Electron-Phonon interactions, MIT & Holographic Massive Gravity

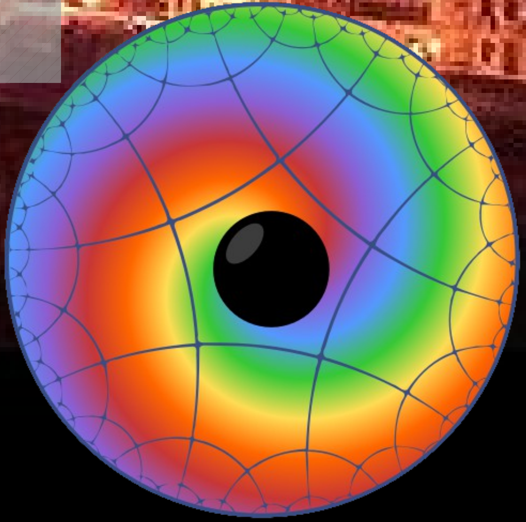
Matteo Baggioli

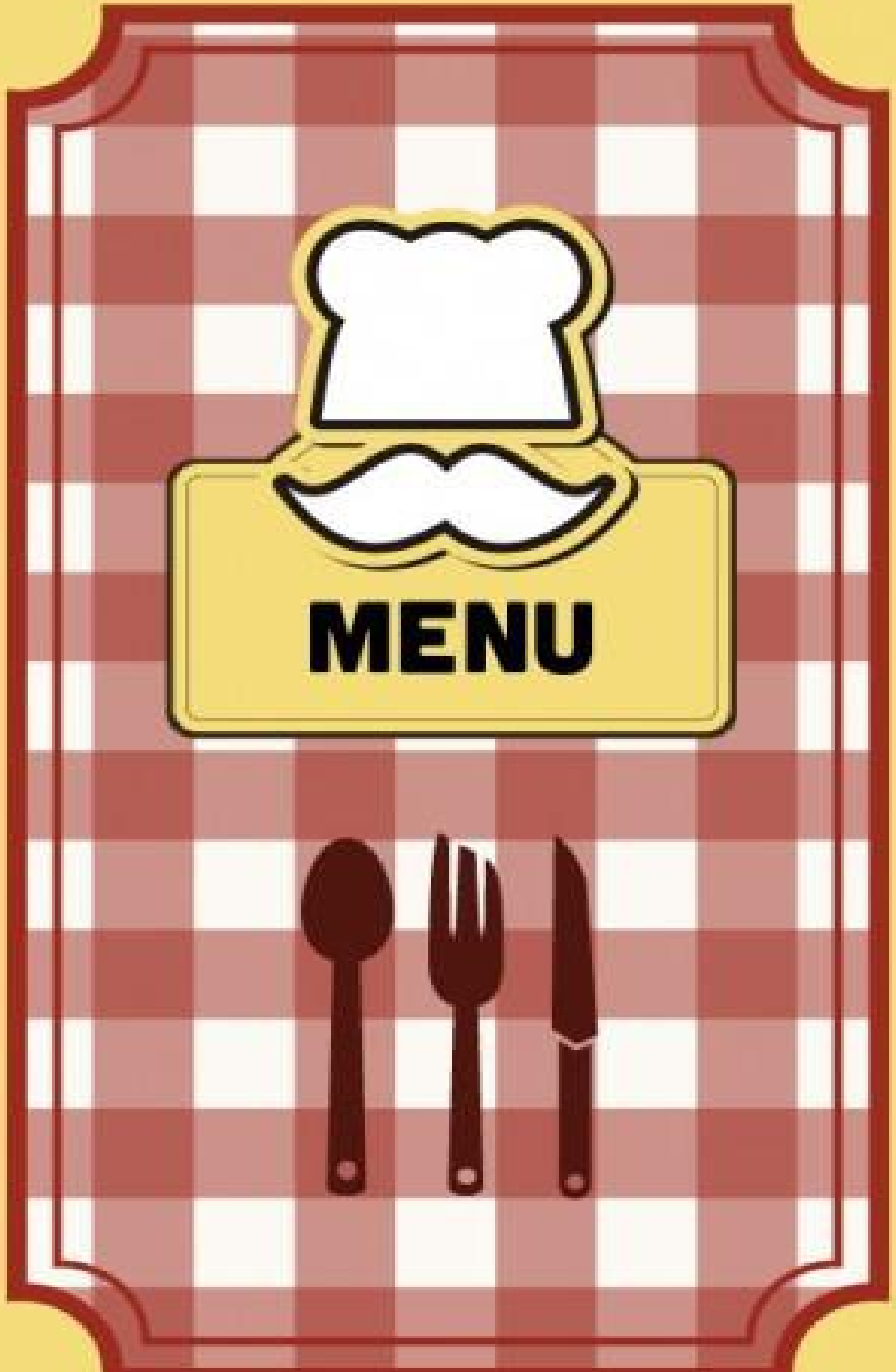
Universitat Autonoma de Barcelona

UAB

Institut de fisica d'Altes Energies IFAE

Gauge Gravity Duality 2015





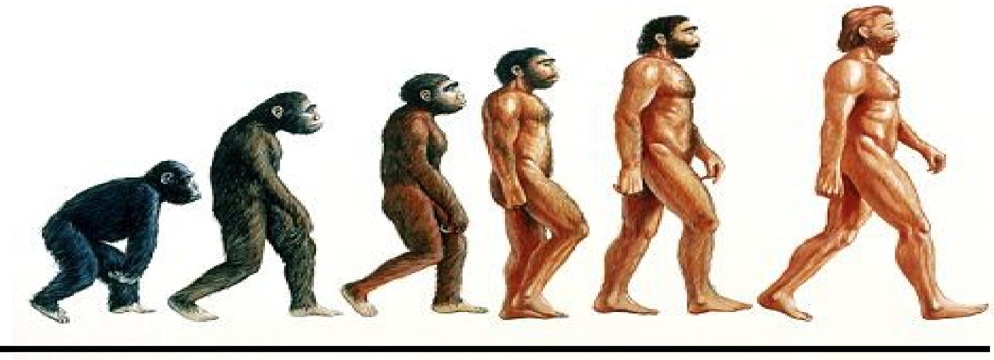
MENU

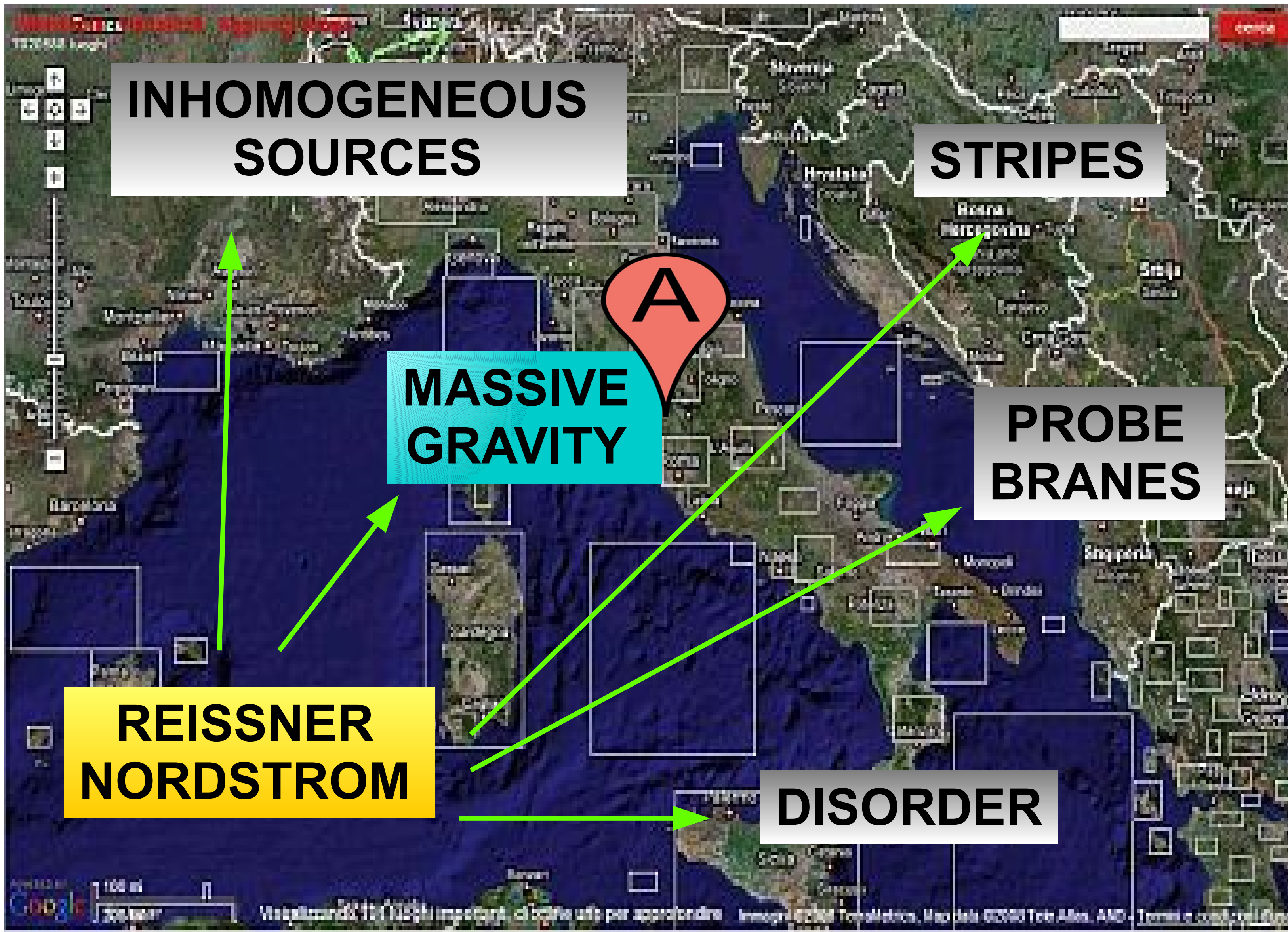
Massive Gravity and effective field theories for phonons in solids

Electron-Phonon Interactions & Holographic Response

Conclusions & future

Holographic





Massive Gravity is a wonderful effective approach, but ...

