The local renormalization group equation in superspace

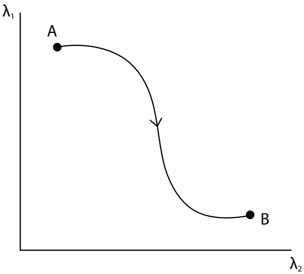
Roberto Auzzi

Università Cattolica, Brescia

April 15, 2015

Based on arXiv:1502.05962 with Boaz Keren-Zur

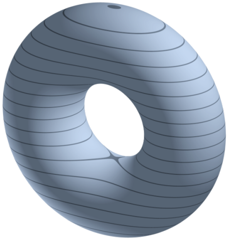
RG flows



Trajectory in the space of theories, induced by a change of scale.

RG and coarse-graining

1. weaker: $a_{IR} < a_{UV}$ where a is a function defined at the conformal fixed point
2. stronger: a is a monotonically decreasing function defined along all the RG flow and stationary at the fixed point
3. strongest: RG is tre gradient flow of an a function: $\beta^I = G^{IJ} \frac{\partial \tilde{a}}{\partial g^J}$



Morse theory may perhaps be used to check option 3 [Gukov 1503.01474]

Tools to study RG flows

- Dispersion Relations
  Dilaton scattering amplitudes
  (Komargodski, Schwimmer 2011)

- Holography (. . .)

- Local Renormalization Group equations
  Wess-Zumino consistency conditions,
  (perturbative proof of a-theorem by Osborn, 1991)

Local renormalization group

Let us start with a fixed point and perturb it with primaries.

$T = T^{\mu}_{\mu}$ controls the change of dynamics under dilatations

T nearby the fixed point can be expanded in a basis of operators of dimension 4:

- Scalar deformations $\mathcal{O}_I$
- Divergences of currents $\partial_\mu J^\mu$
- $\Box \mathcal{O}_a$, where $\mathcal{O}_a$ are primary scalars of dimension 2 (improvement)

The original fixed point can have also global symmetries, which in general will be broken by the perturbations

Sources

W: connected vacuum energy functional; coupling λ^I

$$\langle \mathcal{O}_I \rangle = \frac{1}{\sqrt{-g}} \frac{\delta W}{\delta \lambda^I} \,, \qquad \langle T_{\mu\nu} \rangle = \frac{1}{\sqrt{-g}} \frac{\delta W}{\delta g^{\mu\nu}} \,,$$

Background gauge field for currents and background mass^2 parameters for dimension 2 operators:

$$\frac{1}{\sqrt{-g}} \frac{\delta W}{\delta A^A_\mu} = \langle J^\mu_A \rangle \,, \qquad \frac{1}{\sqrt{-g}} \frac{\delta W}{\delta m^a} = \langle \mathcal{O}_a \rangle \,.$$

Background dilaton field

$$g^{\mu\nu} = e^{2\sigma(x)} \hat{g}^{\mu\nu} \qquad \delta g^{\mu\nu} = 2\sigma(x) \hat{g}^{\mu\nu}$$

is the source of T

$$T = T^\mu_\mu = \frac{2 g^{\mu\nu}}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} W \,.$$

Local RG equation

Let us consider a local Weyl transformation parameterized by $\sigma(x)$, and let us compensate the change of scale with a change in the couplings of the theory:

$$\Delta_\sigma W = \int d^4x\sqrt{-g}\left\{2\sigma g^{\mu\nu}\frac{\delta W}{\delta g^{\mu\nu}} - \sigma\beta^i\frac{\delta W}{\delta\lambda^i}\right.$$

$$-(\sigma\rho_I^A\nabla_\mu\lambda^I - \nabla_\mu\sigma S^A)\frac{\delta W}{\delta A_\mu^A}$$

$$+\left[\sigma\left(m^b(2\delta_b^a - \gamma_b^a) + C^aR + D^a\nabla^2\lambda^I + \frac{1}{2}E_{IJ}^a\nabla_\mu\lambda^I\nabla^\mu\lambda^J\right)\right.$$

$$\left.\left.-\theta_I^a\nabla_\mu\sigma\nabla^\mu\lambda^I + \nabla^2\sigma\eta^a\right]\frac{\delta W}{\delta m^a}\right\} = \mathcal{A}$$

