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Scaling Holographic AC conductivities

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Bibliography

Ongoing work with

Jie Ren (Crete), Fransisco Peña-Benitez, (Crete)

and published work:

J. Ren (Crete), E. Kiritsis (Crete) arXiv: 1503.03481[hep-th]

A. Donos (Durham), B. Gouteraux (Stanford) and E. Kiritsis (Crete) arXiv: 1406.6351[hep-th]

B. S. Kim and C. Panagopoulos (Crete) arXiv:1012.3464 [cond-mat.str-el]

C. Charmousis, B. Gouteraux (Orsay), B. S. Kim and R. Meyer (Crete)
. arXiv:1005.4690 [hep-th]

Introduction

• Conductivity is one of the most important observables in condensed matter systems

• It is relatively easy to define and measure.

• It tells us a lot about the dynamics of charge carriers in a medium

• For low enough voltages it is controlled by the retarded current two-point function.

Strongly correlated electrons

• In metals, although bare electrons are strongly coupled, there are "dressed" (fermionic) quasiparticles that are weakly-coupled and behave as almost free electrons. They are responsible for transporting charge.

• In most materials that are on the border with magnetism (like the cuprates) there are no weakly-coupled quasiparticles.

• Such systems are Mott insulators in some part of the their phase diagram.

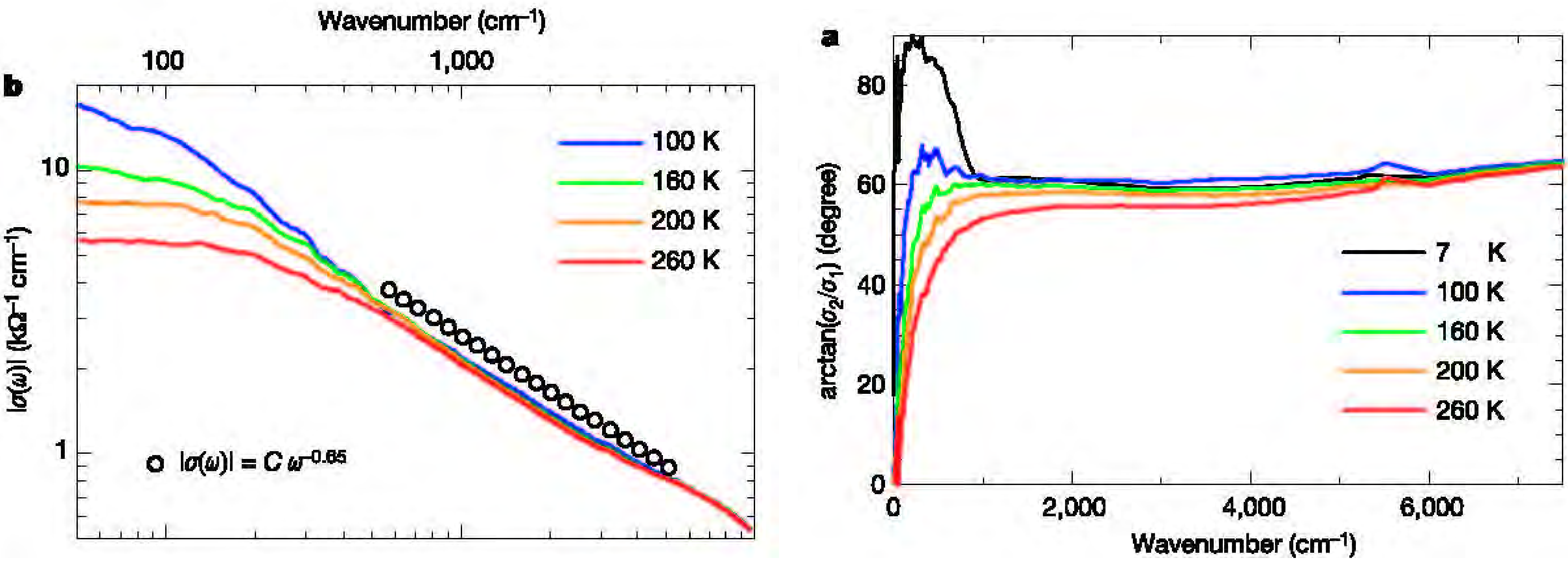
• They have a benchmark linear conductivity in the area above their superconducting dome.

• In some of them the AC Conductivity shows a very simple scaling law,

$$\sigma(\omega) \quad \sim \quad \omega^{-\frac{2}{3}}$$

Van der Marel et al.

Scaling in AC conductivity



Van Der Marel et al.

Insulators

• There are several mechanisms for insulating behavior in condensed matter:

1) Band gap insulators, where the conduction band is empty, and there is therefore a gap that prevents current transport.

2) Anderson localization that is effective in two dimensions and where strong disorder inhibits conduction.

3) Mott localization, where strong onsite interactions localize electrons.

4) A new mechanism at strong coupling: Momentum-dissipating interactions become relevant (strong) in the IR, and they inhibit conduction

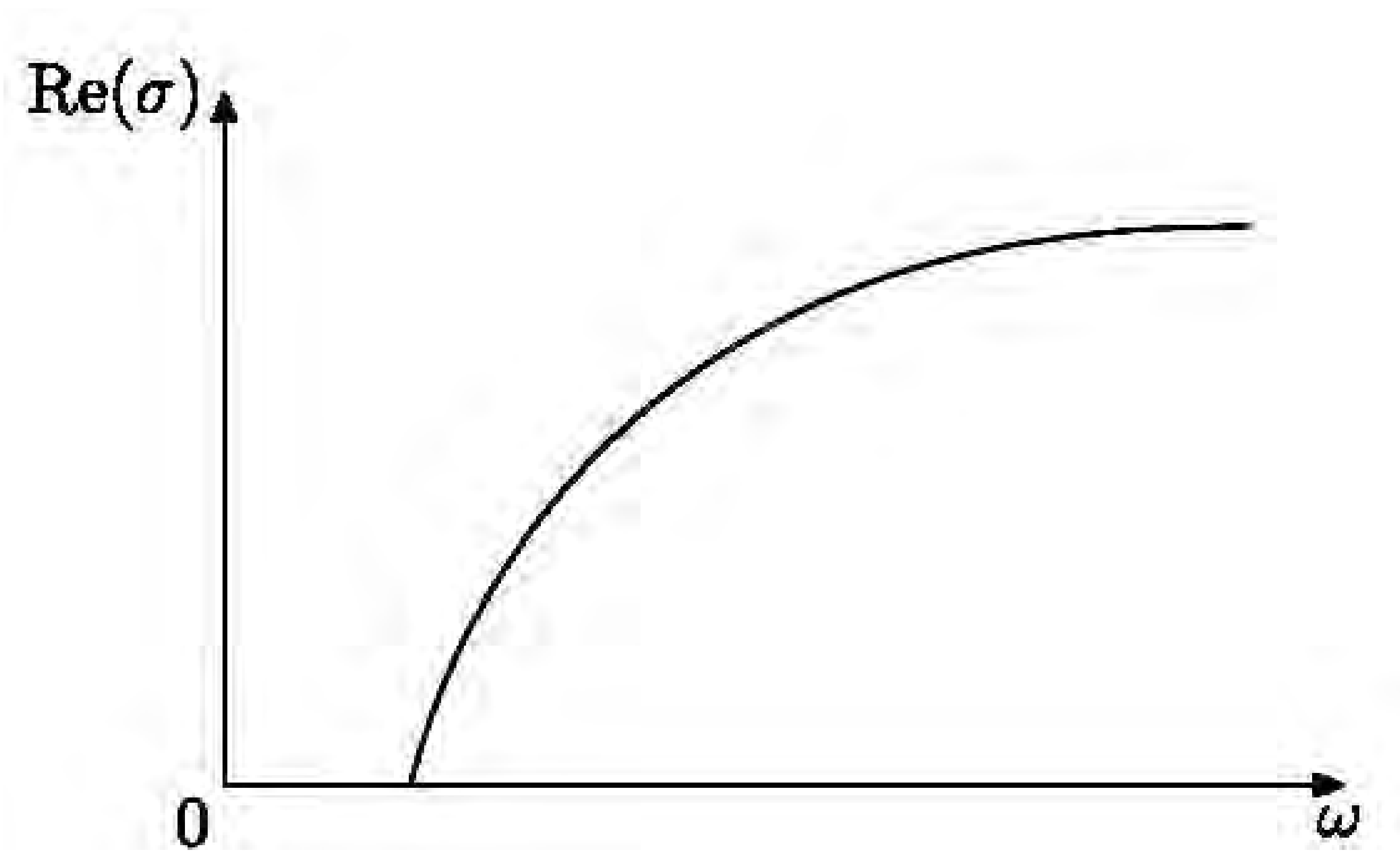
Donos+Hartnoll

Generalized to arbitrary EMD theories

Donos+Gouteraux+Kiritsis

5) A hybrid of (1)+(3) in holographic systems.

Jie Ren et al.


Re(σ)
0
ω

Supersolids

• A supersolid is a generalization of a superfluid state. It is characterized by a spontaneously broken U(1) symmetry which guarantees a superfluid component (a zero frequency δ-function).

• It has (spontaneously) broken translational invariance.

• Therefore the appropriate two-point function has a discretely localized spectral density.

• If probed at a generic non-zero frequency it is non-responsive. If it is probed at zero frequency it behaves as a superfluid.

• They have been theoretically anticipated and studied, especially in the last 2-3 years.

Legget, Fisher+Nelson, Anderson, Nicolis+Penco+Rosen

• There are proposed realizations with cold atoms.

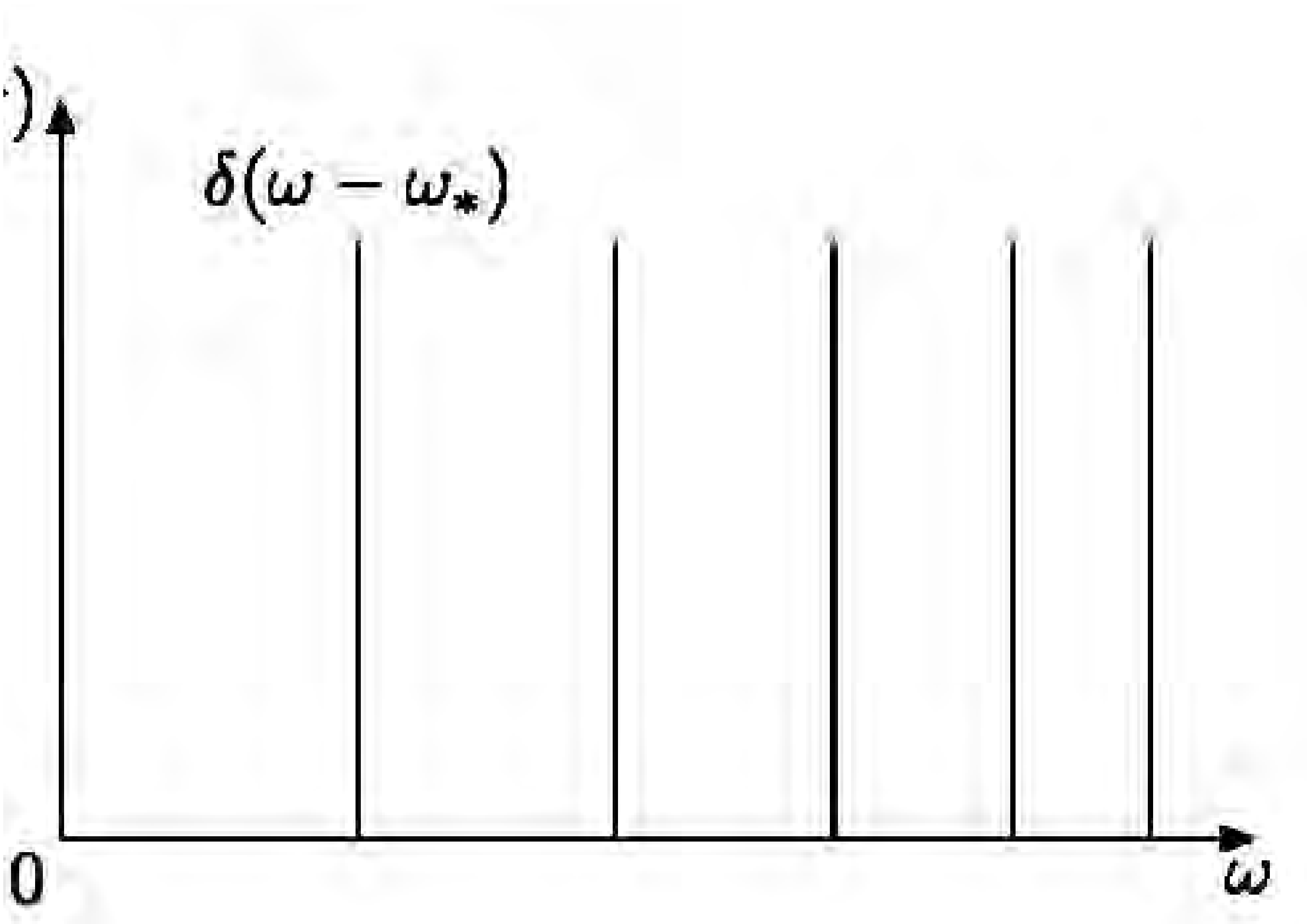
Keilmann+Cirac+Roscilde

• There have been claims for presence in solid Helium^4 as well as a recent refutation.

