Semi-holography and Weyl Semimetals

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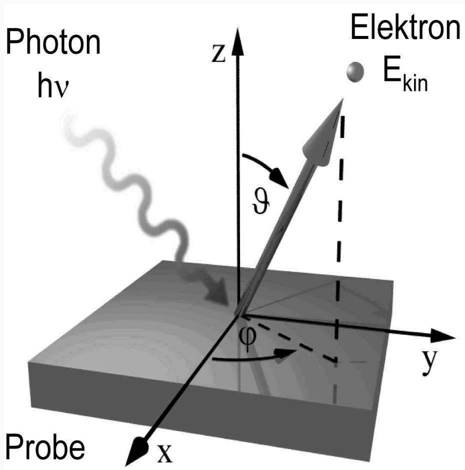
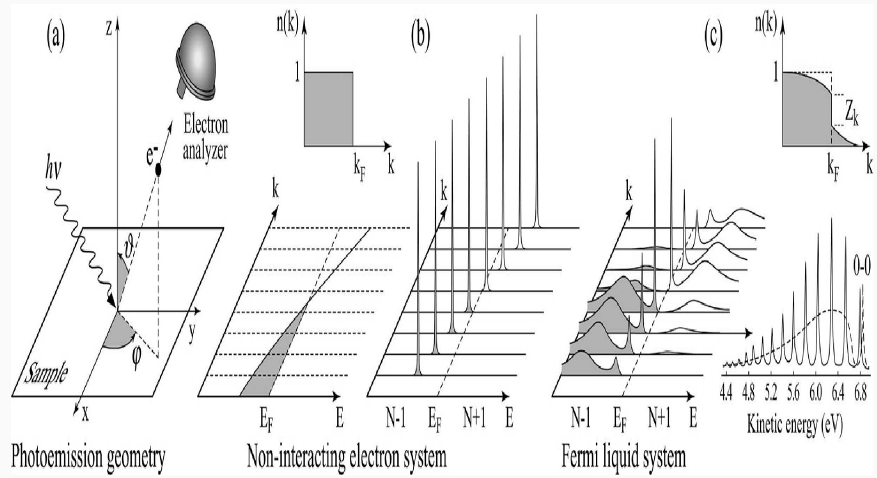
Based on: arXiv:1505.XXXX, w/ P. Betzios, V. Jacobs and H. Stoof

AdS/CMT and ARPES

- Hope to unveil universal features of strongly interacting electron-hole "plasmas"
- e.g. Graphene in 2+1 D with $\alpha_{eff} = \alpha = \frac{e^2}{\epsilon_0 \, v_F} = \frac{300}{137}$ or Weyl semimetals in 3+1 D — three dimensional cousins of Graphene
- Should make contact with experiments: ARPES

ARPES and sum-rules





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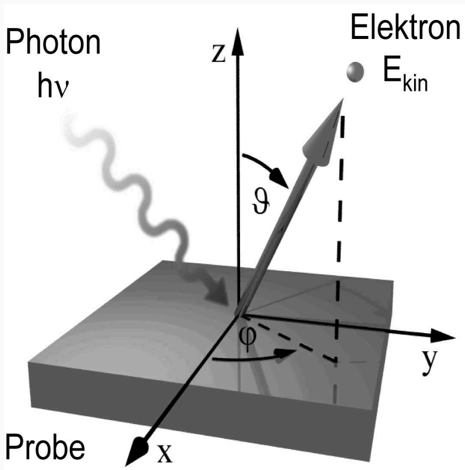