Local RG and operator algebra

When none of the points of the correlators coincide, we get the coordinates of T in the space of composite operators:

$$\langle T \rangle = \beta^I \langle \mathcal{O}_I \rangle + S^A \nabla_\mu \langle J^\mu_A \rangle - \eta^a \nabla^2 \langle \mathcal{O}_a \rangle$$

This equality is valid modulo contact terms for correlators at coincident points

The anomaly $\mathcal{A}$ gives a parametrization of these contact terms

The anomaly

$$\mathcal{A} = \int dx^4 \sqrt{-g} \Big\{ \sigma \Big(\beta_a W^2 + \beta_b E_4 + \frac{\beta_c}{9} R^2$$

$$+ \frac{\chi^e_i}{3} \partial_\mu \lambda^i \partial^\mu R + \frac{\chi^f_{ij}}{6} \partial_\mu \lambda^i \partial^\mu \lambda^j R + \frac{\chi^g_{ij}}{2} \partial_\mu \lambda^i \partial_\nu \lambda^j G^{\mu\nu}$$

$$+ \frac{\chi^a_{ij}}{2} \nabla^2 \lambda^i \nabla^2 \lambda^j + \frac{\chi^b_{ijk}}{2} \partial_\nu \lambda^i \partial^\nu \lambda^j \nabla^2 \lambda^j + \frac{\chi^c_{ijkl}}{4} \partial_\mu \lambda^i \partial^\mu \lambda^j \partial_\nu \lambda^k \partial^\nu \lambda \Big)$$

$$+ \partial^\mu \sigma \left(G_{\mu\nu} w_i \partial^\nu \lambda^i + \frac{Y_i}{3} R \partial_\mu \lambda^i + S_{ij} \partial_\mu \lambda^i \nabla^2 \lambda^j + \frac{T_{ijk}}{2} \partial_\nu \lambda^i \partial^\nu \lambda^j \partial_\mu \lambda^k \right)$$

$$+ \nabla^2 \sigma \left(U_i \nabla^2 \lambda^i + \frac{V_{ij}}{2} \partial_\nu \lambda^i \partial^\nu \lambda^j \right)$$

$$+ \sigma \left(\frac{\kappa_{AB}}{4} F^A_{\mu\nu} F^{B\mu\nu} + \frac{\zeta_{Aij}}{2} F^A_{\mu\nu} \nabla^\nu \lambda^i \right) + \nabla^\mu \sigma (\eta_{AI} F^A_{\mu\nu} \nabla^\nu \lambda^i) \Big\}$$

Wess-Zumino consistency condition

$$\Delta^{CS}(E_4) = 4\sigma E_4 - 8G^{\mu\nu}\nabla_\mu\nabla_\nu\sigma\,, \qquad \Delta^{CS}(R) = 2\sigma R + 6\nabla^2\sigma\,, \qquad \ldots$$

$$[\Delta^{CS}_\sigma, \Delta^{CS}_{\sigma'}] = 0$$

gives several equations the function parameterizing the anomaly $\mathcal{A}$

$$8\partial_i\beta_b - \chi^g_{ij}\beta^j = -\mathcal{L}(w_i)\,,$$

$$\mathcal{L}(w_i) \equiv \beta^j\partial_j w_i + \partial_i\beta^j w_j\,,$$

χ^g_{ij} can be directly computed in weakly-coupled theories (see Jack and Osborn, 1990).