Who are we talking about ??

Spoiling the end

Old School Massive Gravity

Infrared modification of General Relativity = "Large distance" modification

$$S = M_{\rm Pl}^2 \int d^4x \left\{ L_{EH}^{(2)}(h_{\mu\nu}) + \frac{\alpha}{4} h_{\mu\nu}h^{\mu\nu} + \frac{\beta}{4}(h^{\mu}_{\mu})^2 \right\} \qquad \beta = -\alpha = m_G^2.$$

GR **FIERZ PAULI**

BREAKING OF DIFFEOMORPHISM INVARIANCE !!

GRAVITON WITH EXTRA DEGREES OF FREEDOM

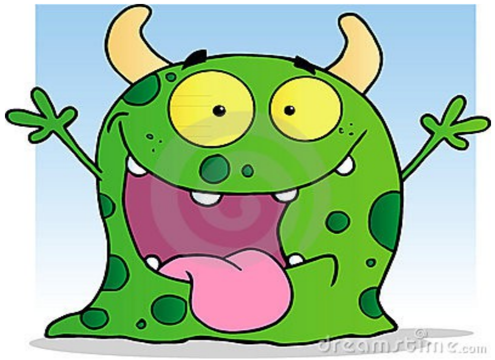
Attention:
- *Ghosts*
- *dVDZ discontinuity*
- *Strong Coupling*

Goldstones ...

Massive Gravity Nightmare

LINEAR LEVEL: OK !

NON LINEAR LEVEL:



BOULWARE DESER GHOST

Recently (probably) solved by :

dRGT MODEL

But this is for the Lorentz Invariant case

And we are doing Condensed Matter ...

Non Relativistic Massive Gravity

$$S^{(2)} = S^{(2)}_{EH} + S_m,$$

arXiv:0802.4379

arXiv:hep-th/0409124

$$L_m = \frac{1}{4}[m_0^2 h_{00}h_{00} + 2m_1^2 h_{0i}h_{0i} - m_2^2 h_{ij}h_{ij} + m_3^2 h_{ii}h_{jj} - 2m_4^2 h_{00}h_{ii}].$$

+ *mass terms in the holographic direction ...*

SUPER HEALTHY THEORIES AT NON LINEAR LEVEL
(provided some weak constraints)

WAY MORE GENERAL THAN dRGT MODEL !!

Not all the choices healthy, need to impose constraints on the masses...

For Example : $m_2^2 \geq 0.$ $m_1^2 \geq 0.$

Residual Gauge Simmetries

One elegant way to avoid fine tuning the masses is thinking of various Residual gauge simmetries...

In several cases this approach does lead to healthy theories...

Example:

$$t \to t + \zeta^0(x^i, t)\,. \longrightarrow m_0 = m_1 = m_4 = 0.$$

Our case: we want to preserve SO(3) and homogeneity And break Just translation in the spatial boundary directions...

More elegant way: COSET constructions

Solids, fluids, superfluids, supersolid

arXiv:1307.0517

Stuckelberg Trick

IDEA : "ENLARGE" THE FIELD CONTENT
RESTORE GAUGE INVARIANCE

introduces a new, Stückelberg field ξ_μ and a new field $\bar{h}_{\mu\nu}$ by writing

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \partial_\mu \xi_\nu(x) + \partial_\nu \xi_\mu(x).$$

Then the linearized theory is invariant under gauge transformations

$$\begin{aligned} \bar{h}_{\mu\nu}(x) &\rightarrow \bar{h}_{\mu\nu}(x) + \partial_\mu \zeta_\nu(x) + \partial_\nu \zeta_\mu(x) \, , \\ \xi_\mu &\rightarrow \xi_\mu - \zeta_\mu . \end{aligned}$$

MASSIVE GRAVITY → **GR + new Sector**

↓

Scalars : GOLDSTONES

Lorentz Violating Scalars and Massive Gravity

Adding additional scalar fields : $\phi^\alpha,\ \alpha = 0, 1, 2, 3,$

GOLDSTONE FIELDS

Breaking Lorentz Invariance = spacetime coordinates dependence

Example:

$$\bar{\phi}^0 = a\Lambda^2 t\,,$$
$$\bar{\phi}^i = b\Lambda^2 x^i,$$

$$S = S_{EH} + S_\phi,$$

To be solution : Lagrangian contains their derivatives only

SHIFT SYM : $\phi^\alpha(x) \to \phi^\alpha(x) + \lambda^\alpha$

Eventually internal Sym :

$SO(3) \quad \phi^i \to \Lambda^i_j \phi^j.$

Avoiding issues with the scalars

In Minkowski background (cosmology for example) these scalars are dangerous

Very hard to neglect their contribution to stress energy tensor
Without violating null energy condition !!

Here we are happy of having them backreacting!

Note that they dont screw up the UV AdS Asymptotics
needed for the dictionary .

HAPPY END :

LOTS OF SUPER HEALTHY LV MASSIVE GRAVITIES
AVAILABLE !

LV Massive Gravity in stuckelberg form

$$S_\phi = \int d^4x \sqrt{-g}\Lambda^4 F(X, V^i, Y^{ij}, Q)$$

This is Way More than A solid

$$X = \frac{1}{\Lambda^4} g^{\mu\nu}\partial_\mu\phi^0\partial_\nu\phi^0,$$

$$V^i = \frac{1}{\Lambda^4} g^{\mu\nu}\partial_\mu\phi^0\partial_\nu\phi^i,$$

$$Y^{ij} = \frac{1}{\Lambda^4} g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j,$$

Even preserving homogeneity
Lot of freedom and opportunities...

Effective Field Theory
Valid at Low energy

The UV cutoff in this theory Λ_{UV} is to be somewhat below Λ

$$\phi^\alpha = \bar{\phi}^\alpha + \pi^\alpha,$$

GOLDSTONES

$$L_\pi = (\partial\pi)^2 + \frac{1}{\Lambda}(\partial\pi)^3 + \dots$$

See: Chiral Perturbation Theory and Pions

Stuckelbergs = Goldstones

Gravity ON : $x^\mu \rightarrow x^\mu + \zeta^\mu(x)$

$$\pi^\alpha(x) \rightarrow \pi^\alpha(x) + \Lambda^2 \zeta^\alpha(x).$$

Stuckelberg Fields Of previous sections ← $\xi^\alpha = \Lambda^{-2} \pi^\alpha$ → **Goldstone Fields**

STUCKELBERGS = GOLDSTONES

Unitary Gauge : $\pi^\alpha = 0$

Usual Massive Gravity with : $m = \dfrac{\Lambda^2}{M_{Pl}}$,

“ WE ARE HIGGSING DIFFEOS “

LV MASSIVE GRAVITY FOR SOLIDS

$$S_\phi = \int d^4x \sqrt{-g}\,\Lambda^4 F(X, V^i, Y^{ij}, Z)$$

arXiv:1307.0517

Note: Simmetries allow for a generic function F

$SO(3)$ → **JUST SCALARS COMING FROM** $Y^{ij} = \frac{1}{\Lambda^4} g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j,$

$Tr[Y], Tr[Y^2], \ldots$

arXiv:1311.5157

$$S_0 = \int_M \sqrt{-g}\left[R - 2\Lambda - \frac{1}{2}\sum_I^{d-1}(\partial\psi_I)^2 - \frac{1}{4}F^2\right] d^{d+1}x$$

A simple holographic model of momentum relaxation

Effective field theory for solids

COMOVING COORDINATES OF THE VOLUME ELEMENTS :

arXiv:hep-th/0512260

TRIPLETS OF SCALARS $\phi^I(\vec{x},t)$

arXiv:1501.03845

GROUND STATE = $\langle\phi^I(x)\rangle = x^I$. = EQUILIBRIUM

HOMOGENEITY AND ISOTROPY OF SOLIDS AT LARGE SCALES → $\phi^I \to \phi^I + a^I\,, \qquad \phi^I \to SO(3)\cdot\phi^I\,,$

