INCOHERENCE and DIFFUSION

The Incoherence of the Incoherence

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The Incoherence of the Incoherence (Arabic: تهافت التهافت *Tahāfut al-Tahāfut*) by Andalusian Muslim polymath and philosopher Ibn Rushd (Averroes) (1126–1198) is an important Islamic philosophical treatise,[1] in which the author defends the use of Aristotelian philosophy within Islamic thought.

It was written in the style of a dialogue against Al-Ghazali's claims in *The Incoherence of the Philosophers* (*Tahāfut al-Falasifa*), which criticized Islamic Neoplatonic thought. Originally written in Arabic, *The Incoherence of the Incoherence* was subsequently translated into many other languages. The book is considered Averroes' landmark; in it, he tries to create harmony between faith and philosophy.

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based upon:

[A. Amoretti, A. Braggio, N. Magnoli, D. Musso hep-th/1411.6631]

Outline

- **Motivations**
  (both theoretical and experimental)

- **Diffusion of charge and heat**
  (framework to study circumstances beyond quasi-particle)

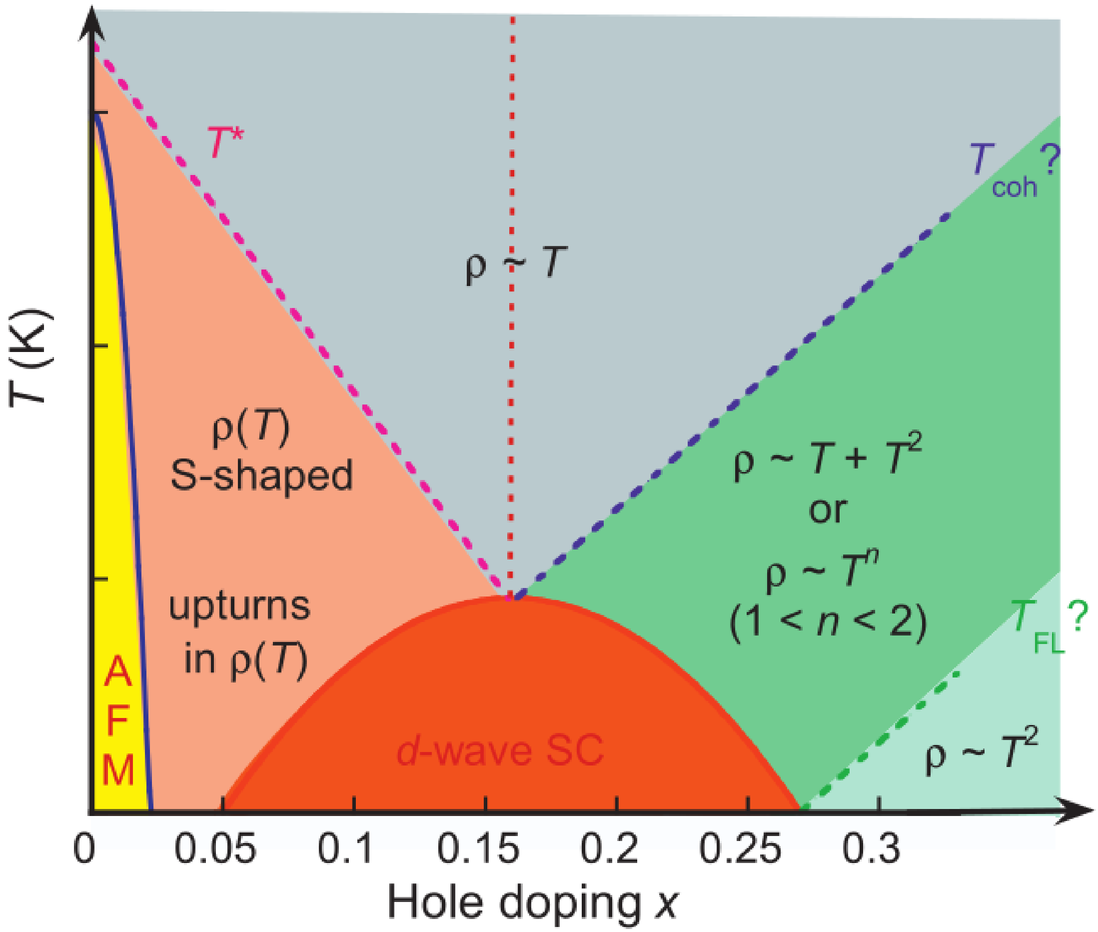
- **Holography without translation invariance**
  (specifically, massive gravity)

