

Neutrino Masses, Mixing and Leptonic CP Violation: Current Status and Future Prospects

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There have been remarkable discoveries in neutrino physics in the last \sim 16 years.

Compellings Evidence for ν -Oscillations

$-\nu_{\text{atm}}$: SK UP-DOWN ASYMMETRY

$\theta_{Z^-}, L/E^-$ dependences of μ -like events

Dominant $\nu_{\mu} \rightarrow \nu_{\tau}$ K2K, MINOS, T2K; CNGS (OPERA)

$-\nu_{\odot}$: Homestake, Kamiokande, SAGE, GALLEX/GNO

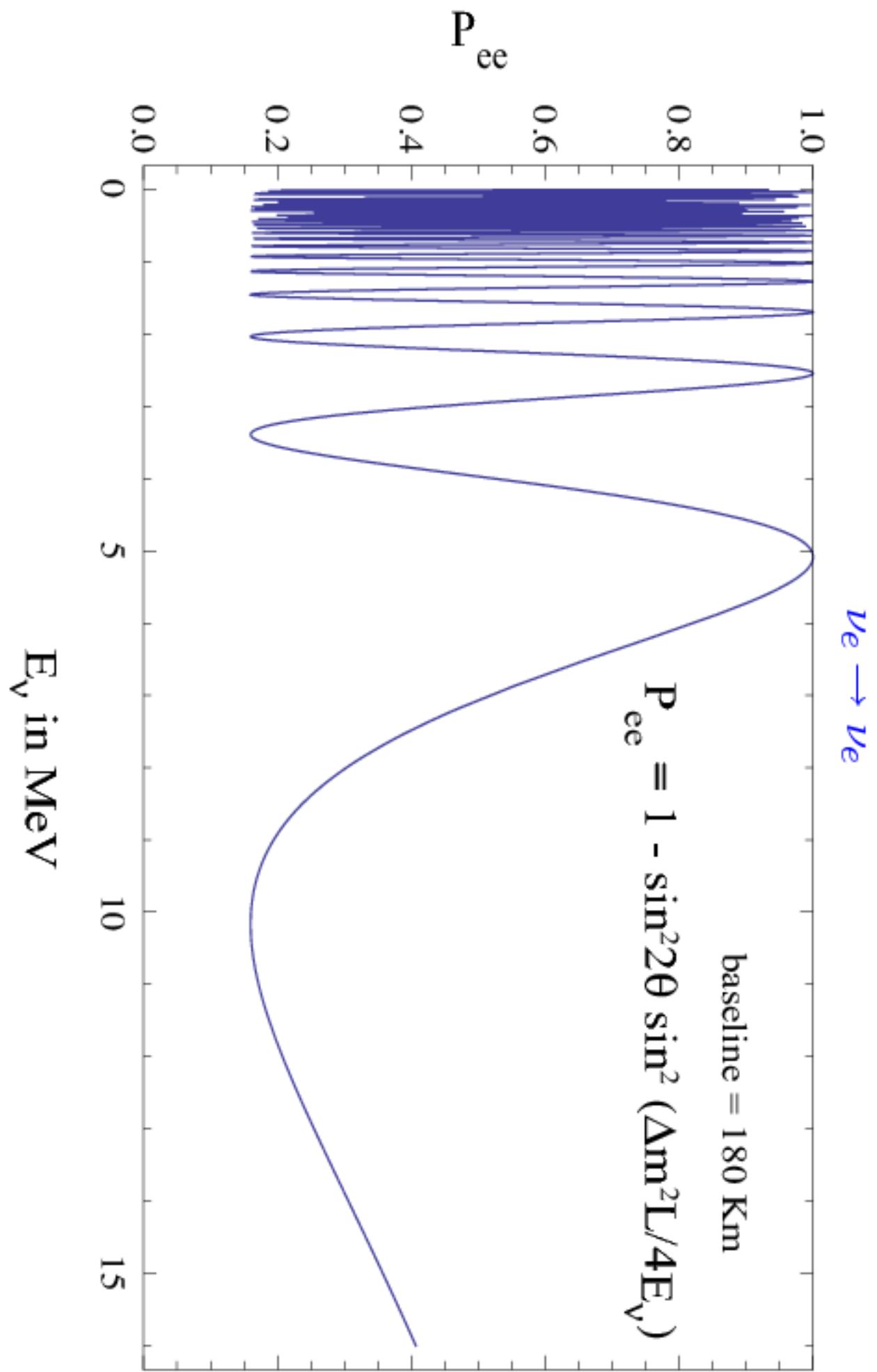
Super-Kamiokande, SNO, BOREXINO; KamLAND

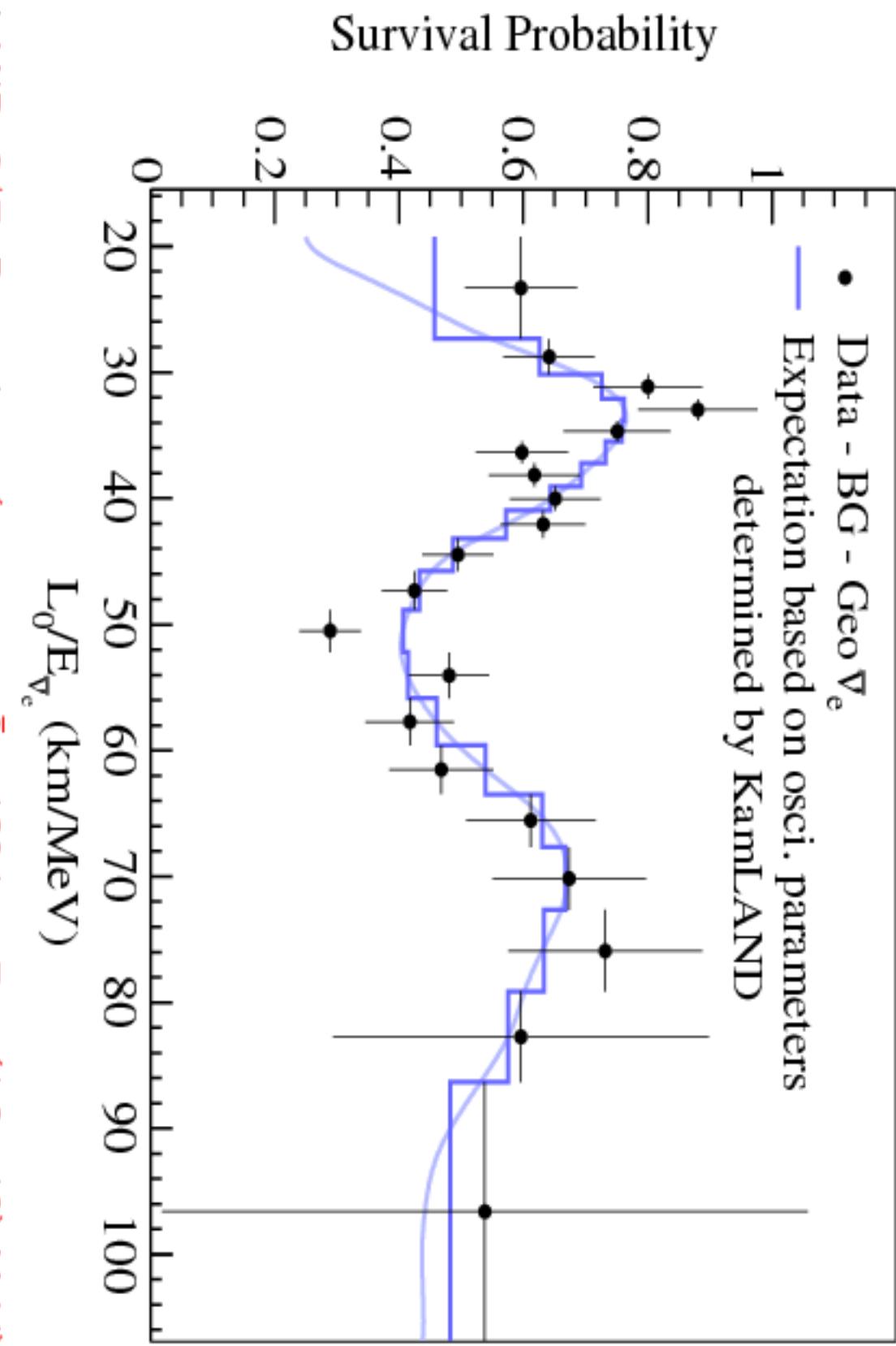
Dominant $\nu_e \rightarrow \nu_{\mu,\tau}$ BOREXINO

$-\bar{\nu}_e$ (from reactors): Daya Bay, RENO, Double Chooz

Dominant $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$

T2K, MINOS (ν_{μ} from accelerators): $\nu_{\mu} \rightarrow \nu_e$





KamLAND: L/E -Dependence (reactor $\bar{\nu}_e$, $L = 180$ km, $E = (1.8 - 10)$ MeV)

Compelling Evidences for ν -Oscillations: ν mixing

$$|\nu_l\rangle = \sum_{j=1}^n U_{lj}^* |\nu_j\rangle, \quad \nu_j : m_j \neq 0; \quad l = e, \mu, \tau; \quad n \geq 3;$$

$$\nu_{l\text{L}}(x) = \sum_{j=1}^n U_{lj} \nu_{j\text{L}}(x), \quad \nu_{j\text{L}}(x) : m_j \neq 0; \quad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;
Z. Maki, M. Nakagawa, S. Sakata, 1962;

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

$\nu_j, m_j \neq 0$: Dirac or Majorana particles.

Data: at least 3 ν s are light: $\nu_{1,2,3}, m_{1,2,3} \lesssim 1$ eV.

We can have $n > 3$ ($n = 4$, or $n = 5$, or $n = 6, \dots$) if, e.g., **sterile** ν_R , $\tilde{\nu}_L$ exist and they mix with the active flavour neutrinos ν_l ($\tilde{\nu}_l$), $l = e, \mu, \tau$.

Two (extreme) possibilities:

- i) $m_{4,5,\dots} \sim 1$ eV;
in this case $\nu_{e(\mu)} \rightarrow \nu_S$ oscillations are possible (hints from LSND and MiniBooNE experiments, re-analyses of short baseline (SBL) reactor neutrino oscillation data ("reactor neutrino anomaly"), data of radioactive source calibration of the solar neutrino SAGE and GALLEX experiments ("Gallium anomaly"));
- ii) $M_{4,5,\dots} \sim (10^2 - 10^3)$ GeV, TeV scale seesaw models; $M_{4,5,\dots} \sim (10^9 - 10^{13})$ GeV, "classical" seesaw models.

We can also have, in principle:

$$m_4 \sim 1 \text{ eV } (\nu_{e(\mu)} \rightarrow \nu_S), \quad m_5 \sim 5 \text{ keV (DM)}, \quad M_6 \sim (10 - 10^3) \text{ GeV (seesaw)}.$$

- Data (relativistic ν 's): ν_l ($\tilde{\nu}_l$) - predominantly LH (RH).