Kim+Chan

• They can be realized holographically

Jie Ren et al.



$\delta(\omega - \omega_*)$
0
ω

The Plan of the rest

- Review of conductivity in Holographic Theories

- AC conductivity and scaling in a holographic non-fermi liquid

- Generic scaling of the holographic AC conductivity

The Wilsonian setup

- Holographic theories are generically RG flows between fixed points.

- The first step: Classification of Scale Invariant/Fixed-point theories (The Wilsonian approach in holography).

- The strategy is to use Effective Holographic Theories (in order to explore all possible QC holographic scale invariant theories with given symmetries.

Charmousis+Gouteraux+Kim+Kiritsis+Meyer (2010)

- Once fixed points are classified for a given bulk action, the conductivity can be studied first "locally" (in the IR fixed point) and then globally (along the whole flow).

Generic Scaling Geometries: A classification of all QC points in holography

Charmousis+Gouteraux+Kim+Kiritsis+Meyer (2010), Gouteraux+Kiritsis (2011), Huisje+Sachdev+Swingle, (2011)

• Assume translation and rotational invariance in space and time.

• The QC points generically break hyperscaling invariance and are characterized by several exponents. Two appear in the metric (z, θ).

• $z \to$ dynamical exponent

• $\theta \to$ hyperscaling violation exponent

$$ds^2 = r^{\frac{2\theta}{d}}\left[\frac{dt^2}{r^{2z}} + \frac{dr^2 + dx_1^2 + dx_2^2 + \cdots + dx_d^2}{r^2}\right]$$

• There is invariance under:

$$x_i \to \lambda\ x_i \quad , \quad t \to \lambda^z\ t \quad , \quad r \to \lambda\ r \quad , \quad ds \to \lambda^{\frac{\theta}{d}}\ ds$$

• The entropy scales as

$$S \sim T^{\frac{d-\theta}{z}}$$

which gives an interpretation to the hyperscaling violation exponent.

• There is a third exponent, associated with the charge density, the conduction exponent ζ:

Gouteraux+Kiritsis, Gouteraux

$$A_t = Q\ r^{\zeta-z}$$

It is also a hyperscaling violation exponent:hyperscaling is valid iff both $\theta = 0$ AND $\zeta = 0$.

Conductivity:basics

• It can be calculated in the linear regime from the correlators of the currents

$$\sigma_{ij}(\omega,\vec{k}) = \frac{1}{i\omega}\int d^p x dt\ e^{-i\omega t - i\vec{k}\cdot\vec{x}}\langle J_i(t,\vec{x})J_j(0,0)\rangle$$

or more generally as the (non-linear) response to an external electric field

$$J_i = \sigma_{ij}\ E_j$$

• Translational invariance and finite density imply a pole at zero frequency for the conductivity.

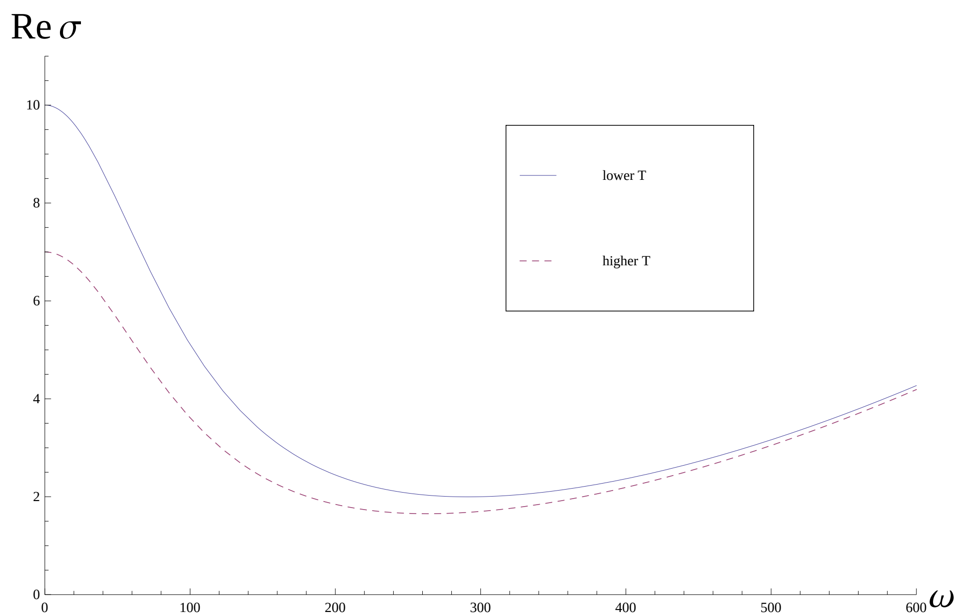
$$\sigma(\omega) \simeq K\left(\delta(\omega) + \frac{i}{\omega}\right) + \cdots \quad , \quad K = \frac{4\rho^2}{\varepsilon + p}$$

• K is the Drude weight.

• Weak scattering over ions or impurities, resolves the zero frequency pole

$$\sigma(\omega) \simeq \frac{K\tau}{1 - i\omega\tau} + \cdots \quad , \quad \frac{1}{\tau} = \text{scattering rate}$$

• This is the so-called Drude peak and defines the response of a metal at low frequencies.



• As $T \to 0$, $\tau \to \infty$. In this limit $\tau \to \infty$ we obtain back the zero frequency pole and δ-function.

General features of holographic conductivities

- When the bulk action of the U(1) gauge field is linear in F^2

$$S_F \sim \int d^4x \sqrt{g}\ Z(\phi)\ F^2$$

the DC conductivity has the schematic form

$$\sigma_{DC} \sim \sigma_{pair} + \sigma_{drag}$$

Blake+Tong, Donos+Gauntlett

- The first term $\sigma_{DC} \sim Z$ was interpreted as a term coming from the pair-production of charge.

Karch+O'Bannon

- It is non-zero even when $Q = 0$.

- It does not contribute to thermal transport.

Donos+Gauntlett

• It is finite, even when there is no momentum dissipation (does not contribute to the Drude weight).

• $\sigma_{drag} \sim \tau$ is due to momentum-dissipating interactions.

• If more than one mechanisms of momentum dissipation are at work,

$$\sigma_{drag} = \sum_I \sigma^I_{drag} \quad , \quad \tau = \sum_I \tau_I$$

(inverse Mathiessen law)

Donos+Gouteraux+Kiritsis

• In the limit of vanishing dissipation, $\tau \to \infty$, it generates the Drude δ-function.

• When charge dynamics is described by the DBI action then

$$\sigma_{DC} = \sqrt{\sigma_{pp}^2 + \sigma_{drag}^2}$$

The AC conductivity in a holographic strange metal

Kim+Kiritsis+Panagopoulos

- The bulk is the AdS Schwarzschild black hole (in light-cone coordinates).

- The charge is described by a standard DBI action coupled to the metric

$$S_{DBI} = \mathcal{N} \int d^5x \sqrt{\det(g+F)}$$

- The ansatz for the ground state is

$$A = (Ey + h_+(u))dx^+ + (b^2Ey + h_-(u))dx^- + (b^2Ex^- + h_y(u))dy,$$

- It is a stationary solution in lightcone coordinates with a nontrivial charge density and a light-cone electric field $F_{+y} = E$

- The DBI equations can be solved exactly and the conductivity computed à la Karch-O'Bannon.

The DC conductivity

• The parameters are E, J^+, T. E is similar to the doping parameter of cuprates.

• They can be combined in one scaling variable t and parameter J.

$$t = \frac{\pi \ell T}{2\sqrt{E}} \quad , \quad J^2 = \frac{J_+^2}{(2\mathcal{N})^2 \sqrt{2} (E)^3} ,$$

and the DC conductivity becomes

$$\sigma_{DC} = \sigma_0 \sqrt{\sigma_{DR}^2 + \sigma_{PP}^2} ,$$

$$\sigma_{DR}^2 = \frac{J^2}{t^2 A(t)} \quad , \quad A(t) = t^2 + \sqrt{1 + t^4} \quad , \quad \sigma_{PP}^2 = \frac{t^3}{\sqrt{A(t)}} .$$

• We will also define the ratio

$$q \equiv \frac{\sigma_{DR}^2}{\sigma_{PP}^2}$$

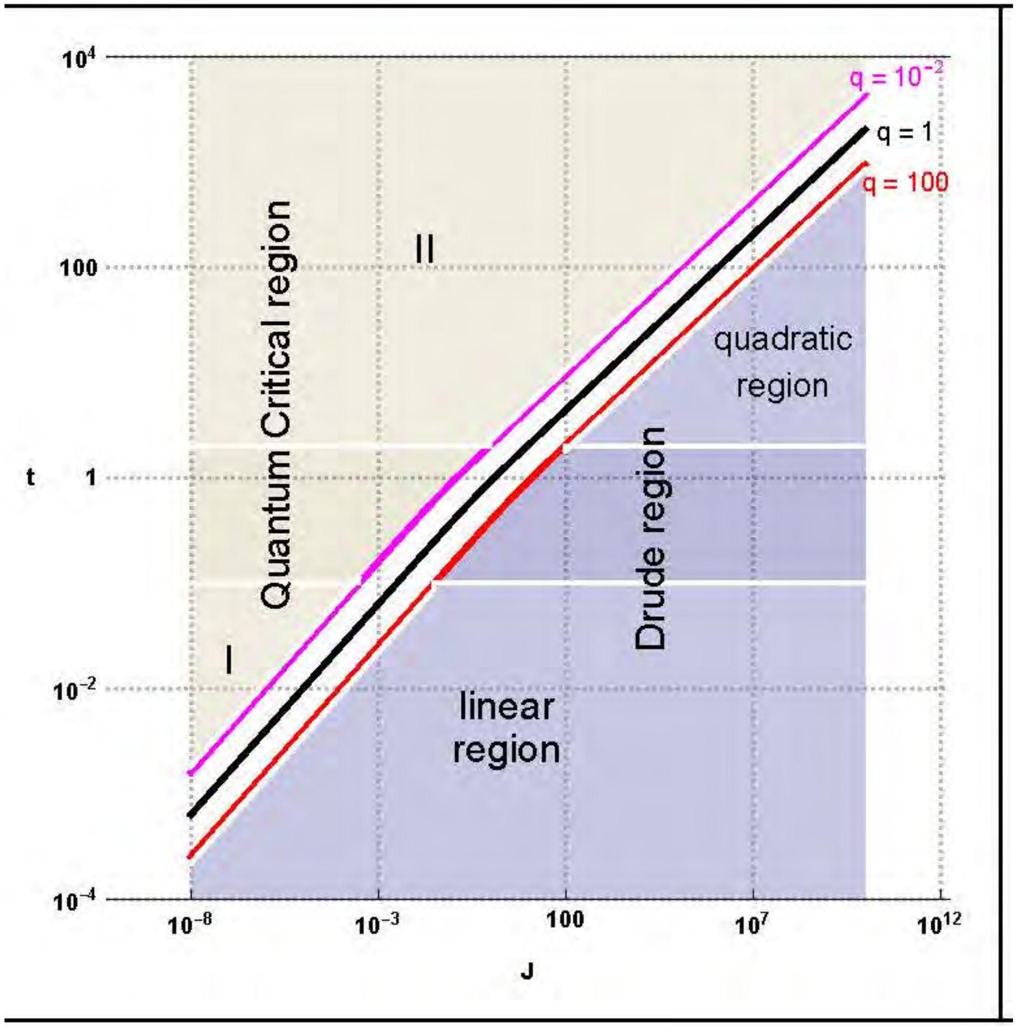
that determines in which regime we are: $q \gg 1$ is the Drude regime, $q \ll 1$ is the pair-production regime.