$a-$function outside criticality

Nearby the fixed point then positivity of χ^g_{ij} follows from the one of χ^a_{ij}, from the consistency condition:

$$2\chi^a_{ij} - \chi^g_{ij} = -(2\partial_I \beta^k \chi^a_{kj} + \beta^k \chi^b_{kij}) + \mathcal{L}(S_{ij})$$

Assuming χ^g_{ij} positive definite, the quantity which decreases then is:

$$\beta^k \partial_k \tilde{a} = \frac{\chi^g_{ij}}{8} \beta^i \beta^j \,, \qquad \tilde{a} = \beta_b + \frac{1}{8} w_i \beta^i$$

this generalizes a outside criticality

This approach does not give a proof of a-theorem because we don't know how to show that χ^g_{ij} is positive

Ambiguity in $a-$function

Correspond to the possibility to add to W a local counterterm

$$\delta W = \int d^4x \sqrt{-g}\, g_{ij} \nabla^{\mu} \lambda^i \nabla_{\mu} \lambda^j$$

which gives the shift:

$$\tilde{a} \rightarrow \tilde{a} + g_{ij} \beta^i \beta^j \,,$$

$$\chi^g_{ij} \rightarrow \chi^g_{ij} + \mathcal{L}(g_{ij})$$

$\tilde{a}$ is a scheme-independent quantity just at criticality

Other consistency conditions

Most of the other consistency conditions give several algebraic relations between the anomaly coefficients

At the end one finds 10 independent functions in the anomaly, constrained by just three relations:

$$8\partial_i \beta_b - \chi^g_{ij}\beta^j = -\mathcal{L}(w_i)\,,$$

$$\mathcal{L}(\eta_{Ai}) = \kappa_{AB} P^B_i + \zeta_{Aij}\beta^j - \chi^g_{ij}(T_A\lambda)^j\,,$$

$$0 = \eta_{Ai}\beta^i + w_i(T_A\lambda)^i\,,$$

[Baume, Keren-Zur, Rattazzi, Vitale 2014]

Local RG and SUSY

One can directly specialize the formalism to the case of a supersymmetric theory [Freedman, Osborn, 1998]

On the other hand in this way we are not using all the information: R-symmetry, holomorphicity

Also in SUSY one can often compute a in term of the R-charge... [Anselmi, Freedman, Grisaru, Johansen, 1997]

... and R-charges from $a-$maximization [Intriligator, Wecht, 2003]

guess for $a-$function outside criticality [Kutasov 2003]

Local RG in superspace

In order to use the consistency conditions in supersymmetric theories in their full power, the local RG equations begs to be put in superspace formalism, with coupling to background supergravity

- several consistency condition which have no analog in the non-SUSY case.
- we derive an analog a-maximization outside criticality
- we find Kahler condition for conformal manifold (in agreement with [Asnin 2009])

Chiral deformations

Nearby a conformal fixed point, perturbed by marginal deformations

$$\int d^4x \lambda^I \mathcal{O}_I(x)$$

Superspace

Neraby superconformal fixed point, perturbed by chiral marginal deformations

$$\int d^4x\, d^2\theta \lambda^I \mathcal{O}_I(z_+) + c.c.$$

e.g. superpotential $\lambda\Phi^3$ coupling, or gauge coupling $\tau W^\alpha W_\alpha$ (in this case it is more convenient to use $\lambda = e^{-S}$ as coupling, in order that symmetries act linearly on it)

The background sources

We promote the couplings to be background fields, $\lambda \to \lambda(x)$
and introduce a background metric $g^{\mu\nu}(x)$
and background gauge fields $A^\mu(x)$.

Superspace

We promote the couplings to be chiral background fields, $\lambda \to \lambda(x)$
and introduce a background of old minimal supergravity
with chiral compensator φ
and background real vector fields $(e^V)_{i\bar{j}} \approx \delta_{i\bar{j}} + \theta\sigma^\mu\bar{\theta}(A_\mu)_{ij}$.