- **Coherent/Incoherent transport**

- **Looking for bounds on diffusivity**

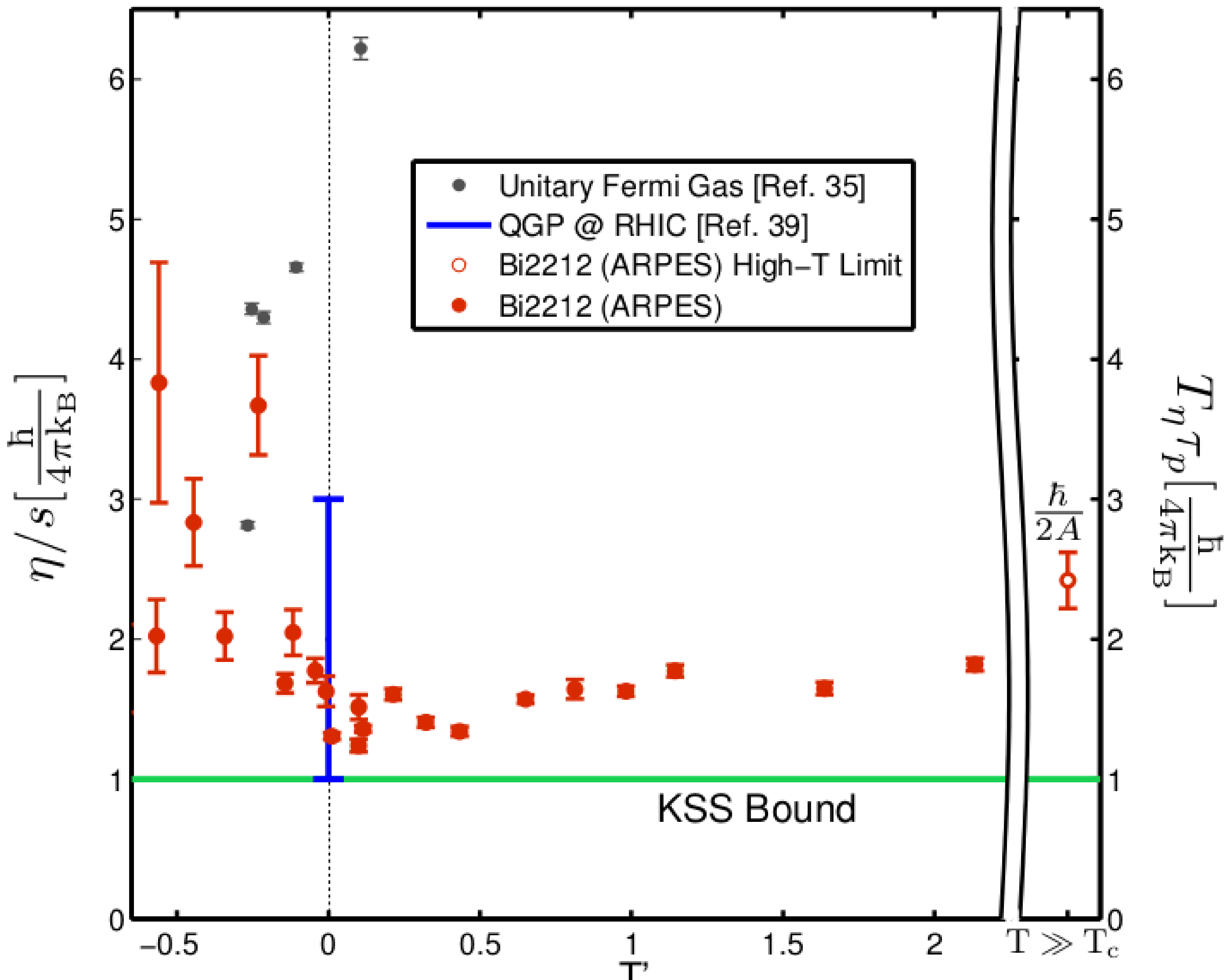
Motivations from Condensed Matter

- High-Tc SUCO's
- Normal phase: *strange metal* & *critical "valley"*
- **Robustness**, both in *(high)* T and in the sense of *universality*



[Hussey, J. Phys: Condens. Matter 20 (2008) 123201]

More widely, strongly coupled systems



[Rameu, Reber, Yang, Akhanjee, Gu, Jonson, cond-mad.str-el/1409.5820]

Diffusion

- Conserved charges
- Thermodynamics
- Low frequency and wave number
- Linear response

Diffusion

$$\frac{d}{dt} n_A = D_{AB} \nabla^2 n_B$$

overdamped modes

Considering a system conserving charge and energy density...

Einstein's relations connecting the thermodynamical and transport quantities to the thermo-electric diffusivities

$$D_+ D_- = \frac{\sigma}{\chi} \frac{\kappa}{c_\rho}$$

$$D_+ + D_- = \frac{\sigma}{\chi} + \frac{\kappa}{c_\rho} + \frac{T(\zeta\sigma - \chi\alpha)^2}{c_\rho \chi^2 \sigma}$$

Also when there are no quasi-particles and regardless of the speed of momentum dissipation

[Hartnoll, JHEP 1502 (2015) 100]

Diffusivity bound

$$D = \frac{\eta}{\mathcal{E} + P}$$

$$\frac{\eta}{s} \geq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

[Policastro, Son, Starinets, JHEP 0209 (2002) 043;
Kovtun, Son, Starinets, Phys. Rev. Lett. 94 (2005) 111601]

Are we secretly introducing model dependence?

