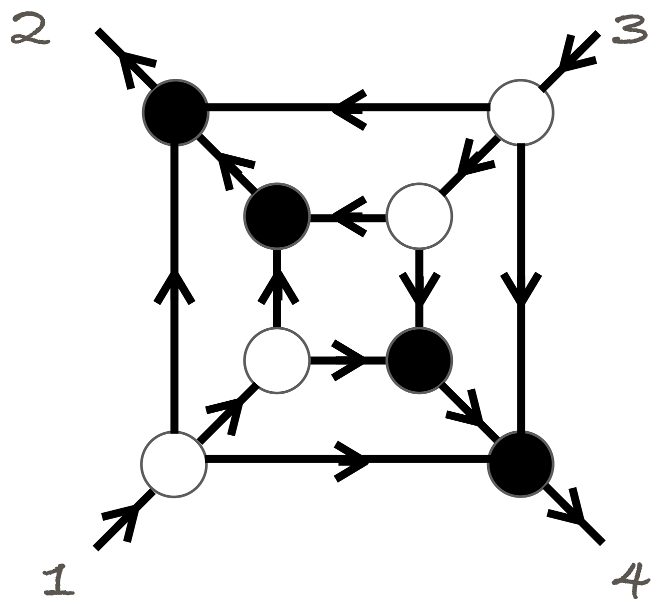
- Keep dimensionality of coupling constant

$$\sum_i \varepsilon_i = 0$$

- Glue the new deformed three-point amplitudes

Does it work?

- Study the simplest case: four particles, one loop, self-interacting massless particle (any spin)
- Several diagrams to be considered



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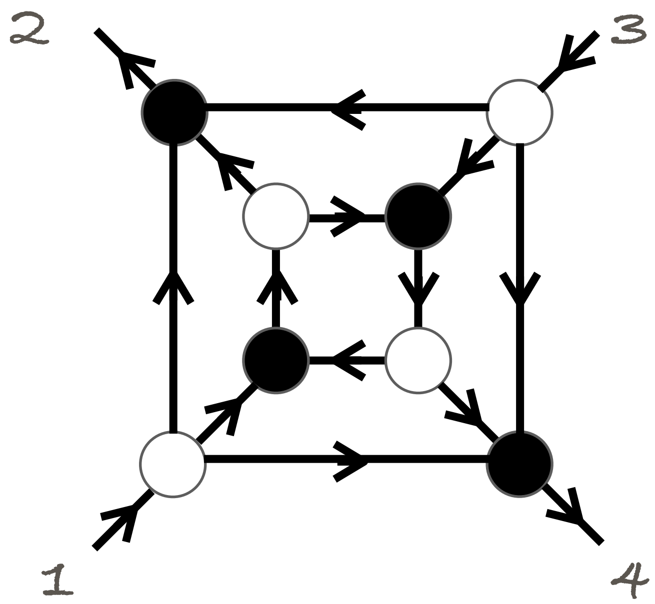