SPATIAL DEFORMATIONS DEGREES OF FREEDOM

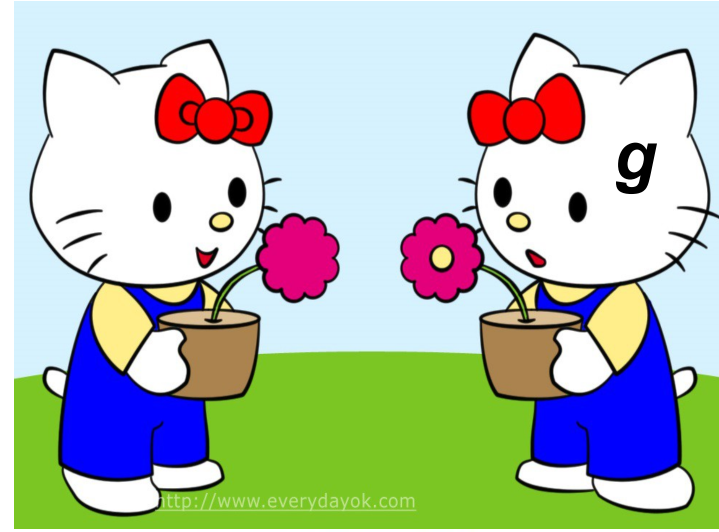
PHONONS

GOLDSTONES BOSONS FOR BROKEN SIMMETRIES

$\phi^a = x^a + \underline{\pi^a}$ describe the three (acoustic) phonons of the solid,

From Simmetry Breaking Pattern → Effective field theory

Find the differences



EFT for Solids : $S_\phi = \int d^4x \; \Lambda^4 F(X, W^i, Y^{ij}, Z)$

LV MGravity: $S_\phi = \int d^4x \sqrt{-g}\,\Lambda^4 F(X, W^i, Y^{ij}, Z)$

RECIPE :

WE ARE JUST GAUGING IT

AND PUT IT IN ANTI DE SITTER !

Getting Concrete: the Model

$$S_{HMG} = M_P^2 \int d^4x \sqrt{-g} \left[\frac{R}{2} + \frac{3}{\ell^2} - m^2 V(X) \right]$$

$$S = S_{HMG} - \frac{M_P^2}{4} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}$$

Generalizing:

arXiv:1311.5157

arXiv:1406.4870

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi^I \partial_\nu \Phi^I$$

$$\bar{\Phi}^I = \alpha \, \delta^I_i \, x^i$$

arXiv:1411.1003

With Oriol Pujolàs

MOMENTUM DISSIPATION
BUT
ENERGY CONSERVATION

Consistency and stability

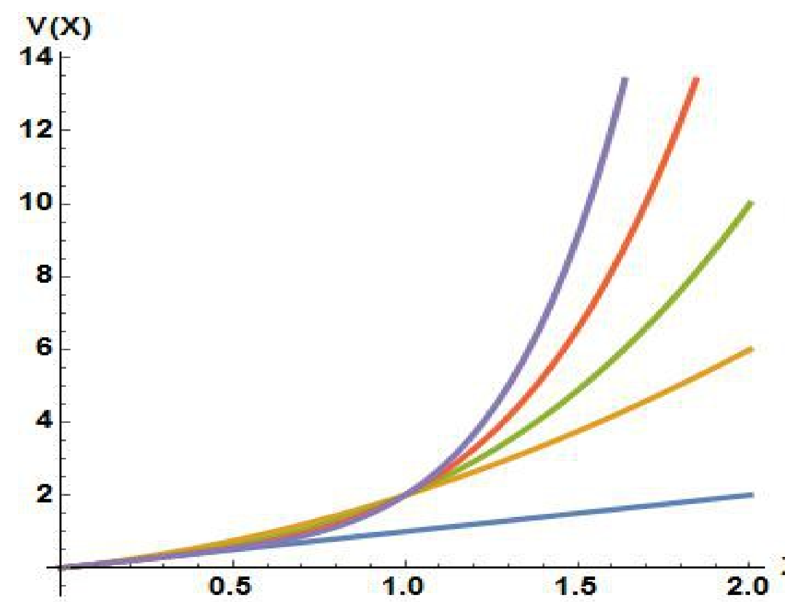
NO GHOSTS : $V'(\bar{X}) > 0\,.$

NO GRADIENT INSTABILITY : $c_S^2 = 1 + \frac{\bar{X} V''(\bar{X})}{V'(\bar{X})} > 0\,.$

AdS UV ASYMPTOTICS : $V(0) = 0.$

WEAKLY COUPLED GOLDSTONES SECTOR :

$V'(0)$ is not too small.



The Physics of the model

GRAVITON + NEW GOLDSTONE DEGREES OF FREEDOM

Operators of the CFT with shift simmetries
“Somehow” related to Phonons and Impurities

$$\Phi^I = \bar{\Phi}^I + \phi^I$$

ELECTROMAGNETIC SECTOR + NEW “PHONONS” SECTOR

GENERIC POTENTIAL FOR THE PHONONS
NON TRIVIAL DYNAMICS
NON ZERO SELF INTERACTIONS

Phonon-Phonon interactions
Electron-Phonon interactions

Where are the phonons?

PHONONS = FLUCTUATIONS OF THE SCALAR FIELDS

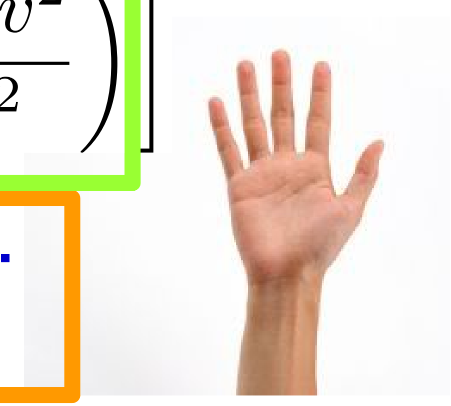
Transversal acoustic phonons

Transversal spatial components of stress tensor Txy

FLUCTUATIONS OF THE PHI FIELDS

→ **ARE THE GOLDSTONES OF THE RELATIVE BROKEN SIMMETRY**

$$f(u) = u^3 \int_u^{u_H} dv \left[\frac{3}{v^4} - \frac{\rho^2}{2\ell^2} - \frac{(m\ell)^2}{v^4} V\left(\frac{\alpha^2 v^2}{\ell^2} \right) \right]$$



We can keep track of the phonon pole in Txy operator...

And measure the elastic response...

With O.Pujolàs

Analytical DC conductivity

Using usual trick: arXiv:1308.4970

$$\sigma_{\mathrm{DC}} = 1 + \frac{\rho^2 u_H^2}{\alpha^2 m^2 V'(\alpha^2 u_H^2)}$$

Quantum Pairs Creation
CFT
Probe Result

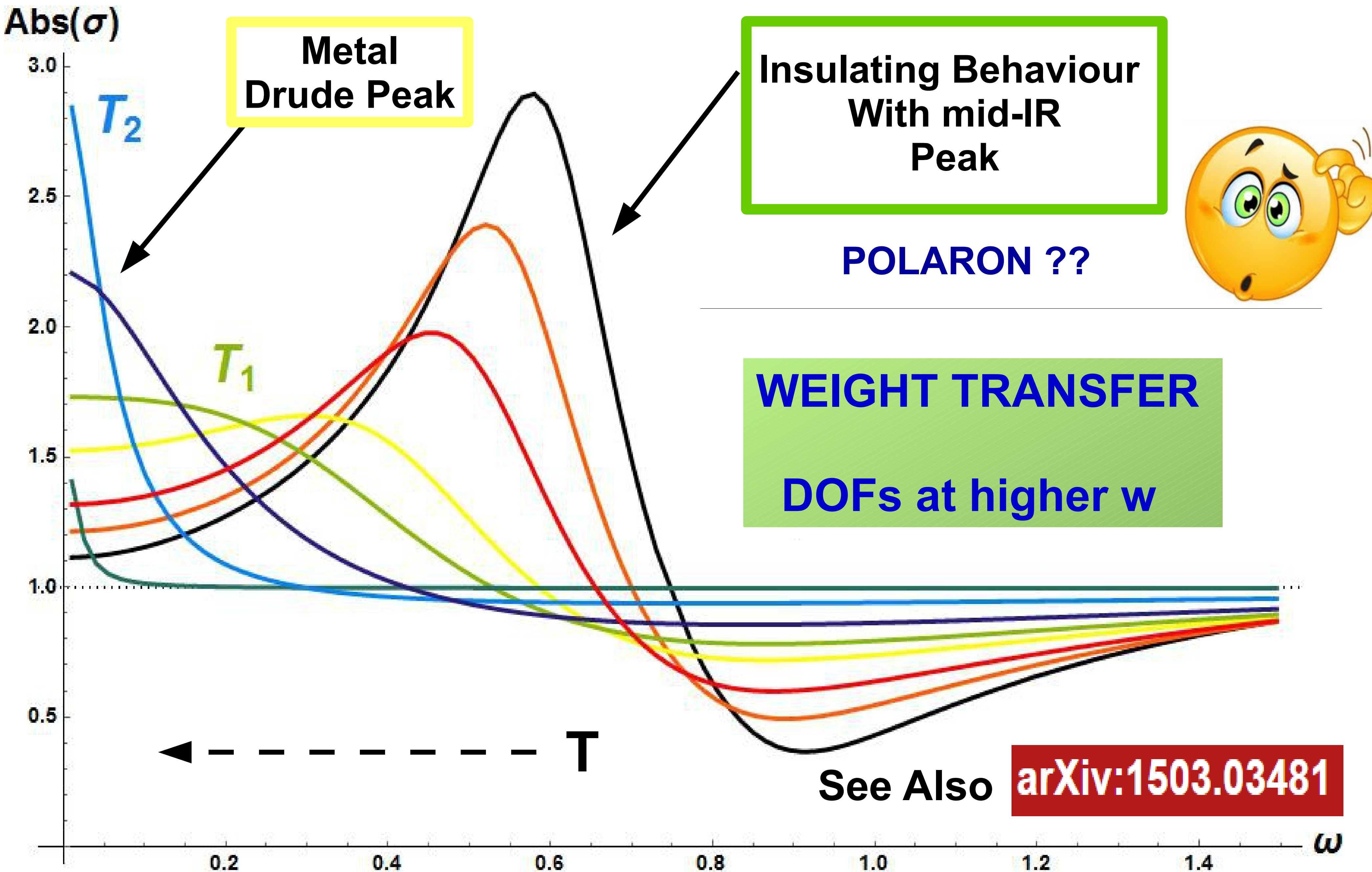
Dissipative Term
”à la Drude”

Possible ways out ...