LOGICAL LEAP

- Dependence on the “Fermi velocity”
- Clarification about the models to which it could be applied

$$D \geq C\frac{\hbar \bar{v}^2}{k_B T}$$

Incoherent regime?

[Hartnoll, JHEP 1502 (2015) 100]

Hints

- **AdS/CFT** → Viscosity bound and universality *(within holography)* of dual Einstein gravity

- **Plank and Heisenberg** → Existence of a minimal length scale set by the *(high)* temperature

  (Exp. on equilibration time) [Bruin, Sakai, Perry, Mackenzie, Science 339, 804 (2013)]

- **Unitary Fermi Gas** → Quantum Mechanical Limitations to Spin Diffusion in the Unitary Fermi Gas

  (computations & experiments) [Sommer, Ku, Roati, Zwierlein, Nature 472, 201–204]

  [Tilman Enss and Rudolf Haussmann Phys. Rev. Lett. 109, 195303]

Tension between Particular and Universal

Robustness calls for ***universal*** mechanisms *however* ***momentum dissipation*** (which is essential to define the DC transport coefficients) appears to be intimately ***model dependent***...

→ ***Can a proper definition of incoherence provide the correct setup?*** → ***"insensitivity to details"***

Context

Conjectural nature of holographic bounds and limitations thereof

[Kovtun, cond-mat.stat-mech/1407.0690;
Cremonini Mod.Phys.Lett. B25 (2011) 1867-1888]