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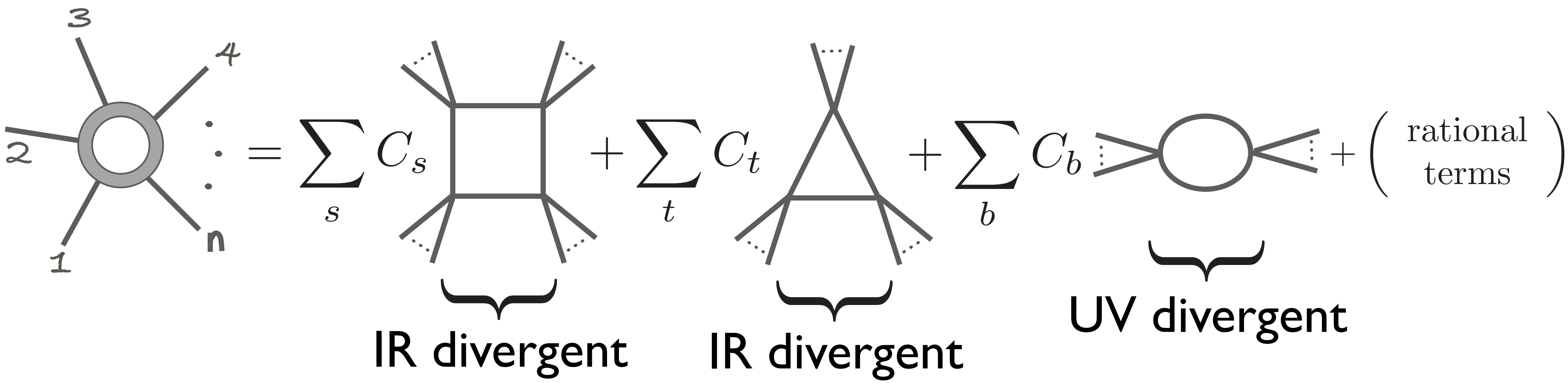
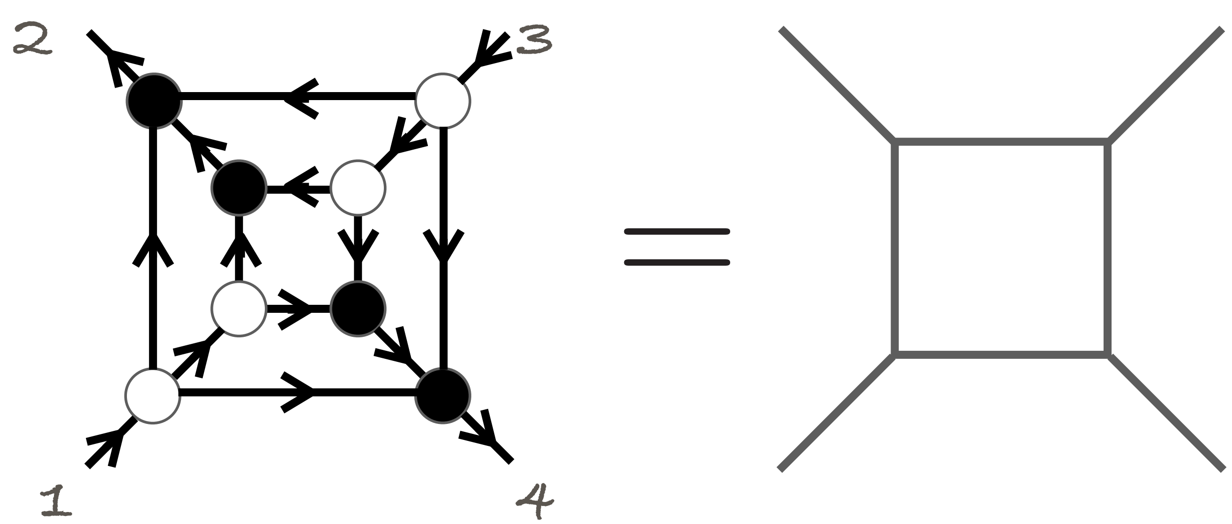
One-loop structure of QFT amplitudes



- Our object must contain all of these contributions



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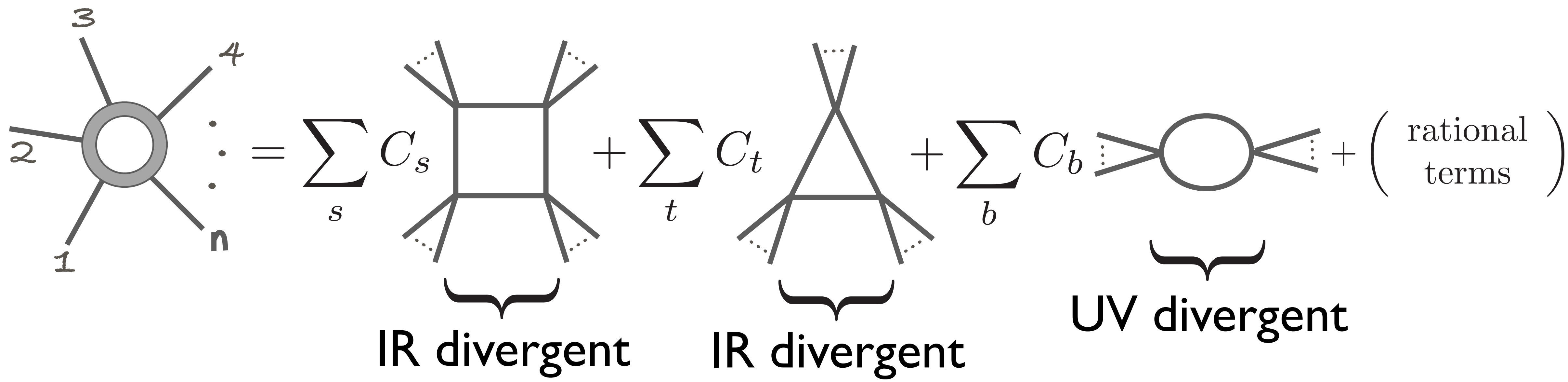