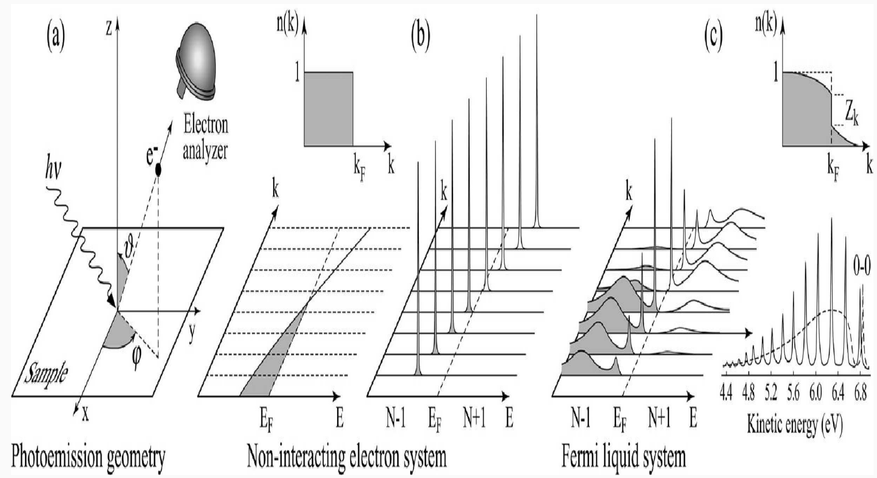

- From the photoemission intensity $I(\omega, k)$ one constructs the retarded Green's function G_R of single particle excitations χ traveling inside the material.
- ARPES sum-rule: $\frac{1}{\pi}\int d\omega \mathrm{Im}[\langle\chi^\dagger\chi\rangle(\omega,k)] = 1, \qquad \forall k,\, T$
  From canonical commutation relations
- Difficult in AdS/CFT, as $\mathcal{O}$ is composite $\Rightarrow$ UV divergences.

Semi-holography **Faulkner, Polchinski '11**

- Consider a material (e.g. semimetal, superconductor,..) at criticality at zero T.
- Perturb the system $\Rightarrow$ excite an elementary electronic d.o.f. χ

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- Perturb the system $\Rightarrow$ excite an elementary electronic d.o.f. χ
- In principle sources to all CFT operators $\mathcal{O}_\Delta$.
- Assume a dominant channel:

$$\mathcal{L} = \overline{\chi}\partial\!\!\!/\chi + g_f(\overline{\chi}\mathcal{O}_\Delta + \overline{\mathcal{O}}_\Delta\chi) + \mathcal{L}_{CFT}(O)$$

- Dyson series: $\langle\overline{\chi}\chi(k)\rangle = \frac{1}{k\!\!\!/ + g\langle\overline{O}_\Delta O_\Delta(k)\rangle}$
- where $\langle\overline{O}_\Delta O_\Delta(k)\rangle \propto k\!\!\!/ k^{2M-1}$ with $\Delta_\pm = \frac{d}{2} \pm M$ mass of the dual fermion Ψ in d+1.
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- A hybrid formulation, not convenient for higher point functions

- A systematic, completely geometric approach:
  Plauschinn, Stoof, Vandoren, U.G. ’11
- Recall $S_f = \int d^{d+1}\sqrt{g}\overline{\Psi}\left(\not{D} - M\right)\Psi + S_\partial$,   d even.
- Decompose the Dirac fermion $\Gamma^z \Psi_\pm = \pm\Psi_\pm$
- Ψ_+ is the source $\Rightarrow$ Ψ_- is the response
- Since $\Gamma^z = \Gamma^{d+1}$ Dirac $\Psi(z,x)$ in the bulk $\Leftrightarrow$ Weyl $\chi = \Psi_+(z_0, x)$ on the boundary.

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- S_∂ from the variational principle: $\delta\Psi_+(z_0, x) = 0$

$$S_\partial = \int_{z=z_0} d^d x\sqrt{h}\left(\overline{\Psi}_+\Psi_- + \mathcal{L}_{UV}[\Psi_+]\right)$$

Contino, Pomarol '04

- In particular one can choose $\mathcal{L}_{UV}[\Psi_+] = Z\overline{\Psi}_+\not{\partial}\Psi_+$
- A particular finite counter-term making a dynamical source

$$S_f = \int d^{d+1}x\sqrt{g}\,\overline{\Psi}\left(\not{D} - M\right)\Psi + \int_{z=z_0} d^d x\sqrt{h}\left(\overline{\Psi}_+\Psi_- + Z\overline{\Psi}_+\not{\partial}\Psi_+\right)$$

- On-shell: effective action for $\chi = \Psi_+(z_0)$

$$Z_{eff}[\chi] = \int \mathcal{D}\chi e^{-\int d^d k\sqrt{h}\overline{\chi}(K_\Psi(z_0,k)+Z\not{k})\chi}$$

where $\Psi_-(z,k) = K_\Psi(z,k)\Psi_+(z,k)$, solves Dirac in the bulk.