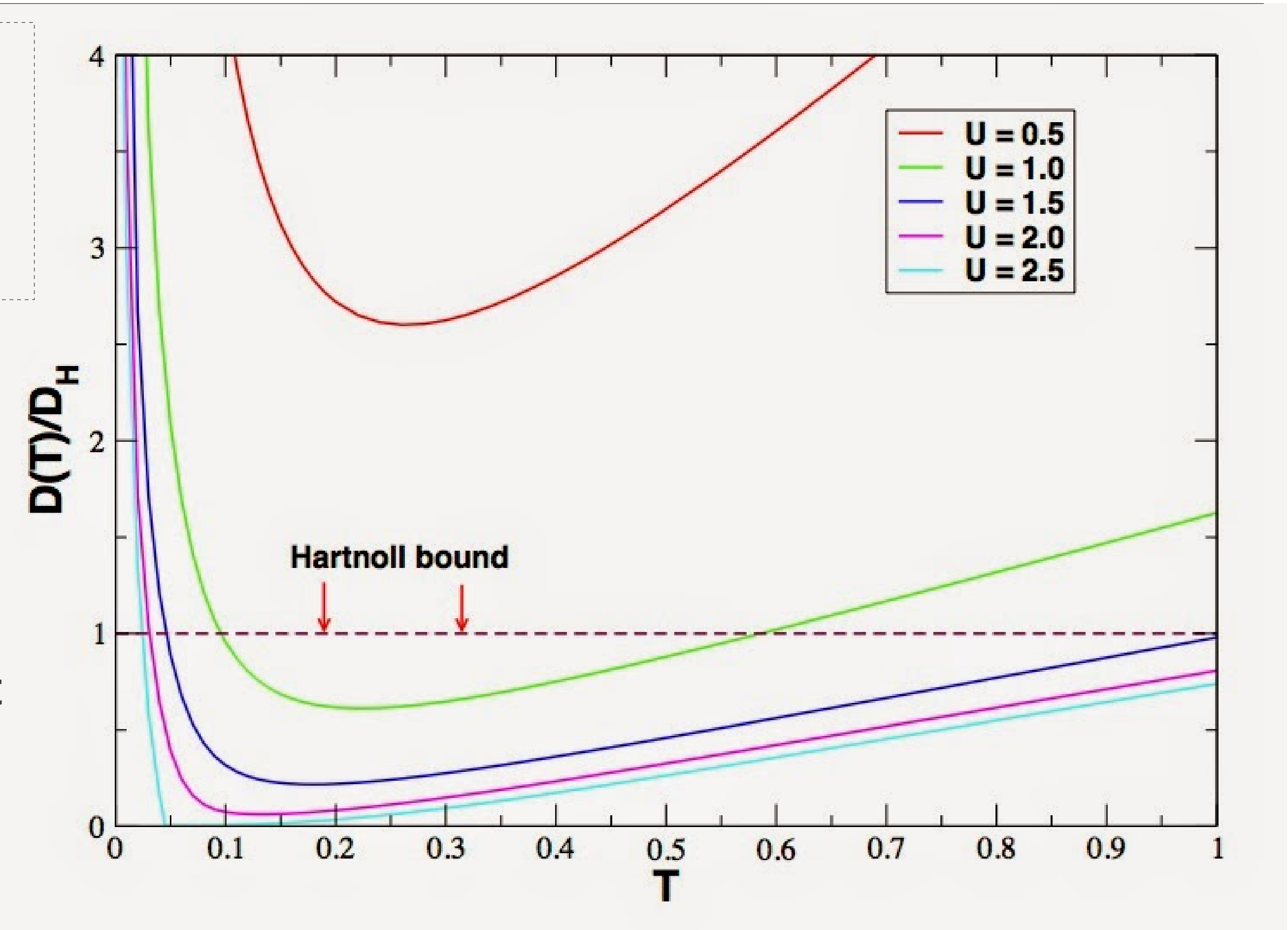
η/s **story...**

Charge diff. const. in the metallic phase of a single-band Hubbard model with DMFT

Absence of a quantum limit to charge diffusion in bad metals?

ATTENTION:

- ***Incoherence*** **as MIR violation**
- **Violation of Hartnoll's bound at coherent/incoherent crossover**
- **Bound satisfied for strongly correlated systems in the high temperature region where the resistivity is close to linear in temperature**



[Pakhira, McKenzie, cond-mat/1409.5662]

*Holographic massive gravity**

[de Rham, Gabadadze, Phys. Rev. D 82, 044020 (2010);
de Rham, Gabadadze, Tolley, Phys. Rev. Lett. 106:231101 (2011)]

$$S = \int d^4x \sqrt{-g} \left[R + 6 + \underline{\mathcal{M}(g)} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right] + S_{\mathrm{c.t.}}$$

[Vegh, hep-th/1301.0537]

$$\mathcal{M}(g) = \alpha\, \mathrm{tr}(\mathcal{K}) + \beta \left[\mathrm{tr}(\mathcal{K})^2 - \mathrm{tr}(\mathcal{K}^2) \right]$$

Theories of massive gravity ***inevitably*** include an auxiliary reference metric

[Hassan, Rosen, Schmidt-May, JHEP 1202 (2012) 026]

$$\mathcal{K}^{\mu}{}_{\nu} = g^{\mu\alpha} f_{\alpha\nu}$$

$$\mathcal{K} \equiv (\sqrt{\mathcal{K}^2})^{\mu}{}_{\nu}$$

$$f_{\mu\nu} = \mathrm{diag}(0, 0, 1, 1)$$

The auxiliary metric is spatially isotropic and homogeneous but breaks spatial diffeomorphisms

** Use responsibly, it may induce instabilities. In case of ghost contact your physicist immediately.*

Bulk

Effective mass of the bulk graviton
(two parameters)

$$m^2(z) = -2\beta - \frac{\alpha}{z}$$

[Vegh, hep-th/1301.0537; Blake, Tong, Phys.Rev. D88 (2013) 10, 106004]

ANSATZ:
homogeneous and isotropic

$$\begin{cases} ds^2 = \dfrac{1}{z^2}\left[-f(z)dt^2 + dx^2 + dy^2 + \dfrac{1}{f(z)}dz^2\right] \\ A = a(z)\,dt \end{cases}$$

SOLUTION

$$\begin{cases} f(z) = 1 - \dfrac{z^3}{z_h^3} + \dfrac{z\alpha}{2} - \dfrac{z^3\alpha}{2z_h^2} + z^2\beta - \dfrac{z^3\beta}{z_h} + \dfrac{z^4\mu^2}{4z_h} - \dfrac{z^3\mu^2}{4z_h} \\ a(z) = \mu\left(1 - \dfrac{z}{z_h}\right) \end{cases}$$

$$L = 1,\ q = 1,\ \kappa = 1/\sqrt{2}$$

The dual Physics

Thermodynamics

$$\begin{cases} T = -\frac{f'(z_h)}{4\pi} = \frac{1}{4\pi}\left(\frac{3}{z_h} + \alpha + z_h\beta - \frac{1}{2}z_h\mu^2\right) \\ \mathcal{S} = \frac{4\pi}{z_h^2} \qquad \rho = \frac{\mu}{z_h} \end{cases}$$

Momentum dissipation rate

$$\tau^{-1} = -\frac{4(\alpha + 2z_h\beta)}{12 + 4z_h\alpha + z_h^2(4\beta + 3\mu^2)}$$

- A thorough thermodynamic and linear response analysis is essential for consistency reasons *(possible pathology of massive gravity...)*
  [Davison, Phys.Rev. D88 (2013) 086003; Blake, Tong, Phys.Rev. D88 (2013) 10, 106004; Amoretti, Braggio, Maggiore, Magnoli, Musso, JHEP 1409 (2014) 160]
- DC transport coefficients are efficiently and analytically computed *(in various circumstances... see **Amoretti**'s talk)*
  [Donos, Gauntlett JHEP 1411 (2014) 081;
  Amoretti, Braggio, Maggiore, Magnoli, Musso, Phys.Rev. D91 (2015) 2, 025002]