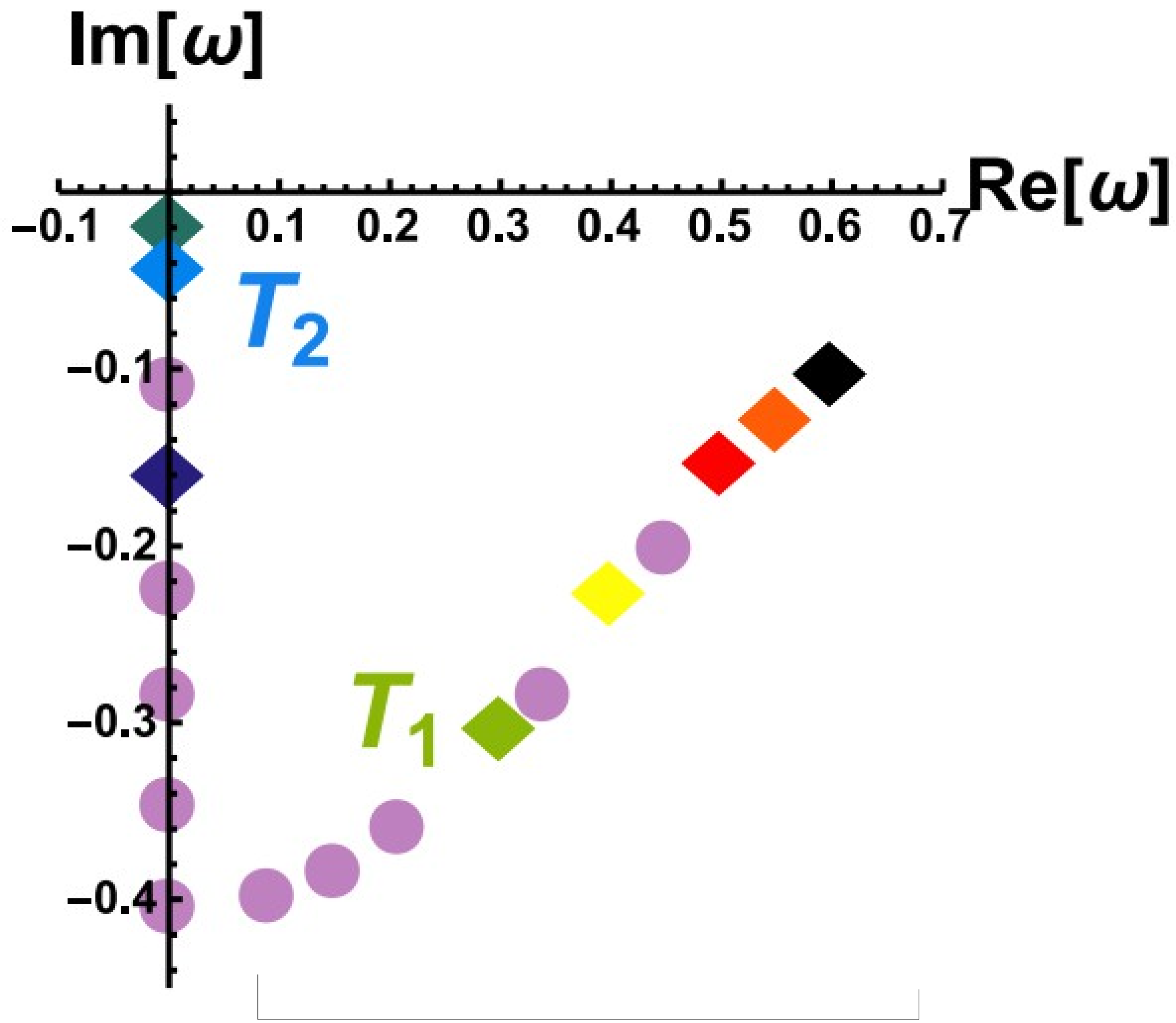
NEWS:
V GENERIC !!

