

LOOKING FOR AN ON-SHELL REGULATOR



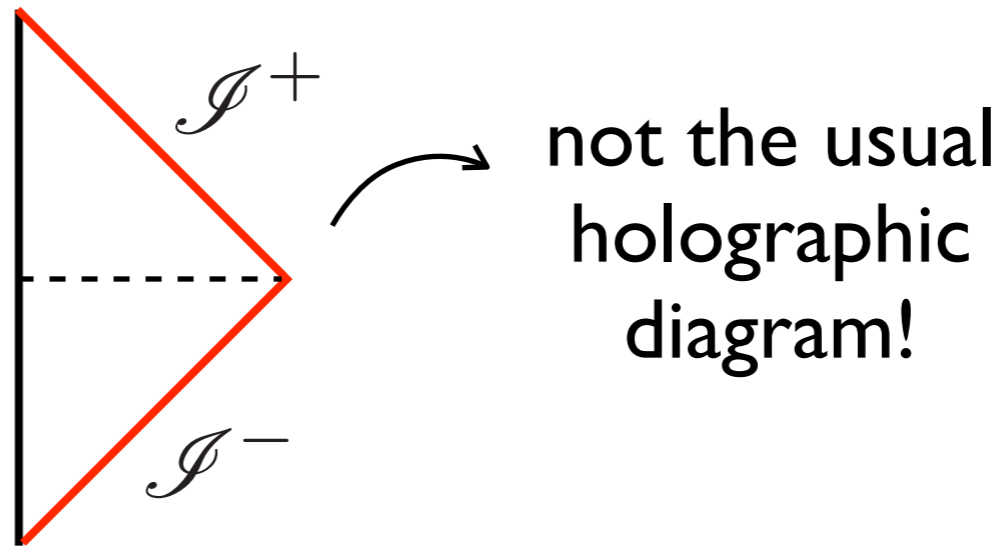
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Holograv 2015 - Firenze



(Based on work with P. Benincasa and D. Gordo)

