Holographic magneto-transport and strange metals

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Based on:

"Universal formulae for thermoelectric transport with magnetic field and disorder", arXiv:1502.02631,

with Daniele Musso.

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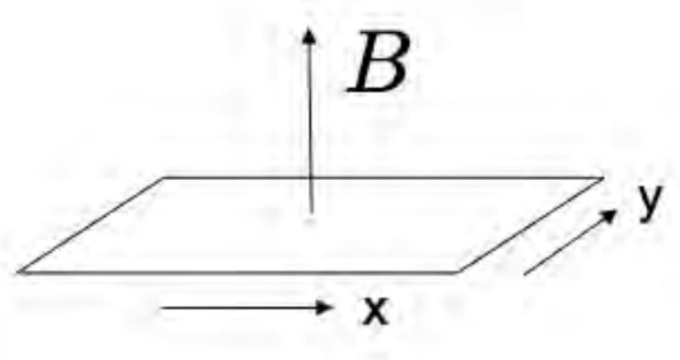
Study the thermo-electric transport properties of a strongly correlated (2+1)-D system immersed in an external magnetic field perpendicular to the plane



Introduction

- Response to an external electric field E_i and thermal gradient $\nabla_i T$

$$\begin{pmatrix} J_i \\ Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & \alpha_{ij} \\ T\alpha_{ij} & \bar{\kappa}_{ij} \end{pmatrix} \begin{pmatrix} E_j \\ -\nabla_j T \end{pmatrix}$$

- transport coefficients are now matrices

$$\sigma_{xx} = \sigma_{yy} \ , \qquad \sigma_{xy} = \sigma_{yx}$$

- There are six independent transport coefficients

Motivations

- Measurements on strange metals are commonly performed at non-zero magnetic field to suppress T_c (and phonons)
- Almost all the transport properties deviate from the Fermi liquid behaviour

	Fermi Liquid	Strange Metals
ρ	T^2	T e.g. Hussey review, '08
$s \equiv \frac{\alpha_{xy}}{\alpha_{xx}}$	T	$s \sim A - BT$ Orbetelli et al. '92
$\tan\theta_H \equiv \frac{\sigma_{xy}}{\sigma_{xx}}$	$\frac{1}{T^2}$	$\frac{1}{T^2}$ e.g. Hussey review, '08
Kohler's rule	$\frac{\Delta\rho}{\rho} \sim \frac{B^2}{\rho^2}$	$\frac{\Delta\rho}{\rho} \sim \tan^2\theta_H$ Harris '92

Motivations

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What can be said in the holographic framework?

see also Blake & Donos '14 Hartnoll & Karch '15 Blake, Donos & Lohitsiri '15

Outline

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Momentum dissipation in holography

1. Inhomogeneous lattices: Horowitz, Santos & Tong '12...
2. Breaking translations to a helical Bianchi VII subgroup Donos & Gauntlett '12...
3. Random-field disorder Hartnoll & Herzog '08...
4. Breaking diffeomorphism in the bulk: Q-Lattices, axions and massive gravity Donos & Gauntlett '13, Vegh '13, Andrade & Withers '13...

We use massive gravity

- simple to solve
- we can obtain general physical statements

Massive gravity and momentum dissipation

- Breaking diffeomorphisms in the bulk by adding a mass term for the graviton

$$S = \int d^4x\sqrt{-g}\left[R - \Lambda - \frac{1}{4}F^2 + \beta\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)\right]$$

where ${\mathcal{K}^2}_\mu^{\ \nu} \equiv f_{\mu\rho}g^{\rho\nu}$, $\mathcal{K} \equiv \sqrt{\mathcal{K}^2}$

- the fixed metric $f_{\mu\nu}$ controls how diffeomorphisms are broken
- Holographic dictionary $\Rightarrow \quad \partial_\mu T^{\mu\nu} \neq 0$
- we want to dissipate momentum but to conserve energy (elastic processes)

$$f_{xx} = f_{yy} = 1\ , \ \text{and zero otherwise}$$

$B = 0$ and momentum dissipation

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- In the hydrodynamic regime ($|\beta| \ll T^2$) a dissipation rate τ^{-1} can be defined Davison, '13

$$\partial_t T^{tt} = 0, \quad \partial_t T^{ti} = \tau^{-1} T^{ti}$$
$$\tau^{-1} \equiv -\frac{\mathcal{S}\beta}{2\pi(\mathcal{E} + P)}$$

- At sufficiently low $|\beta|$ there is a Drude peak in the electric conductivity $\sigma(\omega)$ Vegh, '13