Example: Wess-Zumino model

$$\frac{\delta}{\delta \lambda^{ijk}} \mathcal{W}[\lambda, \bar{\lambda}, Z] = \Phi_i \Phi_j \Phi_k \qquad \frac{\delta}{\delta Z^{i\bar{j}}} \mathcal{W}[\lambda, \bar{\lambda}, Z] = \Phi_i \bar{\Phi}_j$$

- $Z^{i\bar{j}}$ can be interpreted as the source for the kinetic term

  (Necessary because we are working in the holomorphic scheme, and WF renormalization is not absorbed into the dynamical fields)

- Using the notation $Z^{i\bar{j}} \equiv (e^{-V})^{i\bar{j}}$ it can also be interpreted as a background gauge field.

$$Z^{j\bar{i}} \bar{\Phi}_{\bar{i}} \Phi_j \equiv \Phi_j (e^{-V})^{j\bar{i}} \bar{\Phi}_{\bar{i}}$$

The real multiplet containing the Noether current

$$-(e^V T_A)_{\bar{i}j} \frac{\delta}{\delta (e^V)_{\bar{i}j}} \mathcal{W} \quad = \quad \bar{\Phi} e^{-V} T_A \Phi = J_A$$

Weyl invariance

Weyl: The parameter is just a scalar field σ

Superspace

SuperWeyl: The parameter is a chiral superfield

$$\sigma| = \sigma_0 + \frac{2}{3} i\alpha \,,$$

where σ_0 is the Weyl parameter and α is the parameter of local chiral $U(1)$

The local RG generator

$$\Delta_\sigma^W \equiv \int d^4x \; \lambda 2\sigma g^{\mu\nu} \frac{\delta}{\delta g^{\mu\nu}} - \sigma\beta^I \frac{\delta}{\delta\lambda^I} - \sigma P_I^A \nabla^\mu \lambda^I \frac{\delta}{\delta A_\mu^A}$$

Superspace

$$\Delta_\sigma^W \equiv \int d^6z \; \varphi^3 \; \sigma \left(3\frac{\delta}{\delta\varphi} + b^I \frac{\delta}{\delta\lambda^I} \right) + c.c$$

$$+ \int d^8z E^{-1} (\sigma\Gamma^A (e^V T_A)_{\bar{i}j} + \bar{\sigma}\bar{\Gamma}^A (\bar{T}_A e^V)_{\bar{i}j}) \frac{\delta}{\delta(e^V)_{\bar{i}j}}$$

the second term contains both wave function renormalization and background gauge fields

The generator of the R-symmetry off-criticality

$$\Delta^R_\alpha \propto -i\alpha \int d^4x d^2\theta \; b^I \frac{\delta}{\delta\lambda^I} + c.c \\ -i\alpha \int d^4x d^4\theta \left((e^V \Gamma)_{\bar{i}j} - (\bar{\Gamma} e^V)_{\bar{i}j} \right) \frac{\delta}{\delta (e^V)_{\bar{i}j}} + \ldots$$

- To avoid assigning R-charges to the background gauge fields:

$$(e^V \Gamma)_{\bar{i}j} - (\bar{\Gamma} e^V)_{\bar{i}j} = 0$$

- R-symmetry realized linearly on the background fields

$$b^I = 0 \qquad \text{or} \qquad b^I \propto \lambda^I$$

This is the non-renormalization theorem for the superpotential.

(For gauge couplings, $\lambda_G \equiv \exp\left\{-4\pi/g_h^2 + i\Theta/(2\pi)\right\}$, such that the holomorphic β-function is linear in λ)

Holomorphic vs NSZV-like β function

b^I is the holomorphic $\beta-$function
(can be non-zero at conformal fixed point)

$$\langle \mathcal{T} \rangle = -\left(b^I \langle \mathcal{O}_I \rangle - \frac{1}{4} \Gamma^A \bar{\mathcal{D}}^2 \langle J_A \rangle \right) = \beta^I \langle \mathcal{O}_I \rangle$$

where

$$\beta^I = b^I + \Gamma^A (T_A)^I_J \lambda^J \tag{1}$$

is the physical $\beta-$function

Ambiguity in the R-symmetry

- There is some arbitrariness in the definition of Γ^A and b

$$
\begin{aligned}
b^{I'} &= b^I - a^A (T_A \lambda)^I \\
\Gamma^{A'} &= \Gamma^A + a^A
\end{aligned}
$$

- This correspond to change the R-symmetry of the elementary component fields, which are not gauge invariant objects, keeping the dimension of gauge-invariant composite operators unchanged
- The physical β function $\beta^I = b^I + (\Gamma\lambda)^I$ is insensitive to this ambiguity.