Diffusion at high T

$$\mu = 0$$

Simplifies the analysis without spoiling the important features

- Is it possible do distill *intrinsic* momentum conserving contributions?
- Naive limit of big ***beta*** leads to possibly pathological consequences *(for both the dual viewpoints)*
- ***alpha*** crucial to make sense of the high T behavior?

$$m^2(z) = -2\beta - \frac{\alpha}{z}$$

$\alpha = 0$

$$\frac{k_B}{\hbar \bar{v}^2} D_c = \frac{3}{4\pi T} + \mathcal{O}\left(\frac{1}{T^3}\right)$$

$$\frac{k_B}{\hbar \bar{v}^2} D_h = -\frac{\pi T}{\beta} + \frac{3}{8\pi T} + \mathcal{O}\left(\frac{1}{T^3}\right)$$

Unpleasant!

Pleasant! As in the momentum conserving holographic plasma

[*e.g.* Kovtun,Ritz, Phys. Rev. D 78 (2008) 066009]

$$\frac{k_B}{\hbar \bar{v}^2} D_c = \frac{3}{4\pi T} + \frac{3\alpha}{16\pi^2 T^2} + \mathcal{O}\left(\frac{1}{T^3}\right)$$

$$\frac{k_B}{\hbar \bar{v}^2} D_h = -\frac{3}{2\alpha} + \frac{9\beta}{4\pi\alpha^2 T} + \mathcal{O}\left(\frac{1}{T^2}\right)$$

$\alpha \neq 0$

Positive *(positivity of graviton mass)* ***and "UV finite"***

Puzzle?

- ***Coherence*: ** Drude-like pole dominating the long-time dynamics *(over other poles and analytic contributions).* **Momentum is quasi conserved**

- ***Incoherence*: otherwise**

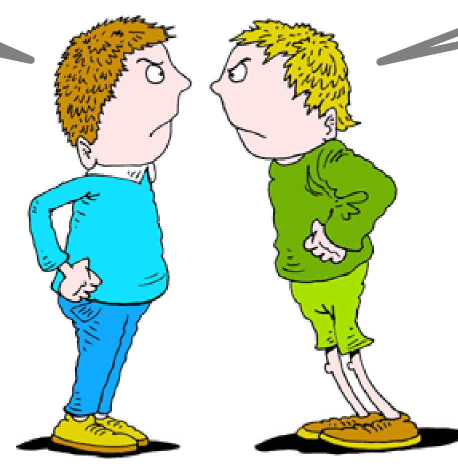
[Hartnoll, JHEP 1502 (2015) 100]

[Davison, Goutéraux, JHEP 1501 (2015) 039]

Momentum must be quickly degraded! Momentum dissipation scale must be at minimum of the order of T!

Criticality is encountered at high T! We want to saturate diffusion bounds in the high T limit!

$$\Gamma \geq k_B T$$



$$\Gamma \ll k_B T$$

Discussion

We should probably not think of the momentum dissipation scale and T as unrelated

But this seems a feature of massive gravity where momentum dissipation is controlled by the parameters *alpha* and *beta* setting the graviton mass

- Massive gravity is too rigid
- There is a way of *phenomenologically* tune the graviton mass and relate it to T
- **Our real focus is on the possibility of saturating a T-dependent diffusivity bound and we want the system to be governed by temperature alone** *(being even agnostic about taking large or small momentum dissipation rate!)*

Incoherence *(stricter version!)*

The extrinsic *(momentum dissipating)* processes **do not** introduce features associated to scales other than the temperature