- The conductivity is as follows:

$$\rho = \begin{cases} (\sigma_0 J)^{-1}\ t & q \gg 1, t \ll 1 \quad \text{linear} \\ \sqrt{2}\sigma_0^{-1} J\ t^2 & q \gg 1, t \gg 1 \quad \text{quadratic} \\ \sigma_0^{-1}\ t^{-3/2} & q \ll 1, t \ll 1 \quad \text{regime I} \\ \sqrt{2}\sigma_0^{-1}\ t^{-1/2} & q \ll 1, t \gg 1 \quad \text{regime II} \end{cases}$$

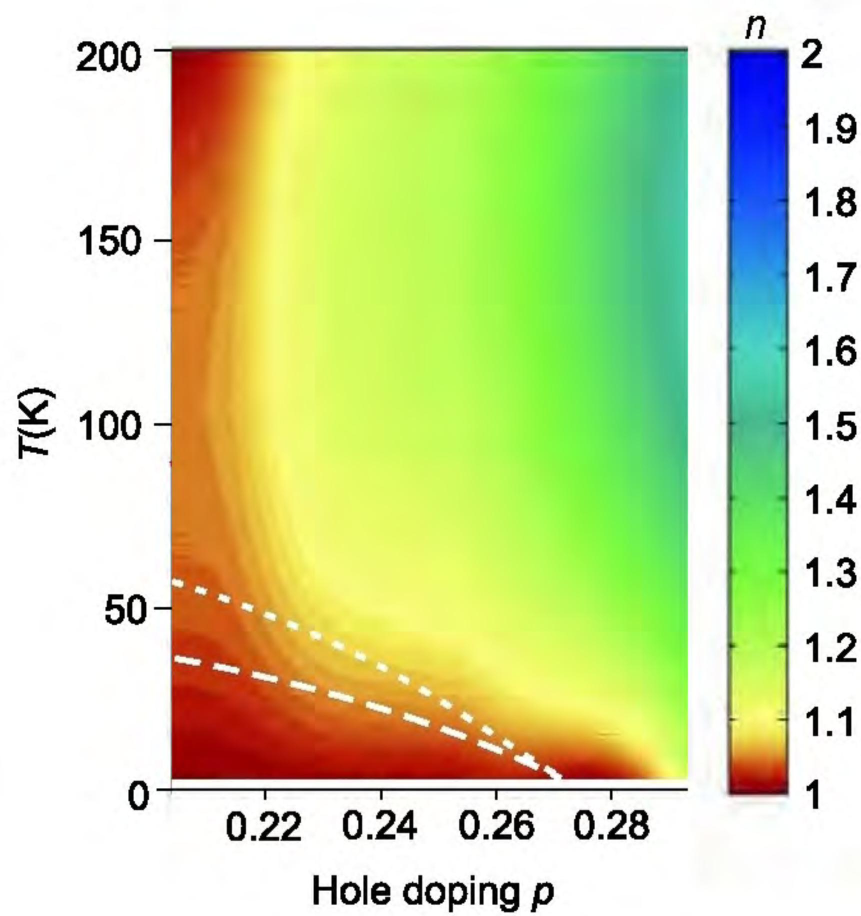
Drude regime (DR) (linear and quadratic)

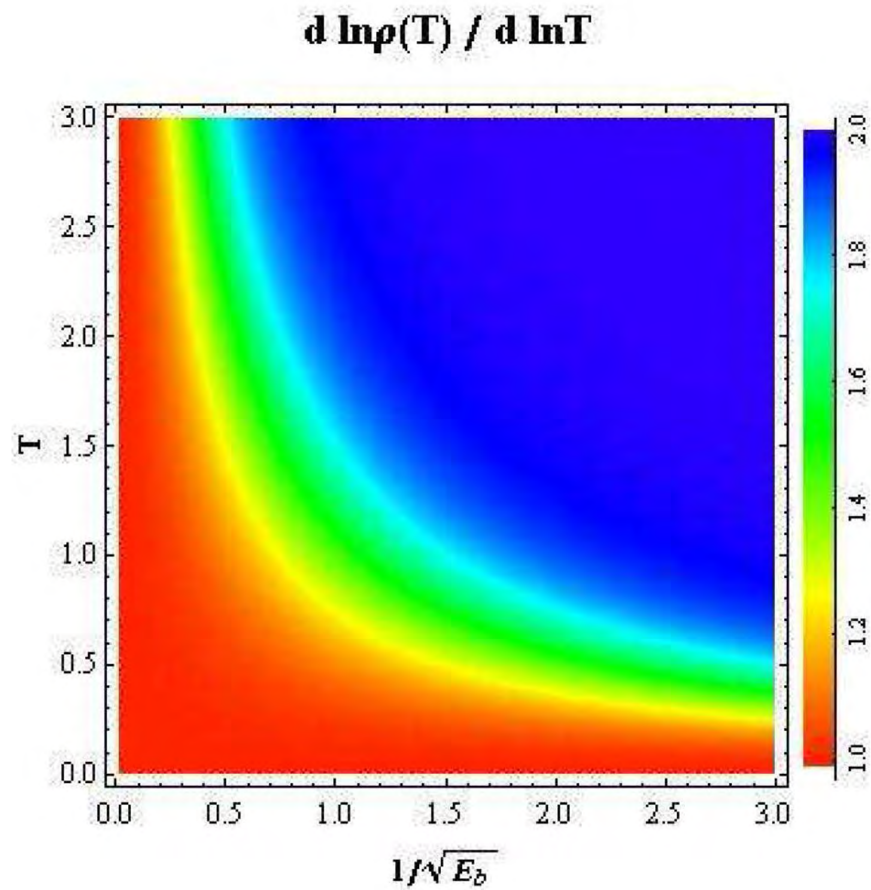
Pair-Production regime (regime I and regime II)



Center: Location of the four regimes in the space of parameters (J, t) log-log scale. The black line $q = 1$, separates the DR respect to the QC regime. The magenta line represents the region with $q = 10^{-2}$ and the red one $q = 100$.

$La_{2-x}Sr_xCuO_4$





Kim+Kiritsis+Panagopoulos

The AC conductivity

- From the AC conductivity equations one can compute the asymptotics

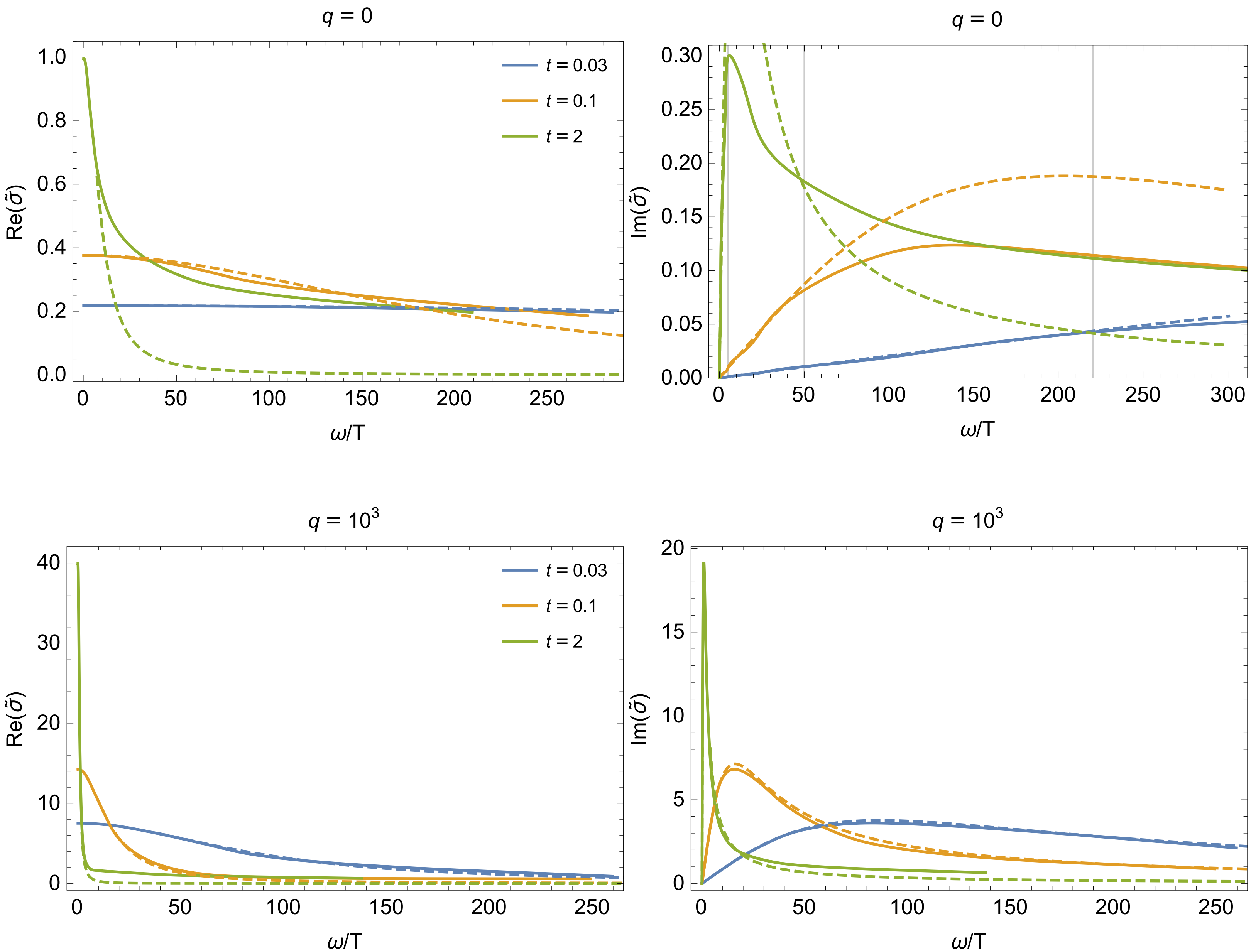
- The large ω behavior

$$\left(b^2\ell\mathcal{N}T\right)^{-1}\sigma(\omega) = i\left(\frac{2\pi}{3}\right)^{7/3}\frac{2}{\Gamma(1/3)\,\Gamma(7/3)}\left(\frac{\omega}{T}\right)^{-1/3}\ e^{i\pi/6}$$