Temperature Dependence

AC conductivity



QNMs and their physics



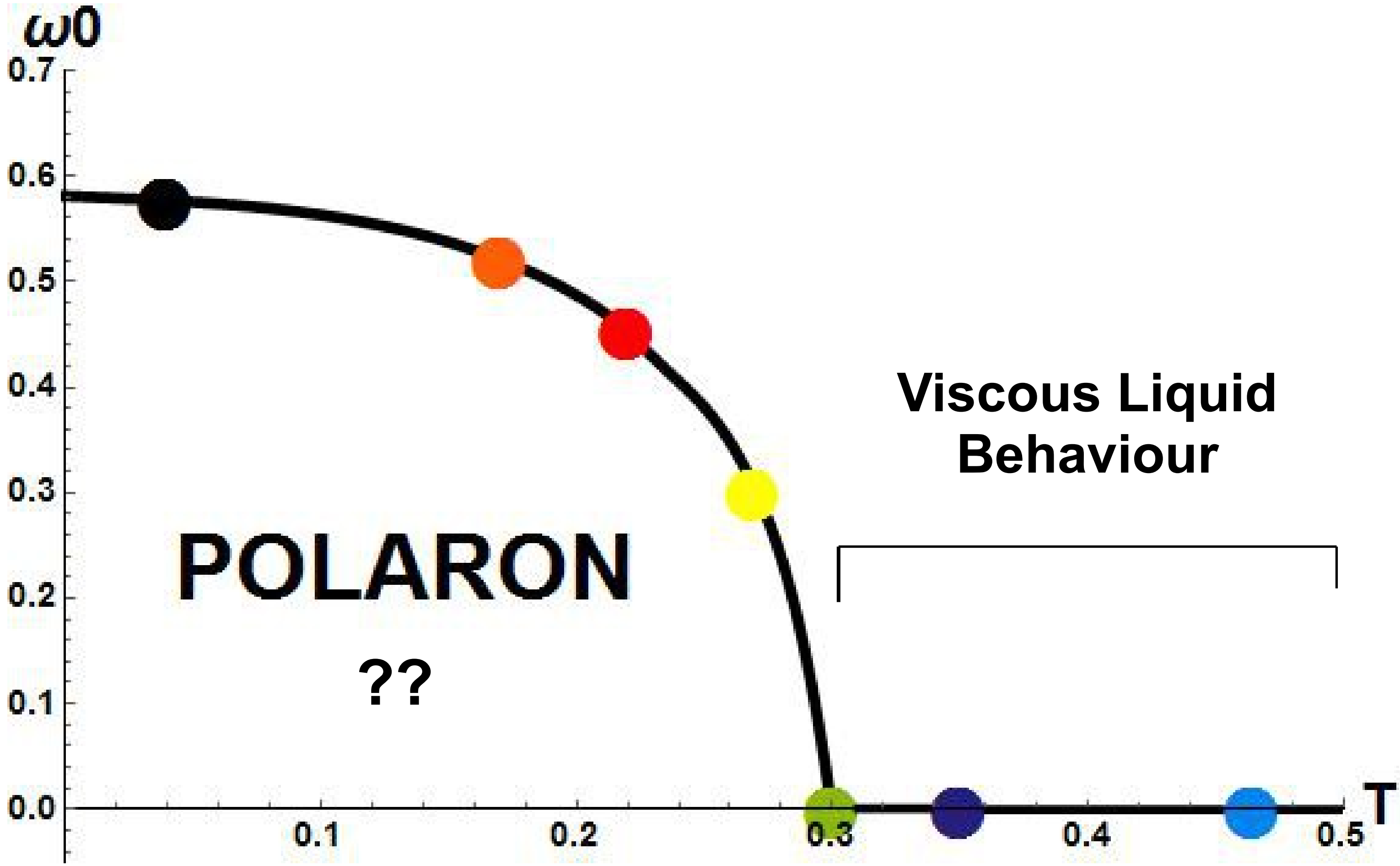
OVERDAMPED REGION

VISCOUS LIQUID BEHAVIOUR

MASSIVE PROPAGATING POLE EMERGING

IT LOOKS LIKE A PINNED CRYSTAL

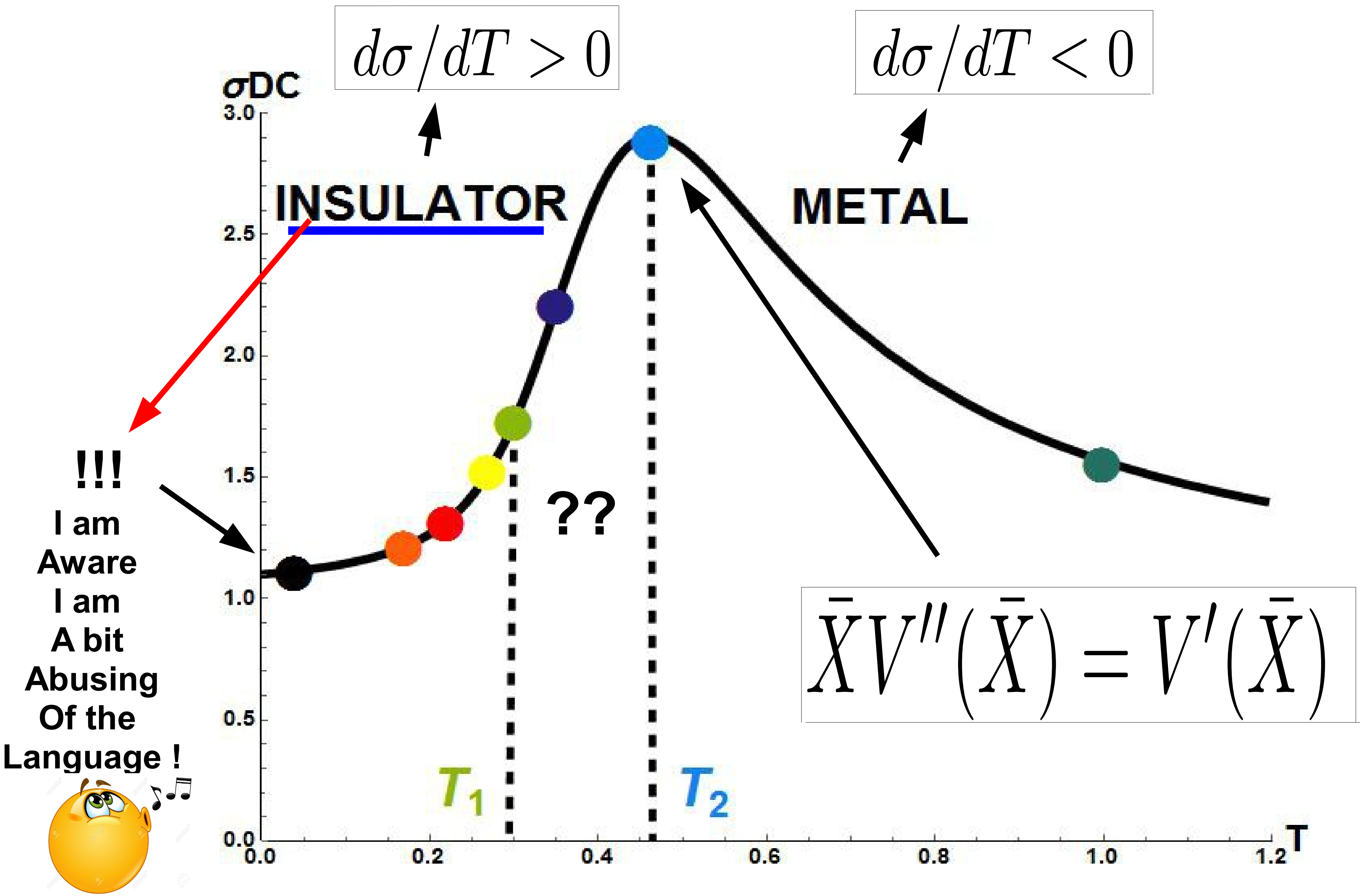
Localized peak formation



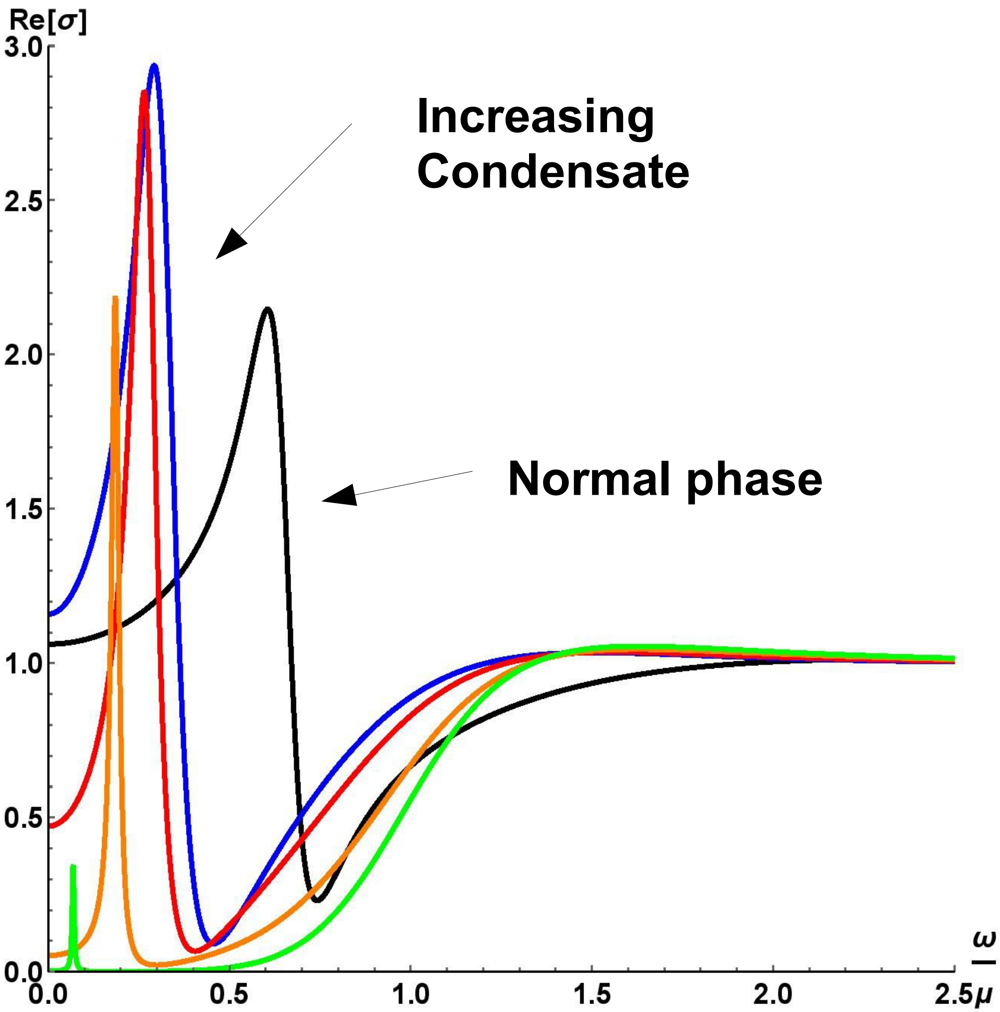
Looks like 2nd order Phase transition (Holographic Superconductors)

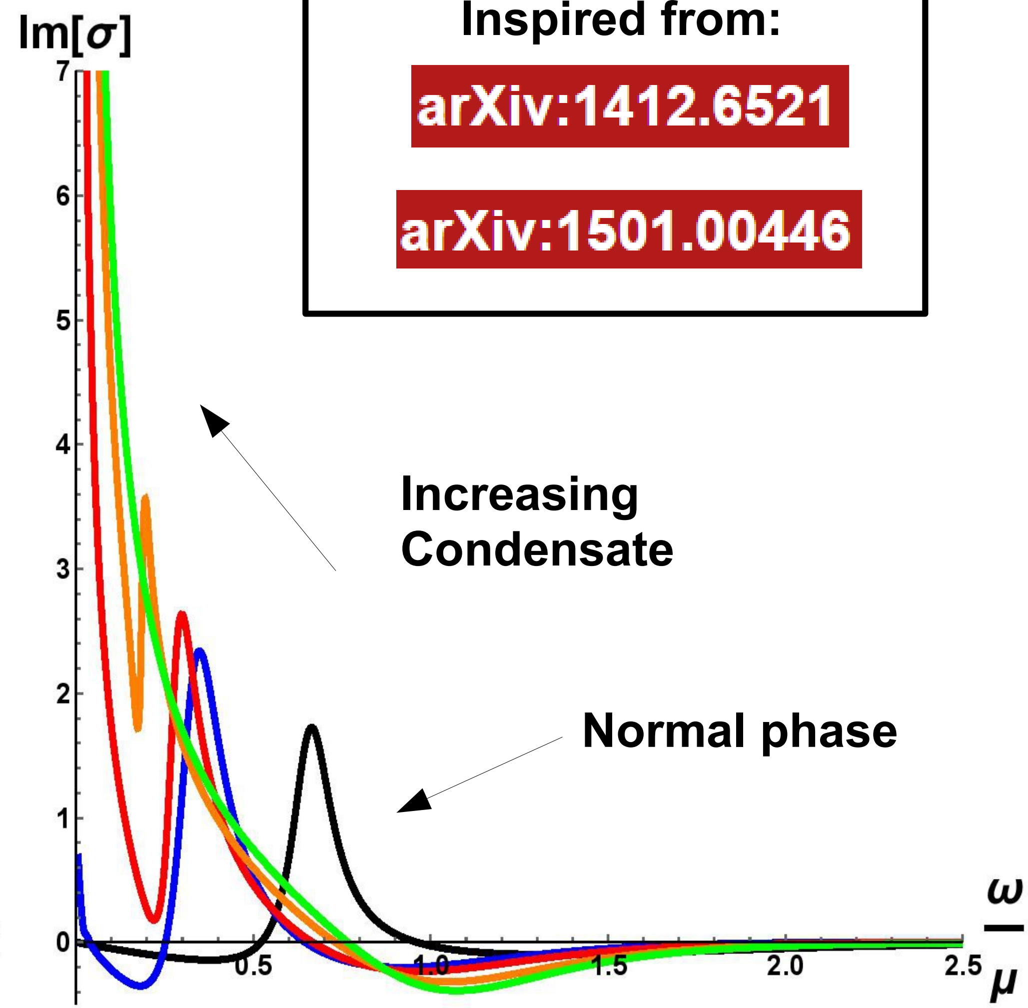
BUT : No Spontaneous Simmetry Breaking , No Condensate

