Thermoelectric Conductivities at Finite Magnetic Field and the Nernst effect

Yunseok Seo

Hanyang University

15 April, 2015

Based on arXiv:1502.05386
in collaboration with Sang-Jin Sin, Keun-Young Kim and Kyung Kiu Kim

Introduction

- Strongly coupled electron systems show many interesting phases
  - Non-Fermi liquid(or Strange metal)
  - High T_c superconuctivity
  - Pseudo-gap phases $\cdots$
- Transport coefficient
  - Response of the electrical current $\vec{J}$ and the heat current $\vec{Q}$ to an applied electric field $\vec{E}$ and a temperature gradient $\vec{\nabla}T$
  - Transport coefficient

$$\begin{pmatrix} \langle J_i \rangle \\ \langle Q_i \rangle \end{pmatrix} = \begin{pmatrix} \sigma_{ij}(\omega) & \alpha_{ij}(\omega)T \\ \bar{\alpha}_{ij}(\omega)T & \bar{\kappa}_{ij}(\omega)T \end{pmatrix} \begin{pmatrix} E_j \\ -(\nabla_j T)/T \end{pmatrix}$$

σ_{ij}: Electric conductivity

α_{ij}, $\bar{\alpha}_{ij}$: Thermoelectric conductivity

$\bar{\kappa}_{ij}$: Thermal conductivity

Introduction

- Transport Coefficients
  - Hall conductivity: σ_{xy}

    Quantum Hall effect (integer or fractional)

    Hall angle: $\tan\theta_H = \frac{\sigma_{xy}}{\sigma_{xx}}$
  - Nernst response: Electric field induced by a thermal gradient, $E_i = -\vartheta_{ij}\nabla_j T$

$$\vartheta_{ij} = -\left(\sigma^{-1}\cdot\alpha\right)_{ij}$$

    $e_N \equiv \vartheta_{yx}$: Nernst signal

    $\nu = e_N/B$: Nernst coefficient
  - Cyclotron resornance
- Performing explicit calculations of strongly interacting transport is extremely hard. We apply holography method to understand such interesting phenomena
- Transport Coefficients from holography
  - DC conductivity with momentum relaxation (Black and Tong, Adrade and Withers, Vegh,....)
  - DC transport coefficient from black hole horizon data (2014, Donos and Gauntlett)
  - AC transport coefficient with momentum relaxation (2014, Keun-Young Kim, Kyung Kiu Kim, YS, Sang-Jin Sin)

arXiv:0502.02631, Amoretti and Musso, arXiv: 0502.03789, Blake and Donos,

arXiv: 0502.04704, Lucas and Sachdev, arXiv: 0502.05386

- General action with metric, $U(1)$ gauge field, dilaton and "axion"

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi_1(\phi)(\partial\chi_1)^2 + \Phi_2(\phi)(\partial\chi_2)^2\right] - V(\phi) - \frac{Z(\phi)}{4}F^2\right]$$

- Ansatz for background

$$ds^2 = -U(r)dt^2 + \frac{1}{U(r)}dr^2 + e^{v_1}dx^2 + e^{v_2}dy^2,$$

$$\chi_1 = k_1 x, \qquad \chi_2 = k_2 y$$

$$A = a(r)dt + \frac{B}{2}(xdy - ydx)$$

  - Horizon at $r = r_h$

$$U \sim T(r - r_h) + \cdots, \qquad a(r) \sim a_0(r - r_h) + \cdots$$
$$v_i \sim v_{i0} + \cdots, \qquad \phi \sim \Phi_h + \cdots \qquad (1)$$

  - Symmetric gauge of the gauge potential

$$\Phi_1(\phi) = \Phi_2(\phi) = \Phi(\phi), \qquad v_1(r) = v_2(r) = v(r), \qquad k_1 = k_2 = k$$

- Fluctuations (2014, Donos and Gauntlett)

$$\delta A_{x_i} = \{-E_{x_i} + \zeta_{x_i} a(r)\}\, t + \delta a_{x_i}(r)$$

$$\delta G_{tx_i} = -\zeta_{x_i} U(r)\, t + \delta g_{tx_i}(r)$$

$$\delta G_{rx_i} = e^{v(r)} \delta g_{rx_i}(r)$$

$$\delta \chi_i = \delta \chi_i(r)$$

  - Time dependent term contains external source
    E_{x_i} : External electric field
    $\zeta_{x_i} = -\nabla_{x_i} T/T$: External thermal gradiant
  - all time dependence in the linearized equations disappears by imposing background equations of motion

