Testing the membrane paradigm holography

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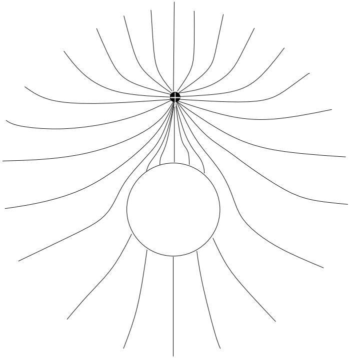
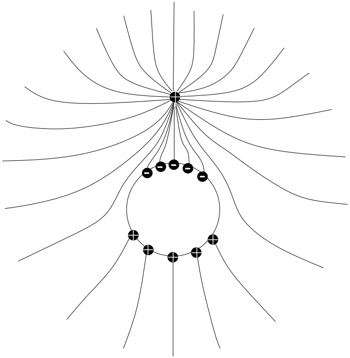
Outline

Membrane paradigm:

[Hanni; Ruffini; Znajek; Damour; Thorne; MacDonald and Price 80´s]

As a set of mental pictures to capture the physics of black holes from an external observer point of view





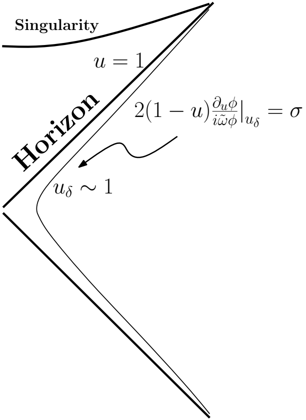
The membrane paradigm - Introduction

- The membrane is charged [Hanni and Ruffini 1973]
- The membrane carries an electric current [Znajek 1978; Damour 1978]
- The evolution of the membrane obeys Navier-Stokes equations with finite shear and bulk viscosity [Damour 1979, 1982]
- Move the membrane to a stretched horizon, a timelike hypersurface a small distance away from the horizon

  [Thorne and MacDonald 1982; Thorne and Price 1986]

The membrane paradigm - Modern definition

The necessary ingredient of the membrane paradigm is the ingoing behavior of the fields near the horizon



The membrane paradigm:

[Iqbal and Liu, 2008]

Replace the interior of a black hole with an **ingoing-like boundary condition** on the horizon/stretched horizon

$$2\,(1-u)\frac{\partial_u \phi}{i\,\tilde{\omega}\,\phi}\Big|_{u_\delta} = \sigma; \quad \text{with} \quad \sigma = \pm 1$$

Which is equivalent to require $C_{\text{out}} = 0$ in the near horizon expansion of the field, where $\tilde{\omega} = \omega/2\pi T$

$$\phi \sim C_{\text{out}}(1-u)^{\frac{i\,\tilde{\omega}}{2}} + C_{\text{in}}(1-u)^{-\frac{i\,\tilde{\omega}}{2}} + \ldots$$

The membrane paradigm - Motivation

What are the **limits of validity** of such approximation scheme?
Is the membrane supposed to live on the horizon or stretched horizon?

1. We provide a general argument

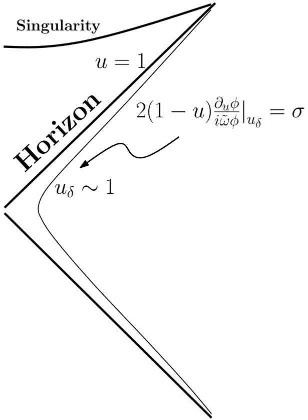
2. Massive QNMs are not captured if the membrane lives on the stretched horizon

3. Hydrodynamic QNMs are reproduced*

Outline

Limits of validity - 1. The general argument

Check that the membrane paradigm boundary condition does not spoil the good ingoing behavior at the horizon:
ingoing wave $\gg$ outgoing wave



For any nonextremal black hole the near horizon expansion of a scalar field:

$$\phi \sim C_{\mathsf{out}}\,(1-u)^{\frac{i\tilde{\omega}}{2}}\,(1+\alpha(1-u)+\dots) \\ +C_{\mathsf{in}}\,(1-u)^{-\frac{i\tilde{\omega}}{2}}\,(1+\beta(1-u)+\dots)$$

insert in the membrane boundary condition to get

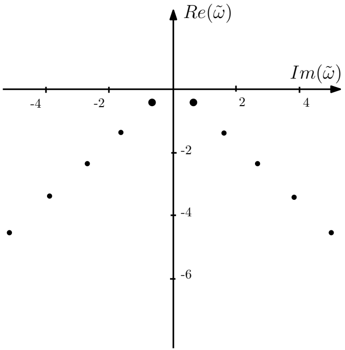
$$\frac{C_{\mathsf{out}}}{C_{\mathsf{in}}} = (1-u_\delta)^{1-i\tilde{\omega}}\ \frac{i\beta}{\tilde{\omega}} + \dots$$

Limits of validity - 1. The general argument

Limits of validity of the membrane paradigm:

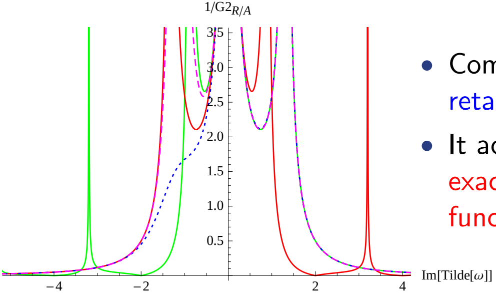
The membrane on a stretched horizon is only valid for

$$C_{\text{out}}/C_{\text{in}} \ll 1 \text{ when } u_\delta \to 1 \quad \Leftrightarrow \quad \mathcal{I}m(\tilde{\omega}) > -1$$



- **Hydrodynamic QNMs** are generically **reproduced***
- **Massive QNMs** are **not reproduced** except possibly for the lowest lying ones

Explicit example illustrating the validity of the argument:
$m = 0$ scalar field in BTZ_3 background $\Rightarrow$ use holography



- Compute the approximated retarded Green's function
- It actually approximates the exact advanced Green's function for $\mathcal{I}m(\tilde{\omega}) < -1$!

- No way to see the poles $\omega = -2\,i\,n$, $k = 0$, with $n = 1, 2, \ldots$ as from the exact retarded Green's function!

Limits of validity - 3. Ex. Hydro QNMs of gravit. pert. on AdS_5 black-brane

- $\mathcal{N}$=4 SYM at finite T

$$ds^2 = -\frac{(\pi T L)^2}{u}(-f(u)dt^2 + d\vec{x}^2) + \frac{L^2}{4u^2 f(u)}du^2, \quad f = 1 - u^2$$

- linearized perturbations (Sound channel):
  $\delta h_{tt},\ \delta h_{tx},\ \delta h_{\vec{x}\vec{x}},\ \delta h_{tu},\ \delta h_{xu},\ \delta h_{uu}$

- Compute the approximated retarded Green's function for the linearized gauge invariant variable

$$Z \sim k^2 \delta h_{tt} + \omega^2 \delta h_{xx} + \dots$$

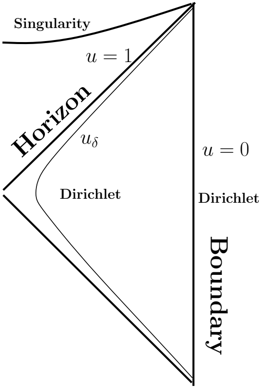
imposing the membrane paradigm boundary condition

$$2\,(1-u)\frac{\partial_u Z}{i\,\tilde{\omega}\,Z}\Big|_{u_\delta} = \sigma$$

Limits of validity - 3. Ex. Hydro QNMs of gravit. pert. on AdS_5 black-brane

- The poles are located in

$$\tilde{\omega}_1 = \pm\sqrt{\frac{2}{3}}\,\tilde{k} + \dots; \qquad \tilde{\omega}_2 = \pm\sqrt{\frac{1}{3}}\,\tilde{k} - \frac{i}{3}\sigma\,\tilde{k}^2 + \dots$$



- Obtain the same leading result from a double-Dirichlet problem in a hydro expansion! [See Jan de Boer's talk]

- Once a Dirichlet boundary condition is fixed on one boundary, the other boundary value must be shifted by Wilson line-like objects

$$\pi^t \sim \int_0^{u_\delta} \delta h_{tu}\, du; \quad \pi^x \sim \int_0^{u_\delta} \delta h_{xu}\, du$$

Limits of validity - 3. Ex. Hydro QNMs of gravit. pert. on AdS_5 black-brane

- Einstein constraint equations on $u_\delta \sim 1$

$$\left(\tilde{\omega}^2 - \frac{1}{3}\tilde{k}^2\right)\pi^x - \frac{1}{2}\tilde{\omega}\,\tilde{k}(1-u_\delta^2)\,\pi^t = 0,$$
$$\sqrt{(1-u_\delta^2)}\left(\tilde{\omega}^2 - \frac{2}{3}\tilde{k}^2\right)\pi^t + \frac{1}{3}\tilde{\omega}\,\tilde{k}\sqrt{(1-u_\delta^2)}\,\pi^x = 0,$$

- when $u_\delta \to 1$ only π^x survives

$$\left(\tilde{\omega}^2 - \frac{1}{3}\tilde{k}^2\right)\pi^x = 0 \quad \Rightarrow \quad \tilde{\omega} = \pm\frac{1}{\sqrt{3}}\,\tilde{k}$$

- Natural interpretation of π^x as the Goldstone mode corresponding to Sound mode excitations!

- With vanishing Dirichlet boundary conditions π^t decouples on the event horizon. Does not correspond to a hydro mode!

Outline

Take home messages

What are the **limits of validity** of the membrane paradigm? Is the membrane supposed to live on the horizon or stretched horizon?

- **1.** We provided a **general argument** showing that the membrane on the stretched horizon is incomplete
- **2.** For **massive QNMs** near horizon details matter and the membrane should be thought of as living on the horizon
- **3.** **Hydrodynamic QNMs** are correctly **reproduced** if one takes good care of the additional timelike Goldstone

Thank you!