"MIT" : from a viscous liquid to a pinned crystal



Holographic Superconductors





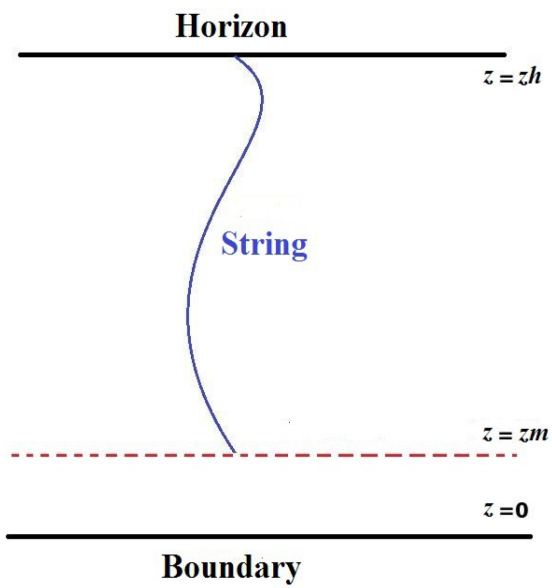
Inspired from:

arXiv:1412.6521

arXiv:1501.00446

With M. Goykhman

Massive Gravity Check In



Idea:
understand what massive gravity Mimicks and test its quality as An effective description for Condensed Matter

Probe String moving In a background with massive graviton

With Danny Brattan

- Additional Drag and Momentum Dissipation
- Non zero entropy at zero temperature
- Decreased mobility
- Heat Capacity
- Temperature Dependent Damping

PRE-CONCLUSIONS

~~AXIONS~~

PHONON

Thanks ...

Conclusions

MASSIVE GRAVITY PROVIDES AN EFFECTIVE
HOLOGRAPHIC THEORY FOR PHONONS IN SOLIDS

PHONONS ARE ENCODED IN THE FLUCTUATIONS OF THE STUCKELBERG
FIELDS WHICH ARE NOTHING ELSE THAN THE GOLDSTONES

SCALARS VEVs PROVIDES ALSO AN EXPLICIT BREAKING
WHICH CAN MIMICK IMPURITIES EFFECTS

MASSIVE GRAVITY PROVIDES
INTERESTING HOLOGRAPHIC FEATURES
LORENTZ VIOLATING MASSIVE GRAVITIES
ARE WAY MORE GENERAL THAN dRGT AND
HAVE STILL A LOT OF POWER TO BE EXPLOITED

IDEA: TEST AND IMPROVE THE QUALITY OF THE ADS-CMT TOOL

To Do

Thermoelectric and Elastic Properties

To understand more the Phonons like nature of the theory

PHONONS DRAG ??

ELASTIC MODULI
HOLOGRAPHIC THEORY
OF ELASTICITY ?

HOLOGRAPHIC
THEORY
FOR SOLIDS


THANK YOU FOR YOUR ATTENTION
YESSSS, FINALLY OVER!
quickmeme.com

BACKGROUND

$$T = -\frac{f'(u_H)}{4\pi} = \frac{6 - \mu^2 u_H^2 - 2m^2 V\left(\alpha^2 u_H^2\right)}{8\pi u_H}$$

$$f(u) = u^3 \int_u^{u_H} dv \, \left[\frac{3}{v^4} - \frac{\rho^2}{2\ell^2} - \frac{(m\ell)^2}{v^4} V\left(\frac{\alpha^2 v^2}{\ell^2}\right)\right]$$

GOLDSTONE SECTOR AND INSTABILITY

$$V'(\bar{X})\partial_\mu \phi^i \partial^\mu \phi^i + \bar{X} V''(\bar{X})(\partial_i \phi^i)^2$$

$$M_\Phi^2(u) = (\partial \bar{a})^2 + \Box \bar{a} = \frac{f(u\partial_u \bar{a})^2}{\ell^2} + \frac{u^2 \partial_u (f \partial_u \bar{a})}{\ell^2}$$

$$\sqrt{V'(\bar{X})}\phi^i \qquad \bar{a}(u) = \frac{1}{2}\log V'(\bar{X})$$

$$M_\Phi^2 \sim -|f'(u)| \frac{V''(\bar{X})}{V'(\bar{X})} \Big|_{\text{horizon}} < 0$$

EOM

$$T_i \equiv u^2 h_{ti} - \frac{\partial_t \phi_i}{\alpha}\ ,\ U_i \equiv f(u)\big[h_{ui} - \frac{\partial_u \phi_i}{\alpha u^2}\big]\ ,\ B_i \equiv b_i - \frac{\phi_i}{\alpha}$$

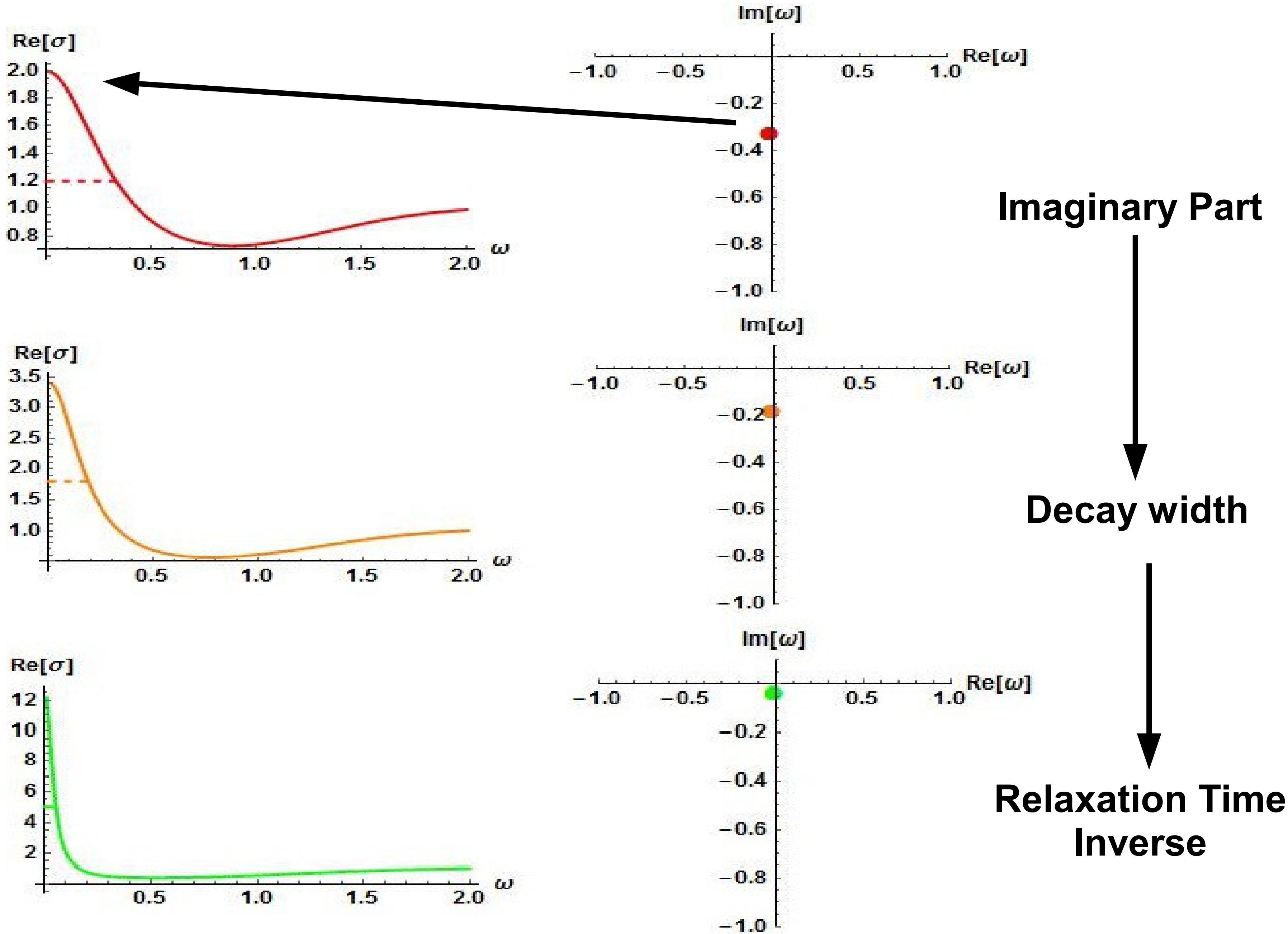
$$\partial_u(f\,\partial_u\, a_i) + \Big[\frac{\omega^2}{f} - k^2 - 2u^2\rho^2\Big]a_i = \frac{i\rho\, u^2(2\bar{m}^2 + k^2)}{\omega}U_i\ - \frac{if\rho\, k^2}{\omega}\partial_u B_i$$

$$\frac{1}{u^2}\partial_u[\frac{fu^2}{\bar{m}^2}\partial_u\big(\bar{m}^2 U_i\big)] + \Big[\frac{\omega^2}{f} - k^2 - 2\bar{m}^2\Big]U_i = -2i\,\rho\,\omega\, a_i + \frac{f' k^2}{u^2}B_i$$