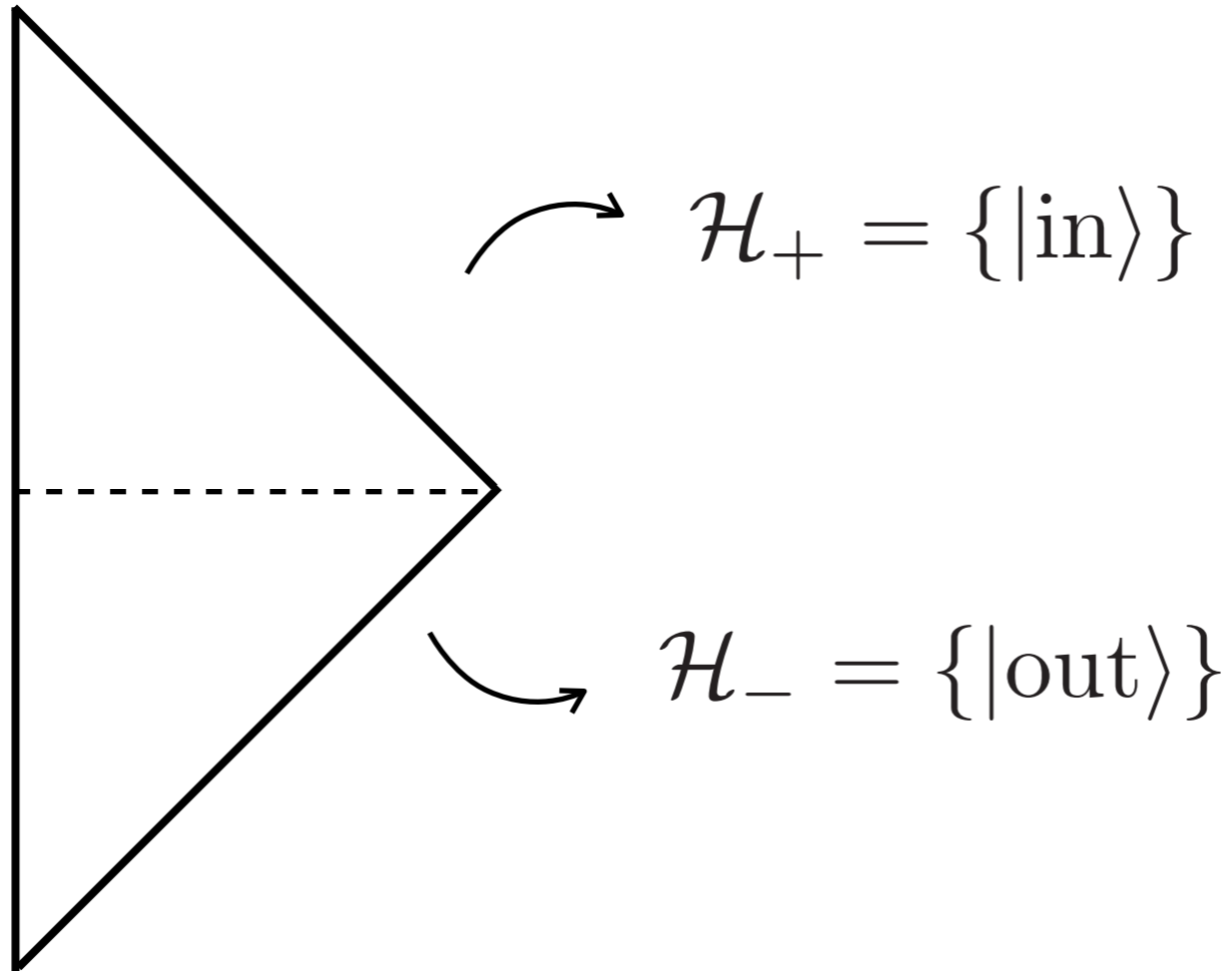
“Holography” without gravity

- Take a QFT in flat Minkowski spacetime



- What information can we gain on the QFT from asymptotic structure?
- Soft theorems, **scattering amplitudes**, ...

Asymptotic structure



$$|\text{out}\rangle = S|\text{in}\rangle$$

**S-matrix is the matrix
of change of basis**

Asymptotic structure

- Asymptotic symmetry group; it defines one-particle states, and it imposes constraints on the form of S
- Unitarity: $S^\dagger S = \mathbb{1}$
- Singularity structure of the S -matrix (e.g. via locality/cluster decomposition pple)

Some restrictions

- **Perturbative approach**
$$M_n^{\text{pert}} = M_n^{\text{tree}} + \sum_{L=1}^{\infty} M_n^L$$

$\propto g^{n-2}$ $\propto g^{n+2L-2}$

$$\left(\begin{array}{c} M_n \\ \lambda \\ \langle \text{in} | S | \emptyset \rangle \end{array} \right)$$
- **Four-dimensional QFTs with only massless particles.**
- **There is a three-point interaction per coupling constant**

Some restrictions

- **Perturbative approach**
$$M_n^{\text{pert}} = \underset{\substack{\text{tree} \\ \text{loop} \\ g^{n-2}}}{M_n^{\text{tree}}} + \sum_{L=1}^{\infty} \underset{\substack{\text{loop} \\ g^{n+2L-2}}}{M_n^L} \left(\begin{array}{c} M_n \\ \text{loop} \\ \langle \text{in} | S | \emptyset \rangle \end{array} \right)$$
- **Four-dimensional QFTs with only massless particles.**
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Goal!

Compute all the perturbative series for the scattering amplitude from asymptotic info

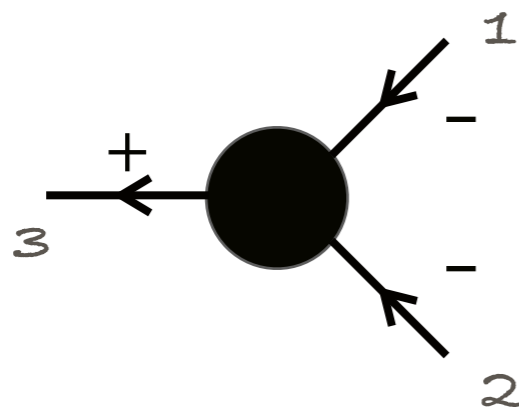
Basic building blocks

- M_3 is fixed by Poincaré invariance (plus allowed singularity structure)