Standard Theory: ν_l , $\tilde{\nu}_l$ - $\nu_L(x)$;

$\nu_L(x)$ form doublets with $l_L(x)$, $l = e, \mu, \tau$:

$$\begin{pmatrix} \nu_L(x) \\ l_L(x) \end{pmatrix} \quad l = e, \mu, \tau.$$

- No (compelling) evidence for existence of (relativistic) ν 's ($\tilde{\nu}$'s) which are predominantly RH (LH): ν_R ($\tilde{\nu}_L$.)

If ν_R , $\tilde{\nu}_L$ exist, must have much weaker interaction than ν_l , $\tilde{\nu}_l$: ν_R , $\tilde{\nu}_L$ - "sterile", "inert".

B. Pontecorvo, 1967

In the formalism of the ST, ν_R and $\tilde{\nu}_L$ - RH ν fields $\nu_R(x)$; can be introduced in the ST as $SU(2)_L$ singlets.

No experimental indications exist at present whether the SM should be minimally extended to include $\nu_R(x)$, and if it should, how many $\nu_R(x)$ should be introduced.

$\nu_R(x)$ appear in many extensions of the ST, notably in $SO(10)$ GUT's.

The RH ν 's can play crucial role

- i) in the generation of $m(\nu) \neq 0$,
- ii) in understanding why $m(\nu) \ll m_l, m_q$,
- iii) in the generation of the observed matter-antimatter asymmetry of the Universe (via leptogenesis).

The simplest hypothesis is that to each $\nu_l L(x)$ there corresponds a $\nu_{lR}(x)$, $l = e, \mu, \tau$.

$$\begin{aligned} S^T + m(\nu) &= 0: L_l = const., \quad l = e, \mu, \tau; \\ L &\equiv L_e + L_\mu + L_\tau = const. \end{aligned}$$

All compelling data compatible with 3- ν mixing:

$$\nu_{l\text{L}} = \sum_{j=1}^3 U_{lj} \nu_{j\text{L}} \quad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary to a good approximation (at least: $|U_{l,n}| \lesssim (<<)0.1$, $l = e, \mu$, $n = 4, 5, \dots$).

ν_j , $m_j \neq 0$: Dirac or Majorana particles.

3- ν mixing: 3-flavour neutrino oscillations possible.

ν_μ , E ; at distance L : $P(\nu_\mu \rightarrow \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \rightarrow \nu_\mu) < 1$

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_l \rightarrow \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$$

Data: the 3 ν s are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 1$ eV.

Three Neutrino Mixing

$$\nu_{iL} = \sum_{j=1}^3 U_{ij} \nu_{jL} .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- $U - n \times n$ unitary:

$$\begin{matrix} n & & 2 & 3 & 4 \\ & & 1 & 3 & 6 \end{matrix}$$

mixing angles:

CP-violating phases:

$$\bullet \nu_j - \text{Dirac: } \frac{1}{2}(n-1)(n-2) \quad 0 \quad 1 \quad 3$$

$$\bullet \nu_j - \text{Majorana: } \frac{1}{2}n(n-1) \quad 1 \quad 3 \quad 6$$

$n = 3$: 1 Dirac and

2 additional CP-violating phases, Majorana phases

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C(\bar{\chi}_k(x))^\top = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

- Invariant under proper Lorentz transformations.
- Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1) : \chi_k(x) \rightarrow e^{i\alpha} \chi_k(x) - \text{impossible}$$

- $\chi_k(x)$ cannot absorb phases.
- $Q_{U(1)} = 0 : Q_{\text{el}} = 0, L_t = 0, L = 0, \dots$
- $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
- $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$ –Dirac, $\chi(x)$ –Majorana

$$\langle 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0\rangle = S_{\alpha\beta}^F(x-y) ,$$

$$\langle 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0\rangle = 0 , \quad \langle 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0\rangle = 0 .$$

$$\langle 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0\rangle = S_{\alpha\beta}^F(x-y) ,$$

$$\langle 0|T(\chi_\alpha(x)\chi_\beta(y))|0\rangle = -\xi^* S_{\alpha\kappa}^F(x-y) C_{\kappa\beta} ,$$

$$\langle 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0\rangle = \xi C_{\alpha\kappa}^{-1} S_{\kappa\beta}^F(x-y)$$

$$U_{CP} \ \chi(x) \ U_{CP}^{-1} = \eta_{CP} \ \gamma_0 \ \chi(x') , \quad \eta_{CP} = \pm i .$$

PMNS Matrix: Standard Parametrization

$$U = V P, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_1}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_3}{2}} \end{pmatrix},$$

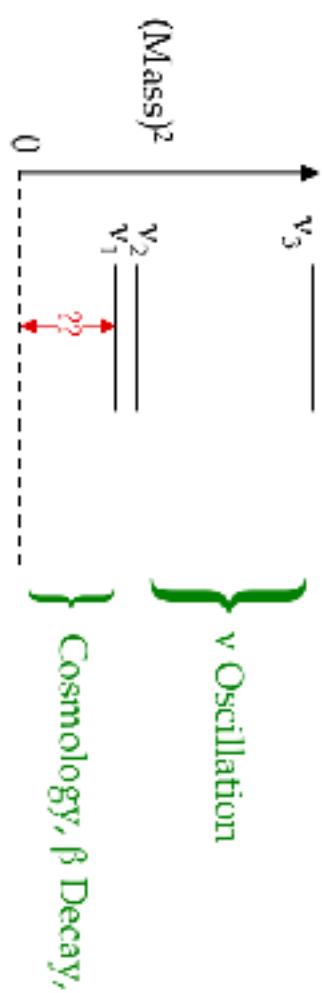
$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, $\theta_{ij} \equiv [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21} , α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2, \dots$
S.M. Bilenky, J. Hosek, S.T.P., 1980
- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$, $\sin^2 \theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m_{31(32)}^2| \cong 2.47$ (2.42) $\times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0234$ (0.0240), NO (IO).

- Fogli et al., Phys. Rev. D86 (2012) 013012, global analysis, b.f.v.: $\sin^2 \theta_{13} = 0.0241$ (0.0244), NO (IO).
- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5}$ eV $^2 > 0$, $\sin^2 \theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m_{31(32)}^2| \cong 2.47$ (2.42) $\times 10^{-3}$ eV 2 , $\sin^2 \theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0234$ (0.0240), NO (IO).
- $1\sigma(\Delta m_{21}^2) = 2.6\%$, $1\sigma(\sin^2 \theta_{12}) = 5.4\%$;
- $1\sigma(|\Delta m_{31(23)}^2|) = 2.6\%$, $1\sigma(\sin^2 \theta_{23}) = 9.6\%$;
- $1\sigma(\sin^2 \theta_{13}) = 8.5\%$;
- $3\sigma(\Delta m_{21}^2)$: $(6.99 - 8.18) \times 10^{-5}$ eV 2 ; $3\sigma(\sin^2 \theta_{12})$: $(0.259 - 0.359)$;
- $3\sigma(|\Delta m_{31(23)}^2|)$: $2.27(2.23) - 2.65(2.60) \times 10^{-3}$ eV 2 ;
 $3\sigma(\sin^2 \theta_{23})$: $0.374(0.380) - 0.628(0.641)$;
- $3\sigma(\sin^2 \theta_{13})$: $0.0176(0.0178) - 0.0295(0.0298)$.

Absolute Neutrino Mass Scale

The Absolute Scale of Neutrino Mass



How far above zero
is the whole pattern?

$$\text{Oscillation Data} \Rightarrow \sqrt{\Delta m_{\text{atm}}^2} < \text{Mass[Heaviest } v_i\text{]}$$

- $\text{sgn}(\Delta m_{\text{atm}}^2) = \text{sgn}(\Delta m_{31(32)}^2)$ not determined

$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 > 0$, normal mass ordering (NO)

$\Delta m_{\text{atm}}^2 \equiv \Delta m_{32}^2 < 0$, inverted mass ordering (IO)

Convention: $m_1 < m_2 < m_3$ - NO, $m_3 < m_1 < m_2$ - IO

$$\Delta m_{31}^2(\text{NO}) = -\Delta m_{32}^2(\text{IO})$$

$$m_1 \ll m_2 < m_3, \quad \text{NH},$$

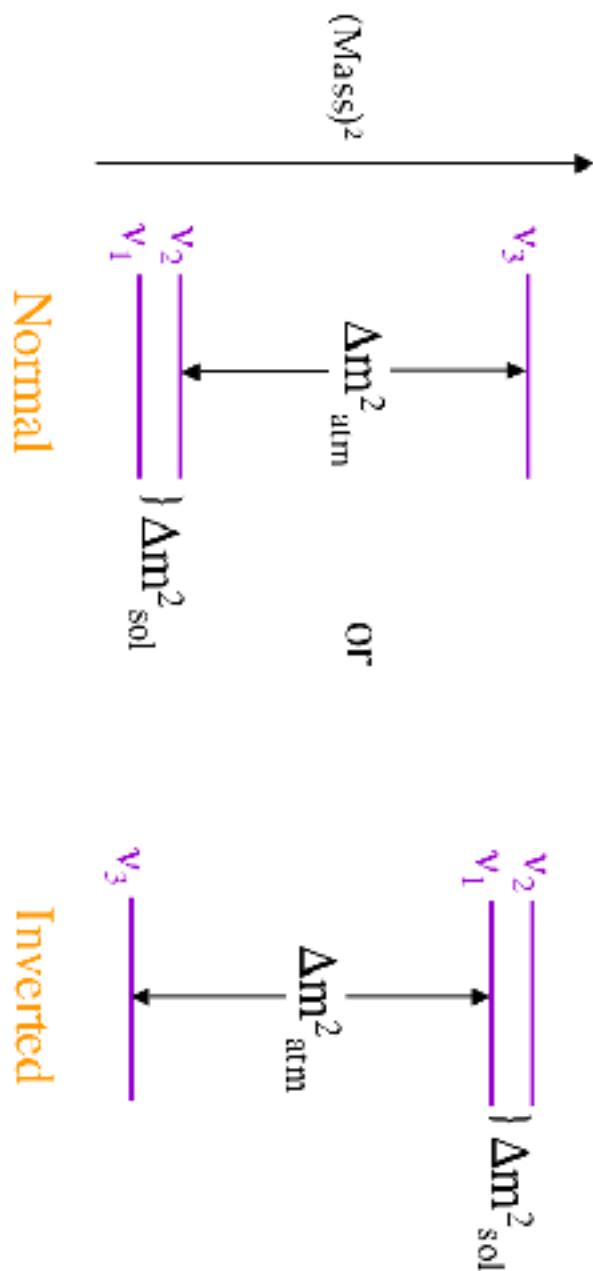
$$m_3 \ll m_1 < m_2, \quad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 >> \Delta m_{\text{atm}}^2, \quad \text{QD}; \quad m_j \gtrsim 0.10 \text{ eV}.$$

$$\bullet m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{31}^2} - \text{NO};$$

$$\bullet m_1 = \sqrt{\frac{m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2}{m_3^2 + \Delta m_{23}^2}}, \quad m_2 = \sqrt{\frac{m_3^2 + \Delta m_{23}^2}{m_3^2 + \Delta m_{23}^2}} - \text{IO};$$