- Number density

$$\rho = \sqrt{-g} Z(\phi) F^{rt} = Z(\phi) e^{v_1(r)} a'(r)$$

- Electric current

$$
\begin{aligned}
J_{x_i} &\equiv Z(\phi)\sqrt{-g}F^{x_i r} \\
&= -Z(\phi)\left\{U(r)(-\epsilon_{ij}B\delta g_{rx_j}(r) + \delta a'_{x_i}(r)) + a'(r)\delta g_{tx_i}\right\}
\end{aligned}
$$

At the boundary ($r \to \infty$), J_{x_i} becomes electric current along x_i direction

- Maxwell equation

$$
\begin{aligned}
0 &= \partial_\mu\left(Z(\phi)\sqrt{-g}F^{x_i\mu}\right) \\
&= \partial_r\left(Z(\phi)\sqrt{-g}F^{x_i r}\right) + \partial_t\left(Z(\phi)\sqrt{-g}F^{x_i t}\right) \\
&= \partial_r J_{x_i} - B\epsilon_{ij}e^{-v(r)}\zeta_{x_j}Z(\phi),
\end{aligned}
$$

- Boundary current

$$
\begin{aligned}
J_{x_i}(\infty) &= J_{x_i}(r_h) + B\epsilon_{ij}\zeta_{x_j}\int_{r_h}^{\infty} dr' e^{-v(r')}Z(\phi(r')) \\
&\equiv J_{x_i}(r_h) + B\epsilon_{ij}\zeta_{x_j}\Sigma_1
\end{aligned}
$$

- Heat current

$$Q_{x_i} \equiv U^2(r)\,\partial_r\left(\frac{\delta g_{tx_i}(r)}{U(r)}\right) - a(r)J_{x_i} \tag{2}$$

At the boundary($r\to\infty$), $Q_{x_i} \to T_{tx_i} - \mu J_{x_i}$: heat current

- Boudary heat current

$$\begin{aligned} Q_{x_i}(\infty) &= Q_{x_i}(r_h) + B\epsilon_{ij}E_{x_j}\int_{r_h}^{\infty} dr' e^{-v(r')}Z(\phi(r')) \\ &- 2B\epsilon_{ij}\zeta_{x_j}\int_{r_h}^{\infty} dr' a(r')e^{-v(r')}Z(\phi(r')) \\ &\equiv Q_{x_i}(r_h) + B\epsilon_{ij}E_{x_j}\Sigma_1 - B\epsilon_{ij}\zeta_{x_j}\Sigma_2. \end{aligned}$$

- Boundary currents J_{x_i}, Q_{x_i} can be written in terms of functions at the horizon and definite integration from black hole horizon to boundary

DC Transport from Horizon Data

- Regularity at the black hole horizon

$$\delta a_{x_i}(r) \sim -\frac{E_{x_i}}{4\pi T}\ln(r-r_h)+\cdots$$

$$\delta g_{tx_i}(r) \sim \delta g^{(0)}_{tx_i} + \delta g^{(1)}_{tx_i}(r-r_h)+\cdots$$

$$\delta g_{rx_i}(r) \sim e^{-v(r_h)}\frac{\delta g^{(0)}_{tx_i}}{U(r_h)}+\cdots$$

$$\delta \chi_{x_i}(r) \sim \chi^{(0)}_{x_i} + \chi^{(1)}_{x_i}(r-r_h)+\cdots$$

- Boundary currents

$$J_{x_i}(\infty) = -(\rho e^{-v_h} + B\epsilon_{ij}e^{-v_h}Z_h)\delta g^{(0)}_{tx_i} + E_{x_i}Z_h + B\epsilon_{ij}\zeta_{x_j}\Sigma_1$$

$$Q_{x_i}(\infty) = -4\pi T\delta g^{(0)}_{tx_i} + B\epsilon_{ij}E_{x_j}\Sigma_1 - B\epsilon_{ij}\zeta_{x_j}\Sigma_2$$