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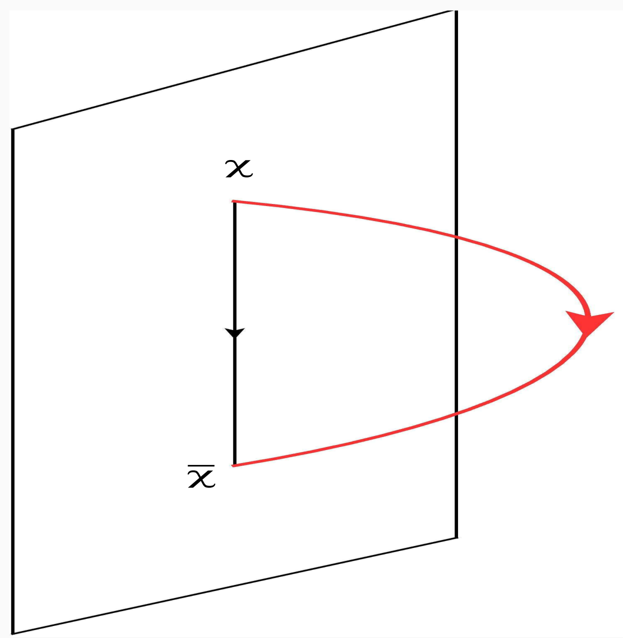
A semi-holograhic Witten diagram:



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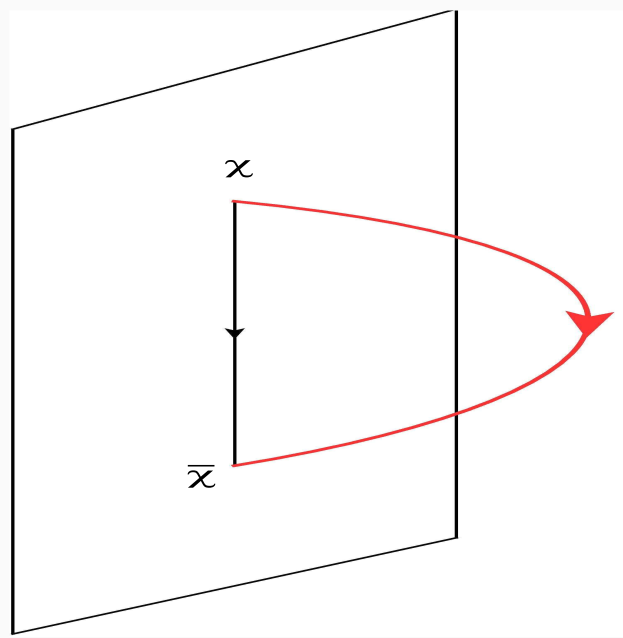

- Immediate generalization: multiple fields

- Consider dynamical χ in the presence of background A_μ^b

$$S_f = \int \overline{\Psi} \left(\not{D} + g_A \not{A} - M \right) \Psi + \int_\partial \left(\overline{\Psi}_+ \Psi_- + Z \overline{\Psi}_+ (\not{\partial} + \not{A}^b) \Psi_+ \right)$$

$$\Psi(z,k) = K_{\Psi,+}(z,k)\Psi_+(z_0,k),\, A_M(z,k) = K^A_{M,\nu}(z,k)A^{b,\nu}(z_0,k)$$

- Consider dynamical χ in the presence of background A^b_μ

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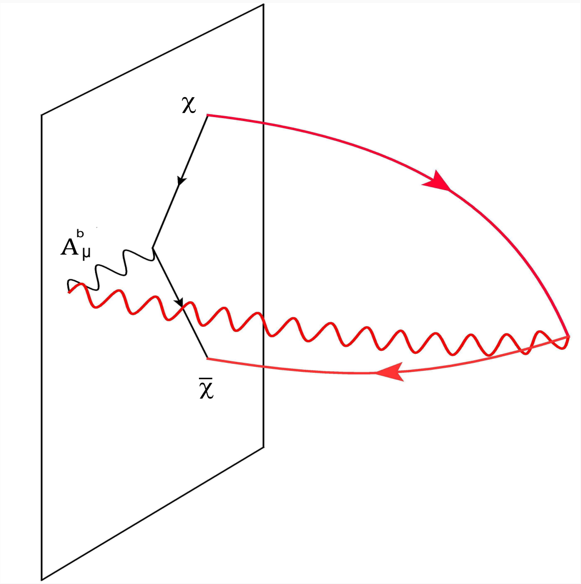
A Semi-holographic Witten diagram first order in A^b:



- Crucial for the Ward Identity:
  $\partial^x_\mu \langle J^\mu_{tot}(x) \overline{\chi}(x_1) \chi(x_2) \rangle = \langle \overline{\chi}\chi \rangle \delta(\vec{x} - \vec{x}_1) - \langle \overline{\chi}\chi \rangle \delta(\vec{x} - \vec{x}_2)$
  with $J^\mu_{tot} = J^\mu_{CFT} + Z\, \bar{\chi} \gamma^\mu \chi$
- A completely geometric proof of the Ward identity

- Suppose the Weyl fermion χ couples to both a background field A^b_μ and a 4D CFT through $\mathcal{O}_\Delta$ and J_{CFT}.
- The effective action is

$$S_{eff}[\chi, A^b_\mu(q)] = \int d^4k \left\{ \chi^\dagger(k) G_\chi^{-1} \chi(k) + A^b_\mu(q) \chi^\dagger(k) \Sigma^\mu \chi(k+q) \right\},$$

- Full propagator: $G_\chi^{-1}(k) = Z\not{k} + g_f \not{k} k^{2M-1}$
- Full vertex:

$$\Sigma^\mu = Z\,\gamma^\mu + g_A\, A(M)(k+q)^{M+\frac{1}{2}}\, k^{M+\frac{1}{2}} q \gamma^\mu I_1(k,q) + \frac{\not{k}}{k} \gamma^\mu \frac{\not{k}+\not{q}}{k+q} I_2(k,q)$$

with $A(m) = 2^{1-2M}/\Gamma[M+\frac{1}{2}]^2$, and

$$I_1(k,q) = \int_{z_0}^{\infty} dz z^2 K_1(q\,z) K_{M+\frac{1}{2}}((k+q)z) K_{M+\frac{1}{2}}(k\,z)$$

$$I_2(k,q) = \int_{z_0}^{\infty} dz z^2 K_1(q\,z) K_{M-\frac{1}{2}}((k+q)z) K_{M-\frac{1}{2}}(k\,z)$$

Summary:

$$Z_{eff}[\chi, A_b] = \int D\chi e^{-\int A_b \cdot J_\chi + Z\chi \not{k} \chi} \langle e^{-\int A_b \cdot J_{CFT} + \chi O} \rangle_{CFT}$$

$$= \int D\chi e^{-\int \chi^\dagger G_\chi^{-1} \chi + A_\mu^b \cdot \chi^\dagger \Sigma^\mu \chi}$$

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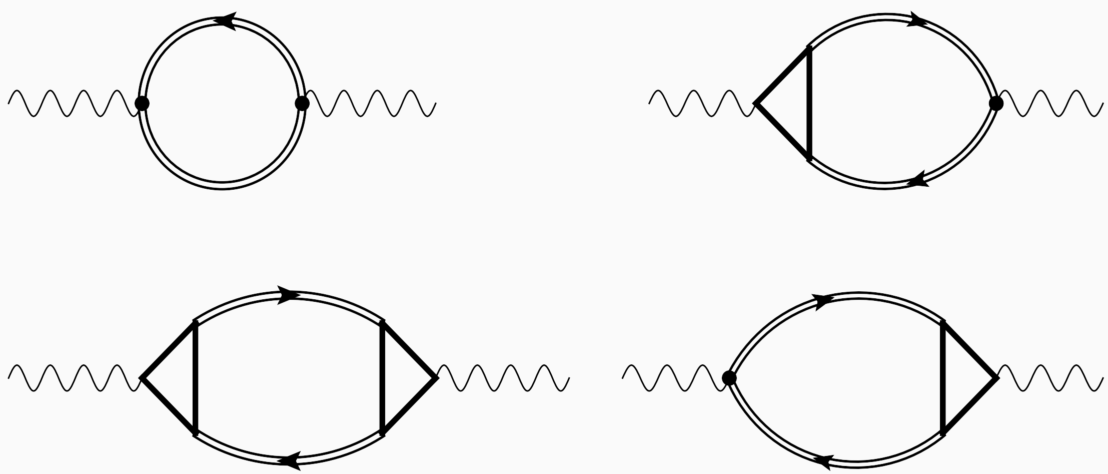
Conductivity: $\sigma(\omega) = \frac{\delta^2}{\delta A_b \delta A_b} Z_{eff}$:



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$$
\begin{aligned}
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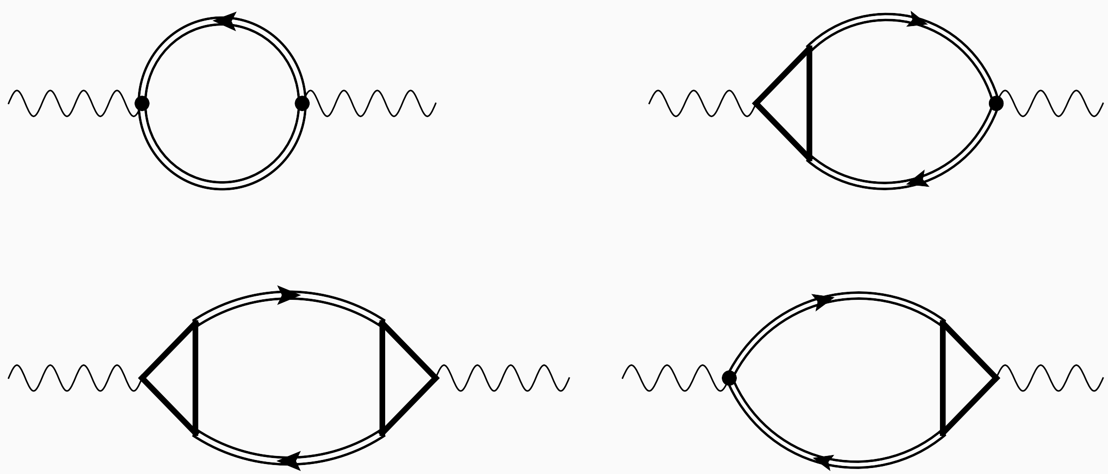

- In the IR: $\sigma = a_1\,\omega + a_2\,\omega^{2-2M} + a_3\,\omega^{3-4M}$
- In the UV: $\sigma = b_1\,\omega + a_2\,\omega^{2M} + a_3\,\omega^{4M-2}$
- $a_i\,,\,b_i$ fixed by $M\,,\,g_f\,,\,Z$ and g_A.

Dissecting the vertex

- Recall the ordinary QED vertex for Dirac fermions:
  $\Sigma^\mu(q) = \gamma^\mu F_1(q^2) - \frac{1}{2m}[\gamma^\mu, \gamma^\nu] q_\nu F_2(q^2)$
  $F_1(q^2) = 1 + \mathcal{O}(e^2), F_2(q^2) = \mathcal{O}(e^2)$.
- $\mu = 0$ term $\Rightarrow$ charge renormalization: $e \to e\, F_1(0)$
- $\mu = i$ term $\Rightarrow$ magnetic moment: $\vec{\mu}_e = \frac{e}{2m}\,(1 + F_2(0))\,\vec{\sigma}$.

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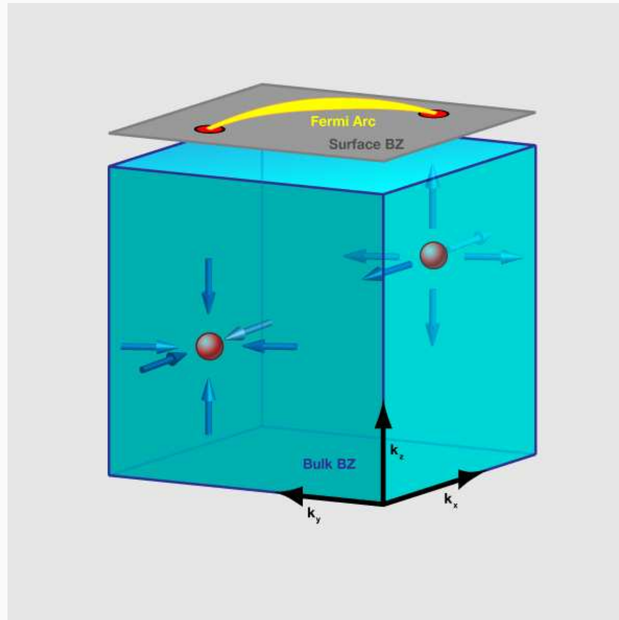
- What are the charge renormalization and magnetic moment for a Weyl fermion coupled to CFT?

Dissecting the vertex

For the Weyl fermion coupled to CFT one finds:

- Charge renormalization: $e \to e + g_A\, p^{2M-1} \frac{\pi\, \mathsf{S}ec(\pi M)}{2^{2M}\Gamma[M+\frac{1}{2}]^2]}$
- Anomalous magnetic moment: $\vec{\mu}_e = p^0\, p^{2M-1} \frac{(2M-1)\pi\, \mathsf{S}ec(\pi M)}{2^{2M+2}\Gamma[M+\frac{1}{2}]^2]} \vec{\sigma}$
- No ordinary magnetic moment for Weyl fermions.