The local RG equation

$$\Delta_\sigma^{SW} \mathcal{W} = \int d^4x \mathcal{A}_\sigma^{SW}$$

How do we find the generalized Weyl anomaly?

- Write the most general function of the sources with the right dimensions.
- Eliminate as many anomaly terms as possible
  by adding local terms to the action.

The super Weyl anomaly

$$
\begin{aligned}
\mathcal{A}_\sigma^{SW} \quad = \quad & \int d^6 z \phi^3 \sigma \left((c-a)(W_{\alpha\beta\gamma})^2 + \kappa_{AB} W^{A\alpha} W^B_\alpha \right) + \text{c.c.} \\
& + \int d^8 z E^{-1} \sigma \Big(a(G^2 + 2R\bar{R}) + bR\bar{R} + \chi_{I\bar{J}} G^{\alpha\dot{\alpha}} \nabla_\alpha \lambda^I \bar{\nabla}_{\dot{\alpha}} \bar{\lambda}^{\bar{J}} \\
& \qquad + \xi^1_{[IJ]\bar{K}} \nabla_{\alpha\dot{\alpha}} \lambda^I \nabla^\alpha \lambda^J \bar{\nabla}^{\dot{\alpha}} \bar{\lambda}^{\bar{K}} + \xi^2_{[\bar{I}\bar{J}]K} \nabla_{\alpha\dot{\alpha}} \bar{\lambda}^{\bar{I}} \bar{\nabla}^{\dot{\alpha}} \bar{\lambda}^{\bar{J}} \nabla^\alpha \lambda^K \\
& \qquad + \zeta^1_{AI} W^{A\alpha} \nabla_\alpha \lambda^I + \zeta^2_{A\bar{I}} \bar{W}^A_{\dot{\alpha}} \bar{\nabla}^{\dot{\alpha}} \bar{\lambda}^{\bar{I}} \\
& \qquad + \epsilon^1_{I\bar{J}} \nabla^2 \lambda^I \bar{\nabla}^2 \lambda^{\bar{J}} + \epsilon^2_{IJ\bar{K}} \nabla^\alpha \lambda^I \nabla_\alpha \lambda^J \bar{\nabla}^2 \bar{\lambda}^{\bar{K}} \\
& \qquad + \epsilon^3_{\bar{I}\bar{J}K} \bar{\nabla}_{\dot{\alpha}} \bar{\lambda}^{\bar{I}} \bar{\nabla}^{\dot{\alpha}} \bar{\lambda}^{\bar{J}} \nabla^2 \lambda^K + \epsilon^4_{IJ\bar{K}\bar{L}} \nabla^\alpha \lambda^I \nabla_\alpha \lambda^J \bar{\nabla}_{\dot{\alpha}} \bar{\lambda}^{\bar{K}} \bar{\nabla}^{\dot{\alpha}} \bar{\lambda}^{\bar{L}} \\
& \qquad + \eta^1_I R \nabla^2 \lambda^I + \eta^2_{\bar{I}} \bar{R} \bar{\nabla}^2 \bar{\lambda}^{\bar{I}} + \eta^3_{IJ} R \nabla^\alpha \lambda^I \nabla_\alpha \lambda^J + \eta^4_{\bar{I}\bar{J}} \bar{R} \bar{\nabla}_{\dot{\alpha}} \bar{\lambda}^{\bar{I}} \bar{\nabla}^{\dot{\alpha}} \bar{\lambda}^{\bar{J}} \Big) + \text{c.c.} \\
& + \int d^8 z E^{-1} \mathcal{D}^\alpha \sigma \left(w^1_{\bar{I}} G_{\alpha\dot{\alpha}} \bar{\nabla}^{\dot{\alpha}} \bar{\lambda}^{\bar{I}} + w^2_I R \nabla_\alpha \lambda^I + u^1_{I\bar{J}} \nabla_\alpha \lambda^I \bar{\nabla}^2 \bar{\lambda}^{\bar{J}} \right) + \text{c.c.} + \mathcal{A}^{GW}_\sigma
\end{aligned}
$$