- Transport coefficients

$$\hat{\sigma}^{ij} = \frac{\partial J_{x_i}(\infty)}{\partial E_{x_j}} = -(\rho e^{-v_h} + B\epsilon_{ij} e^{-v_h} Z_h)\frac{\partial \delta g^{(0)}_{tx_i}}{\partial E_{x_j}} + Z_h\,\delta_{ij}$$

$$\hat{\alpha}^{ij} = \frac{1}{T}\frac{\partial J_{x_i}(\infty)}{\partial \zeta_{x_j}} = -(\rho e^{-v_h} + B\epsilon_{ij} e^{-v_h} Z_h)\frac{1}{T}\frac{\partial \delta g^{(0)}_{tx_i}}{\partial \zeta_{x_j}} + \epsilon_{ij}\frac{B}{T}\Sigma_1$$

$$\hat{\bar{\alpha}}^{ij} = \frac{1}{T}\frac{\partial Q_{x_i}(\infty)}{\partial E_{x_j}} = -4\pi\frac{\delta g^{(0)}_{tx_i}}{\partial E_{x_j}} + \epsilon_{ij}\frac{B}{T}\Sigma_1$$

$$\hat{\bar{\kappa}}^{ij} = \frac{1}{T}\frac{\partial Q_{x_i}(\infty)}{\partial \zeta_{x_j}} = -4\pi\frac{\delta g^{(0)}_{tx_i}}{\partial \zeta_{x_j}} - \epsilon_{ij}\frac{B}{T}\Sigma_2$$

- Einstein equations at the horizon

$$\delta g^{(0)}_{tx} = \frac{e^{v_h}}{B^2 Z_h + k^2 e^{v_h}\Phi_h}\left[B\rho e^{-v_h}\delta g^{(0)}_{ty} - BZ_h E_y - \rho E_x - 4\pi e^{v_h} T\zeta_x\right]$$

$$\delta g^{(0)}_{ty} = \frac{e^{v_h}}{B^2 Z_h + k^2 e^{v_h}\Phi_h}\left[-B\rho e^{-v_h}\delta g^{(0)}_{tx} - BZ_h E_x - \rho E_y - 4\pi e^{v_h} T\zeta_y\right]$$

DC Transport from Horizon Data

- Electric transport coefficients

$$\sigma^{xx} = \sigma^{xy} = \frac{e^{v_h} k^2 \Phi_h \left(\rho^2 + B^2 Z_h^2 + e^{v_h} k^2 Z_h \Phi_h\right)}{B^2 \rho^2 + \left(B^2 Z_h^2 + e^{v_h} k^2 \Phi_h\right)^2}$$

$$\sigma^{xy} = -\sigma^{yx} = \frac{B\rho \left(\rho^2 + B^2 Z_h^2 + 2 e^{v_h} k^2 Z_h \Phi_h\right)}{B^2 \rho^2 + \left(B^2 Z_h^2 + e^{v_h} k^2 \Phi_h\right)^2}$$

- Thermoelectric transport coefficients : $\hat{\alpha}^{ij} - \epsilon^{ij} \frac{B}{T} \Sigma_1$

$$\alpha^{xx} = \alpha^{yy} = \frac{4\pi e^{2v_h} k^2 \rho \Phi_h}{B^2 \rho^2 + \left(B^2 Z_h^2 + e^{v_h} k^2 \Phi_h\right)^2}$$

$$\alpha^{xy} = -\alpha^{yx} = \frac{4\pi e^{v_h} B \left(\rho^2 + B^2 Z_h^2 + e^{v_h} k^2 Z_h \Phi_h\right)}{B^2 \rho^2 + \left(B^2 Z_h^2 + e^{v_h} k^2 \Phi_h\right)^2}$$

- Thermal transport coefficients: $\hat{\bar{\kappa}}^{ij} + \epsilon^{ij} \frac{B}{T} \Sigma_2$

$$\bar{\kappa}^{xx} = \bar{\kappa}^{yy} = \frac{16\pi^2 T e^{2v_h} \left(B^2 Z_h + e^{v_h} k^2 \Phi_h\right)}{B^2 \rho^2 + \left(B^2 Z_h^2 + e^{v_h} k^2 \Phi_h\right)^2}$$

$$\bar{\kappa}^{xy} = -\bar{\kappa}^{yx} = \frac{16\pi^2 T e^{2v_h} B\rho}{B^2 \rho^2 + \left(B^2 Z_h^2 + e^{v_h} k^2 \Phi_h\right)^2}$$