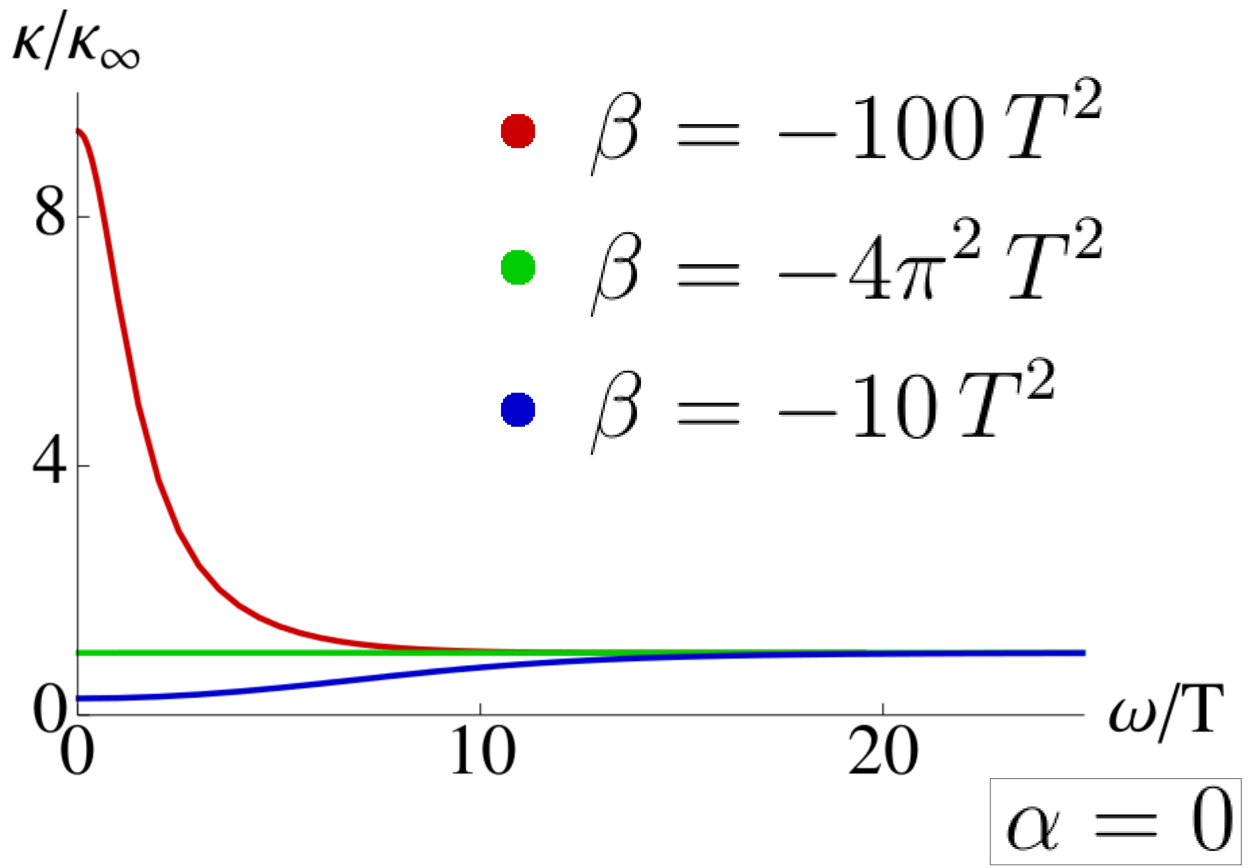
If massive gravity was rigid by itself, we are making the analysis even stiffer *(i.e. adding a constraint)*. However, we can hope of being able to grasp the essential features of a system governed by temperature alone

- *Nice relation with optical transport coefficients*
- *Nice relation with symmetry enhancement*

[Davison, Goutéraux, JHEP 1501 (2015) 039]