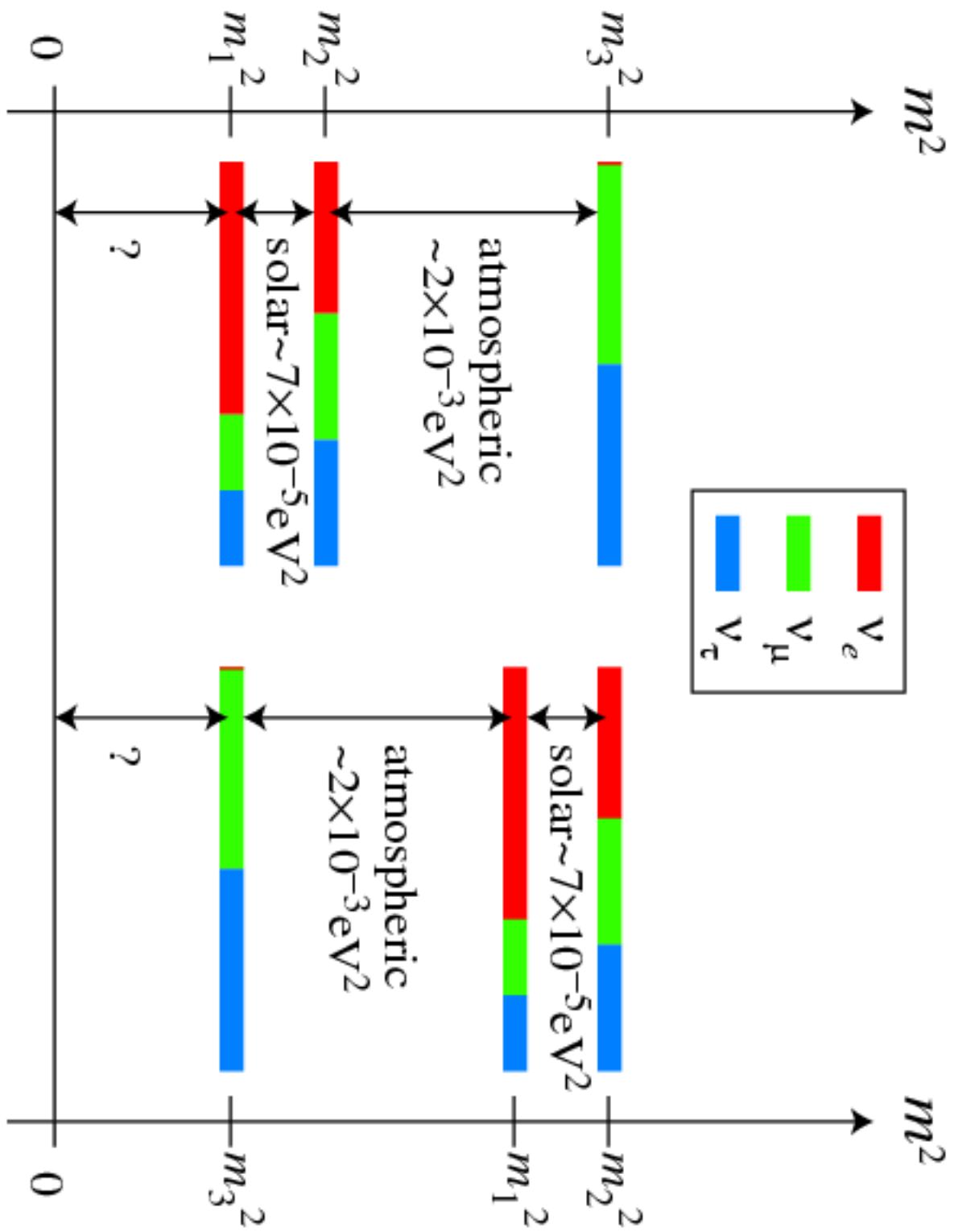
The $(\text{Mass})^2$ Spectrum



$$\Delta m_{\text{sol}}^2 \approx 7.6 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{\text{atm}}^2 \approx 2.4 \times 10^{-3} \text{ eV}^2$$

Are there *more* mass eigenstates, as LSND suggests, and MiniBooNE recently hints?

3



- Dirac phase δ : $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A_{CP}^{(ll')} \propto J_{CP} \propto \sin \theta_{13} \sin \delta$;

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

Current data: $|J_{CP}| \lesssim 0.040$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$:
 $J_{CP} \cong -0.030$.

- Majorana phases α_{21}, α_{31} :

– $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P., 1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

- $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay depends on α_{21}, α_{31} ;
- $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
- BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

Future Progress

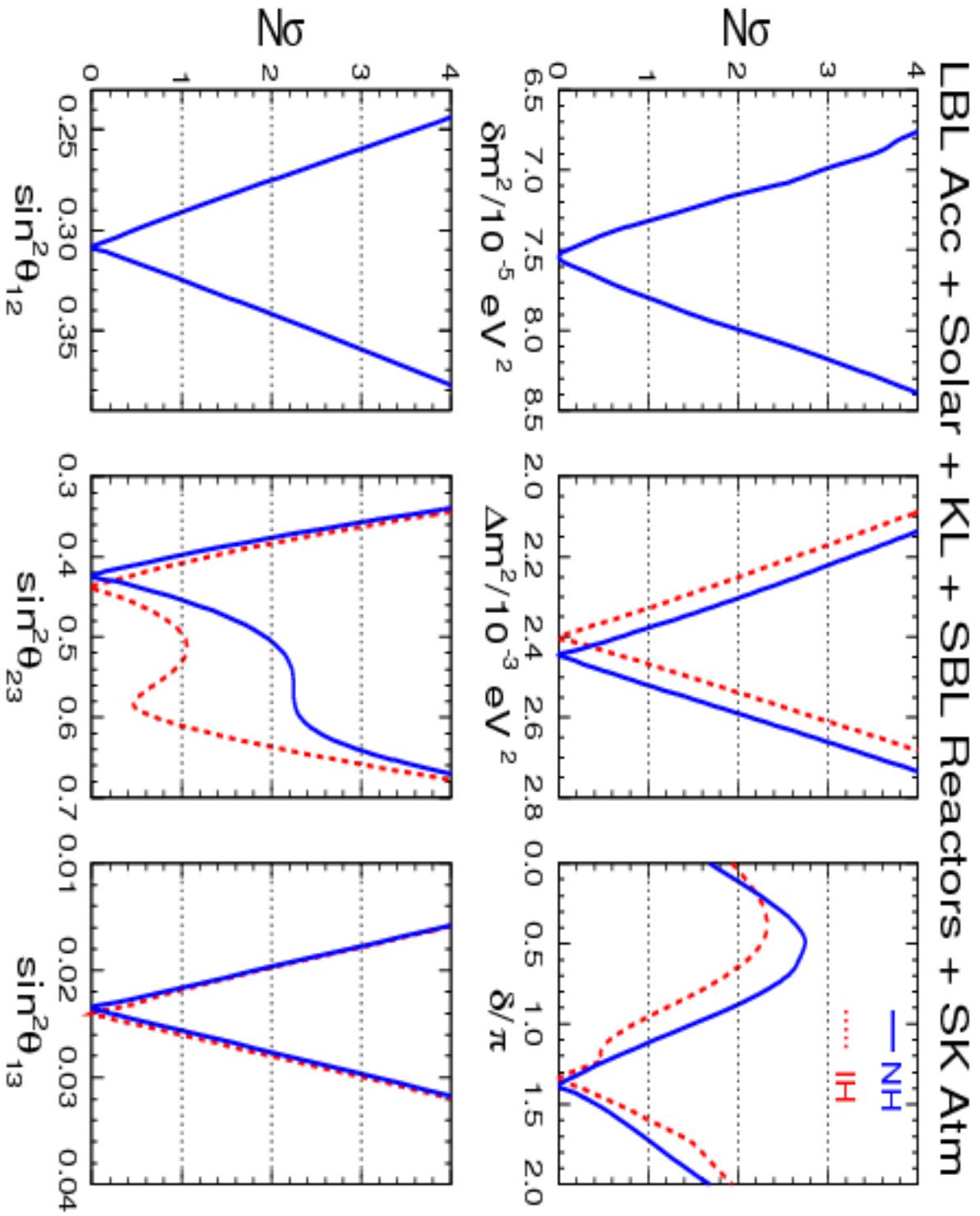
- Determination of the nature - Dirac or Majorana, of ν_j .
- Determination of $\text{sgn}(\Delta m_{\text{atm}}^2)$, type of ν - mass spectrum
 - $m_1 \ll m_2 \ll m_3$, NH,
 - $m_3 \ll m_1 < m_2$, IH,
- $m_1 \cong m_2 \cong m_3$, $m_{1,2,3}^2 >> \Delta m_{\text{atm}}^2$, QD; $m_j \gtrsim 0.10$ eV.
- Determining, or obtaining significant constraints on, the absolute scale of ν_j - masses, or $\min(m_j)$.
- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21} , α_{31} (Majorana)?
- High precision determination of Δm_{\odot}^2 , θ_{12} , Δm_{atm}^2 , θ_{23} , θ_{13}
- Searching for possible manifestations, other than ν -oscillations, of the non-conservation of L_l , $l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. decays.