- Hall angle (2014 Black and Donos)

$$\tan\theta_H = \frac{\sigma^{xy}}{\sigma^{xx}} = \frac{B\rho\left(\rho^2 + B^2 Z_h^2 + 2e^{v_h} k^2 Z_h \Phi_h\right)}{e^{v_h} k^2 \Phi_h \left(\rho^2 + B^2 Z_h^2 + e^{v_h} k^2 Z_h \Phi_h\right)}$$

- Nernst signal

$$\begin{aligned} e_N &= -(\sigma^{-1}\cdot\alpha)^{yx} \\ &= \frac{4\pi e^{2v_h} k^2 Z_h^2 \Phi_h B}{\rho^4 + 2e^{v_h} k^2 \rho^2 Z_h \Phi_h + Z_h^2\left(B^2\rho^2 + e^{2v_h} k^4 \Phi_h^2\right)} \end{aligned}$$

  - Has maximum at

$$k_{\max}^2 = \frac{e^{-v_h}\rho\sqrt{\rho^2 + B^2 Z_h^2}}{Z_h \Phi_h}$$

  - $\rho \to 0$ limit

$$e_N\Big|_{\rho=0} = \frac{4\pi B}{k^2\Phi_h} = \frac{B}{T}\tau_L, \qquad \left(\tau_L = \frac{4\pi}{k^2\Phi_h} T\right)$$

- Dyonic black hole solution with momentum relaxation

$\phi = \text{const.} \quad , \quad \Phi_1 = \Phi_2 = 1 \quad , \quad V(\phi) = -\frac{6}{L^2} \quad , \quad Z(\phi) = 1$

$$S_0 = \frac{1}{16\pi G}\int_M d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - \frac{1}{2}\sum_{I=1}^{2}(\partial\chi_I)^2\right] - \frac{1}{8\pi G}\int_{\partial M} d^3x\sqrt{-\gamma}K$$

$$U(r) = r^2 - \frac{\beta^2}{2} - \frac{m_0}{r} + \frac{\mu^2 + q_m^2}{4}\frac{r_h^2}{r^2} \ , \ e^{v_1(r)} = e^{v_2(r)} = r^2 \ ,$$

$$k_1 = k_2 = \beta \quad , \quad a(r) = \mu\left(1 - \frac{r_h}{r}\right) \quad , \quad B = q_m r_h$$

$$m_0 = r_h^3\left(1 + \frac{\mu^2 + q_m^2}{4r_h^2} - \frac{\beta^2}{2r_h^2}\right)$$

$$T = T_H = \frac{U'(r_h)}{4\pi} = \frac{1}{4\pi}\left(3r_h - \frac{\mu^2 + q_m^2 + 2\beta^2}{4r_h}\right)$$

- Electric conductivities

$$\sigma^{xx} = r_h^2\beta^2 \cdot \frac{B^2 + r_h^2\left(\mu^2+\beta^2\right)}{r_h^2\mu^2B^2 + \left(B^2 + r_h^2\beta^2\right)^2}$$

$$\sigma^{xy} = r_h\mu B \cdot \frac{B^2 + r_h^2\left(\mu^2+2\beta^2\right)}{r_h^2\mu^2B^2 + \left(B^2 + r_h^2\beta^2\right)^2}$$

- Thermoelectric conductivities

$$\alpha^{xx} = \frac{4\pi r_h^5\beta^2\mu}{r_h^2\mu^2B^2 + \left(B^2 + r_h^2\beta^2\right)^2}$$

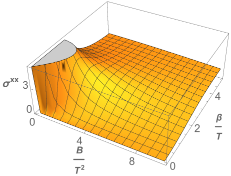
$$\alpha^{xy} = 4\pi r_h^3 B \cdot \frac{B^2 + r_h^2\left(\mu^2+\beta^2\right)}{r_h^2\mu^2B^2 + \left(B^2 + r_h^2\beta^2\right)^2}$$

- Thermal conductivities

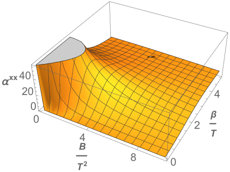
$$\bar{\kappa}^{xx} = 16\pi r_h^4 T \cdot \frac{B^2 + r_h^2\beta^2}{r_h^2\mu^2B^2 + \left(B^2 + r_h^2\beta^2\right)^2}$$

$$\bar{\kappa}^{xy} = \frac{16\pi^2 r_h^5\mu T B}{r_h^2\mu^2B^2 + \left(B^2 + r_h^2\beta^2\right)^2}$$