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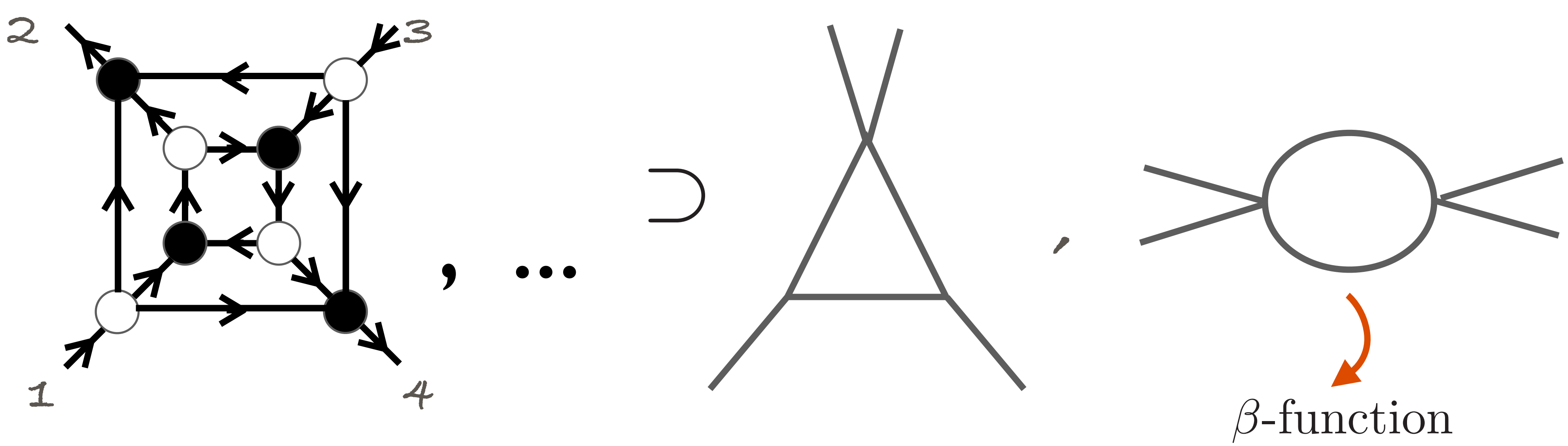
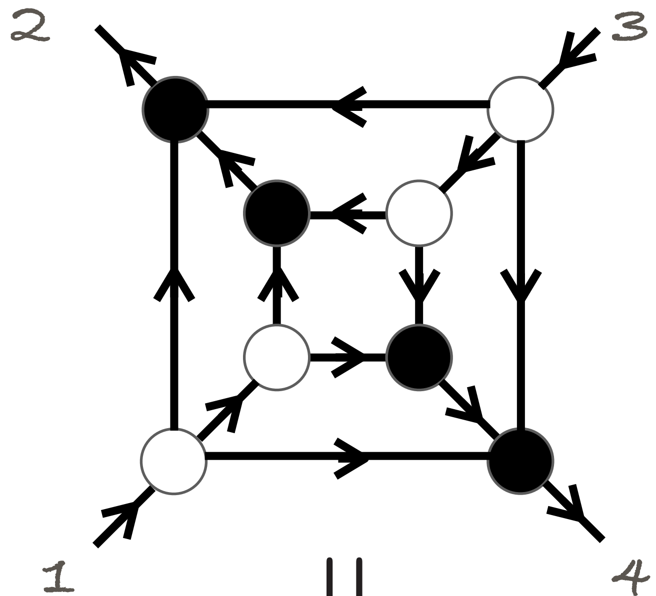


Box and IR divergencies



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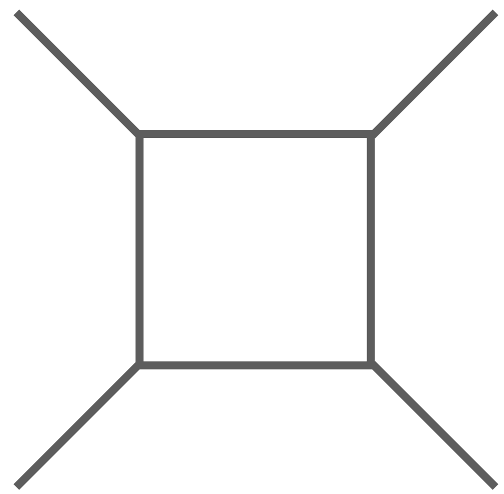