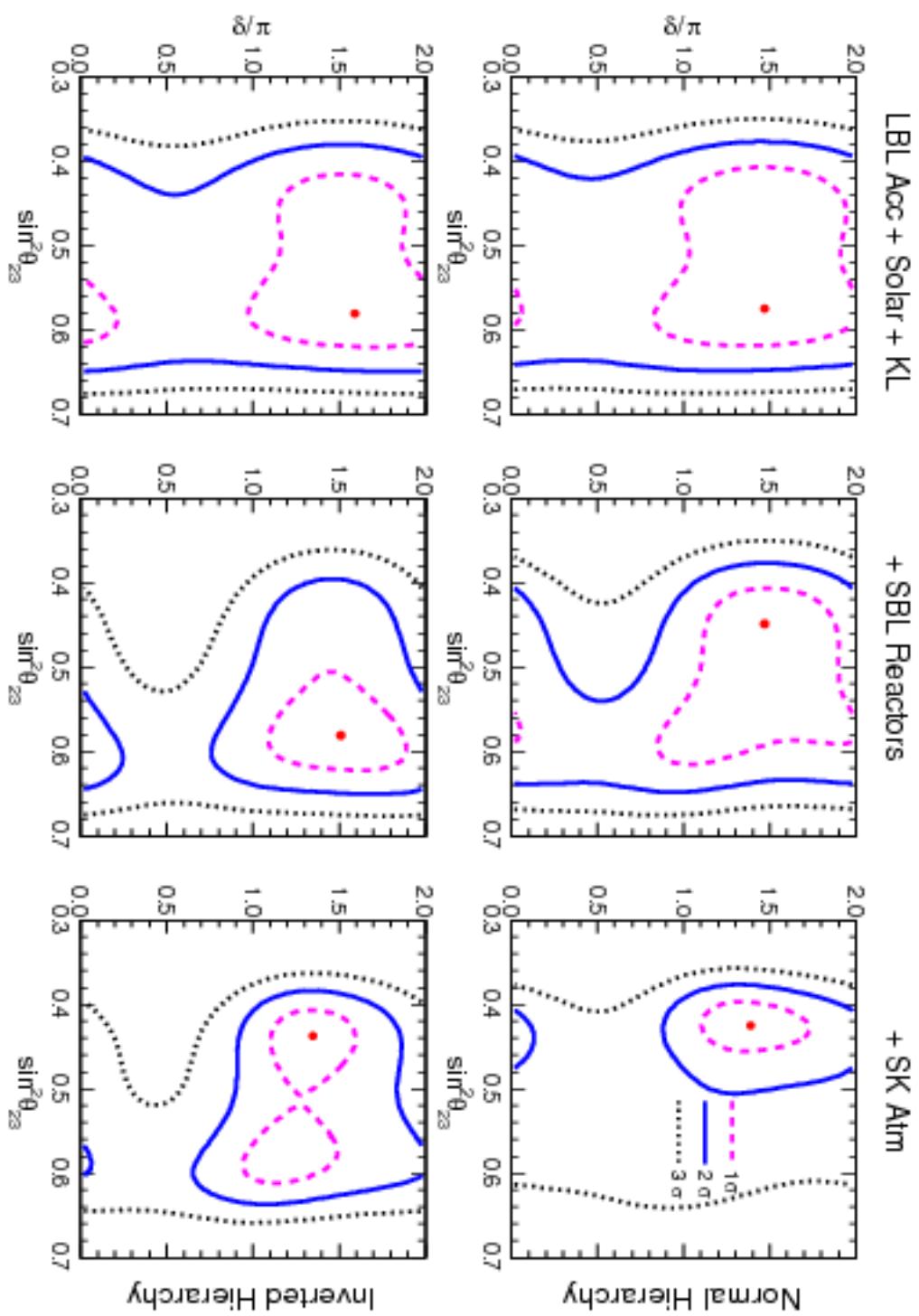
- Understanding at fundamental level the mechanism giving rise to the ν – masses and mixing and to the L_i –non-conservation. Includes understanding
 - the origin of the observed patterns of ν -mixing and ν -masses ;
 - the physical origin of CPV phases in U_{PMNS} ;
 - Are the observed patterns of ν -mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?
 - Is there any relations between q –mixing and ν – mixing? Is $\theta_{12} + \theta_c = \pi/4$?
 - Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?
 - Is there any correlation between the values of CPV phases and of mixing angles in U_{PMNS} ?
- Progress in the theory of ν -mixing might lead to a better understanding of the origin of the BAU.
 - Can the Majorana and/or Dirac CPV in U_{PMNS} be the leptogenesis CPV parameters at the origin of BAU?

The next most important steps are:

- determination of the nature - Dirac or Majorana, of massive neutrinos.
- determination of the neutrino mass hierarchy;
- determination of the absolute neutrino mass scale (or $\min(m_j)$);
- determination of the status of the CP symmetry in the lepton sector.

Hints for Dirac CP Violation: $\delta \cong 3\pi/2$





Large $\sin \theta_{13} \cong 0.15$ (Daya Bay, RENO) + $\delta = 3\pi/2$ - far-reaching implications:

- For the searches for CP violation in ν -oscillations; for the b.f.v. one has $J_{CP} \cong -0.030$;
- Important implications also for the "flavoured" leptonogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

If all CPV, necessary for the generation of BAU is due to δ , a necessary condition for reproducing the observed BAU is

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.09$$

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}, \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP α_{21}, α_{31}

CP-Invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}), \quad l \neq l' \stackrel{\text{V}}{=} e, \mu, \tau$$

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

$$l = l': \quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-Invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

3 ν -mixing:

$$A_{\text{CP}}^{(ll')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}), \quad l \neq l' \stackrel{\text{V}}{=} e, \mu, \tau$$

$$A_{\text{T}}^{(ll')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}), \quad l \neq l' \stackrel{\text{V}}{=} e, \mu, \tau$$

$$A_{\text{T(CP)}}^{(e,\mu)} = A_{\text{T(CP)}}^{(\mu,\tau)} = -A_{\text{T(CP)}}^{(e,\tau)}$$

In vacuum: $A_{CP(T)}^{(e,\mu)} = J_{CP} F_{osc}^{vac}$

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

$$F_{osc}^{vac} = \sin(\frac{\Delta m_{21}^2}{2E}L) + \sin(\frac{\Delta m_{32}^2}{2E}L) + \sin(\frac{\Delta m_{13}^2}{2E}L)$$

In matter: Matter effects violate

$$CP: \quad P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$$

$$CPT: \quad P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

P. Langacker et al., 1987

Can conserve the T-invariance (**Earth**)

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

In matter with constant density: $A_T^{(e,\mu)} = J_{CP}^{\text{mat}} F_{osc}^{\text{mat}}$

$$J_{CP}^{\text{mat}} = J_{CP}^{\text{vac}} R_{CP}$$

R_{CP} does not depend on θ_{23} and δ ; $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

Rephasing Invariants Associated with CPVP

Dirac phase δ :

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21} , α_{31} :

$$\begin{aligned} S_1 &= \text{Im} \{ U_{e1} U_{e3}^* \}, & S_2 &= \text{Im} \{ U_{e2} U_{e3}^* \} && (\text{not unique}); \quad \text{or} \\ S'_1 &= \text{Im} \{ U_{\tau 1} U_{\tau 2}^* \}, & S'_2 &= \text{Im} \{ U_{\tau 2} U_{\tau 3}^* \} \end{aligned}$$

J.F. Nieves and P. Pal, 1987, 2001

G.C. Branco et al., 1986

J.A. Aguilar-Saavedra and G.C. Branco, 2000

CP-violation: both $\text{Im} \{ U_{e1} U_{e3}^* \} \neq 0$ and $\text{Re} \{ U_{e1} U_{e3}^* \} \neq 0$.

S_1 , S_2 appear in $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay.

In general, J_{CP} , S_1 and S_2 are independent.

- March 8, 2012, Daya Bay: 5.2σ evidence for $\theta_{13} \neq 0$,
 $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$.

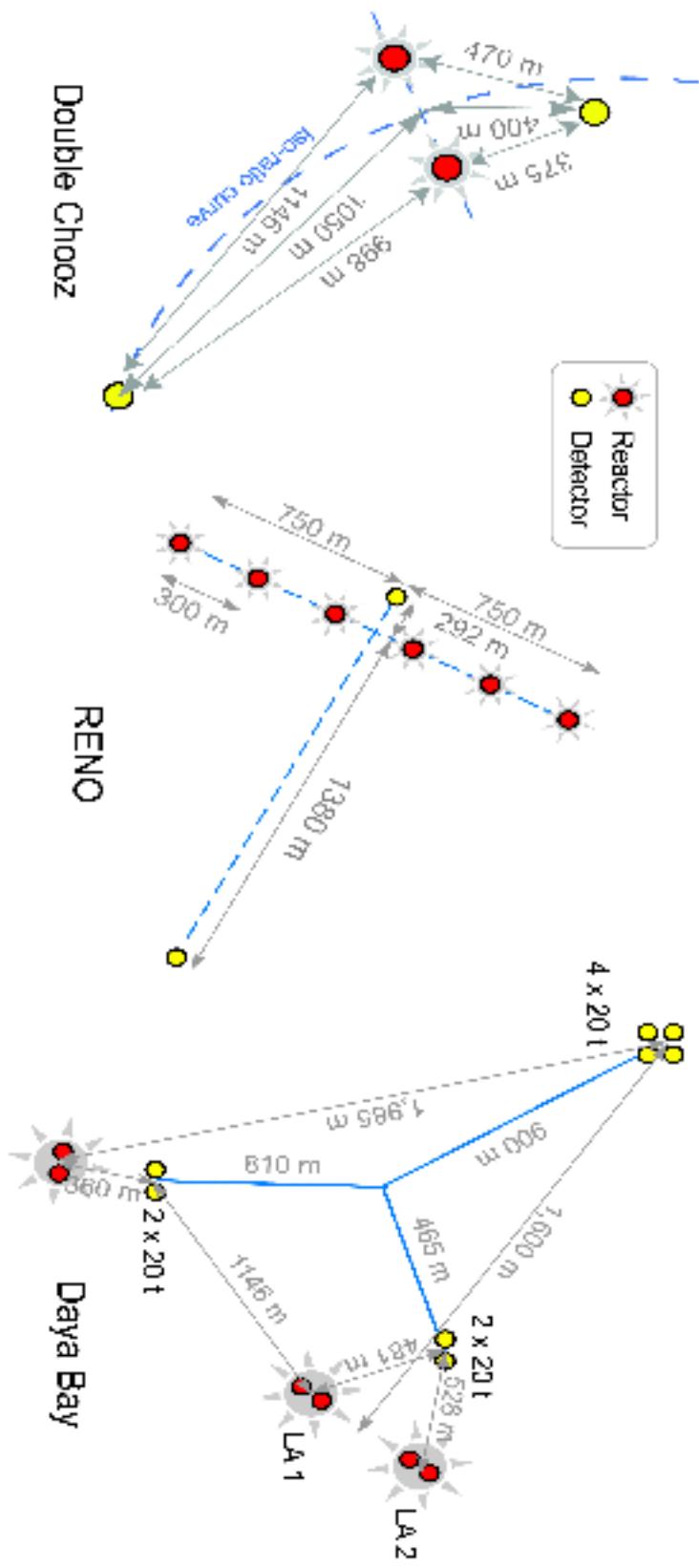
- April 4, 2012, RENO: 4.9σ evidence for $\theta_{13} \neq 0$,
 $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$.

- Nu'2012 (June 4-9, 2012), T2K, Double Chooz: 3.2σ
and 2.9σ evidence for $\theta_{13} \neq 0$.

- Daya Bay, 23/08/2013:
 $\sin^2 2\theta_{13} = 0.090 \pm 0.009$.

- RENO, 12/09/2013 (TAUP 2013):
 $\sin^2 2\theta_{13} = 0.100 \pm 0.010$ (*stat.*) ± 0.012 .

$$P^{3\nu}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P^{3\nu}(\theta_{13}, \Delta m_{31}^2(32); \theta_{12}, \Delta m_{21}^2) \cong \\ 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{31}^2(32)}{4E} L \right), \text{ no dependence on } \theta_{23}, \delta.$$



M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]



T2K: Search for $\nu_\mu \rightarrow \nu_e$ oscillations

T2K: first results March 2011 (2 events);
June 14, 2011 (6 events): evidence for $\theta_{13} \neq 0$ at 2.5σ ;
July, 2013 (28 events).