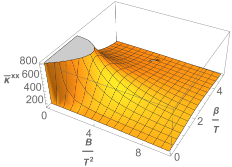
- DC transport coefficients



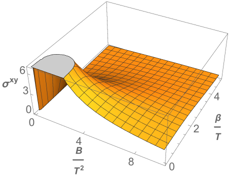
(a) σ^{xx}



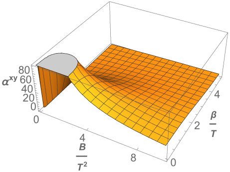
(b) α^{xx}



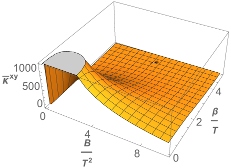
(c) $\bar{\kappa}^{xx}$



(d) σ^{xy}



(e) α^{xy}



(f) $\bar{\kappa}^{xy}$

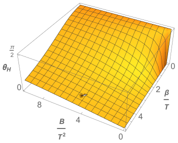
Figure : β/T and B/T dependence of DC conductivities at fixed $\mu/T = 4$

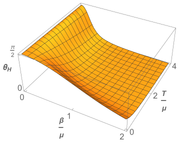
Dyonic Black Hole with Momentum Relaxation

DC transport coefficients

- Hall angle

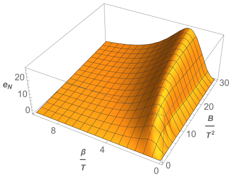
$$\tan\theta_H \equiv \frac{\sigma^{xy}}{\sigma^{xx}} = \frac{\mu B}{r_h \beta^2} \cdot \frac{B^2 + r_h^2(\mu^2 + 2\beta^2)}{B^2 + r_h^2(\mu^2 + \beta^2)}$$

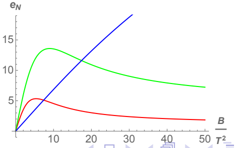




- Nernst signal

$$e_N = \frac{4\pi r_h^2 \beta^2 B}{\mu^2 B^2 + r_h^2(\mu^2 + \beta^2)^2}$$





- Fluctuations

$$\delta A_i(t,r) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} a_i(\omega, r)\,,$$
$$\delta g_{ti}(t,r) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} r^2 h_{ti}(\omega, r)\,,$$
$$\delta \chi_i(t,r) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \psi_i(\omega, r)\,,$$

- Linearized equations of motion

$$\begin{aligned}
\frac{q_m^2 h_{ti}}{r^4 U} + \epsilon_{ij}\frac{i\omega q_m a_j}{r^4 U} + \frac{\beta^2 h_{ti}}{r^2 U} + \frac{i\beta\omega\psi_i}{r^2 U} - \frac{\mu a_i'}{r^4} - \frac{4h_{ti}'}{r} - h_{ti}'' &= 0 \\
\epsilon_{ij}\frac{iU q_m a_j'}{r^4\omega} + \frac{i\beta U\psi_i'}{r^2\omega} + \epsilon_{ij}\frac{i\mu q_m h_{tj}}{r^4\omega} + \frac{\mu a_i}{r^4} + h_{ti}' &= 0 \\
\frac{U' a_i'}{U} + \epsilon_{ij}\frac{i\omega q_m h_{tj}}{U^2} + \frac{\mu h_{ti}'}{U} + \frac{\omega^2 a_i}{U^2} + a_i'' &= 0 \\
\frac{U'\psi_i'}{U} - \frac{i\beta\omega h_{ti}}{U^2} + \frac{\omega^2\psi_i}{U^2} + \frac{2\psi_i'}{r} + \psi_x'' &= 0
\end{aligned}$$

- Near horizon($r \to r_h$)

$$h_{ti} = (r-1)^{\nu_\pm +1}(h_{ti}^{(I)} + h_{ti}^{(II)}(r-1) + \cdots),$$
$$a_i = (r-1)^{\nu_\pm}(a_i^{(I)} + a_i^{(II)}(r-1) + \cdots),$$
$$\psi_i = (r-1)^{\nu_\pm}(\psi_i^{(I)} + \psi_i^{(II)}(r-1) + \cdots$$