Incoherence in holographic massive gravity: **spectral** *standpoint*

- Drude-like peak
- Flat & featureless
- Depletion region



Strict incoherence is a "delicate" circumstance which seems to require *tuning!*

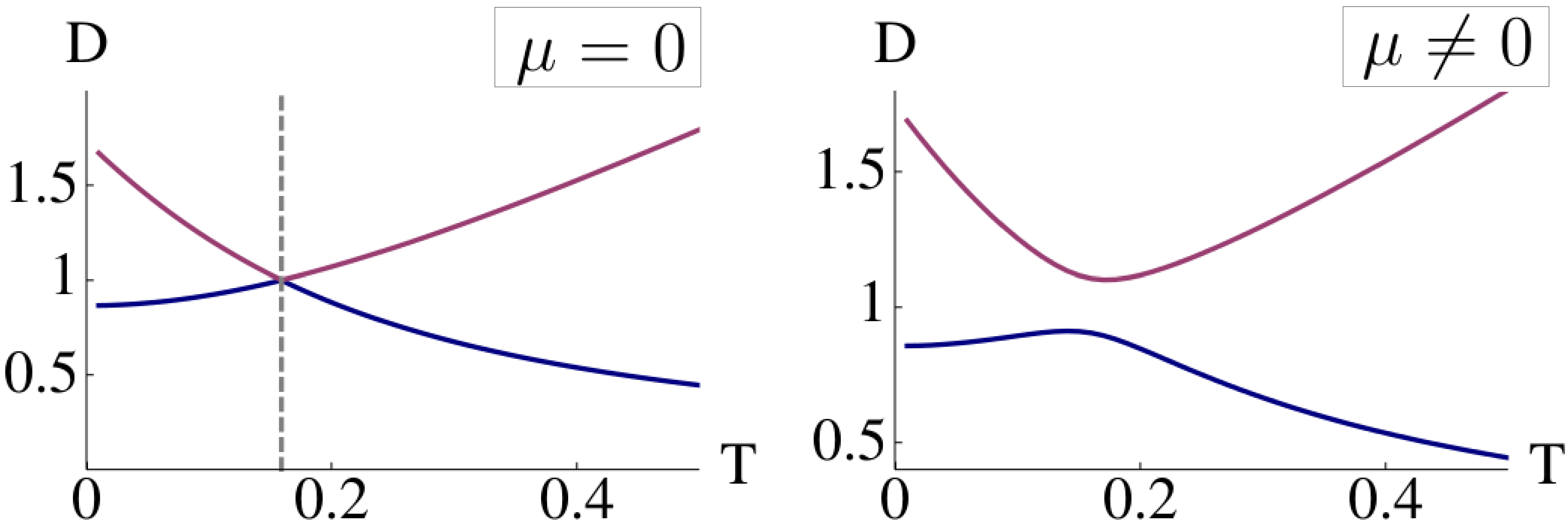
↓

We repeated the spectral analysis along the lines of

[Davison, Goutéraux, JHEP 1501 (2015) 039]

Undesirable because it contrasts the sought for robustness ...

Incoherence in holographic massive gravity: **diffusion** *standpoint*



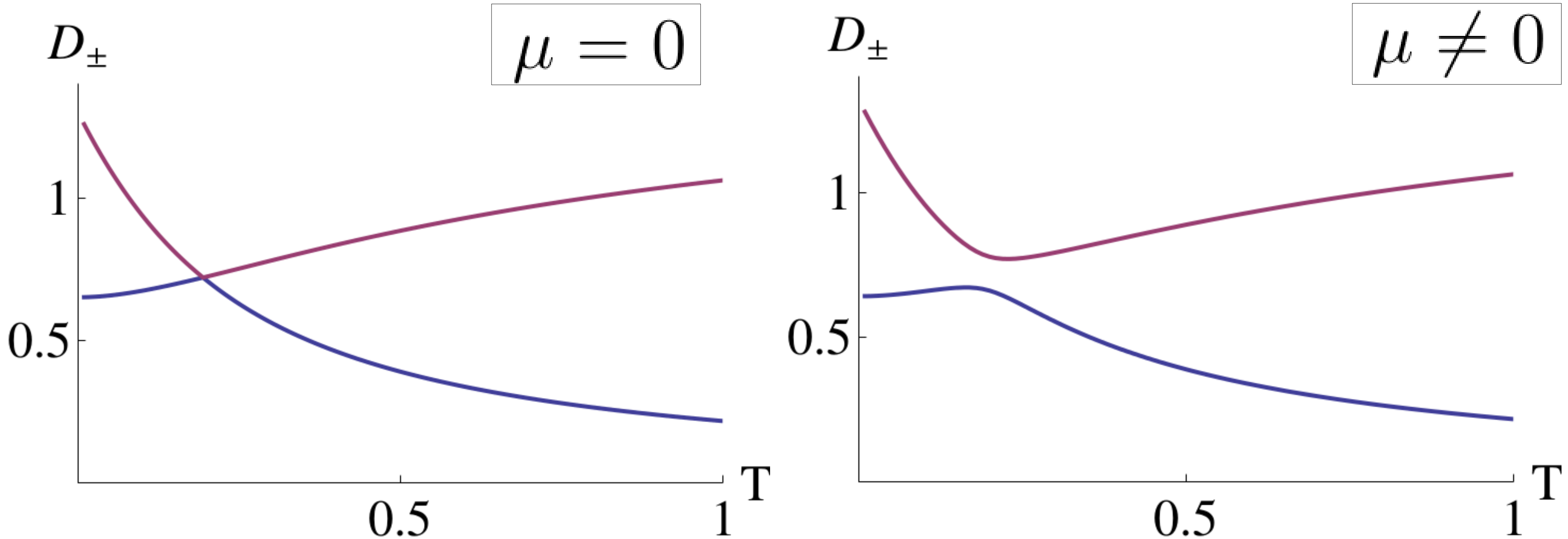
$\alpha = 0$

β fixed

Again, strict incoherence is a delicate circumstance. However...

...we possibly have direct and quantitative control of a genuinely incoherent state!

… with ***alpha*** as well



$$\alpha = -1$$
$$\beta = -1$$

- We notice the “regularizing” effect of ***alpha*** in the high-T region
- Diffusion constants at zero temperature are finite
- Impossibility of having incoherence *(at least in its strict version)* at finite chemical potential

Incoherence, a closer look

No scales other than T
(incoherence)

→ Featureless optical response

→ Equality of all diffusion const.

$$D_c = D_h$$

For any *fixed* T, constraint on the mass parameters:

$$12\beta = \alpha^2 - 8\pi T\alpha - 48\pi^2 T^2$$

In agreement with the enhanced symmetry point of Davison, Goutéraux, JHEP 1501 (2015) 039