The anomaly coefficients a, b, $\chi_{I\bar{J}} \ldots$ are functions of the couplings which are the subject of our investigation. $\mathcal{A}^{GW}_\sigma$ is a non-gauge invariant piece and is related to chiral anomalies.

The Wess-Zumino consistency conditions

$$[\Delta^{SW}_{\sigma_2}, \Delta^{SW}_{\sigma_1}]\mathcal{W} \;=\; \Delta^{SW}_{\sigma_2}\mathcal{A}^{SW}_{\sigma_1} - \Delta^{SW}_{\sigma_1}\mathcal{A}^{SW}_{\sigma_2} \;=\; 0$$

- This constraint leads to differential equations relating between the anomaly coefficients.
- In the original formulation, these equations were used to prove the irreversibility of the RG flow in perturbation theory.
- There was a previous attempt to write the Weyl consistency conditions in superspace, which missed many of the crucial ingredients (Grosse, 07)

Chiral anomalies

The Weyl-flavor Wess-Zumino consistency condition (Keren-Zur, 2014)

$$[\Delta_\sigma^{SW}, \Delta_\Lambda^G]\mathcal{W} \;=\; \Delta_\sigma^{SW}\mathcal{A}_\Lambda^G - \Delta_\Lambda^G\mathcal{A}_\sigma^{SW} \;=\; 0$$

If there are no chiral anomalies $\mathcal{A}_\sigma^{SW}$ is gauge invariant; otherwise it picks up some non-gauge invariant terms:

$$\mathcal{A}_\sigma^{GW} = 2k_{ABC}\int d^8 z E^{-1}\,\sigma\Big(\Gamma^C\nabla^\alpha V^A W_\alpha^B - \partial_{\bar{i}}\Gamma^C V^A \bar{W}_{\dot\alpha}^B \bar{\nabla}^{\dot\alpha}\bar{\lambda}^{\bar{i}})\Big) + \mathrm{c.c.}$$

This was addressed only for anomalous $U(1)$

The consistency conditions

$$
\begin{aligned}
0 &= \Delta^{SW}_{\sigma_2} \mathcal{A}^{SW}_{\sigma_1} - \Delta^{SW}_{\sigma_1} \mathcal{A}^{SW}_{\sigma_2} \\
&= \int d^8 z E^{-1} \sigma_{[1} \bar{\sigma}_{2]} G^{\alpha\dot{\alpha}} G_{\alpha\dot{\alpha}} \left(\bar{\beta}^{\bar{I}} \partial_{\bar{I}} a - \beta^I \partial_I \bar{a} \right) \\
&\quad + \int d^8 z E^{-1} \sigma_{[1} \nabla_\alpha \sigma_{2]} G^{\alpha\dot{\alpha}} \bar{\nabla}_{\dot{\alpha}} \bar{\lambda}^{\bar{I}} \left(\partial_{\bar{I}} a - 2\beta^J \chi_{J\bar{I}} + \ldots \right) \\
&\quad + \int d^8 z E^{-1} \sigma_{[1} \nabla_\alpha \sigma_{2]} W^A_\alpha \left(2\kappa_{AB} \Gamma^B + \ldots \right) \\
&\quad + \ldots
\end{aligned}
$$

The consistency conditions may contain total derivatives, e.g.

$$
\int d^8 z E^{-1} \sigma_{[1} \mathcal{D}^\alpha \sigma_{2]} W^A_\alpha \Upsilon_A
$$

The consistency conditions are defined up to two new functions Υ_A and Ω_I, not anomaly coefficients.