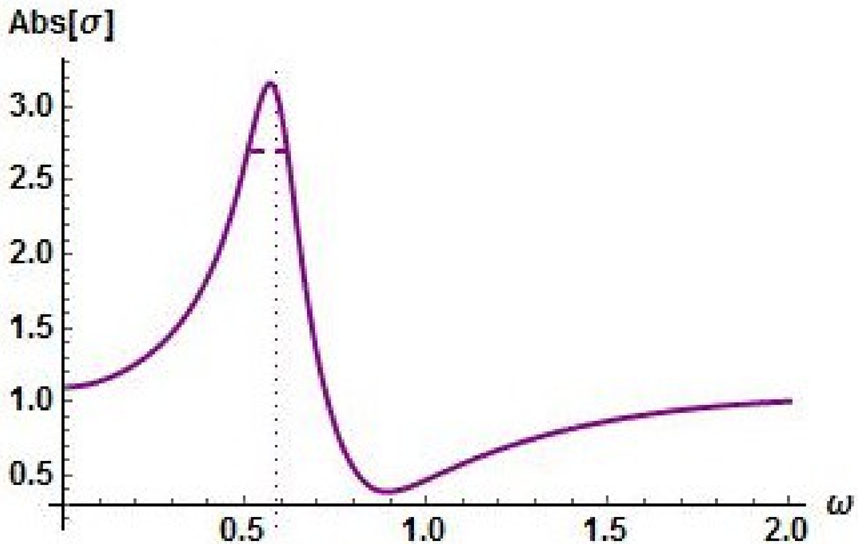
$$k\Big\{u^2\partial_u\Big(\frac{f}{u^2}\,\partial_u\, B_i\Big) + \Big[\frac{\omega^2}{f} - k^2 - 2\bar{m}^2\Big]B_i = -2\frac{\bar{m}'}{\bar{m}}U_i\Big\}$$

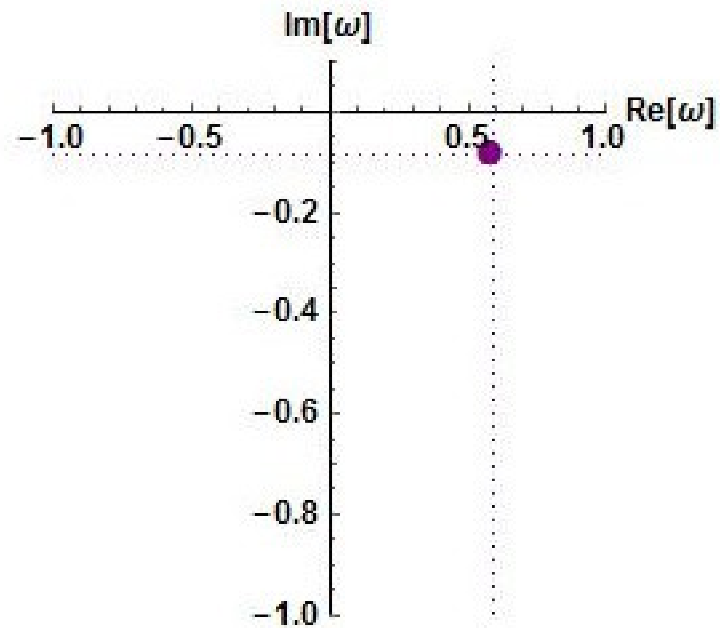
$$\bar{m}^2(u) = \alpha^2 m^2 V'(\alpha^2 u^2) \qquad\qquad \sigma(\omega) = \frac{\partial_u a_j}{i\,\omega\, a_j}\Big|_{u\to 0}$$