An application: Weyl semimetals



- 3D cousins of Graphene: A "gapless semiconductor" where the valence and conduction bands touch at separate points in the Brilloin momentum cell
- Conjectured to exist since Abrikosov and Beneslavskii, '71

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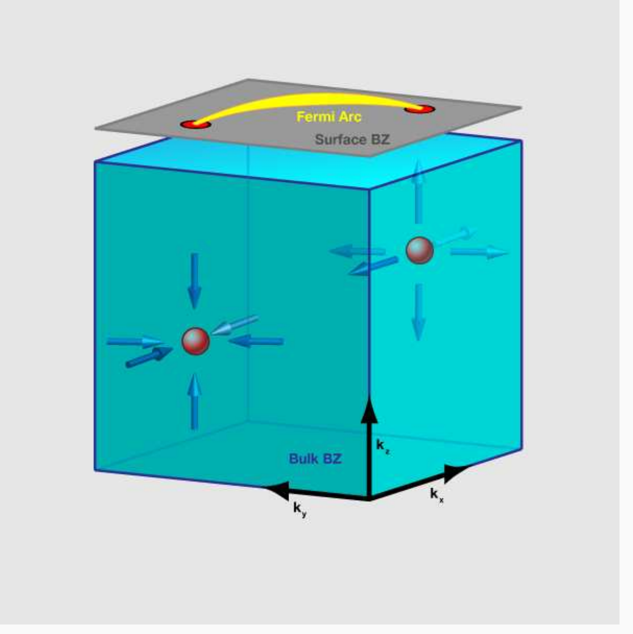

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- Explicit proposals Wan et al. '11, Witczak-Krempa and Kim '12, Chen and Hermele '12, Turner and Vishwanath '13, Vafek and Vishwanath '13, Volovik '09
- Realization with TaAs (analyzed by ARPES): Xu et al '15

Outlook

- Conductivity in detail $\Rightarrow$ fix paramaters g_f, g_A, Z by fitting ARPES
- Finite T and μ
- Anomalous transport in Weyl semimetals
- More general semi-holography: couple χ to more than one $\mathcal{O}_\Delta$
- More general semi-holography: dynamical A^b_μ on the boundary
- Other applications: e.g. single-particle excitations coupled to an order parameter at non-trivial fixed points.

THANK YOU !

The Ward identity

- $\partial_\mu^x \langle J^\mu_{CFT}(x)\overline{O}(x_1)O(x_2)\rangle =$
  $iq\langle\overline{O}(x_1)O(x_2)\rangle\delta(\vec{x}-\vec{x}_1) - iq\langle\overline{O}(x_1)O(x_2)\rangle\delta(\vec{x}-\vec{x}_2)$

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A geometric proof:

- $\langle\overline{O}(x_1)O(x_2)\rangle = S_\Psi[\Psi(z_0) = \delta(\vec{z}-\vec{x}_1), \overline{\Psi}(z_0) = \delta(\vec{z}-\vec{x}_2), A_M(z_0) = 0]$
- The action S_Ψ is invariant under
  $\Psi(z,\vec{z}) \to e^{iq\alpha(z,\vec{z})}\Psi(z,\vec{z}), \overline{\Psi}(z,\vec{z}) \to e^{-iq\alpha(z,\vec{z})}\overline{\Psi}(z,\vec{z}),$
  $A_M(z,\vec{z}) \to A_M(z,\vec{z}) + \partial_M\alpha(z,\vec{z})$
- However the boundary conditions are NOT:
  $0 = iqS_\Psi[\Psi \to \delta(\vec{z}-\vec{x}_1), \overline{\Psi} \to \delta(\vec{z}-\vec{x}_2), A_M \to 0]\delta(\vec{x}-\vec{x}_1)$
  $-iqS_\Psi[\Psi \to \delta(\vec{z}-\vec{x}_1), \overline{\Psi} \to \delta(\vec{z}-\vec{x}_2), A_M \to 0]\delta(\vec{x}-\vec{x}_2)$
  $-\partial^x_\mu S_\Psi[\Psi \to \delta(\vec{z}-\vec{x}_1), \overline{\Psi} \to \delta(\vec{z}-\vec{x}_2), A_M \to \delta(\vec{z}-\vec{x})]\delta(\vec{z}-\vec{x})$

  Q.E.D.