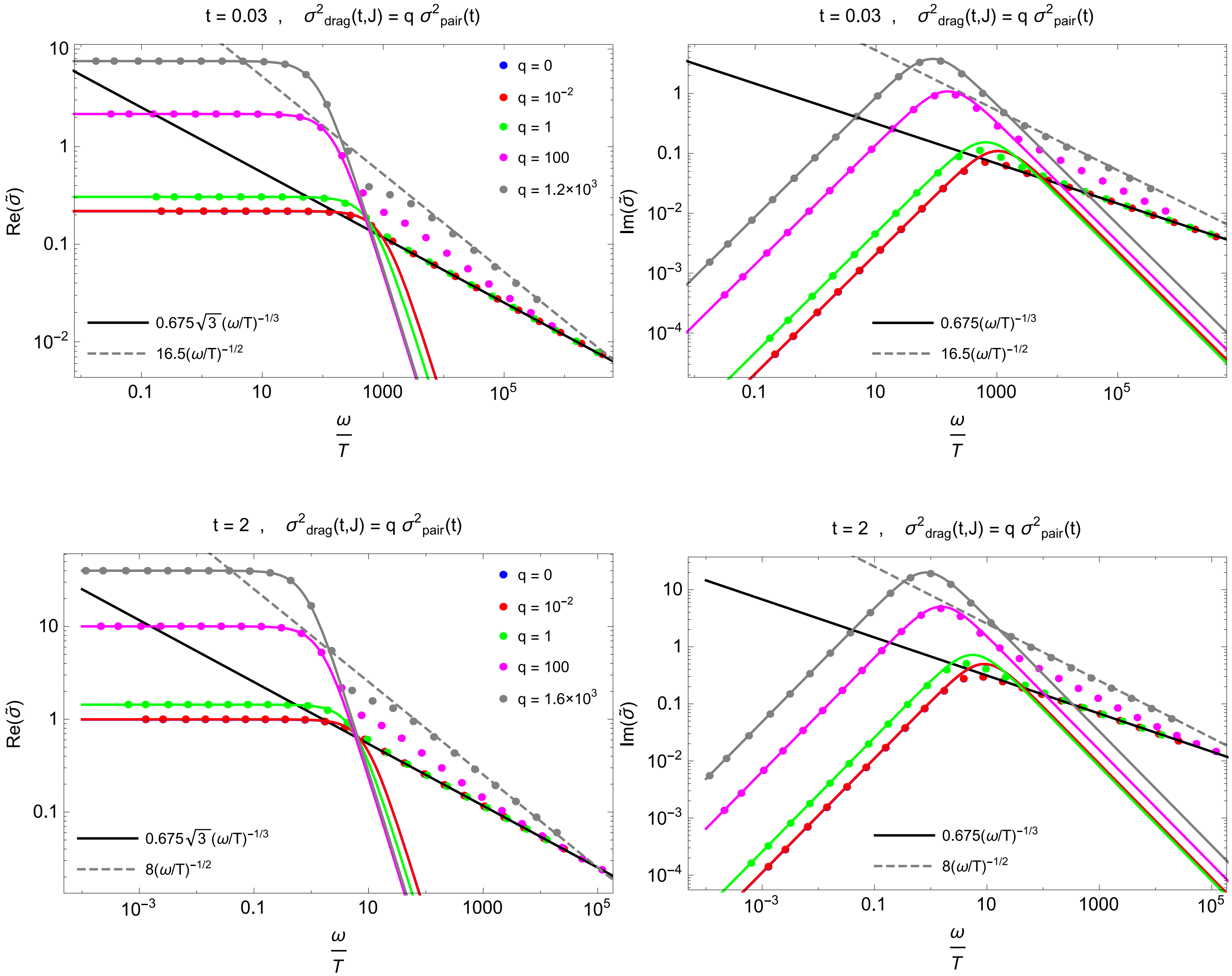
- The generalized relaxation time defined as

$$\sigma(\omega) \approx \sigma_{DC}\left(1 + i\tau\omega + \mathcal{O}(\omega^2)\right)\,,$$

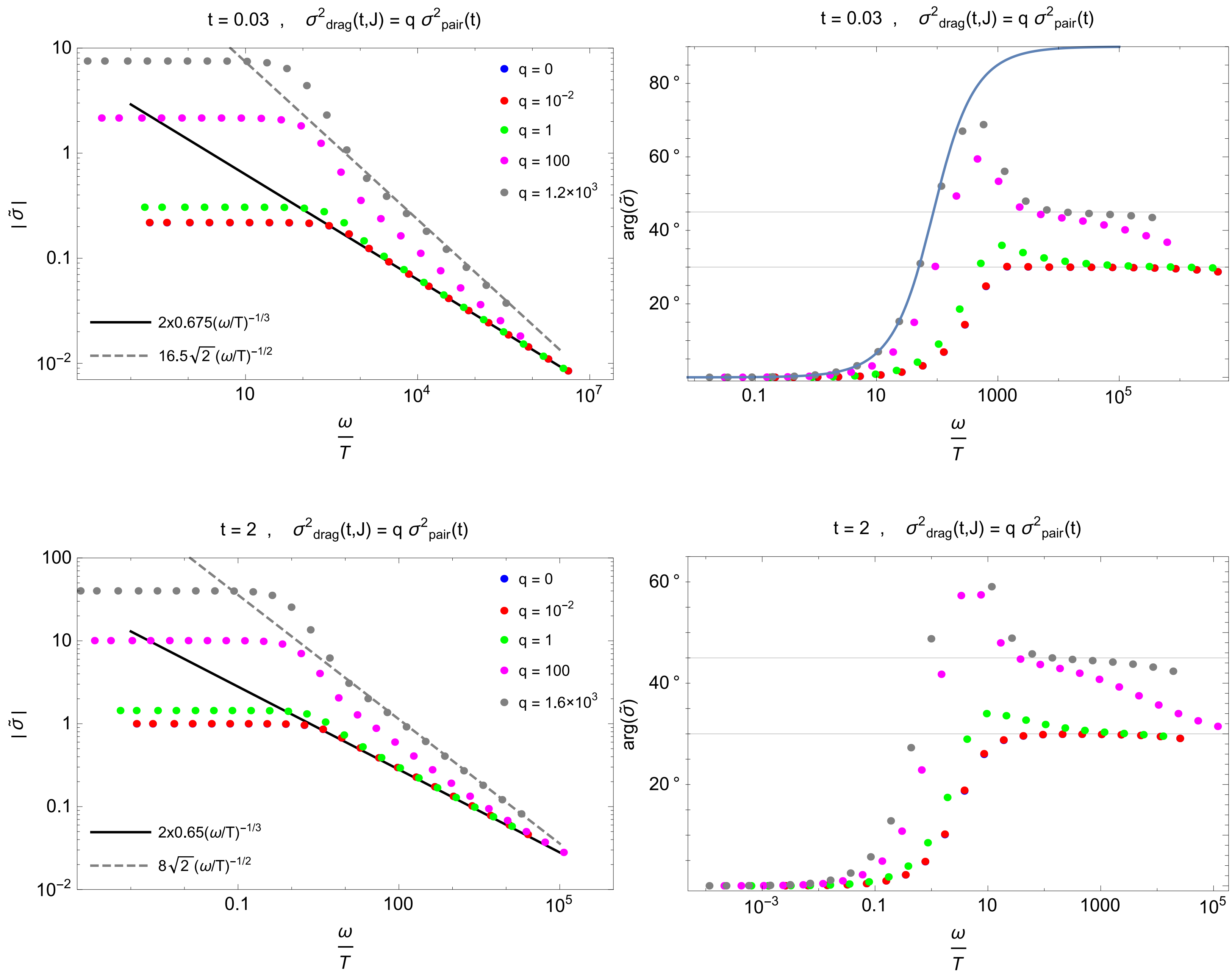
can be computed analytically but has a complicated formula.



Conductivity for three different temperatures in the PP (up) and in the DR (down) regimes. The left figures show the real part of the conductivity while the right figures the imaginary part of the conductivity. Continues lines represent the numerical data, dashed lines correspond to a fit to the Drude peak formula.



The colored continuous lines show the Drude fitting using the analytic computation of the generalized relaxation time. Straight black and dashed gray lines show the UV and the intermediate power law behavior of the AC conductivity.



Absolute value (left) and argument (right) part of the conductivity. The blue continuous line shows the Drude fit using the analytic computation of the generalized relaxation time. Straight black and dashed gray lines show the UV and the intermediate power law behavior of the AC conductivity.

Lessons learned

• When the conductivity is dominated by the drag mechanism (momentum dissipation) there is a clear Drude peak.

• When the conductivity is dominated by the pair-production mechanism there is no Drude peak.

• There is an associated scaling tail in the conductivity (here $\sigma \sim \omega^{-\frac{1}{3}}$). This always survives beyond the Drude peak as it falls off slower than ω^{-1}.

• It has the feature seen in experiment by Van der Marel et al.: Constant phase matching the power falloff.

• These properties seem to hold more generally and beyond the system under study.

General Scaling

Charmousis+Gouteraux+Kiritsis+Kim+Meyer
Gouteraux

Consider EMD solutions with general scaling exponents z, θ, ζ. (unbroken U(1)) and gapless charge excitations.

• The conductivity is obtained by solving for the fluctuations of the gauge field, $\delta A_i = a_i(r) e^{i\omega t}$.

$$\frac{1}{Z}\sqrt{\frac{g_{rr}}{g_{tt}}}\partial_r\left(Z\sqrt{\frac{g_{tt}}{g_{rr}}}a_i'\right) + \left[\frac{g_{rr}}{g_{tt}}\omega^2 - \frac{Q^2}{Z g_{xx}^2}\frac{g_{rr}}{}\right] a_i = 0$$

• By a field and coordinate redefinitions it can be mapped into a Schrödinger problem

$$-\psi'' + V_{eff}\ \psi = \omega^2 \psi \quad , \quad V_{eff}(x) = V_1(x) + Q^2 V_2(x)$$

$$V_1(x) \sim \frac{1}{x^2} \quad , \quad V_2 \sim \frac{1}{x^{2a}} \quad , \quad x \to \infty$$

$$a \geq 1$$

• The Charge density is "supporting" the IR geometry. Q^2 is fixed in terms of z, θ. Both terms in the potential contribute at the same order.

$$|\sigma| \sim \omega^m \quad , \quad Arg(\sigma) \simeq -\frac{\pi\ m}{2}$$

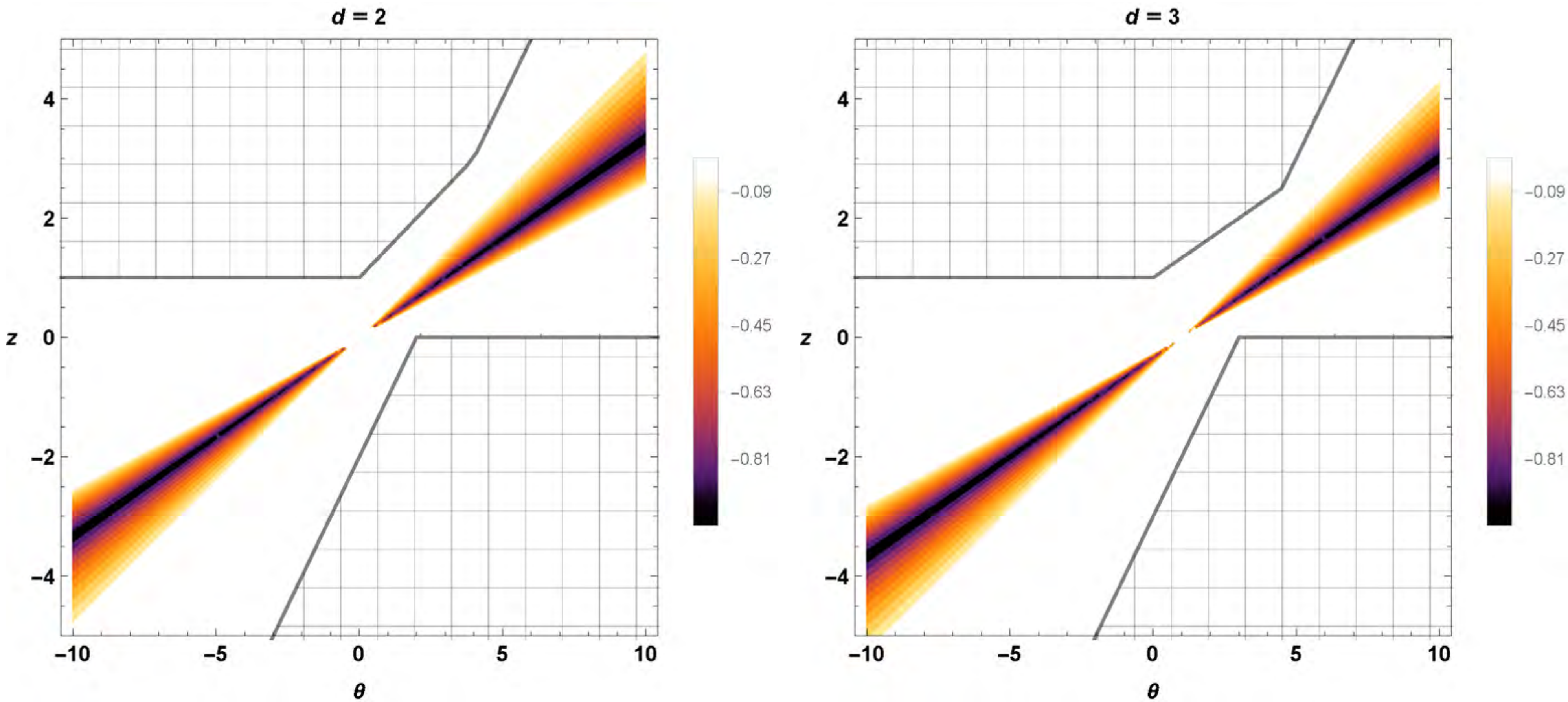
$$m = \frac{2(z-1)+d-\theta}{z}$$

• In his case m is always positive.