$\nu_\pm = \pm i4\omega/(-12 + q_m^2 + 2\beta^2 + \mu^2)$

- Near boundary($r \to \infty$)

$$h_{ti} = h_{ti}^{(0)} + \frac{1}{r^2}h_{ti}^{(2)} + \frac{1}{r^3}h_{ti}^{(3)} + \cdots,$$
$$a_i = a_i^{(0)} + \frac{1}{r}a_i^{(1)} + \cdots,$$
$$\psi_i = \psi_i^{(0)} + \frac{1}{r^2}\psi_i^{(2)} + \frac{1}{r^3}\psi_i^{(3)} + \cdots$$

- Quadratic action at the boundary

$$S^{(2)}_{\rm ren} = \frac{\mathcal{V}}{2}\int_0^\infty \frac{d\omega}{2\pi}\left(-\mu\bar{a}^{(0)}_i h^{(0)}_{ti} - m_0\bar{h}^{(0)}_{ti}h^{(0)}_{ti} + \bar{a}^{(0)}_i a^{(1)}_i - 3\bar{h}^{(0)}_{ti}h^{(3)}_{ti} + 3\bar{\psi}^{(0)}\psi^{(3)}\right)$$
$$= \frac{\mathcal{V}}{2}\int_0^\infty \frac{d\omega}{(2\pi)}\left[\bar{J}^a\mathbb{A}_{ab}J^b + \bar{J}^a\mathbb{B}_{ab}R^b\right]$$

- Near boundary solution (2010, Kaminsky et. al.)

$$\Phi^a(r) \equiv \Phi^a_i(r)c^i \to \mathbb{S}^a_i c^i + \cdots + \frac{\mathbb{O}^a_i c^i}{r^{\delta_a}} + \cdots$$
$$\equiv J^a + \cdots + \frac{R^a}{r^{\delta_a}} + \cdots$$

$$c^I = (\mathbb{S}^{-1})^I{}_a J^a$$
$$R^a = \mathbb{O}^a{}_I c^I = \mathbb{O}^a{}_I(\mathbb{S}^{-1})^I{}_b J^b$$

- Retared Green's function

$$S^{(2)}_{\rm ren} = \frac{\mathcal{V}}{2}\int_0^\infty \frac{d\omega}{(2\pi)}\bar{J}^a\left[\mathbb{A}_{ab} + \mathbb{B}_{ac}\mathbb{O}^c{}_I(\mathbb{S}^{-1})^I{}_b\right]J^b$$
$$= \frac{\mathcal{V}}{2}\int_0^\infty \frac{d\omega}{(2\pi)}\bar{J}^a G_{ab}J^b$$

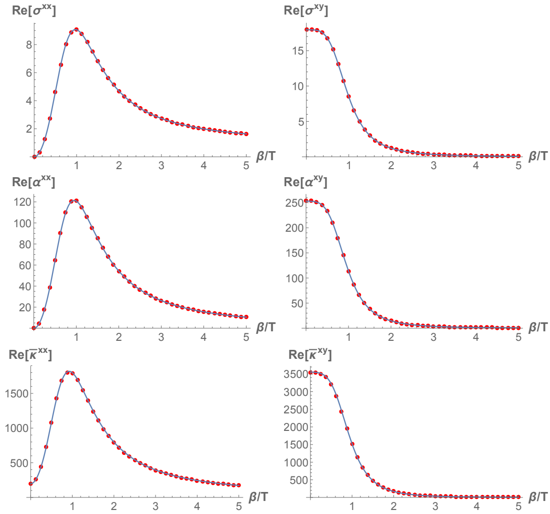
- Transport coefficients and retared Green's function

$$\begin{pmatrix} \langle J_i \rangle \\ \langle T_{ti} \rangle \end{pmatrix} = \begin{pmatrix} G^{ij}_{JJ} & G^{ij}_{JT} \\ G^{ij}_{TJ} & G^{ij}_{TT} \end{pmatrix} \begin{pmatrix} a^{(0)}_j \\ h^{(0)}_{tj} \end{pmatrix},$$

$$\begin{pmatrix} \langle J_i \rangle \\ \langle Q_i \rangle \end{pmatrix} = \begin{pmatrix} \hat{\sigma}^{ij} & \hat{\alpha}^{ij} T \\ \hat{\bar{\alpha}}^{ij} T & \hat{\bar{\kappa}}^{ij} T \end{pmatrix} \begin{pmatrix} E_j \\ -(\nabla_j T)/T \end{pmatrix}$$
$$= \begin{pmatrix} \hat{\sigma}^{ij} & \hat{\alpha}^{ij} T \\ \hat{\bar{\alpha}}^{ij} T & \hat{\bar{\kappa}}^{ij} T \end{pmatrix} \begin{pmatrix} i\omega(a^{(0)}_j + \mu h^{(0)}_{tj}) \\ i\omega h^{(0)}_{tj} \end{pmatrix}$$