DIMENSIONAL REGULARIZATION

$$\sim \frac{1}{\epsilon^2} - \frac{1}{2\epsilon}\left(\log(-\frac{s}{\mu}) + \log(-\frac{t}{\mu})\right) - \frac{\pi^2}{3} + \frac{1}{2}\log(-\frac{s}{\mu})\log(-\frac{t}{\mu})$$

$||$

$$st\, M^{\text{tree}}_{4,\text{def}} \int \bigwedge_{i=1}^{4} d\zeta_{i,i+1}\zeta_{i,i+1}^{2\varepsilon_{i,i+1}-1}(1-\zeta_{i,i+1})^{-2(\varepsilon_{i,i+1}+\bar{\varepsilon}_{i,i+1})}$$

$$\Gamma = \left\{\zeta \in \mathbb{C}^4 \,\middle|\, -\frac{s}{u}\zeta_{12}\zeta_{34} - \frac{t}{u}\zeta_{23}\zeta_{41} = \frac{\zeta_{i,i+1}}{\zeta^*_{i,i+1}}\right\} \equiv \mathbb{R}^4 \subset \mathbb{C}^4$$

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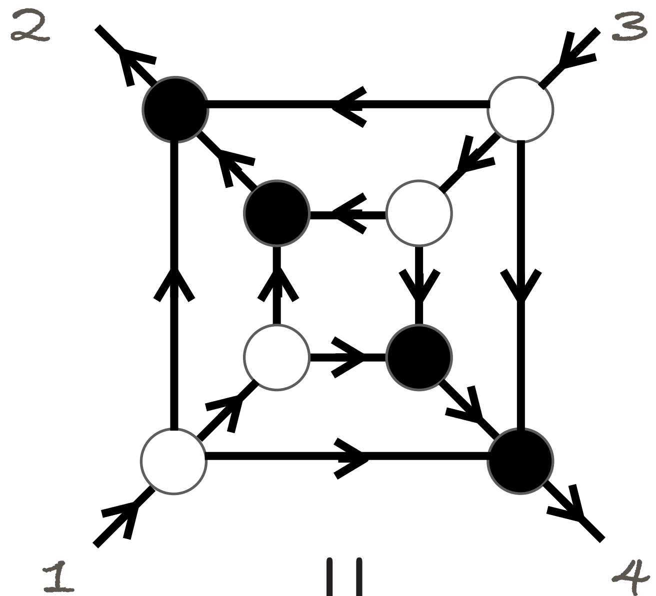

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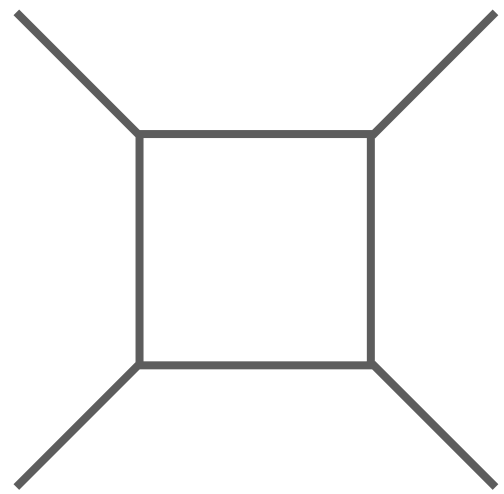

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$\|$

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$$\sim \frac{1}{\epsilon^2} - 3\frac{\log\left(-\frac{s}{u}\right) + \log\left(-\frac{t}{u}\right)}{2\epsilon} + \frac{1}{2}\log\left(-\frac{s}{u}\right)^2 + \frac{1}{2}\log\left(-\frac{t}{u}\right)^2 + 2\log\left(-\frac{s}{u}\right)\log\left(-\frac{t}{u}\right) - \frac{56\pi^2}{3}$$

$$(\varepsilon_{12} = \varepsilon_{23} = \epsilon\,, \quad \varepsilon_{34} = \varepsilon_{41} = 2\epsilon\,, \quad \bar{\varepsilon}_{i,i+1} = \varepsilon_{i,i+1})$$

Perspective

- A general prescription for regulating certain QFT observables has been proposed. Everything is done on-shell

- More particles, more theories, more loops (contours!)
- UV divergencies(?!)