• The case where the IR geometry is AdS$_2$ can be obtained for $z \to \infty$ giving $m = 2$.

• For hyperscaling violating semilocal geometries, we must take, $\theta \to \infty$, $z \to \infty$ with $\frac{\theta}{z} = -\eta$ fixed.

$$m \quad = \quad 2+\eta \quad > \quad 0$$



Contour plots to illustrate the region in the parameter space where the exponent m takes negative values for the single charged model. Left: Conductivity in the charged case for $d = 2$. Right: Conductivity in the charged regime for $d = 3$. The allowed values for the parameters are bounded by the gray mesh. The negative values for m are outside the permitted region.

• The Charge density is a probe in the IR geometry. The charge term is subleading or absent.

$$|\sigma| \sim \omega^m \quad , \quad Arg(\sigma) \simeq -\frac{\pi \ m}{2}$$

$$m = \left|\frac{z + \zeta - 2}{z}\right| - 1\,,$$

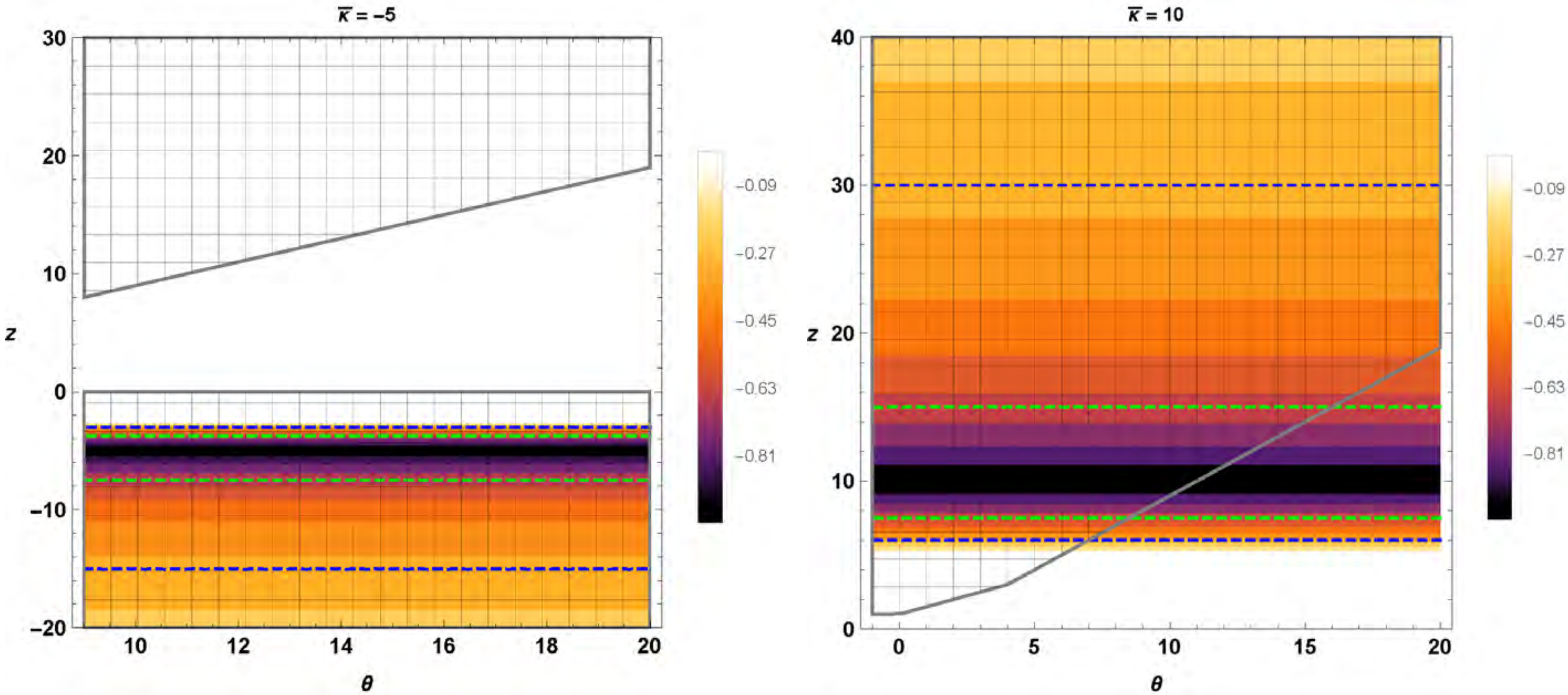
• It allows for negative values in m but always $m \geq -1$ (unitarity bound)

• For an AdS_2 IR geometry the exponent can be obtained by an $z \to \infty$ giving $m = 0$.

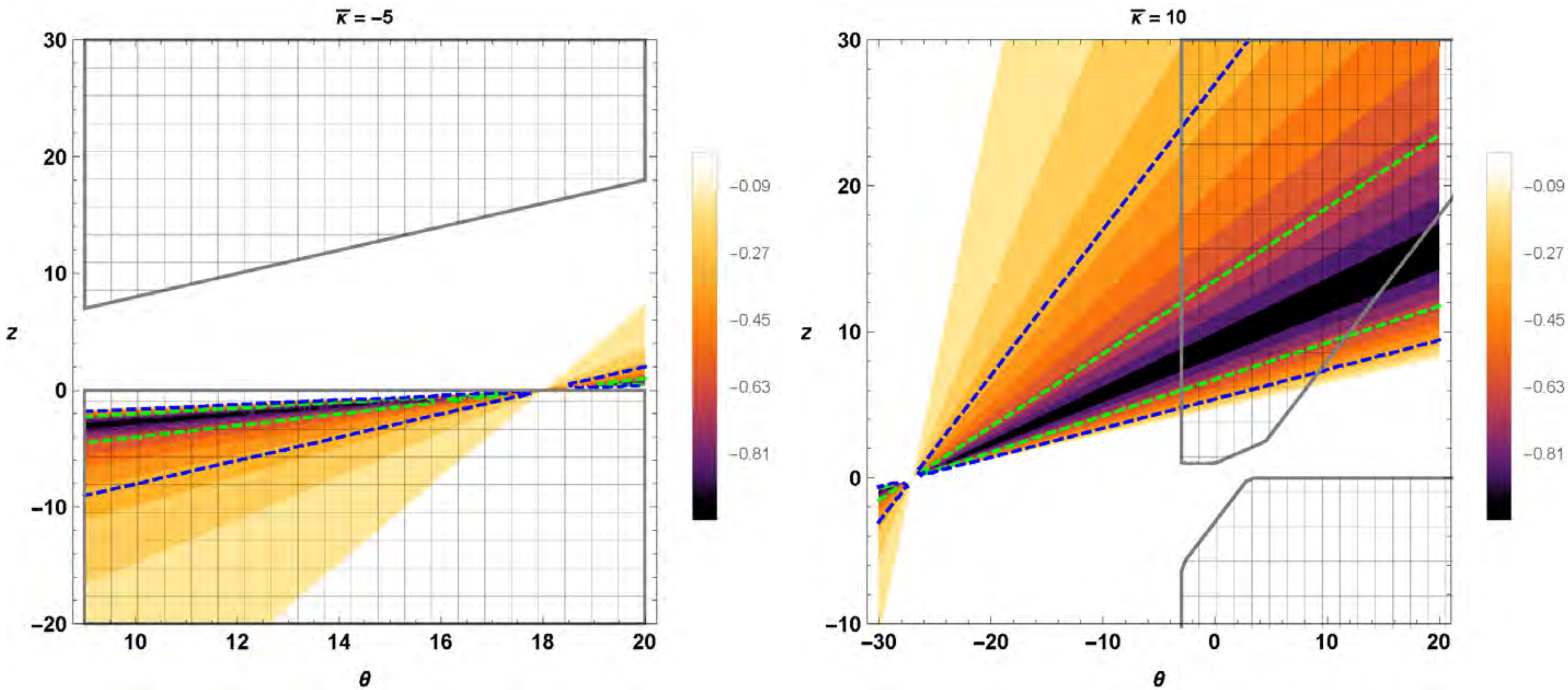
• For hyperscaling violating semilocal geometries we must take $\theta \to \infty$, $z \to \infty$ with $\frac{\theta}{z} = -\eta$ fixed and obtain

$$m \quad = \quad \frac{d-2}{d}\,\eta \quad > 0$$

• Finally, for the gauge field conformal case we obtain $m = 0$ when $d = 2$.



Contour plots to illustrate the region in the parameter space where the exponent m takes negative values for the probe charge density case. The plots above correspond to $d = 2$. Left: m for $\bar{\kappa} = -5$. Right: m for $\bar{\kappa} = 10$. The allowed values for the parameters are bounded inside the gray mesh. Green dashed lines are the contour levels where $m = -2/3$ and blue dashed lines $m = -1/3$.



Contour plots to illustrate the region in the parameter space where the exponent m takes negative values for the probe charge density case. The plots above correspond to $d = 3$ Left: m for $\bar{\kappa} = -5$. Right: m for $\bar{\kappa} = 10$. The allowed values for the parameters are bounded inside the gray mesh. Green dashed lines are the contour levels where $m = -2/3$ and blue dashed lines $m = -1/3$.

Outlook

• The scaling of the holographic AC conductivity is a generic phenomenon

• It is controlled generically by the pair-production mechanism and seems independent of the momentum dissipation mechanism

• It always dominates the Drude peak tail for sufficiently large ω.

• It has the general properties observed in cuprates

• The relevant exponent can be negative only if the charge density does not backreact on the geometry.

• It is important to analyze all this in concrete and complete RG Flows with and without U(1) symmetry breaking.

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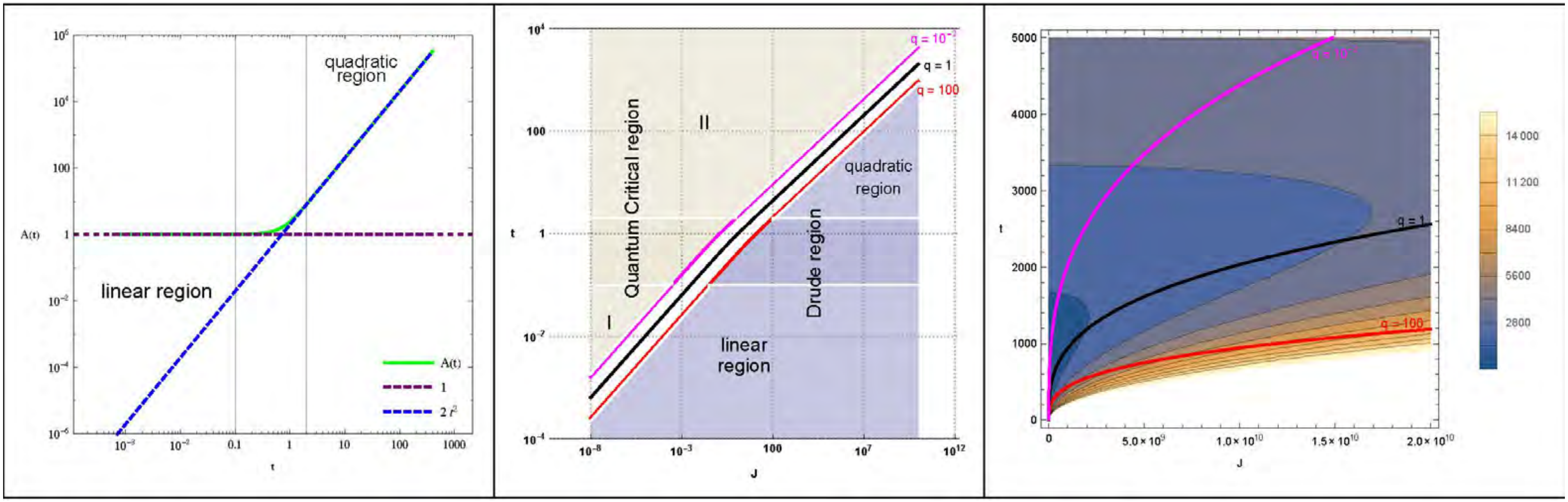
THANK YOU

The four regimes of the model



Left: Comparison of the function $A(t)$ with its low and high t behaviour.
Center: Location of the four regimes in the space of parameters (J, t) log-log scale. The black line $q = 1$, separates the DR respect to the QC regime. The magenta line represents the region with $q = 10^{-2}$ and the red one $q = 100$.