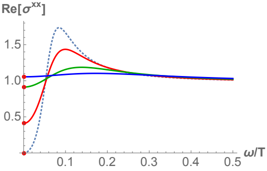
$$\begin{pmatrix} \sigma^{ij} & \alpha^{ij} T \\ \bar{\alpha}^{ij} T & \bar{\kappa}^{ij} T \end{pmatrix} = \begin{pmatrix} -\frac{iG^{ij}_{JJ}}{\omega} & \frac{i(\mu G^{ij}_{JJ} - G^{ij}_{JT})}{\omega} \\ \frac{i(\mu G^{ij}_{JJ} - G^{ij}_{TJ})}{\omega} & -\frac{i(G^{ij}_{TT} - G^{ij}_{TT}(\omega=0) - \mu(G^{ij}_{JT} + G^{ij}_{TJ} - \mu G^{ij}_{JJ}))}{\omega} \end{pmatrix}$$
$$- \frac{B}{T} \begin{pmatrix} 0 & \Sigma_1 \epsilon^{ij} \\ \Sigma_1 \epsilon^{ij} & \Sigma_2 \epsilon^{ij} \end{pmatrix}.$$

- Comparison to the DC conductivities from horizon data

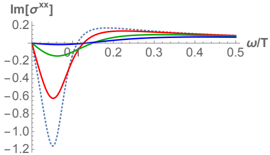


Dyonic Black Hole with Momentum Relaxation
AC transport coefficients

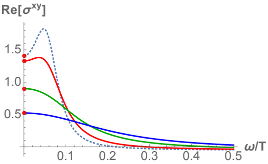
- Electric conductivities



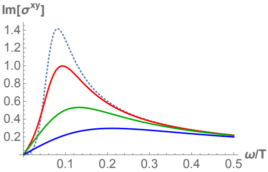
(a) Re σ^{xx}



(b) Im σ^{xx}



(c) Re σ^{xy}



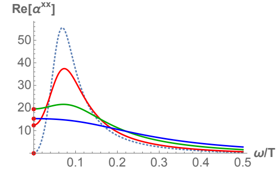
(d) Im σ^{xy}

Figure : β dependence of electric conductivities: $\beta/T = 0,\ 0.5,\ 1,\ 1.5$(dotted, red, green, blue) with $\mu/T = 1,\ B/T^2 = 3$

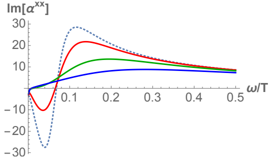
Dyonic Black Hole with Momentum Relaxation

AC transport coefficients

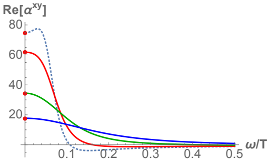
- Thermolectric conductivities



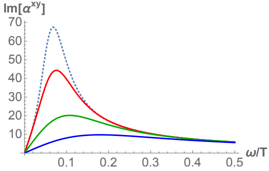
(a) Re α^{xx}



(b) Im α^{xx}



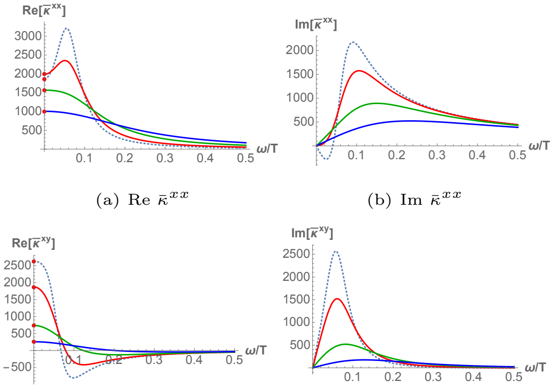
(c) Re α^{xy}



(d) Im α^{xy}

Figure : β dependence of thermoelectric conductivities: $\beta/T = 0,\ 0.5,\ 1,\ 1.5$(dotted, red, green, blue) with $\mu/T = 1,\ B/T^2 = 3$