$B = 0$ and momentum dissipation

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- The DC electric conductivity σ_{DC} splits into two parts Blake & Tong, '13

$$\sigma_{DC} = \sigma_{\mathsf{ccs}} + \frac{\rho^2 \tau}{\mathcal{E} + P}$$

- The thermal $\bar{\kappa}_{DC}$ and thermoelectric α_{DC} DC conductivity are affected only by the Drude part A.A. et al., '14

$$\alpha_{DC} = \frac{\mathcal{S} \rho \tau}{\mathcal{E} + P} \qquad \bar{\kappa}_{DC} = \frac{\mathcal{S}^2 T \tau}{\mathcal{E} + P}$$

Switch on B

- modify the gauge field A in order to introduce a magnetic field perpendicular to the xy plane

$$A = (\mu - \rho z)\, dt + Bx\, dy$$

- a background black-brane solution can be found and consequently the thermodynamics can be defined in terms of the horizon radius z_h ($g_{tt}(z_h) = 0$):

$$T = -\frac{z_h^2 \left(B^2 z_h^2 + \mu^2\right) - 2\left(\beta z_h^2 + 3\right)}{8\pi z_h}\,, \qquad \mathcal{S} = \frac{2\pi}{z_h^2}$$

$$\rho = \frac{\mu}{z_h}\,, \qquad \mathcal{E} + P = T\mathcal{S} + \mu\rho$$

DC thermo-electric response

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- in a system with a $U(1)$ gauge field A and a killing vector ∂_t you can define two radially conserved quantities (independent on the radial AdS coordinate z) Donos & Gauntlett, '14

- concerning the DC response, these two quantities can be identified with the electric current J^i and the heat current $Q^i \equiv T^{ti} - \mu J^i$ at the conformal boundary $z = 0$

- due to their radial independence we can express these quantities in terms of horizon data (thermodynamics)

DC thermo-electric response

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- having $J^i(z_h)$ and $Q^i(z_h)$ we can compute the DC transport

$$\begin{pmatrix} J_i \\ Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & \alpha_{ij} \\ T\alpha_{ij} & \bar{\kappa}_{ij} \end{pmatrix} \begin{pmatrix} E_j \\ -\nabla_j T \end{pmatrix}$$

four quantities determine the six transport coefficients

$$\sigma_{\text{ccs}}\ , \qquad \rho \qquad \frac{\tau}{\mathcal{E}+P}\ , \qquad \mathcal{S}$$

$$\sigma_{xx} = \frac{\mathcal{E}+P}{\tau}\,\frac{\rho^2 + \sigma_{\text{ccs}}\left(B^2\sigma_{\text{ccs}} + \frac{\mathcal{E}+P}{\tau}\right)}{B^2\rho^2 + \left(B^2\sigma_{\text{ccs}} + \frac{\mathcal{E}+P}{\tau}\right)^2}$$

$$\sigma_{xy} = \rho B\,\frac{\rho^2 + \sigma_{\text{ccs}}\left(B^2\sigma_{\text{ccs}} + 2\,\frac{\mathcal{E}+P}{\tau}\right)}{B^2\rho^2 + \left(B^2\sigma_{\text{ccs}} + \frac{\mathcal{E}+P}{\tau}\right)^2}$$

$$\alpha_{xx} = \rho\,\mathcal{S}\,\frac{\mathcal{E}+P}{\tau}\,\frac{1}{B^2\rho^2 + \left(B^2\sigma_{\text{ccs}} + \frac{\mathcal{E}+P}{\tau}\right)^2}$$

$$\alpha_{xy} = \mathcal{S}B\,\frac{\rho^2 + \sigma_{\text{ccs}}\left(B^2\sigma_{\text{ccs}} + \frac{\mathcal{E}+P}{\tau}\right)}{B^2\rho^2 + \left(B^2\sigma_{\text{ccs}} + \frac{\mathcal{E}+P}{\tau}\right)^2}$$

$$\bar{\kappa}_{xx} = \frac{\mathcal{S}^2 T\left(B^2\sigma_{\text{ccs}} + \frac{\mathcal{E}+P}{\tau}\right)}{B^2\rho^2 + \left(B^2\sigma_{\text{ccs}} + \frac{\mathcal{E}+P}{\tau}\right)^2}$$

$$\bar{\kappa}_{xy} = \frac{B\rho\,\mathcal{S}^2 T}{B^2\rho^2 + \left(B^2\sigma_{\text{ccs}} + \frac{\mathcal{E}+P}{\tau}\right)^2}$$