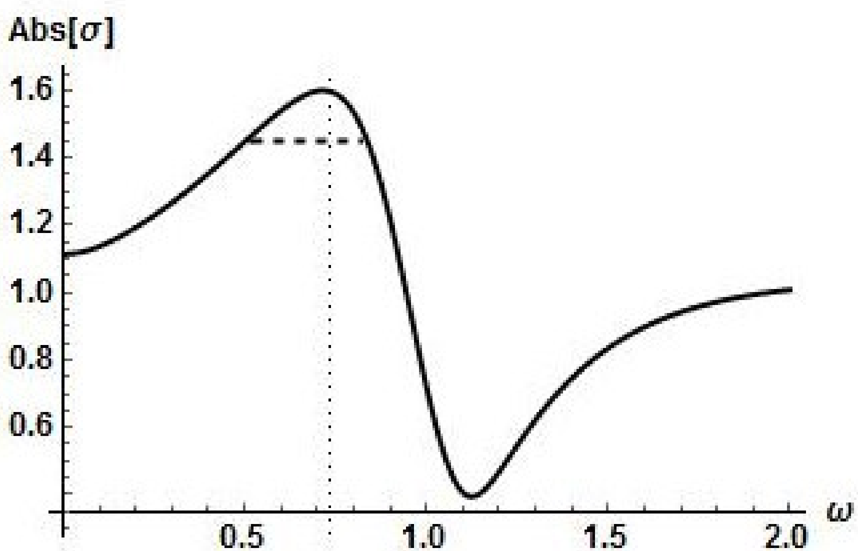
Quasinormal Modes I

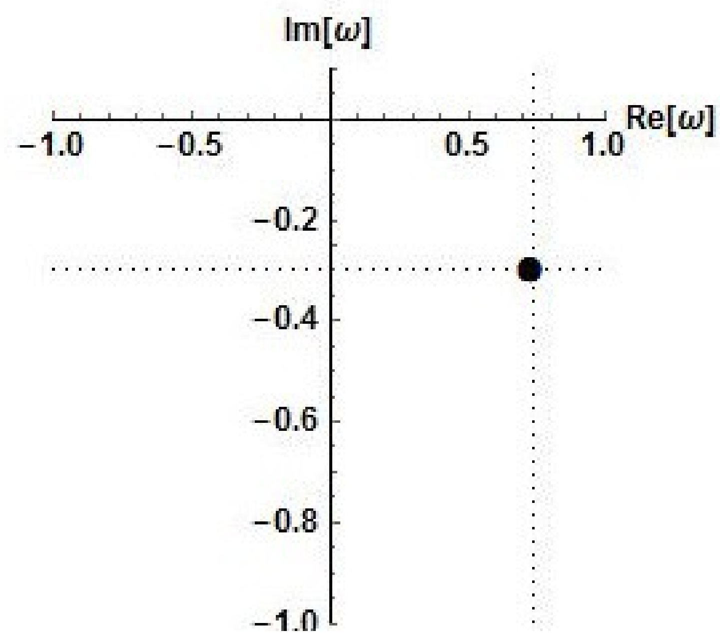


Quasinormal Modes II

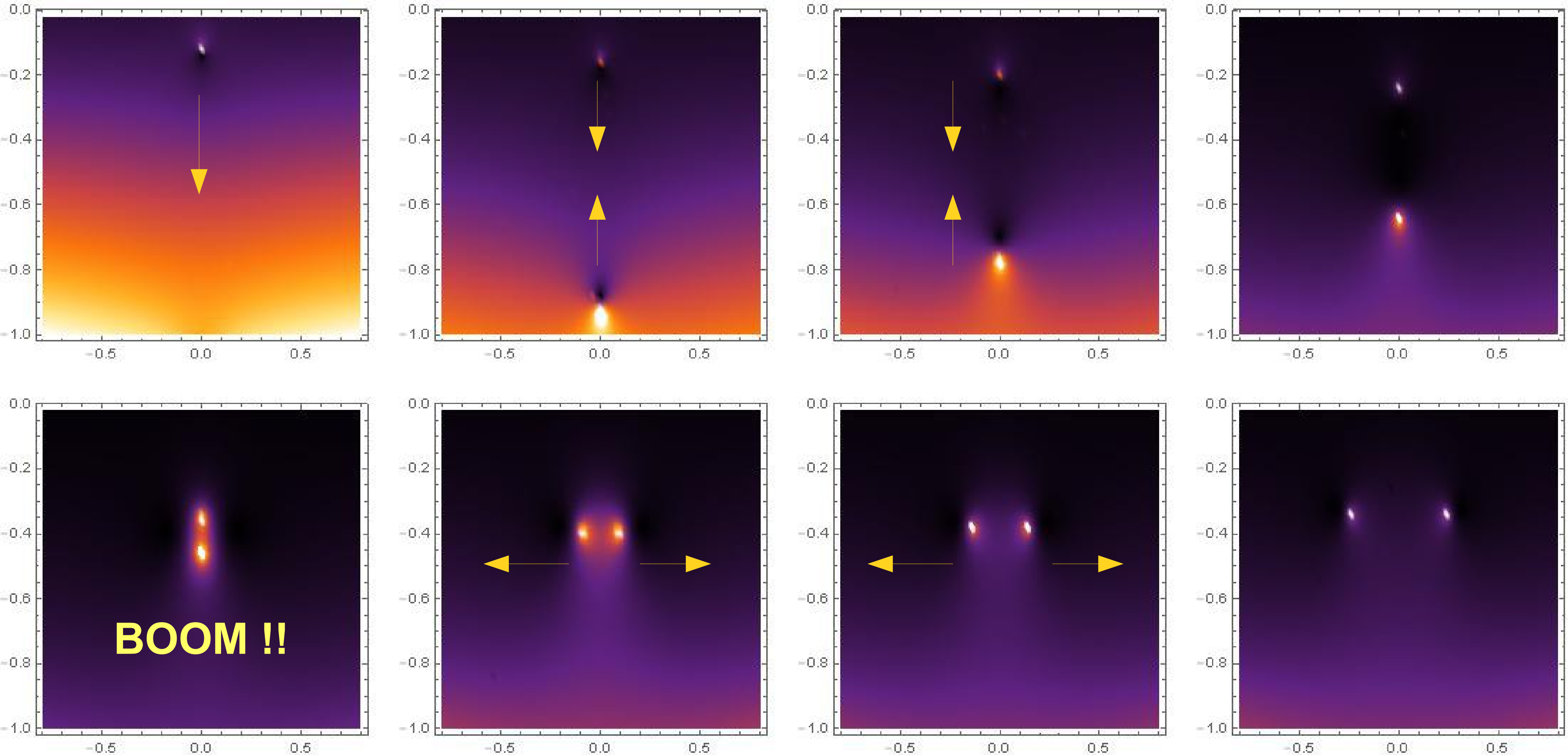








Quasinormal Modes IV



COHERENT – INCOHERENT TRANSITION ← **arXiv:1411.1062**

&

LOCALIZED PROPAGATING EXCITATION ← **NEW**

Non-Relativistic Effective field theories

arXiv:hep-ph/9311264

arXiv:hep-ph/9609466

SOLIDS :

relevant effective fields are the components of the vector $\vec{\xi}(x) = (\xi^1(x), \xi^2(x), \xi^3(x))$,

which specifies the displacement of the material from the position in the ground state.

$$\mathcal{L}_2 = \frac{1}{2}\rho_0 \dot{\xi}_a \dot{\xi}_a - \frac{1}{2}\mu\, \partial_a \xi_b \partial_a \xi_b - \frac{1}{6}(\mu + 3K)\partial_a \xi_a \partial_b \xi_b + l_0\, \xi_a \xi_a + O(p^4)$$

$$l_0 = 0\ , \quad k_1 = -\mu\ , \quad k_2 = \frac{2}{3}\mu - K\ .$$

Theory of Sound Waves in Solids

Phonons as goldstone fields

COVARIANT FORMULATION :

$$z_a(t, \vec{x}) = x_a - \xi_a(t, \vec{x})$$

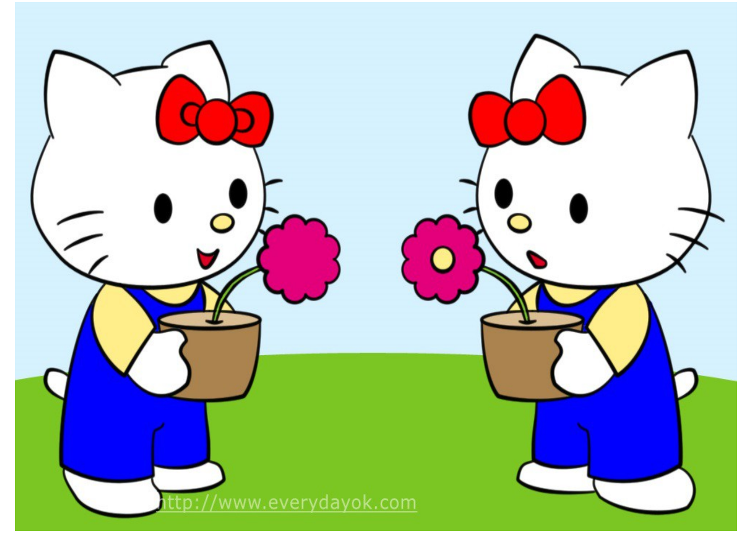
$$\mathcal{L}_{eff} = \sqrt{|g|}\, F(\partial z, \nabla \partial z, \ldots)_g\ .$$

If only 1st derivatives :

$$H_{ab} = g^{\mu\nu} \partial_\mu z_a \partial_\nu z_b$$

$$\mathcal{L}_{eff} = \sqrt{|g|}\, c^2 f(H)$$

Find the differences



Effective theories for phonons in solids :

COVARIANT FORMULATION :

$$z_a(t,\vec{x}) = x_a - \xi_a(t,\vec{x})$$

$$\mathcal{L}_{eff} = \sqrt{|g|}\, F(\partial z, \nabla\partial z, \ldots)_g \quad .$$

If only 1st derivatives :

$$H_{ab} = g^{\mu\nu}\partial_\mu z_a \partial_\nu z_b$$

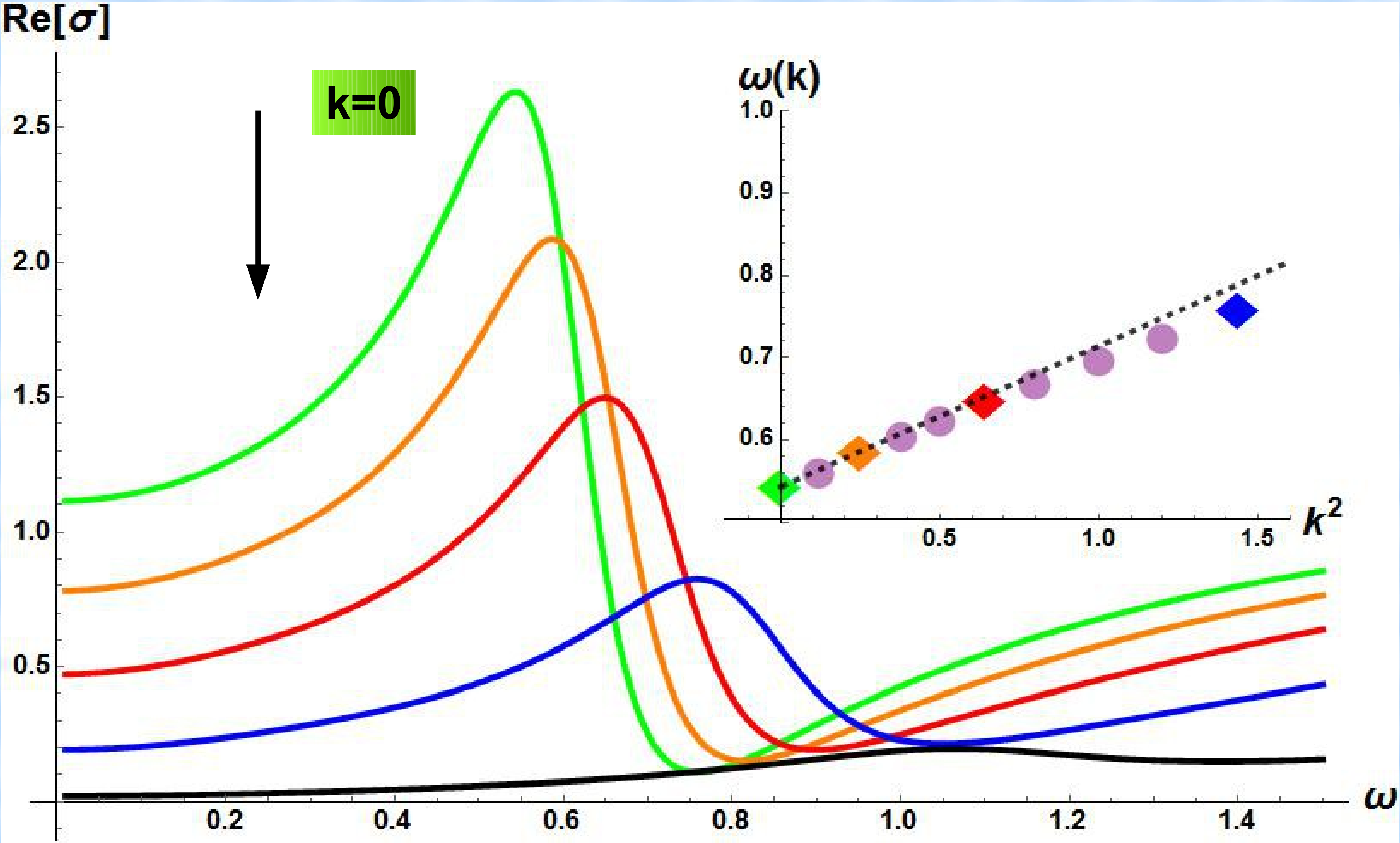
$$\mathcal{L}_{eff} = \sqrt{|g|}\, c^2 f(H)$$

PREVIEW – RECIPE :

WE ARE JUST GOING TO GAUGE IT

AND PUT IT IN ANTI DE SITTER !

EMERGED PROPAGATING "(quasi) PARTICLE" $\omega = \omega_0 + i\Gamma$



Quasinormal Modes Imprints