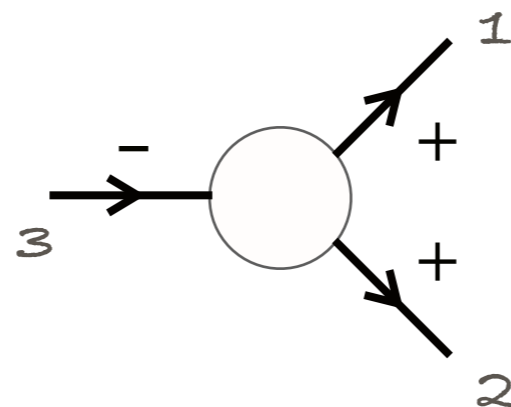
Notice

- Non-perturbative result
- $M_3 \neq 0$ only for complex momenta

- Massless particle in four dimensions $|\text{in}\rangle = |p, h\rangle$



$$h_1 + h_2 + h_3 < 0$$



$$h_1 + h_2 + h_3 > 0$$

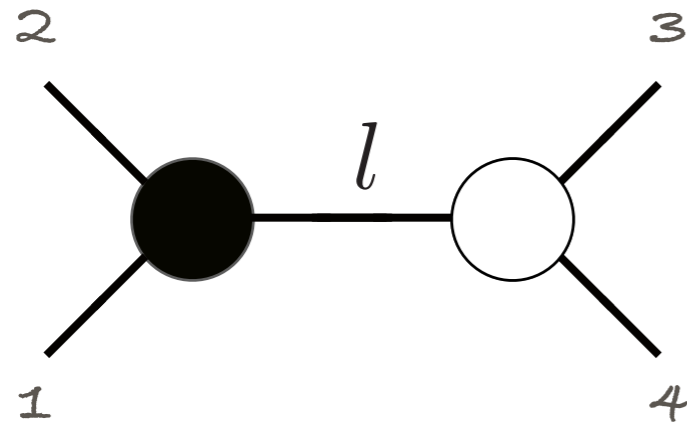
$$\sim g \delta(p_1 + p_2 + p_3)$$

$$(p_1 \cdot p_2)^{\pm(h_3 - h_2 - h_1)}$$

$$(p_2 \cdot p_3)^{\pm(h_1 - h_2 - h_3)}$$

$$(p_3 \cdot p_1)^{\pm(h_2 - h_3 - h_1)}$$

Gluing three-point amplitudes



$$= \int_{\mathbb{R}^4 \subset \mathbb{C}^4} d^4 l \delta^+(l^2) M_3(1, 2, l) M_3(-l, 3, 4)$$

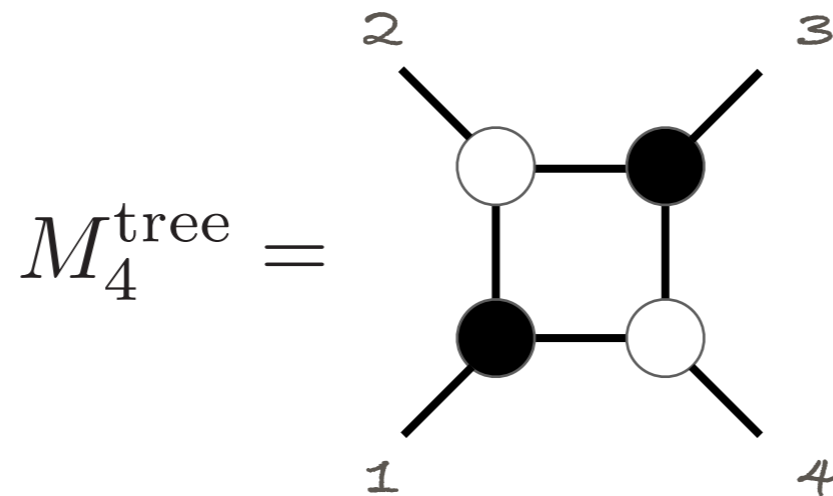
- Any desired diagram can be built in this way. They are called on-shell diagrams.
- What is it computing? (What contour?)