For $|\Delta m_{23}^2| = 2.4 \times 10^{-3}$ eV 2 , $\sin^2 2\theta_{23} = 1$, $\delta = 0$, NO
(IO) spectrum:

$$\sin^2 2\theta_{13} = 0.14 \text{ (1.7), best fit.}$$

This value is by a factor of ~ 1.6 (1.9) bigger than the
value obtained in the Daya Bay and RENO experiments.

$$P_m^{3\nu}(\nu_\mu \rightarrow \nu_e) = P_m^{3\nu}(\theta_{13}, \Delta m_{31}^2(32), \theta_{12}, \Delta m_{21}^2, \theta_{23}, \delta).$$

Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \ll 1$ and $\sin^2 \theta_{13} \ll 1$:

$$P_m^{3\nu\ man}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = -\alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{man} \frac{2E}{\Delta m_{31}^2}.$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$

Predictions for the CPV Phase δ

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_\odot \cong \frac{\pi}{5.4}$, $\theta_{23} = \theta_{\text{atm}} \cong \frac{\pi}{4}(?)$, $\theta_{13} \cong \frac{\pi}{20}$

$$U_{\text{PMNS}} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \pi/4 - 0.20$, $\theta_{13} \cong 0 + \pi/20$, $\theta_{23} \cong \pi/4 - 0.10$.
- U_{PMNS} due to new approximate symmetry?

A Natural Possibility (vast literature):

$$U = U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \psi) Q(\phi, \varphi) U_{\text{TBM}, \text{BM}, \text{LC}} P(\alpha_{21}, \alpha_{31}),$$

with

$$U_{\text{TBM}} = \begin{pmatrix} \frac{\sqrt{2}}{3} & \frac{\sqrt{1}}{3} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \psi)$ - from diagonalization of the l^- mass matrix;
- $U_{\text{TBM}, \text{BM}, \text{LC}}$ $P(\alpha_{21}, \alpha_{31})$ - from diagonalization of the ν mass matrix;
- $Q(\phi, \varphi)$, - from diagonalization of both the l^- and ν mass matrices.

$U_{\text{TBM(BM)}}$: Groups A_4 , S_4 , T' , ... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552;
S. King and Ch. Luhn, arXiv:1301.1340)

• $U_{\text{LC(BM)}}$: alternatively $U(1)$, $L' = L_e - L_\mu - L_\tau$

S. T.P., 1982

- U_{TBM} : $s_{12}^2 = 1/3$, $s_{23}^2 = 1/2$, $s_{13}^2 = 0$; $s_{13}^2 = 0$ must be corrected; if $\theta_{23} \neq \pi/4$, $s_{23}^2 = 0.5$ must be corrected
- U_{BM} : $s_{12}^2 = 1/2$, $s_{23}^2 = 1/2$, $s_{13}^2 = 0$; $s_{13}^2 = 0$, $s_{12}^2 = 1/2$ and possibly $s_{23}^2 = 1/2$ must be corrected.
- U_{LC} : $s_{12}^2 = 1/2$, $s_{13}^2 = 0$, s_{23}^2 - free parameter;
 $s_{13}^2 = 0$ and $s_{12}^2 = 1/2$ must be corrected.

None of the symmetries leading to U_{TBM} , U_{BM} or other approximate forms of U_{PMNS} can be exact.

Which is the correct approximate symmetry, i.e., approximate form of U_{PMNS} (if any)?

In the two cases of U_ν given by U_{TBM} , or U_{BM} , the requisite corrections of some of the mixing angles are small and can be considered as perturbations to the corresponding symmetry values.

Depending on the symmetry leading to $U_{\text{TBM,BM}}$ and on the form of U_{lep} , one obtains different experimentally testable predictions for the sum of the neutrino masses, the neutrino mass spectrum, the nature (Dirac or Majorana) of ν_j and the CP violating phases in the neutrino mixing matrix. Future data will help us understand whether there is some new fundamental symmetry behind the observed patterns of neutrino mixing and Δm_{ij}^2 .

Predictions for δ

Assume:

- $U_{PMNS} = U_{\text{lept}}^\dagger(\theta_{ij}^\ell, \psi) Q(\phi, \varphi) U_{\text{TBM}, \text{BM}} P(\alpha_{21}, \alpha_{31})$,
- U_{lept}^\dagger - minimal, such that
 - i) $\sin \theta_{13} \cong 0.16$; BM: $\sin^2 \theta_{12} \cong 0.31$;
 - ii) $\sin^2 \theta_{23}$ can deviate significantly (by more than $\sin^2 \theta_{13}$) from 0.5 (b.f.v. = 0.42-0.43).

From i), ii) + $m_e << m_\mu << m_\tau$:

$$U_{\text{lept}}^\dagger(\theta_{ij}^\ell, \psi) = R_{12}(\theta_{12}^\ell) R_{23}(\theta_{23}^\ell), \quad Q(\phi, \varphi) = \text{diag}(1, e^{i\phi}, 1)$$

Leads to $\delta = \delta(\theta_{12}, \theta_{23}, \theta_{13})$ - new sum rules for δ !

For U_{TBM} :

$$\cos \delta = \frac{\tan \theta_{23}}{3 \sin 2\theta_{12} \sin \theta_{13}} [1 + (3 \sin^2 \theta_{12} - 2)(1 - \cot^2 \theta_{23} \sin^2 \theta_{13})]$$

For $U_{\text{TBM}} + \text{b.f.v.}$ of $\theta_{12}, \theta_{23}, \theta_{13}$:

$$\delta \cong 3\pi/2 \text{ or } \pi/2 \quad (\delta = 266^\circ \text{ or } \delta = 94^\circ)$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevilla, arXiv:1302.

T' model of lepton flavour: U_{TBM} , $\delta \cong 3\pi/2$ or $\pi/2$.

I. Girardi, A. Meroni, S.T.P., M. Spinrath, arXiv:1312.1966

For U_{BM} :

$$\cos \delta = -\frac{1}{2 \sin \theta_{13}} \cot 2\theta_{12} \tan \theta_{23} (1 - \cot^2 \theta_{23} \sin^2 \theta_{13}).$$

For $U_{\text{BM}} + \text{b.f.v.}$ of $\theta_{12}, \theta_{23}, \theta_{13}$:

$$\delta \cong \pi$$

Determining the ν –Mass Hierarchy ($\text{sgn}(\Delta m_{\text{atm}}^2)$)

- Reactor $\bar{\nu}_e$ Oscillations in vacuum (JUNO, RENO50).
- Atmospheric ν experiments: subdominant $\nu_{\mu(e)} \rightarrow \nu_{\alpha(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations (matter effects) (HK, ORCA, PINGU (IceCube), INO).
- LBL ν –oscillation experiments (T2K, NO $_\nu$ A; LBNO, LBNE, ν –factory); designed to search also for CP violation.
- ${}^3\text{H}$ β -decay Experiments (sensitivity to 5×10^{-2} eV) (NH vs IH).
- $(\beta\beta)_{0\nu}$ –Decay Experiments; ν_j – Majorana particles (NH vs IH).
- Cosmology: $\sum_j m_j$ (NH vs IH).
- Atomic Physics Experiments: RENP.

Reactor $\bar{\nu}_e$ Oscillations in vacuum

$$P_{\text{NO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left(1 - \cos \frac{\Delta m_A^2 L}{2E_\nu} \right) - \frac{1}{2} \cos^4 \theta_{13} \sin^2 2\theta_\odot \left(1 - \cos \frac{\Delta m_\odot^2 L}{2E_\nu} \right)$$
$$+ \sin^2 2\theta_{13} \sin^2 \theta_\odot \sin \frac{\Delta m_\odot^2 L}{4E_\nu} \sin \left(\frac{\Delta m_A^2 L}{2E_\nu} - \frac{\Delta m_\odot^2 L}{4E_\nu} \right),$$

$$P_{\text{IO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left(1 - \cos \frac{\Delta m_A^2 L}{2E_\nu} \right) - \frac{1}{2} \cos^4 \theta_{13} \sin^2 2\theta_\odot \left(1 - \cos \frac{\Delta m_\odot^2 L}{2E_\nu} \right)$$
$$+ \sin^2 2\theta_{13} \cos^2 \theta_\odot \sin \frac{\Delta m_\odot^2 L}{4E_\nu} \sin \left(\frac{\Delta m_A^2 L}{2E_\nu} - \frac{\Delta m_\odot^2 L}{4E_\nu} \right),$$

$\theta_\odot = \theta_{12}$, $\Delta m_\odot^2 = \Delta m_{21}^2 > 0$; $\sin^2 \theta_{12} \leq 0.36$ at 3σ ;

$\Delta m_A^2 = \Delta m_{31}^2 > 0$, NO spectrum,

$\Delta m_A^2 = \Delta m_{23}^2 > 0$, IO spectrum

The reactor $\bar{\nu}_e$ detected via

$$\bar{\nu}_e + p \rightarrow e^+ + n.$$

The visible energy of the detected e^+ :

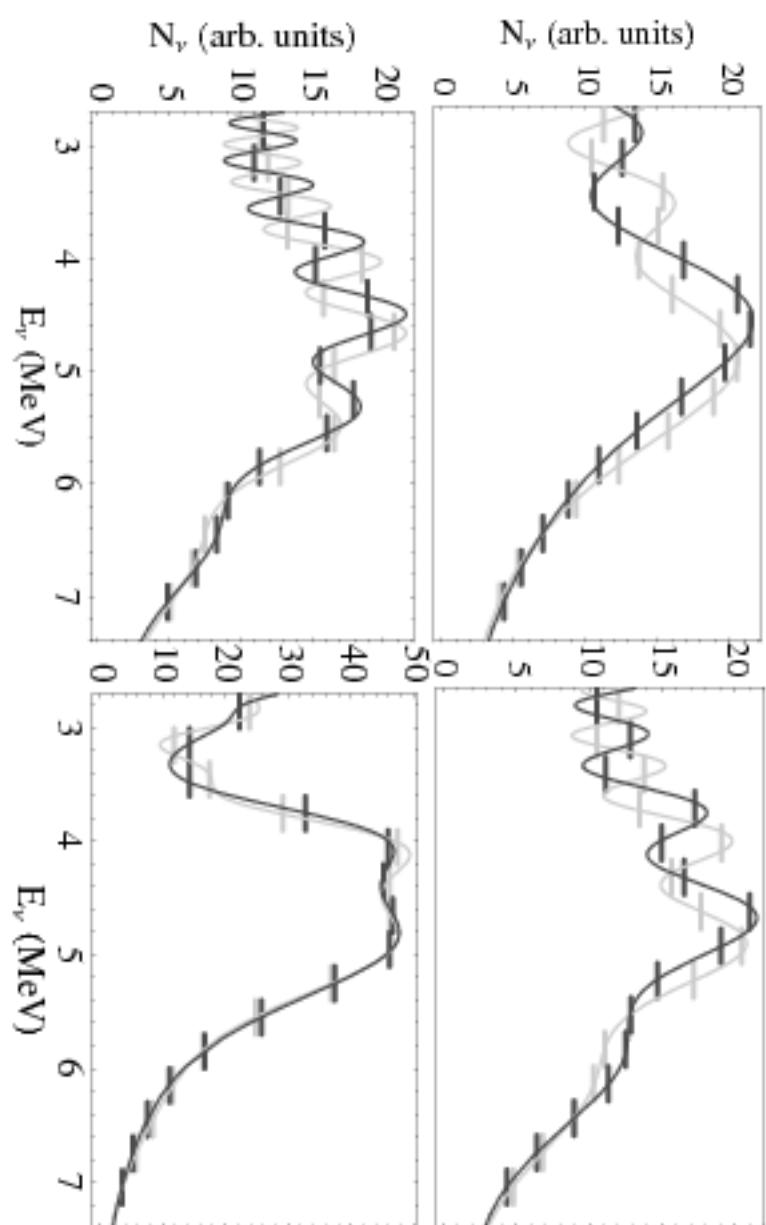
$$E_{vis} = E + m_e - (m_n - m_p) \simeq E - 0.8 \text{ MeV}.$$