DC thermo-electric response

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- The dissipation rate τ^{-1} can be rigorously defined only in the hydro regime [Davison & Gouteraux, '15] ($\beta \ll T^2$, $B \ll \rho^2$, $\rho \ll T^2$) and has the same form as in the $B = 0$ case

$$\tau = -\frac{\mathcal{S}\beta}{2\pi(\mathcal{E} + P)}$$

- The transport coefficients are compatible with Q-lattices Blake, Donos & Lohitsiri '15
- With some assumptions the transport coefficients can be obtained from the memory matrix formalism Lucas & Sachdev, '15

Holography and hydrodynamic

- Contrary to σ_{ij}, α_{ij} and $\bar{\kappa}_{ij}$ are not equivalent to the hydro analysis of Hartnoll et al. '07
- Exact self duality: $\rho \leftrightarrow B$, $\sigma_{\text{ccs}} \leftrightarrow 1/\sigma_{\text{ccs}}$

$$\sigma_{xx},\ \sigma_{xy},\ \alpha_{xx},\ \alpha_{xy},\ \overline{\kappa}_{xx},\ \overline{\kappa}_{xy}$$
$$\updownarrow$$
$$\rho_{xx}, -\rho_{xy}, -\vartheta_{xy}, -\vartheta_{xx},\ \kappa_{xx}, -\kappa_{xy}$$

where $\hat{\rho} = \hat{\sigma}^{-1}$ is the resistivity matrix, $\hat{\theta} \equiv -\hat{\rho} \cdot \hat{\alpha}$ is the Nernst coefficient matrix and $\hat{\kappa} = \hat{\bar{\kappa}} - T\hat{\alpha} \cdot \hat{\rho} \cdot \hat{\alpha}$

Holographic strange metals phenomenology

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Phenomenological temperature scalings in strange metals

We need 4 phenomenological inputs to predict the scalings of all the 6 transport coefficients

- Blake & Donos, '14:

$$\sigma_{\text{ccs}} \sim \frac{\sigma^0_{\text{ccs}}}{T}, \qquad \sigma_D \equiv \frac{\rho^2 \tau}{\mathcal{E}+P} \sim \frac{\sigma^0_D}{T^2}$$

and $\sigma^0_D \ll \sigma^0_{\text{ccs}}$, reproduces the correct scaling for the resistivity and the hall angle:

$$\rho_{xx} \sim T \;, \qquad \tan\theta_H \equiv \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$$

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What about the other transport coefficients?

Proposal

$$\sigma_{\text{ccs}} \sim \frac{\sigma^0_{\text{ccs}}}{T}, \quad \sigma_D \sim \frac{\sigma^0_D}{T^2}, \quad \rho \sim \rho_0, \quad \sigma^0_D \ll \sigma^0_{\text{ccs}}, \quad \mathcal{S} \sim \mathcal{S}_0 T^\delta$$

To fix the scaling exponent δ we need phenomenological inputs which are free form spurious interactions (phonons effects): transverse conductivities do the game!

- $\kappa_{xy} \sim \frac{1}{T}$ Zhang et al., '00, Matusiak et al., '09

$$\Rightarrow \mathcal{S} \sim \mathcal{S}_0 T, \text{ and } L_{xy} \equiv \frac{\sigma_{xy}}{T \kappa_{xy}} \sim T$$

in accordance with calorimetric measurements Loram et al.,

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Magneto-resistance

$$\frac{\Delta\rho}{\rho} \equiv\sim \sigma^0_{\text{ccs}}\,\sigma^0_D \left(\frac{B}{\rho_0}\right)^2 \left(\frac{\sqrt{\rho_0}}{T}\right)^3 - 2\sigma^{0\,2}_D \left(\frac{B}{\rho_0}\right)^2 \left(\frac{\sqrt{\rho_0}}{T}\right)^4$$

Experiments: T^{-n} with $n \sim 3.5 - 3.9$ Harris '92

Seebeck coefficient

$$s \equiv \frac{\alpha_{xy}}{\alpha_{xx}} \sim \frac{\mathcal{S}_0\,\sigma^0_D}{\rho_0\,\sigma^0_{\text{ccs}}} - \frac{\mathcal{S}_0\sigma^{0\,2}_D}{\rho_0\sigma^{0\,2}_{\text{ccs}}}\,\frac{\sqrt{\rho_0}}{T}$$

Experiments: $A - BT$ Orbetelli et al., '92

Possible $1/T$ correction at high-T? Kim et al., '04 What about phonon drag?

Conclusions

- The six thermoelectric transport coefficients are functions of four quantities: possibility to be predictive!
- Discrepancies between holography, hydrodynamic and memory matrix: can self-duality do the game?
- At finite density thermodynamics and transport are intimately related
- To get phenomenological insight we need data clean from spurious effects: working directly with experimentalists!
- Does the magnetic field play a role in criticality?

Thank you