Gluing three-point amplitudes

- Using unitarity (via BCFW techniques), [ARKANI-HAMED, BOURJAILY, CACHAZO, GONCHAROV, POSTNIKOV, TRNKA '12] showed how to represent any amplitude of planar $N=4$ SYM with on-shell diagrams

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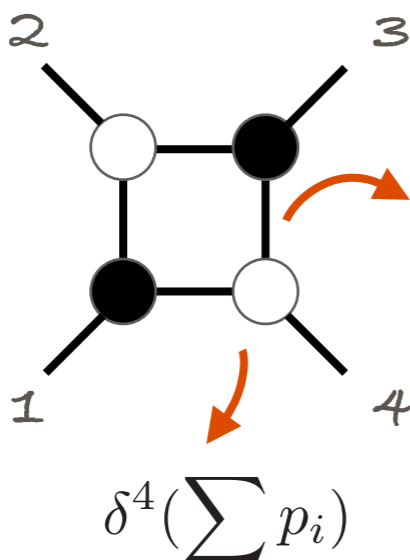
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$$M_{6, \text{NMHV}}^{\text{tree}} =$$

The image shows three on-shell diagrams for the tree-level 6-point NMHV amplitude in planar N=4 SYM. Each diagram is a planar graph with 6 external legs labeled 1 to 6. The vertices are black or white circles. The first diagram has a central square with a triangle on top. The second diagram has a central square with a triangle on the right. The third diagram has a central square with a triangle on the left.

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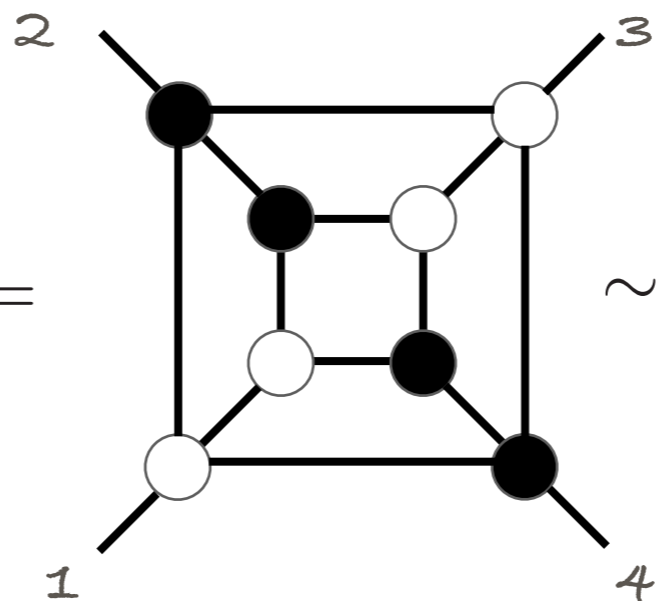
$$M_4^{\text{tree}} = \int d^4l \delta^+(l^2) \delta^4(\sum p_i) \sim \int d\zeta_I$$


$$\# \zeta_I |_{\text{free}} = 3I - 4(V - 1)$$

I : # (internal lines)
 V : # (3-pt vertices)

Gluing three-point amplitudes

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$$M_4^{L=1} = \text{Diagram} \sim \int d^4 \zeta_I$$


What are we computing

$$M_n^L = \int \prod_{k=1}^L d^4 l_k F(p_i, l_k) = \int (\text{integrand})$$

- To integrate or not to integrate
- Contour of integration?
- Need to regulate! What can the regulator be?

First answer given by [FERRO, LUKOWSKI, MENEGHELLI, PLEFKA, STAUDACHER '12], inspired by integrability techniques

An on-shell regulator?

[BENINCASA,
E.C., GORDO '13]