Irreversibility of the RG flow

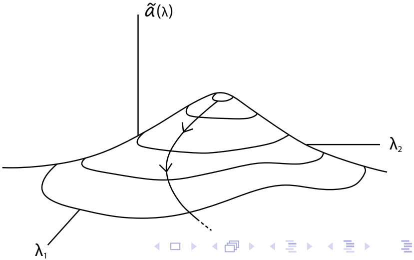
$$\tilde{a} \equiv a + \bar{w}_I \beta^I$$

Reality of $\tilde{a}$ follows from a consistency conditions: $\widetilde{a} = \bar{\widetilde{a}}$

$$\partial_I \widetilde{a} \;=\; 2\bar{\beta}^{\bar{J}} \chi_{I\bar{J}} - 2\beta^J(\partial_J \bar{w}_I - \partial_I \bar{w}_J)$$

$$\mu \frac{d}{d\mu}\tilde{a} \;=\; \beta^I \partial_I \tilde{a} + \bar{\beta}^{\bar{J}} \partial_{\bar{J}} \tilde{a} \;=\; 2\beta^I \bar{\beta}^{\bar{J}}(\chi_{I\bar{J}} + \bar{\chi}_{\bar{J}I})$$

- If $\chi_{I\bar{J}} + \bar{\chi}_{\bar{J}I}$ is positive definite then $\tilde{a}$ **changes monotonously**



a-max at fixed point: traditional derivation

[Intriligator, Wecht, 03]

1. Guess an R-symmetry R^f.
2. Define a linear combination with other unbroken $U(1)$ symmetries.

$$R_a^f = R^f + a^A T_A$$

   a^A a set of coefficients, the index A runs over the unbroken $U(1)$s.
3. Find the coefficients a using the equations which must be satisfied by the correct R symmetry

$$9Tr[(R_a^f)^2 T_A] - Tr[T_A] \quad = \quad 0$$

Derived from component expression:

$$\partial_\mu (J^\mu)_A = \frac{k_A}{384\pi^2}\left(R\tilde{R} + \frac{8}{3} F_R \tilde{F}_R\right)$$

(equality of $U(1)_A \times$ (Gravity)^2 and $U(1)_A \times U(1)_R^2$ anomaly coefficients)

a-maximization from consistency conditions

$$2\kappa_{AB}\Gamma^B + 3k_{ABC}\Gamma^B\Gamma^C + \zeta^1_{AI}\beta^I + w_{\bar{I}}(e^V T_A e^{-V}\bar{\lambda})^{\bar{I}}$$
$$= \Upsilon_A + \Omega_I(T_A\lambda)^I$$

κ , ζ and w are anomaly coefficients; Ω is a function, which appear also in another consistency condition:

$$\chi_{I\bar{J}} - \partial_I w_{\bar{J}} - 2i\xi^1_{[IK]\bar{J}}\beta^K + \zeta^1_{AI}\partial_{\bar{J}}\Gamma^A = -\partial_{\bar{J}}\Omega_I \,.$$

Υ_A is holomorphic function of chiral coupling; we can argue that it is zero in asymptotically free theories
At the fixed point, considering unbroken symmetries, this reduces to

$$2\kappa_{AB}\Gamma^B + 3k_{ABC}\Gamma^B\Gamma^C = \Upsilon_A$$

a-maximization: comparison

Using $R = \frac{2(\Gamma^A T_A)-1}{3}$ the a-extremization is given by

$$Tr[T_A T_B]\Gamma^B - Tr[T_A T_B T_C]\Gamma^B \Gamma^C = 0$$

Let's compare this to the consistency condition

$$2\kappa_{AB}\Gamma^B + 3k_{ABC}\Gamma^B\Gamma^C + \zeta^1_{AI}\beta^I + w_{\bar{I}}(e^V T_A e^{-V}\bar{\lambda})^{\bar{I}}$$
$$= \Upsilon_A + \Omega_I(T_A\lambda)^I$$