Right: Contour Plot of the DC conductivity as a function of the scaling charge density and the temperature.

Overview of the solutions

• The gravitational action:

$$S = M^3 \int d^5x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z_1(\phi)}{4}F_1^2 - \frac{Z_2(\phi)}{4}F_2^2\right]$$

$$\text{with} \qquad F^i_{\mu\nu} \equiv \partial_\mu A^i_\nu - \partial_\nu A^i_\mu$$

and

$$V(\phi) \to_{IR} \sim V_0\, e^{-\delta\phi}, \quad Z_1(\phi) \underset{IR}{\sim} Z_{10}\, e^{\gamma_1\phi}, \quad Z_2(\phi) \underset{IR}{\sim} Z_{20}\, e^{\gamma_2\phi}.$$

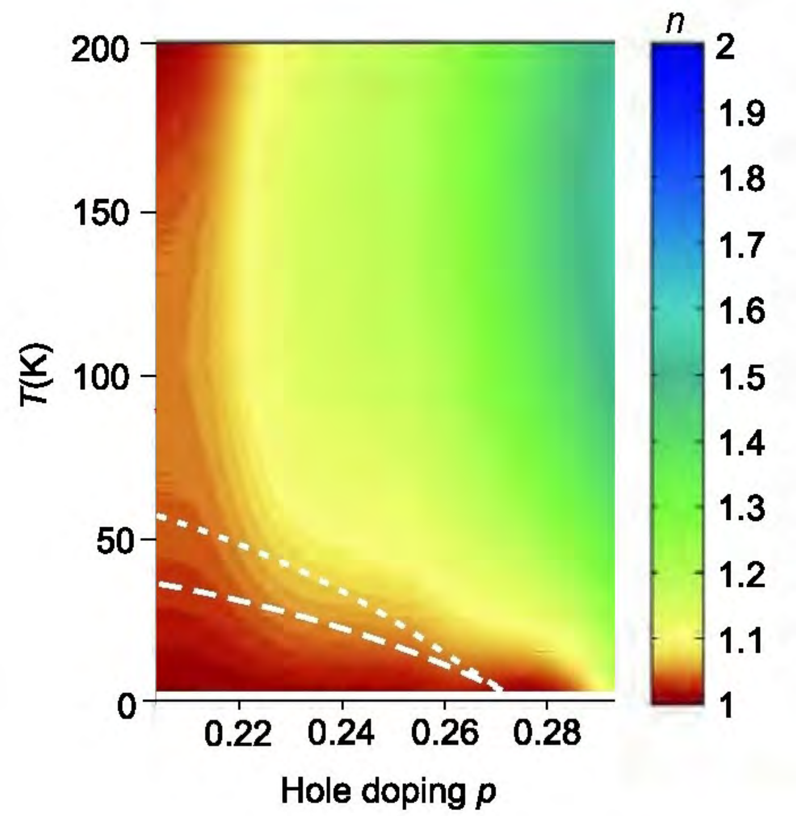
• The metric ansatz has helical symmetry and we parametrize the scaling solutions as :

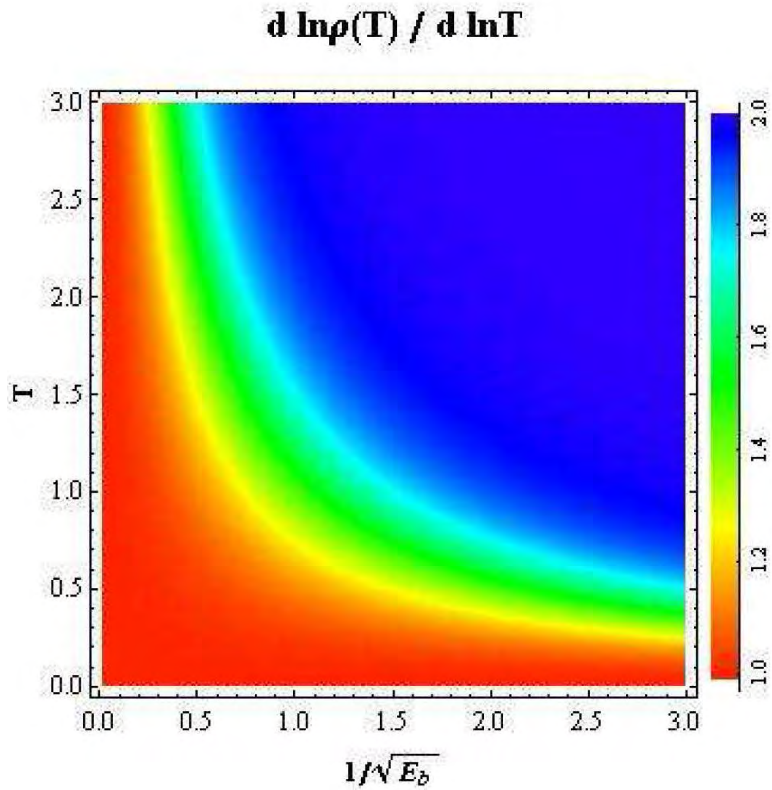
$$ds^2 = r^{\frac{2\theta}{3}}\left[-\frac{\mathrm{d}t^2}{z^{2z}} + \frac{L^2\mathrm{d}r^2 + {\omega_1}^2}{r^2} + \frac{1}{r^{2z_2}}\left({\omega_2}^2 + \frac{\lambda}{r^2}{\omega_3}^2\right)\right] \quad , \quad A_1 \sim r^{\zeta - z}$$

where

$$\omega_1 = \mathrm{d}x_1\,, \omega_2 = \cos(kx_1)\mathrm{d}x_2 + \sin(kx_1)\mathrm{d}x_3\,, \quad \omega_3 = \sin(kx_1)\mathrm{d}x_2 - \cos(kx_1)\mathrm{d}x_3$$

Critical lines vs critical points in Cuprates





Kim+Kiritsis+Panagopoulos

Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$.

R.A. Copper et. al. 2009

Breaking Translation Invariance

• Charge lattices lead to non-linear (gravitational) PDEs that are in general very difficult to solve.

Horowitz+Santos+Tong (2012), Lin+Liu+Wu+Xiang (2013), Donos+Kiritsis (2013)

• Some other string mechanisms of translation invariance breaking (momentum dissipation) are simpler to analyze:

♠ An effective "phenomenological" treatment of momentum dissipation using massive gravity.

Vegh (2013), Davison (2013), Blake+Tong (2013), Davison+Schalm+Zannen (2013)

♠ Interactions of charge with a bulk sector that carries most of the energy (DBI probe approximation)

Karch+O'Bannon (2007), Charmousis+Gouteraux+Kiritsis+Kim+Meyer (2010)

♠ Interactions with string axions that model a kind of homogeneous disorder.

Andrade+Withers (2013), Gouteraux (2013), Donos+Gauntlett (2013)

♠ The use of random field disorder in holography.

Hartnoll+Herzog (2008), Davison+Schalm+Zaanen (2013), Lucas+Sachdev+Schalm (2014)

♠ Study of saddle points with helical symmetry.

Kachru et al. (2012), Donos+Gauntlett (2012), Donos+Hartnoll (2012)

DC conductivity

• The general structure of the holographic DC conductivity at strong coupling:

$$\sigma_{\mathrm{DC}} = \sqrt{(\sigma_{DC}^{pp})^2 + (\sigma_{DC}^{drag})^2} \quad , \quad \sigma_{\mathrm{DC}} = \sigma_{DC}^{pp} + \sigma_{DC}^{drag}$$

• σ^{pp} is the pair-creation contribution: it exists also when $Q = 0$.

• σ^{drag} is the "drag" contribution. It originates from the force and dissipation a fractionalized charge (a string) feels as it is moving in the strongly coupled medium.

Karch+O'Bannon, (2007)

Ground-states with helical symmetry

Work with A. Donos and B. Gouteraux, to appear

- The gravitational action:

$$S = M^3 \int d^5x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z_1(\phi)}{4} F_1^2 - \frac{Z_2(\phi)}{4} F_2^2 \right]$$

$$\text{with} \qquad F^i_{\mu\nu} \equiv \partial_\mu A^i_\nu - \partial_\nu A^i_\mu$$

and

$$V(\phi) \to_{IR} \sim V_0 \, e^{-\delta\phi} \,, \quad Z_1(\phi) \underset{IR}{\sim} Z_{10} \, e^{\gamma_1 \phi} \,, \quad Z_2(\phi) \underset{IR}{\sim} Z_{20} \, e^{\gamma_2 \phi} \,.$$

- The metric ansatz has helical symmetry and we parametrize the scaling solutions as :

$$ds^2 = r^{\frac{2\theta}{3}} \left[-\frac{dt^2}{z^{2z}} + \frac{L^2 dr^2 + \omega_1{}^2}{r^2} + \frac{1}{r^{2z_2}} \left(\omega_2{}^2 + \frac{\lambda}{r^2} \omega_3{}^2 \right) \right] \quad , \quad A_1 \sim r^{\zeta - z}$$

where

$$\omega_1 = dx_1 \,, \omega_2 = \cos(kx_1) dx_2 + \sin(kx_1) dx_3 \,, \quad \omega_3 = \sin(kx_1) dx_2 - \cos(kx_1) dx_3$$

Anisotropic saddle points

(a) IR marginal current ($\zeta = \theta - 2 - 2z_2$)

$$S_a \sim T^{\frac{2z_2+2-\theta}{z}}$$

vanishes at $T \to 0$ • For the DC conductivity

$$\sigma^a_{DC} \sim T^{\frac{\zeta-2}{z}}$$

• σ_{DC} along the helical axis vanishes always at $T = 0$ (power insulators)

• For the IR limit of AC conductivity we find

$$\sigma^a_{AC}(T=0) \sim \omega^{\frac{\theta-2z_2-4}{z}}$$

• This is the same power as the DC conductivity and $\sigma^a_{AC}(T=0) \to 0$ as $\omega \to 0$.

• (b) IR Irrelevant current ($z = \frac{3}{2} z_2$).

$$S_b \sim T^{\frac{(2z_2+2-\theta)}{z}} \quad ,$$

• For the DC conductivity

$$\sigma_{DC}^{b} \sim T^{\frac{\zeta-2}{z}}$$

• It is sometimes diverging (metal) and sometimes vanishing (insulator).

• σ_{DC} comes always for the pair creation term. Therefore in the metallic case we expect no Drude peak (incoherent metals) .

• For the AC conductivity at zero T:

$$\sigma_{AC}^{a}(T=0) \sim \omega^{\min\{n_1,n_2\}}$$

$$n_1 = -1 + \left|\frac{-4+2\zeta+3z_2}{3z_2}\right| \quad , \quad n_2 = -1 + \frac{1}{3}\sqrt{\frac{4\theta^2+96z_2-76\theta z_2+217z_2^2}{z_2^2}}$$

• Whenever the system is an insulator, n_1 dominates and the the AC and DC exponents are equal.

• When n_2 dominates, the system is metallic with a decaying low-frequency AC tail. A $\delta(\omega)$ may bridge this behavior.