- Deform helicities of three-point amplitudes

$$h_i \rightarrow h_i + \varepsilon_i$$

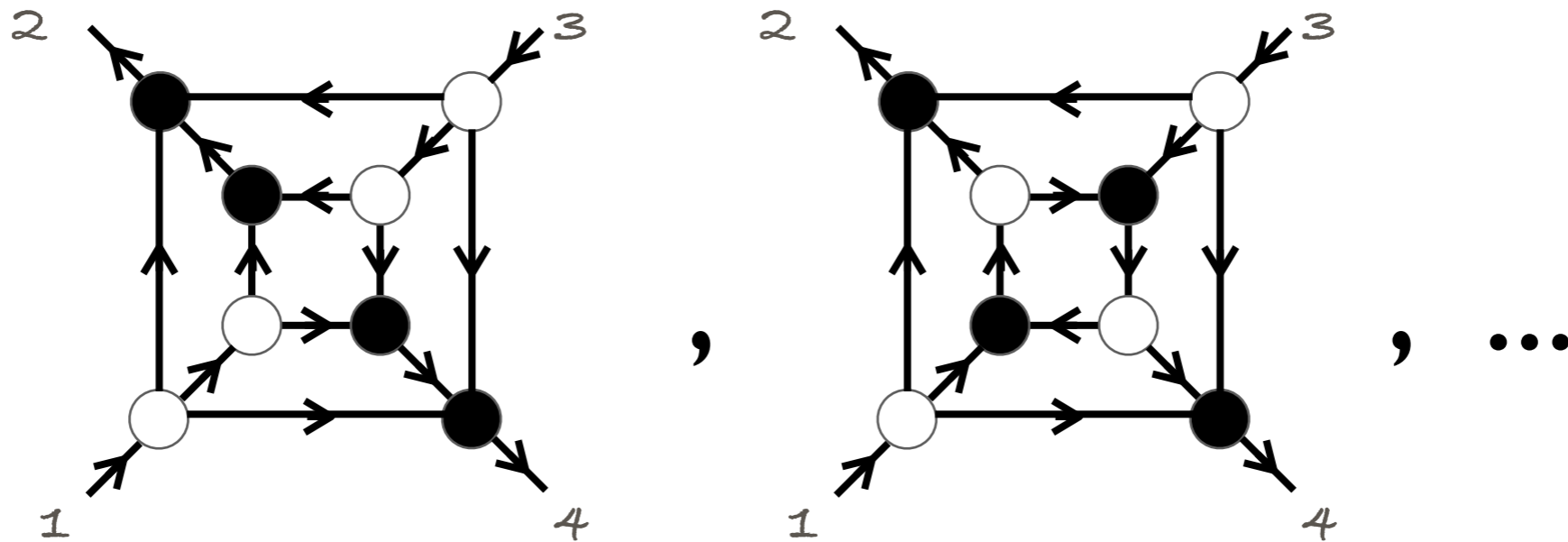
- Keep dimensionality of coupling constant

$$\sum_i \varepsilon_i = 0$$

- Glue the new deformed three-point amplitudes

Does it work?

- Study the simplest case: four particles, **one loop**, self-interacting massless particle (any spin)
- Several diagrams to be considered



One-loop structure of QFT amplitudes

The diagram shows the decomposition of a one-loop amplitude. On the left, a circle with a thick grey border has external lines labeled 1, 2, 3, 4, ..., n. This is equal to the sum of three terms:

- $\sum_s C_s$ multiplied by a square loop diagram with four external lines, labeled "IR divergent".
- $+$ $\sum_t C_t$ multiplied by a triangle loop diagram with three external lines, labeled "IR divergent".
- $+$ $\sum_b C_b$ multiplied by a circle loop diagram with two external lines, labeled "UV divergent".

 The sum is followed by a plus sign and a bracketed term "rational terms".

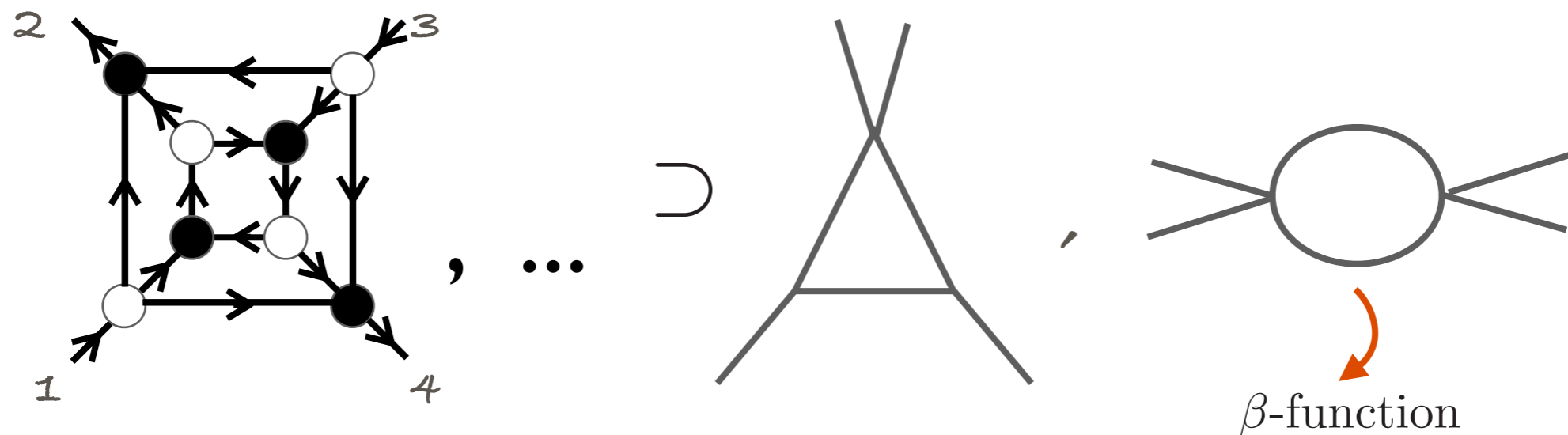
- Our object must contain all of these contributions