Our derivation extends the usual one in two ways:
1) It is valid out of criticality
2) It is valid for non-conserved $U(1)$ currents (which are explicitly broken by the background couplings)

$\tilde{a}$ off-criticality

Combining a few consistency conditions and setting $\tilde{\Omega}_I = \Omega_I - \bar{w}_I$:

$$\partial_I \tilde{a} = \partial_I \left(\frac{1}{128\pi^2} Tr[\gamma^2] - \frac{1}{192\pi^2} Tr[\gamma^3] - \beta^I \tilde{\Omega}_I \right)$$

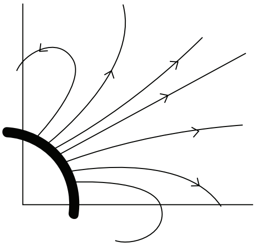
$$\tilde{a} = \frac{1}{128\pi^2} Tr[\gamma^2] - \frac{1}{192\pi^2} Tr[\gamma^3] - \beta^I \tilde{\Omega}_I + const$$

Similar expression were assumed (without proof) in early studies of the *a*-function for SUSY gauge theories e.g.(Freedman and Osborn 1998, Kutasov 2003, Jack and Osborn 2013).

Here $\tilde{\Omega}_I$ is not a Lagrange multiplier as in Kutasov 2003

Conformal manifolds

- The parameter space may contain manifolds of fixed points (Leigh, Strassler, 95) .
- The manifold can be studied using the local RG consistency conditions.
- Characterized by $\beta^I = \partial_J \beta^I = 0$
- This simplifies the consistency conditions.



Conformal manifolds – more simplifications

- Due to the Ward identity, some of the operators sourced by the λ are not primary

$$\mathcal{O}_I (T_A \lambda)^I \propto \bar{D}^2 J_A$$

- The coordinates on the manifold must correspond to primary operators.
- Primary operators are annihilated by the special conformal transformation.
- We find:

$$\partial_I \Gamma^A = 0 \qquad \forall\ \mathcal{O}_I \text{ primary}$$

Further simplifications of the CC.

Conformal manifolds

- The conformal manifold admits a metric known as the **Zamolodchikov metric**

$$\mathcal{G}_{I\bar{J}} \quad \propto \quad \langle \mathcal{O}_I(z_{1+})\bar{\mathcal{O}}_{\bar{J}}(z_{2-})\rangle x_{2\bar{1}}^6$$

- On this manifold the metric is proportional to anomaly coefficients

$$\mathcal{G}_{I\bar{J}} \propto \chi_{I\bar{J}} + \bar{\chi}_{\bar{J}I}$$

- The combination of several consistency conditions on the manifold proves that the Zamolodchikov metric is Kahler:

$$\partial_K \mathcal{G}_{I\bar{J}} = \partial_I \mathcal{G}_{K\bar{J}}$$

and its complex conjugate.
Already shown by (Asnin 2009) with different methods.

The main points of the talk

The local RG equation is a basic tool in the study of RG flows, which has not been fully exploited.

We defined the equation in superspace,
and initiated the study of the consistency conditions.

Results so far:

- An analog of a-maximization off-criticality
- An expression for a off-criticality
- Kahler conditions for the metric on conformal manifolds

Future directions

- There are several other consistency equations whose physical meaning should be explored.
- What about using new minimal SUGRA ? will it give equivalent consistency conditions ?
- Can we use it to constrain accidental symmetries?
- Gradient flow ? Topology of the coupling manifold and of the conformal manifolds? [Gukov 1503.01474]
- Extended supersymmetry
- Other dimensions
- Dig deeper and explore the potential of this formalism....

Thank you