• When the system is metallic and n_1 dominates, it can have:

1. Diverging AC frequency power tail and match the DC scaling

2. Diverging AC frequency power tail without matching the DC scaling

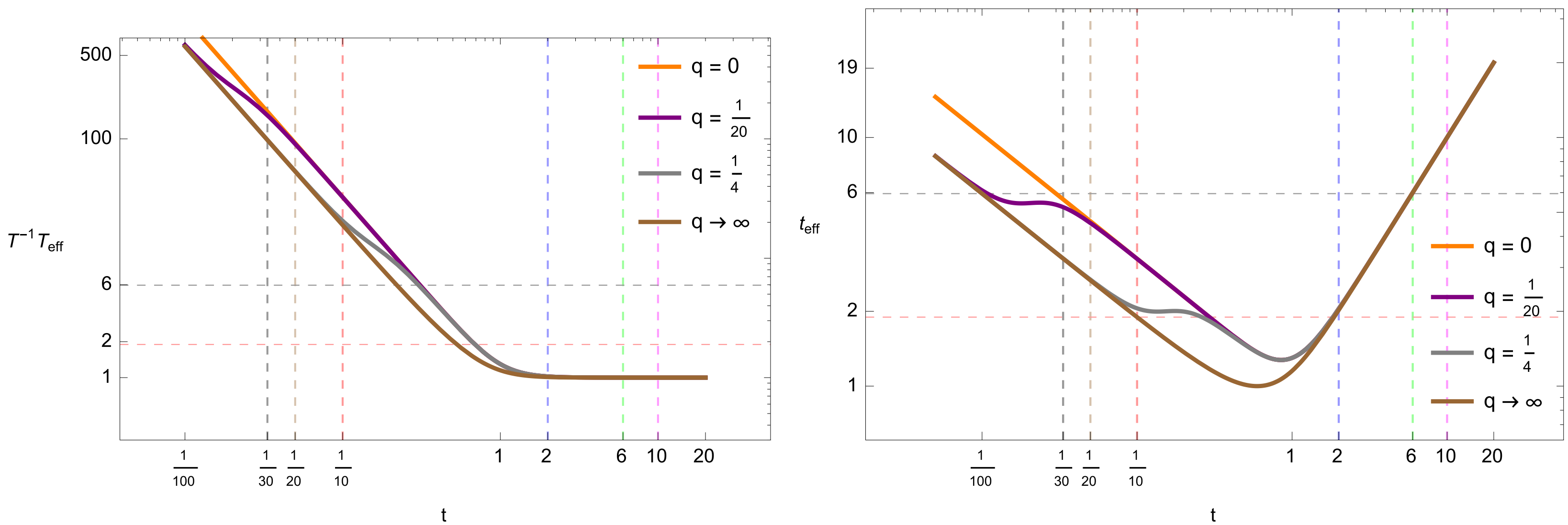
3. Decaying AC frequency power tail.

• Whenever $\sigma_{AC} \sim \omega^0$ then $\sigma_{DC} \sim T^0 \quad or \quad \sigma_{DC} \sim T^{-2}$

The effective temperature

The effective temperature on the brane is not t for non-trivial E:

$$t_{eff} = t\sqrt{\frac{J^2 A(t)^2\sqrt{A(t)-2t^2} + t^5\left(t^2A(t)+2t^4+3\right)}{2\sqrt{2}t^3\left(t^5\sqrt{A(t)}+J^2\right)}}$$



Dependence of the world-volume temperature T_{eff} with the parameters J and t, the effective temperature is always bigger than the background temperature T.

Holographic gapped systems at finite density

- There are two ways that a system can be insulating at T=0:

♠ Charged excitations are gapped. In that case the conductivity is non-zero only above the gap.

♠ There is no gap but the limit $\omega \to 0$ gives a vanishing conductivity. In both cases the operator/mechanism that breaks translation invariance, is relevant in the IR.

- Gapped holographic systems are known at zero charge density, and they are in use as models of YM.

Witten, '98, Gursoy+Kiritsis+Nitti, '07, Nishioka+Ryu+Takatanagi, '10

- At finite density, many saddle-points with discrete spectrum were found, in the classification of QC points in EMD Theories.

Charmousis+Gouteraux+Kiritsis+Kim+Meyer, '10, McGreevy+Balasubramanian, '10

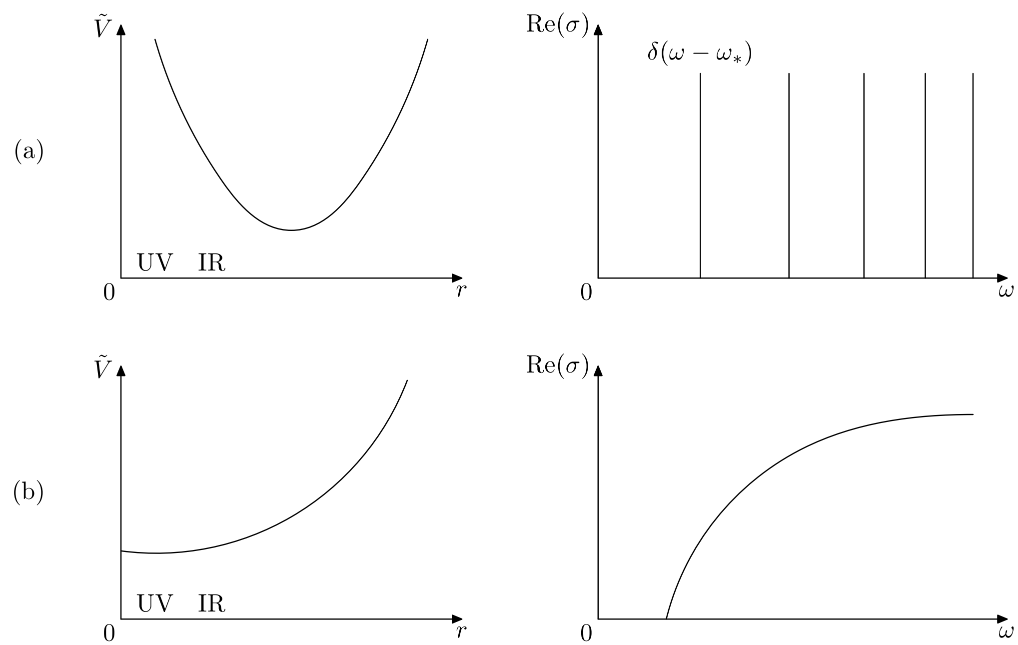
$$S = \int d^4x \sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{Z(\phi)}{4}F^2\right] \quad , \quad V \sim e^{-\delta\phi} \quad , \quad Z \sim e^{\gamma\phi}$$

• The conductivity is obtained by solving for the fluctuations of the gauge field, $\delta A_i = a_i(r) e^{i\omega t}$.

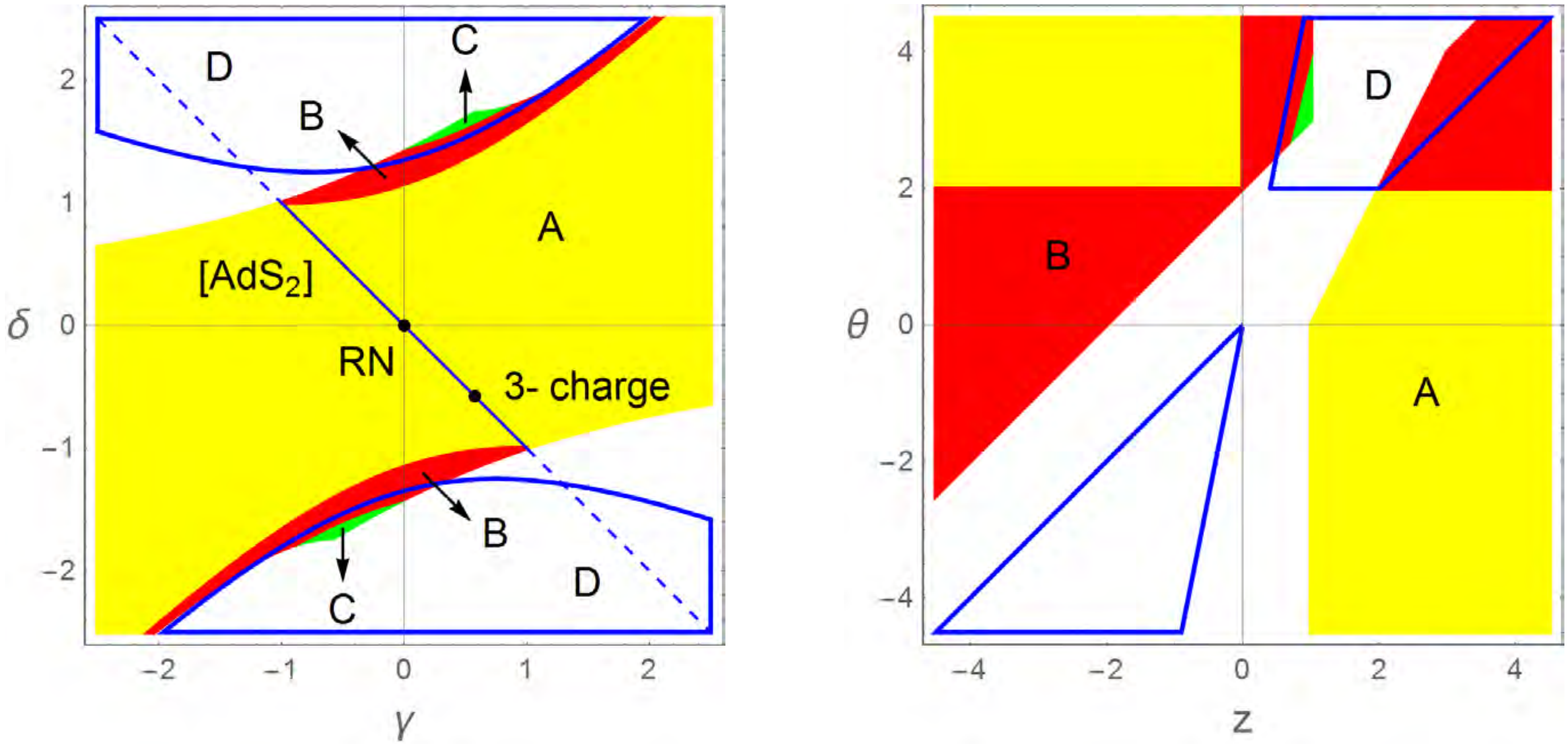
$$\frac{1}{Z}\sqrt{\frac{g_{rr}}{g_{tt}}}\partial_r\left(Z\sqrt{\frac{g_{tt}}{g_{rr}}}a_i'\right) + \left[\frac{g_{rr}}{g_{tt}}\omega^2 - \frac{Q^2\ g_{rr}}{Z g_{xx}^2}\right] a_i = 0$$

• By a field and coordinate redefinitions it can be mapped into a Schrödinger problem

$$-\psi'' + V_{eff}\ \psi = \omega^2 \psi$$



The parameter space



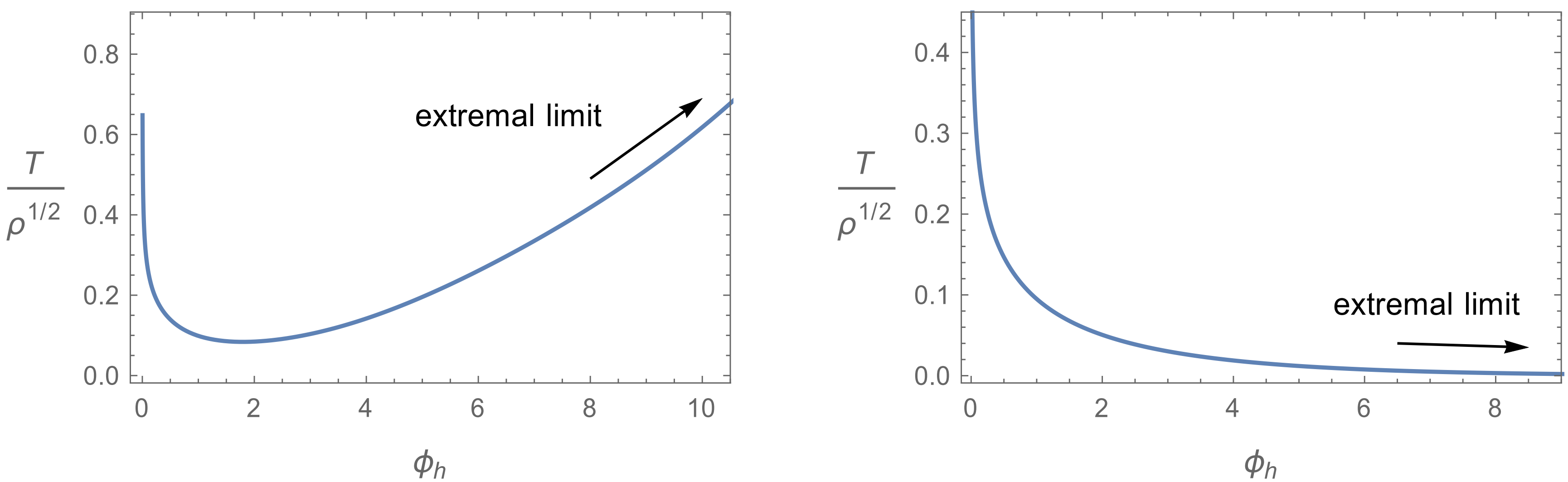
The regions A, B, and C are the parameter space constrained by the Gubser criterion. In region A (yellow), the extremal limit is at $T \to 0$, and the current-current correlator is gapless. In region B (red), the extremal limit is at $T \to \infty$, and the current-current correlator is gapped. In region C (green), the extremal limit is at $T \to \infty$, and the current-current correlator is gapless. Region D (enclosed by blue boundaries) is holographically unreliable.