The diagram shows an equality between two square loop diagrams. On the left, a square loop with four external lines (labeled 1, 2, 3, 4) contains four internal vertices, each represented by a black dot. Arrows indicate the flow of particles between these vertices. On the right, a simple square loop with four external lines is shown, representing the same diagrammatic structure without the internal vertices.

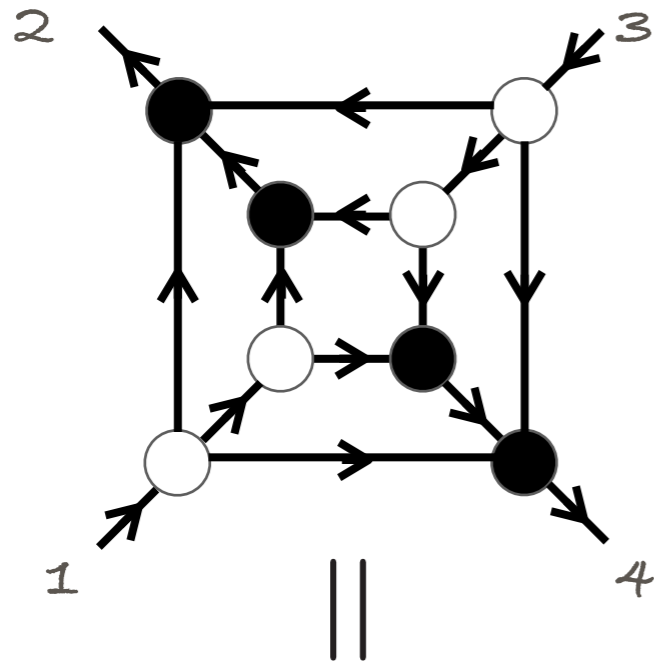
One-loop structure of QFT amplitudes

$$\begin{aligned}
 & \text{Circle with } n \text{ external legs } = \sum_s C_s \text{ (square loop)} + \sum_t C_t \text{ (triangle loop)} + \sum_b C_b \text{ (bubble loop)} + \left(\begin{array}{c} \text{rational} \\ \text{terms} \end{array} \right) \\
 & \text{Labels: IR divergent, IR divergent, UV divergent}
 \end{aligned}$$

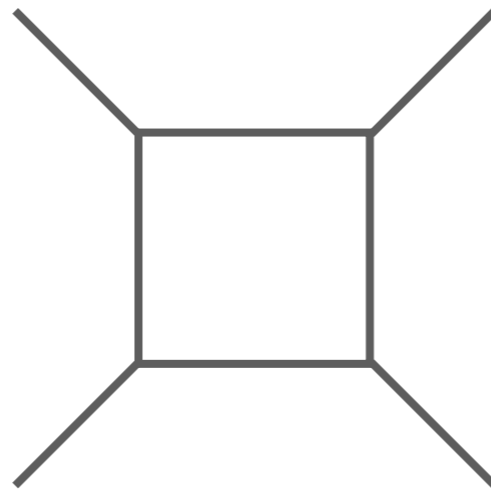
- Our object must contain all of these contributions



Box and IR divergencies



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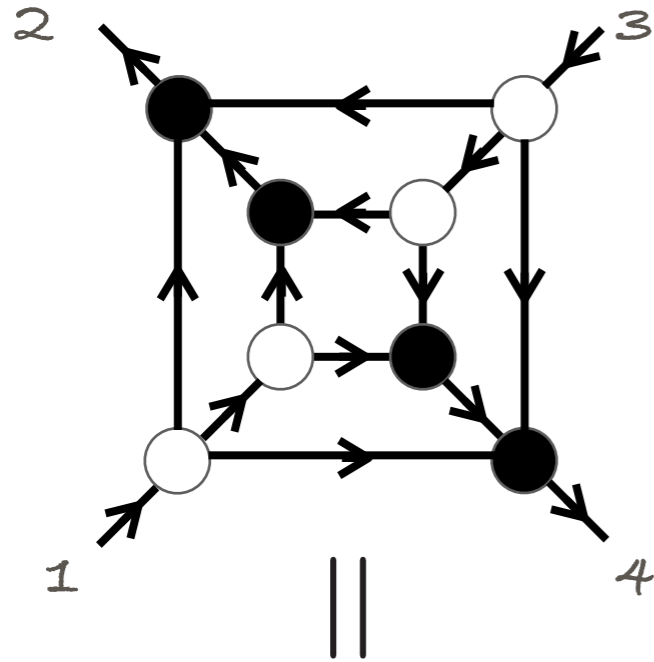
DIMENSIONAL REGULARIZATION