The measured event rate spectrum vs. L/E_m :

$$N(L/E_m) = \int R(E, E_m) \Phi(E) \sigma(\bar{\nu}_e p \rightarrow e^+ n; E) P_{ee}^{NO(IO)} dE.$$

$$|P_{\text{NO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) - P_{\text{IO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e)| \propto \sin^2 2\theta_{13} \cos 2\theta_{12}$$

$$\cos 2\theta_{12} \cong 0.38; \quad 3\sigma : \cos 2\theta_{12} \geq 0.28; \quad \sin^2 2\theta_{13} \cong 0.09.$$



M. Piai, S.T.P., 2001

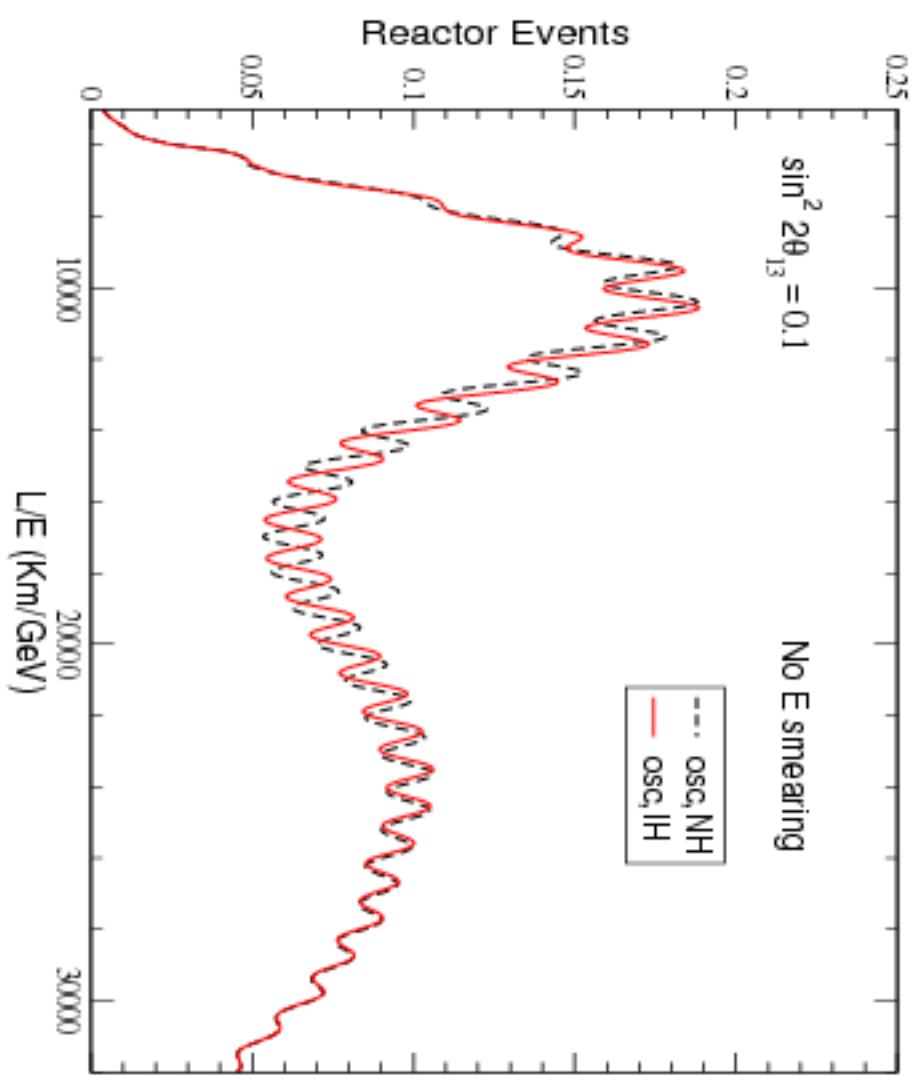
$\sin^2 \theta_{13} = 0.05$, $\Delta m_{21}^2 = 2 \times 10^{-4}$ eV 2 ; $\Delta m_A^2 = 1.3$; 2.5 ; 3.5×10^{-3} eV 2

$L = 20$ km, $\Delta E_\nu = 0.3$ MeV.

$\Delta m_{21}^2 = 2 \times 10^{-4}$ eV 2 ; $L = 20$ km;

$\Delta m_{21}^2 = 7.6 \times 10^{-5}$ eV 2 ; $L \cong 53$ km.

NO – light grey; IO – dark grey



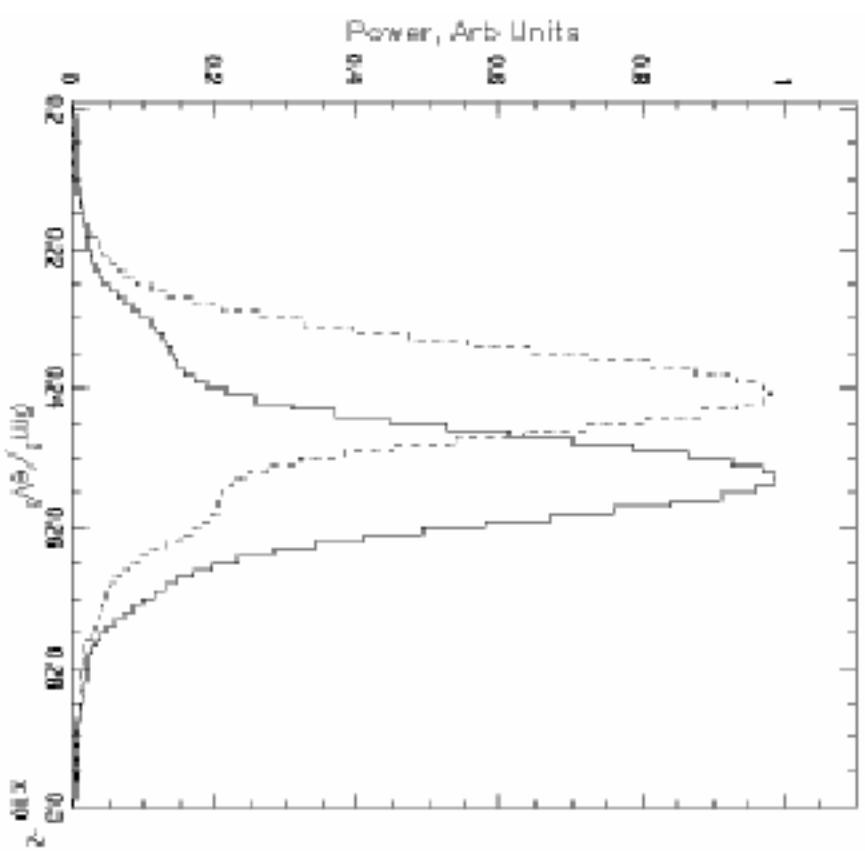
Fourier Analysis:

$$NO : \cos^2 \theta_{12} \sin^2 \Delta + \sin^2 \theta_{12} \sin^2(\Delta - \Delta_{21}),$$

$$IO : \sin^2 \theta_{12} \sin^2 \Delta + \cos^2 \theta_{12} \sin^2(\Delta - \Delta_{21}),$$

$$\Delta \equiv \Delta_{31}(NO) = |\Delta_{32}(IO)|;$$

$$\sin^2 \theta_{12} \cong 0.31, \quad \cos^2 \theta_{12} \cong 0.69.$$



Very challenging; requires:

- energy resolution $\sigma/E_{\text{vis}} \lesssim 3\%/\sqrt{E_{\text{vis}}}$;
- relatively small energy scale uncertainty;
- relatively large statistics ($\sim (300 - 1000)$ kT GW yr);
- relatively small systematic errors;
- subtle optimisations (distance, number of bins, effects of “interfering distant” reactors).

Two experiments planned with $L \cong 50$ km: Juno (20 kT, approved), RENO50 (18 kT). Can measure also $\sin^2 \theta_{12}$, Δm_{21}^2 and $|\Delta m_{31}^2|$ with remarkably high precision. Can be used for detection of Geo, solar, SN neutrinos as well.

Atmospheric Neutrino Experiments on $\text{sgn}(\Delta m_{31}^2)$

Atmospheric ν experiments

Subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations in the Earth.

$$P_{3\nu}(\nu_e \rightarrow \nu_\mu) \cong P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong s_{23}^2 P_{2\nu}, P_{3\nu}(\nu_e \rightarrow \nu_\tau) \cong c_{23}^2 P_{2\nu},$$
$$P_{3\nu}(\nu_\mu \rightarrow \nu_\mu) \cong 1 - s_{23}^4 P_{2\nu} - 2c_{23}^2 s_{23}^2 [1 - R_e (e^{-ik} A_{2\nu}(\nu_\tau \rightarrow \nu_\tau))],$$

$P_{2\nu} \equiv P_{2\nu}(\Delta m_{31}^2, \theta_{13}; E, \theta_n; N_e)$; 2- ν $\nu_e \rightarrow \nu'_\tau$ oscillations in the Earth,
 $\nu'_\tau = s_{23} \nu_\mu + c_{23} \nu_\tau$; $\Delta m_{21}^2 \ll |\Delta m_{31(32)}^2|$, $E_\nu \gtrsim 2$ GeV;

κ and $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau) \equiv A_{2\nu}$ are known phase and 2- ν amplitude.

NO: $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ matter enhanced, $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ – suppressed

IO: $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ matter enhanced, $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ – suppressed

No charge identification (SK, HK, IceCube-PINGU, ANTARES-ORCA); event rate (DIS regime): $[2\sigma(\bar{\nu}_l + N \rightarrow l^- + X) + \sigma(\bar{\nu}_l + N \rightarrow l^+ + X)]/3$

Neutrino Oscillations in Matter

When neutrinos propagate in matter, they interact with the background of electrons, protons and neutrinos, which generates an effective potential in the neutrino Hamiltonian: $H = H_{vac} + V_{eff}$.