The finite temperature picture

• The gapped systems above are at finite density, and have therefore a zero-frequency δ-function: They are perfect conductors.

• Note also that we are at $T = 0$. In this respect as far as the δ-function is concerned they resemble real metals at $T = 0$.

• Up to $T = T_{min}$ this is the only saddle-point for the system. For $T > T_{min}$ there are also two black holes that are competing at the same temperature.



• At $T = T_c > T_{min}$ there is a first order phase transition to the large black hole phase that is a gapless plasma phase. This is very similar to the confinement-deconfinement phase transition in gauge theories.

Adding Momentum dissipation

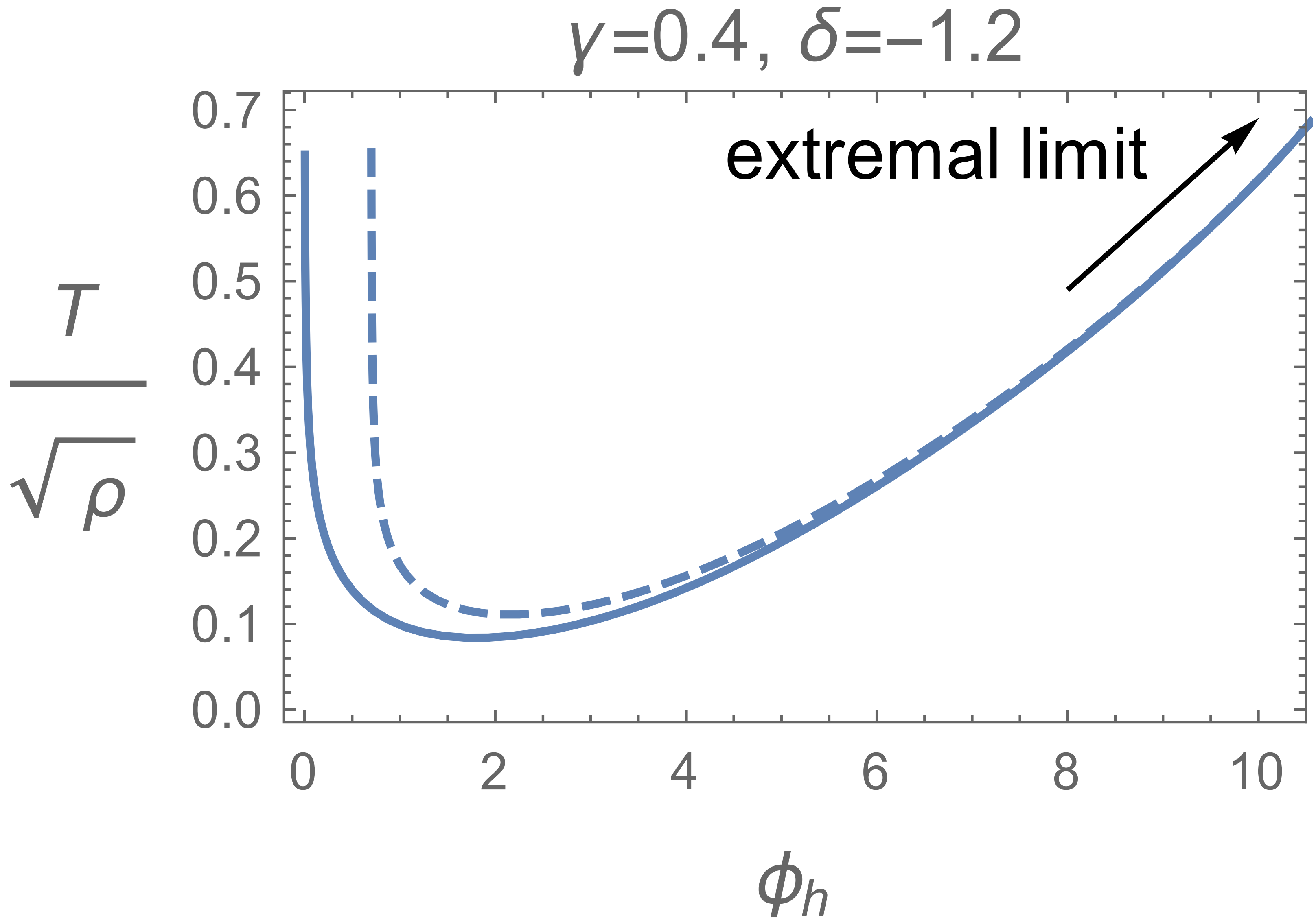
- We use axions as a source of momentum dissipation.

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{Y(\phi)}{2}\sum_{i=1}^{2}(\partial\psi_i)^2\right],$$

$$V(\phi) \sim e^{-\delta\phi}, \qquad Z(\phi) \sim e^{\gamma\phi}, \qquad Y(\phi) \sim e^{\lambda\phi}.$$

$$\psi_1 = kx \quad , \quad \psi_2 = k\ y$$

- We also choose λ in the region where the axions are "irrelevant" in the IR.


γ=0.4, δ=−1.2
extremal limit
T/√ρ
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
2
4
6
8
10
φh

• Momentum dissipation is expected to remove the zero frequency δ-function but:

(a) We are in the T=0 geometry as long as $T < T_c$

(b) The axions are irrelevant in the deep IR.

• A detailed analysis of the Drude weight is needed.

• The fluctuation equations for the conductivity involve not only δA_i but also the axions.

• There is a direct formula for the DC conductivity in the presence of a regular (non-extremal) horizon.

$$\sigma_{\text{DC}} = Z_h + \frac{q^2}{k^2 (g_{xx})_h Y_h} \equiv Z_1 + \frac{q^2}{k^2} Z_2$$

Gouteraux, Donos+Gauntlett

• There is also a direct formula for the Drude weight:

$$\Pi(\omega) = fH\lambda_1' - \frac{q}{k^2} f Z_2 \left(\frac{Z_1}{Z_2}\right)' \lambda_2, \quad , \quad \partial_r \Pi = \mathcal{O}(\omega^2)$$

$$\lambda_1 = \frac{Z_1}{H}\left(a_x - \frac{q}{k^2}\frac{Z_2}{Z_1} b_x\right), \qquad \lambda_2 = \frac{Z_2}{H}\left(q a_x + b_x\right).$$

• It can be shown that $\Pi(0)$ is the Drude weight. We can then establish that:

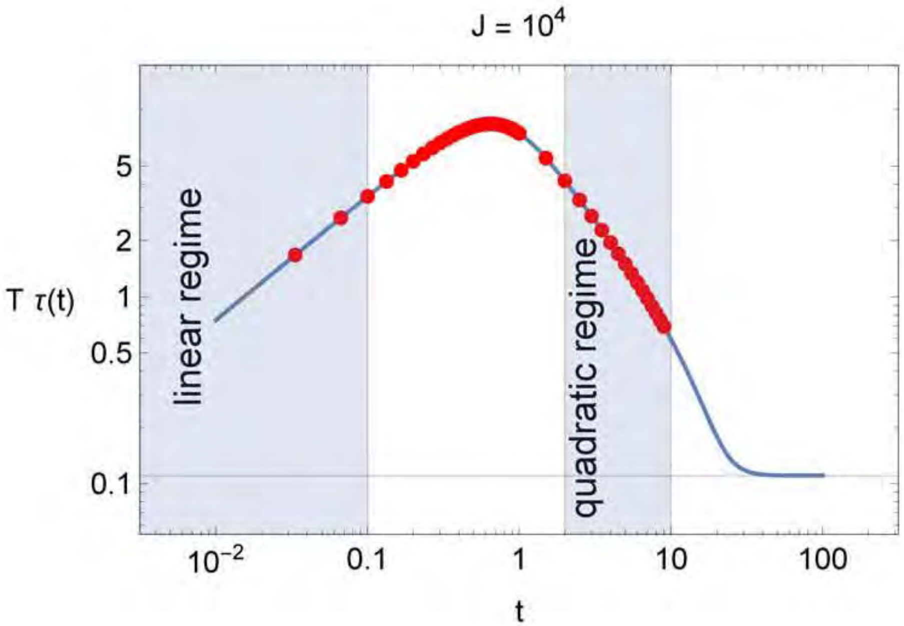
(a) when $\sigma_{DC} \to \infty$ at extremality then there is a zero-frequency δ-function, and this happens in the gapless geometries.

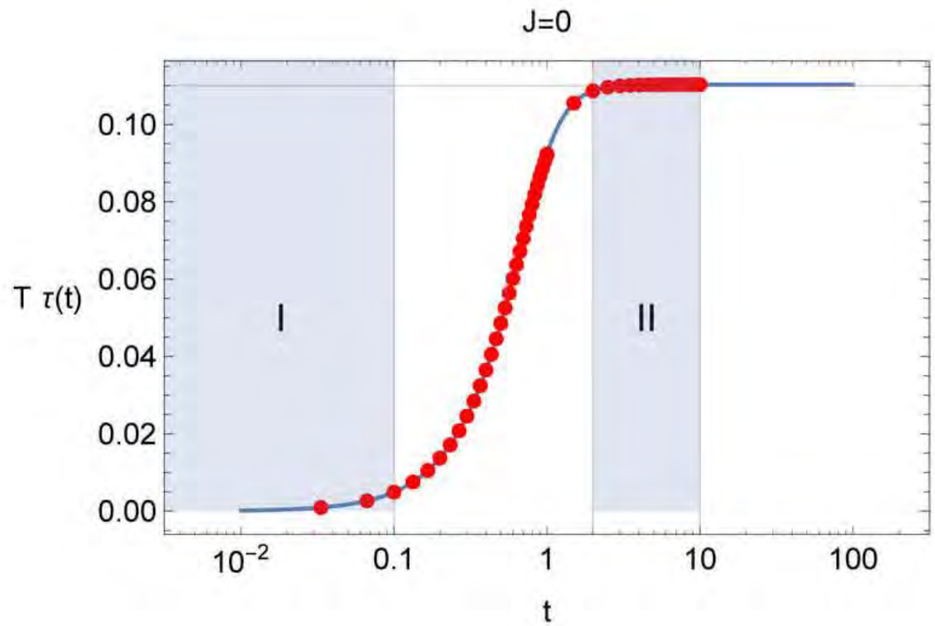
(b) when $\sigma_{DC} \to 0$ at extremality, then there is no zero-frequency δ-functions and this happens for the gapped geometries.

• We have therefore found holographic system with a charged spectrum that is gapped and discrete.

• These are insulators that share properties of both band-gap insulators and Mott insulators.

generalized relaxation time





The generalized relaxation time τ as a function of the scaling temperature variable t.The left plot shows the behaviour of the dimensionless quantity Tτ (T is the temperature) deep in the Drude regime (DR). The right plot shows the behavior in the pair-production regime at zero density. Dots show numerical data and the continuous line is the analytic formula obtained in the text using perturbation theory .

Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 1 minutes
- Introduction 2 minutes
- Strongly Correlated Electrons 6 minutes
- Insulators 8 minutes
- Supersolids 9 minutes
- The plan for the rest 9 minutes
- The Wilsonian Program 10 minutes
- Generic Scaling Geometries: A classification of all QC points in holography 12 minutes