$$\sim \frac{1}{\epsilon^2} - \frac{1}{2\epsilon} \left(\log\left(-\frac{s}{\mu}\right) + \log\left(-\frac{t}{\mu}\right) \right) - \frac{\pi^2}{3} + \frac{1}{2} \log\left(-\frac{s}{\mu}\right) \log\left(-\frac{t}{\mu}\right)$$

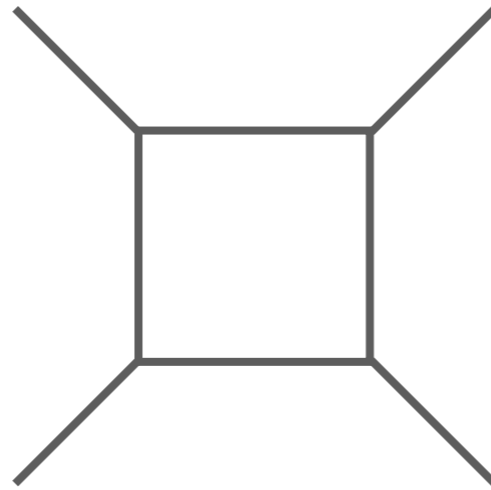
$$st M_{4,\text{def}}^{\text{tree}} \int \bigwedge_{i=1}^4 d\zeta_{i,i+1} \zeta_{i,i+1}^{2\varepsilon_{i,i+1}-1} (1 - \zeta_{i,i+1})^{-2(\varepsilon_{i,i+1} + \bar{\varepsilon}_{i,i+1})}$$

$$\Gamma = \left\{ \zeta \in \mathbb{C}^4 \mid -\frac{s}{u} \zeta_{12} \zeta_{34} - \frac{t}{u} \zeta_{23} \zeta_{41} = \frac{\zeta_{i,i+1}}{\zeta_{i,i+1}^*} \right\} \equiv \mathbb{R}^4 \subset \mathbb{C}^4$$

Box and IR divergencies



=



DIMENSIONAL REGULARIZATION

$$\sim \frac{1}{\epsilon^2} - \frac{1}{2\epsilon} \left(\log\left(-\frac{s}{\mu}\right) + \log\left(-\frac{t}{\mu}\right) \right) - \frac{\pi^2}{3} + \frac{1}{2} \log\left(-\frac{s}{\mu}\right) \log\left(-\frac{t}{\mu}\right)$$

$$st M_{4,\text{def}}^{\text{tree}} \int \prod_{i=1}^4 d\zeta_{i,i+1} \zeta_{i,i+1}^{2\epsilon_{i,i+1}-1} (1 - \zeta_{i,i+1})^{-2(\epsilon_{i,i+1} + \bar{\epsilon}_{i,i+1})}$$

$$\sim \frac{1}{\epsilon^2} - 3 \frac{\log\left(-\frac{s}{u}\right) + \log\left(-\frac{t}{u}\right)}{2\epsilon} + \frac{1}{2} \log\left(-\frac{s}{u}\right)^2 + \frac{1}{2} \log\left(-\frac{t}{u}\right)^2 + 2 \log\left(-\frac{s}{u}\right) \log\left(-\frac{t}{u}\right) - \frac{56\pi^2}{3}$$

$$(\epsilon_{12} = \epsilon_{23} = \epsilon, \quad \epsilon_{34} = \epsilon_{41} = 2\epsilon, \quad \bar{\epsilon}_{i,i+1} = \epsilon_{i,i+1})$$

Perspective

- A general prescription for regulating certain QFT observables has been proposed.
Everything is done on-shell
- More particles, more theories, more loops
(contours!)
- UV divergencies(?!)