This modifies the neutrino mixing since the eigenstates and the eigenvalues of $H = H_{vac} + V_{eff}$ are different, leading to a different oscillation probability w.r.t to that in vacuum.

Typically the matter background is not CP and CPT symmetric, e.g., the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations violate CP and CPT symmetries.

$$P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

$$\sin^2 2\theta_{13}^m, \Delta M_{31}^2 \text{ depend on the matter potential}$$
$$V_{eff} = \sqrt{2} G_F N_e,$$

For antineutrinos V_{eff} has the opposite sign:

$$V_{eff} = - \sqrt{2} G_F, N_e.$$

$\Delta m_{31}^2 > 0$ (NO): $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ matter enhanced,
 $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ suppressed

$\Delta m_{31}^2 < 0$ (IO): $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ matter enhanced,
 $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ suppressed

$$\sin^2 2\theta_{13}^m = \frac{\tan^2 2\theta_{13}}{(1 - \frac{N_e}{N_e^{res}})^2 + \tan^2 2\theta_{13}},$$

$$\cos 2\theta_{13}^m = \frac{1 - N_e/N_e^{res}}{\sqrt{(1 - \frac{N_e}{N_e^{res}})^2 + \tan^2 2\theta_{13}}},$$

$$N_e^{res}$$

$$= \frac{\Delta m_{31}^2 \cos 2\theta_{13}}{2E\sqrt{2}G_F}$$

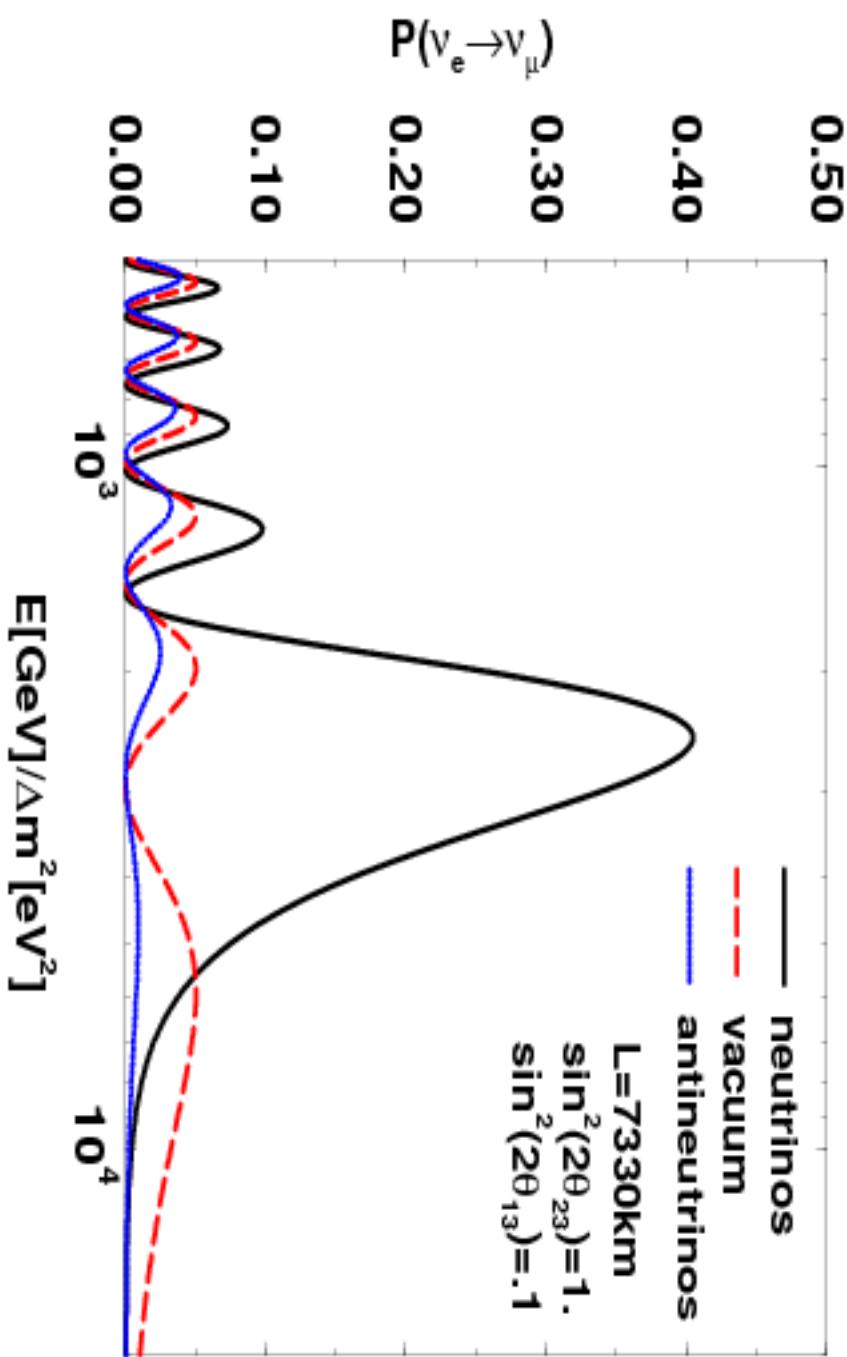
$$6.56 \times 10^6 \frac{\Delta m^2 [\text{eV}^2]}{E[\text{MeV}]} \cos 2\theta \text{ cm}^{-3} \text{ N}_A,$$

$$\frac{\Delta M_{31}^2}{2E} \equiv \frac{\Delta m_{31}^2}{2E} \left((1 - \frac{N_e}{N_e^{res}})^2 \cos^2 2\theta_{13} + \sin^2 2\theta_{13} \right)^{\frac{1}{2}}$$

\approx

For $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$: $N_e \rightarrow (-N_e)$.

Earth matter effect in $\nu_\mu \rightarrow \nu_e$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (MSW)



$\Delta m^2 = 2.5 \times 10^{-3} \text{ eV}^2$, $E^{\text{res}} = 6.25 \text{ GeV}$; $P^{3\nu} = \sin^2 \theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu}$;
 $N_e^{\text{res}} \cong 2.3 \text{ cm}^{-3} \text{ N_A}$; $L_m^{\text{res}} = L^\nu / \sin 2\theta_{13} \cong 6250 / 0.32 \text{ km}$; $2\pi L / L_m \cong 0.75\pi (\neq \pi)$.

HyperKamiokande (10SK), IceCube-PINGU, ANTARES-ORCA;

Iron Magnetised detector: INO

INO: 50 or 100 kt (in India); ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^-); not designed to detect ν_e and $\bar{\nu}_e$ induced events.

IceCube at the South Pole: PINGU

PINGU: 50SK; ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^- , no μ charge identification); Challenge: $E_\nu \gtrsim 2$ GeV (?)

ANTARES in Mediteranian sea: ORCA

Water-Cerenkov detector: Hyper Kamiokande (10SK)

Sensitivity depends critically on θ_{23} , the "true" hierarchy.

J. Bernabeu, S. Palomares-Ruiz, S. T.P., 2003

$$P(\nu_\mu \rightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

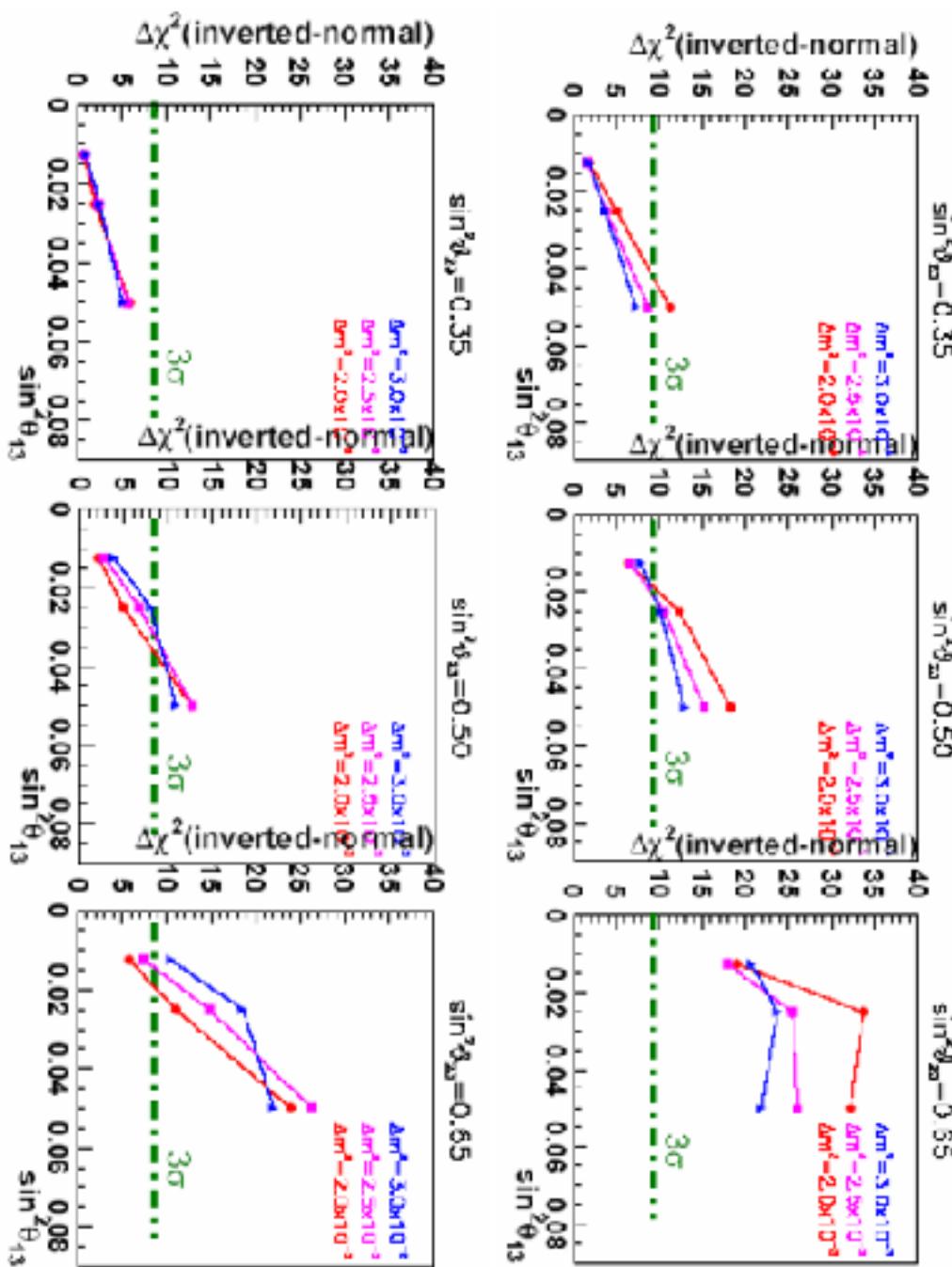
No charge identification (SK, HK, PINGU, ORCA); event rate (DIS regime):