- Thermal conductivities

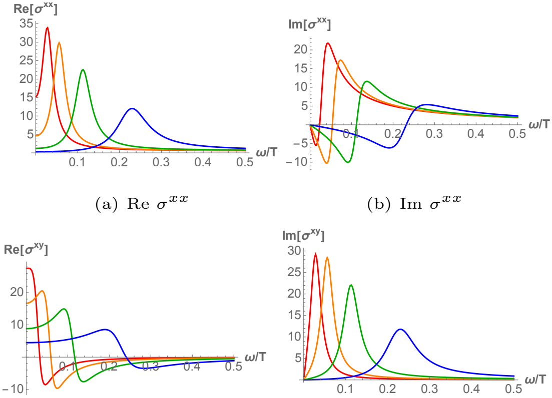


(a) Re $\bar{\kappa}^{xx}$ (b) Im $\bar{\kappa}^{xx}$

(c) Re $\bar{\kappa}^{xy}$ (d) Im $\bar{\kappa}^{xy}$

Figure : β dependence of thermal conductivities: $\beta/T = 0,\ 0.5,\ 1,\ 1.5$(dotted, red, green, blue) with $\mu/T = 1,\ B/T^2 = 3$

- Magnetic field dependence on the electric conductivity



(a) Re σ^{xx} (b) Im σ^{xx}

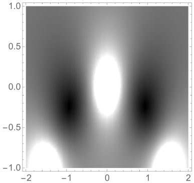
(c) Re σ^{xy} (d) Im σ^{xy}

Figure : B dependence of electric conductivities:

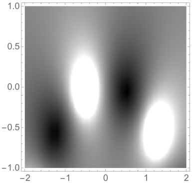
$B/T^2 = 0.5,\ 1,\ 2,\ 4$(red,orange,green,blue) with $\beta/T = 1/2,\ \mu/T = 4$

- Cyclotron pole

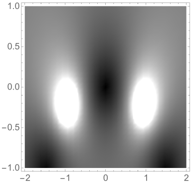
$$\sigma^{\pm} = \sigma^{xy} \pm i\sigma^{xx}$$



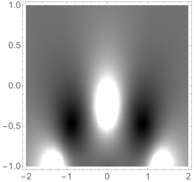
(a) $q_m = 0, \mu = 12.5$



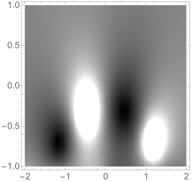
(b) $q_m = 8.9, \mu = 8.9$



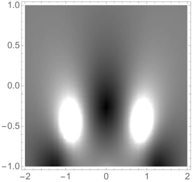
(c) $q_m = 12.5, \mu = 0$



(d) $q_m = 0, \mu = 12.5$



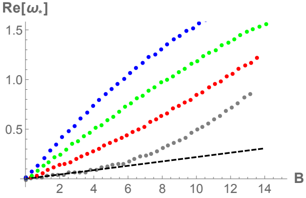
(e) $q_m = 8.9, \mu = 8.9$



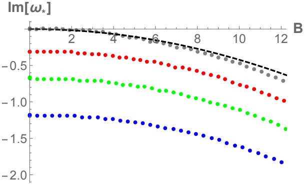
(f) $q_m = 12.5, \mu = 0$

- Hydrodynamic analysis($\tau_{imp} = \infty$)

$$\omega_* \equiv \omega_c - i\gamma = \frac{\rho B}{\mathcal{E}+\mathcal{P}} - i\frac{B^2}{g^2(\mathcal{E}+\mathcal{P})} \equiv \omega_c^0 - i\gamma^0$$



(a) Re $\omega_*(=\omega_c)$



(b) Im $\omega*(= -\gamma)$

$$\omega_* = \omega_c - i\gamma = \omega_c^0 + c_1\beta^2 B - i(\gamma^0 + c_2\beta^2)$$
$$\sim \omega_c^0 + \frac{B}{\tau_{imp}} - i\left(\gamma^0 + \frac{1}{\tau_{imp}}\right) \qquad for\ \beta << \mu$$

Summary and Discussion

- We derive general DC transport coefficients in general background with momentum relaxation by requiring regularity condition at the black hole horizon
- We apply our general formula to the dyonic black hole background and calculation DC transport coefficient
- We develop prescription to calculate AC transport coefficient and the results are consistent with DC calculations
- We investigae AD trnasport coefficienst and calculate the cyclotron pole beyond hydrodynamic limit

Thank you !!!