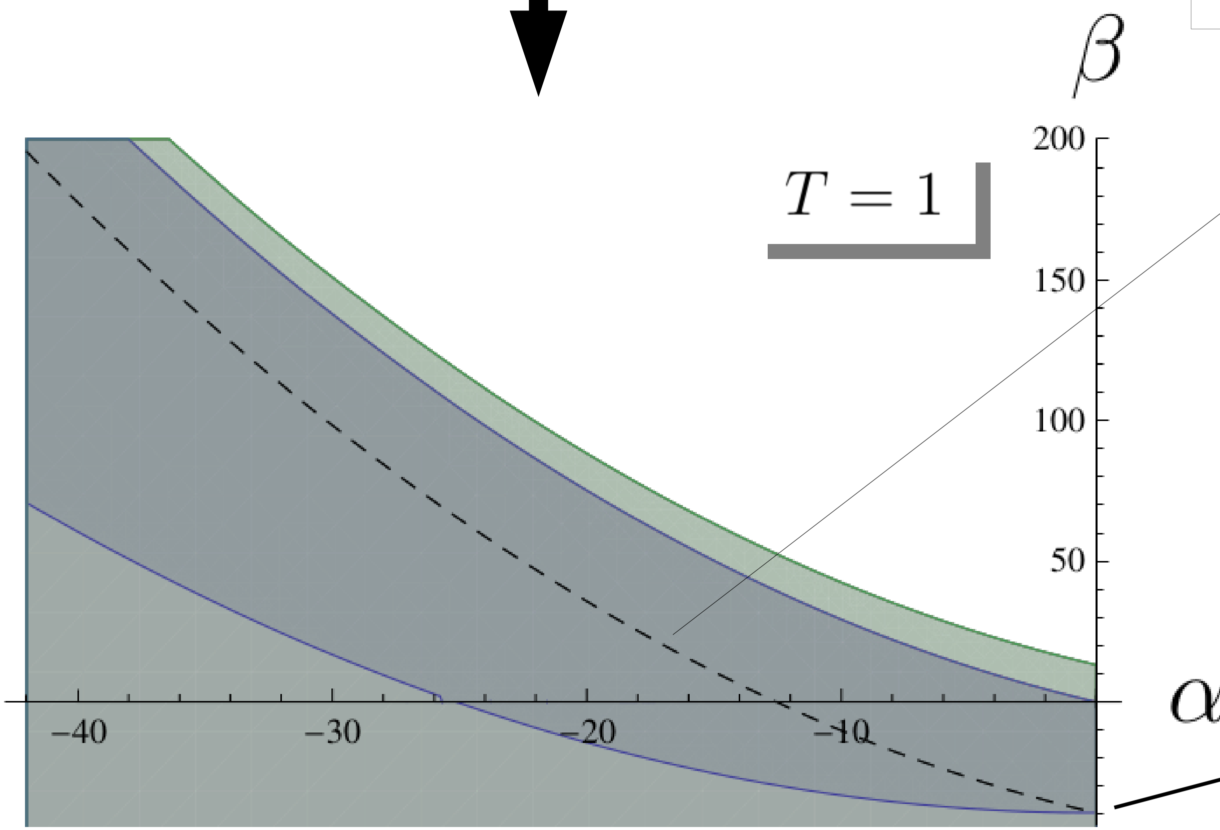
$$D_c = D_h = \frac{6}{12\pi T - \alpha}$$

Physical requirements can guide us in further constraining the mass parameters

Chasing bounds...

- **Reality** *and* **positivity** *of the momentum dissipation rate*
- **Positivity** *of black hole and graviton masses*

"incoherent line" $D_c = D_h$



No minimal "*physical*" value of the incoherent diffusivity?

Special points:

- ***alpha* = 0 ?**
- ***beta* = 0 ?**

The analysis is performed at fixed T but can be repeated for any value of T

Interesting questions

- Could the simplest *heat transport* provide a diffusion bound?

- Necessity of ***alpha*** for both UV physics and IR bounds?

- Relation between boundary and bulk physical soundness *(bottom-up cure for massive gravity pathologies...)*?

[Alberte, Khmelnitsky, Phys.Rev. D91 (2015) 4, 046006]

- Possibility of playing phenomenologically with the radial profile of the fiducial metric?

[Blake, Tong, Phys.Rev. D88 (2013) 10, 106004;
Marolf, Class.Quant.Grav. 31 (2014) 015008]

Future prospect

- Analysis in other models (e.g Q-lattices); different renormalization for the momentum dissipating device

[Donos, Gauntlett, JHEP 1404 (2014) 040]

- Seeking for **robust** incoherence and check the “goodness” of the attained (*even though delicate*) incoherence

- Diffusivity and magnetic field *(many good theoretical and experimental reasons to do that... see **Amoretti**'s talk)*

[Hayes, Breznay, Helm, Moll, cond-mat.str-el/1412.6484;
Amoretti, Musso, hep-th/1502.02631]

THANKS!

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