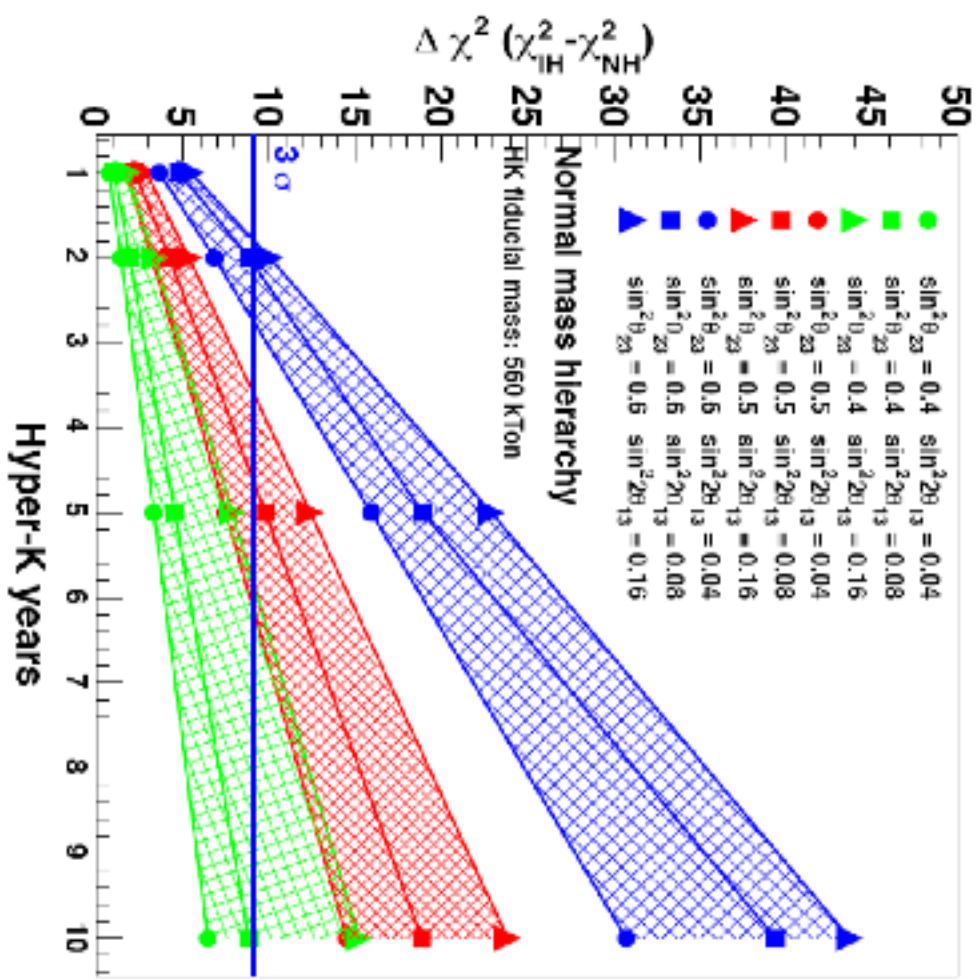
$$[2\sigma(\nu_l + N \rightarrow l^- + X) + \sigma(\bar{\nu}_l + N \rightarrow l^+ + X)]/3$$

Water-Cerenkov detector, 1.8 MTy ($\text{HK} = 10\text{SK}$)

Critical dependence on θ_{23} , "true hierarchy".

T. Kajita et al., 2004





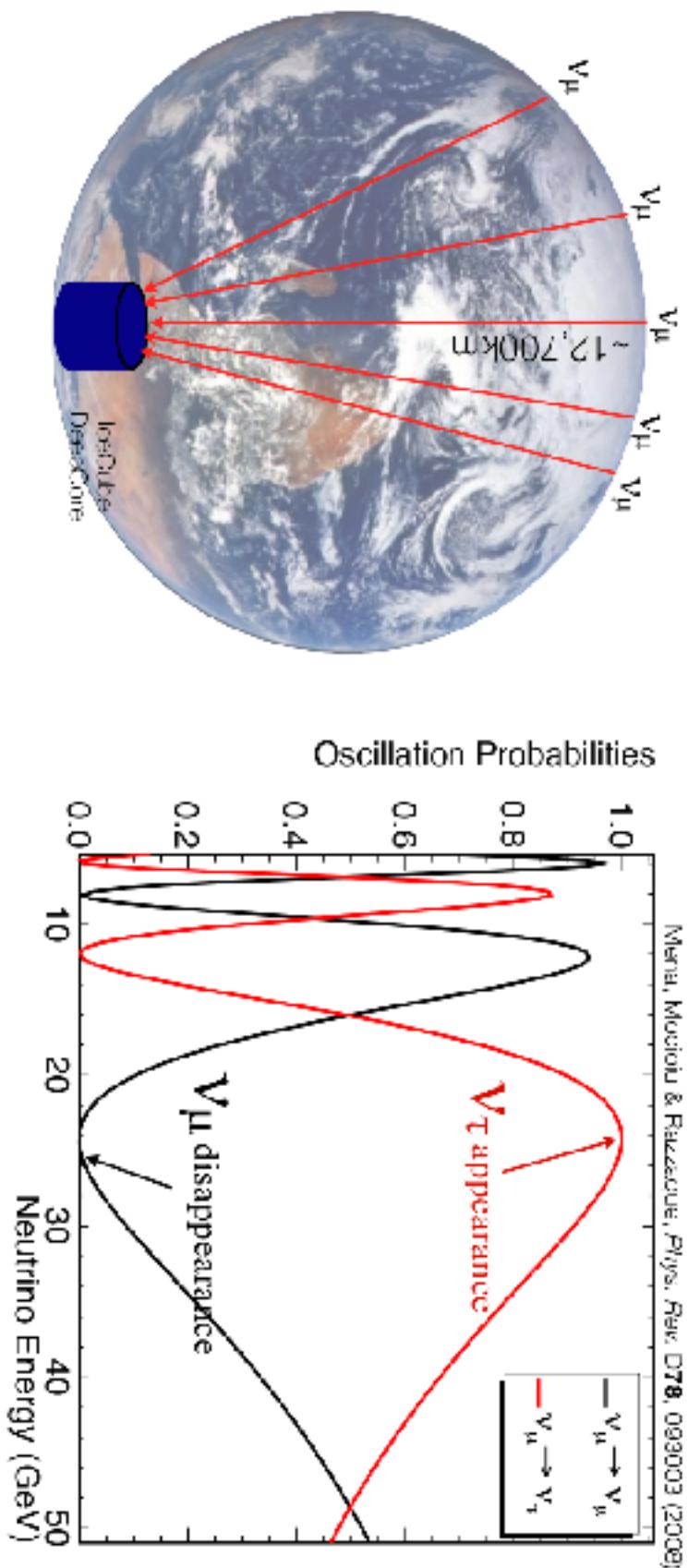
Sensitivity to the neutrino mass hierarchy from HK atmospheric neutrino data. θ_{23} and θ_{13} are assumed to be known as indicated in the figure.

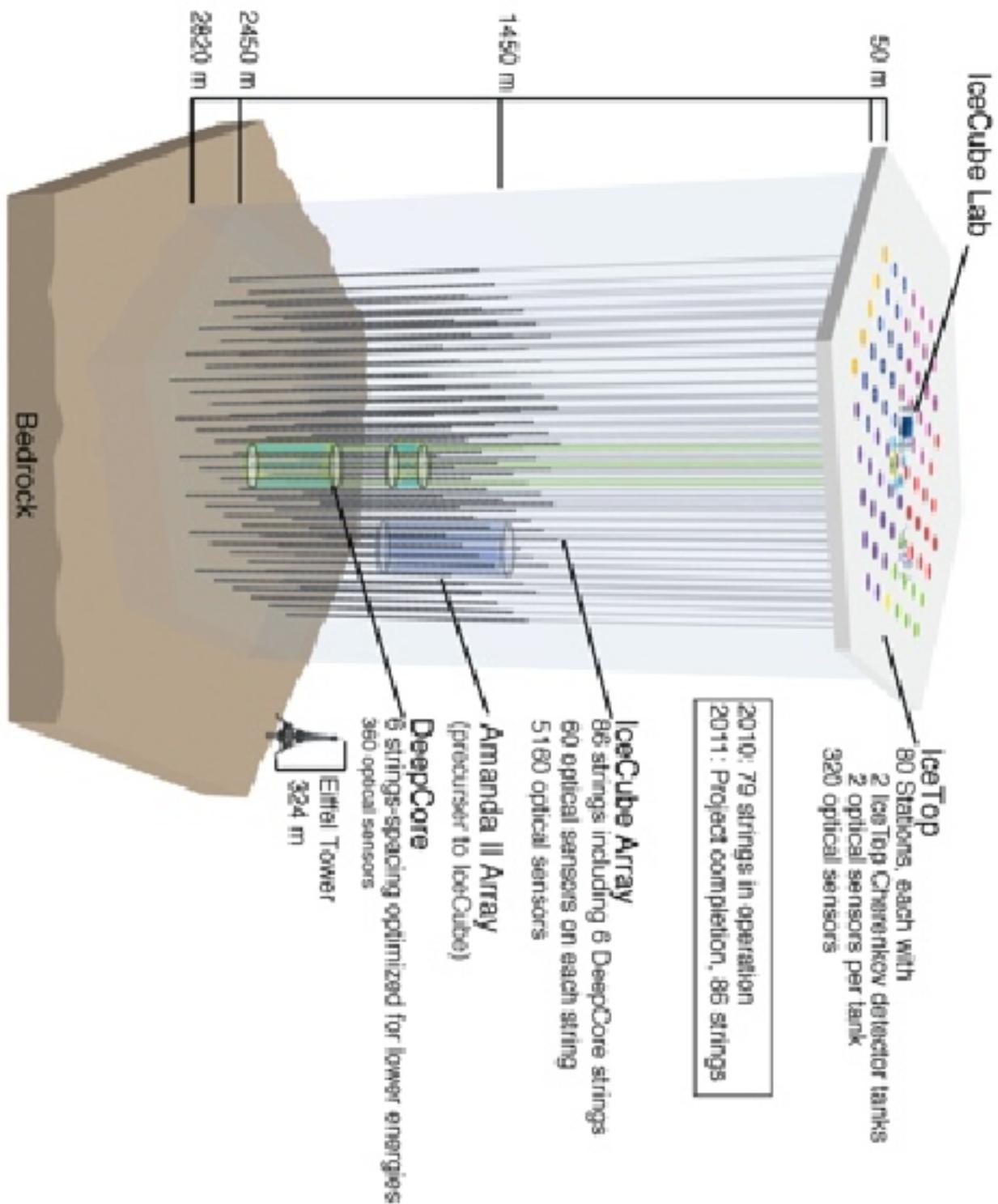
K. Abe et al. [Letter of intent: Hyper-Kamiokande Experiment], arXiv:1109.3262.

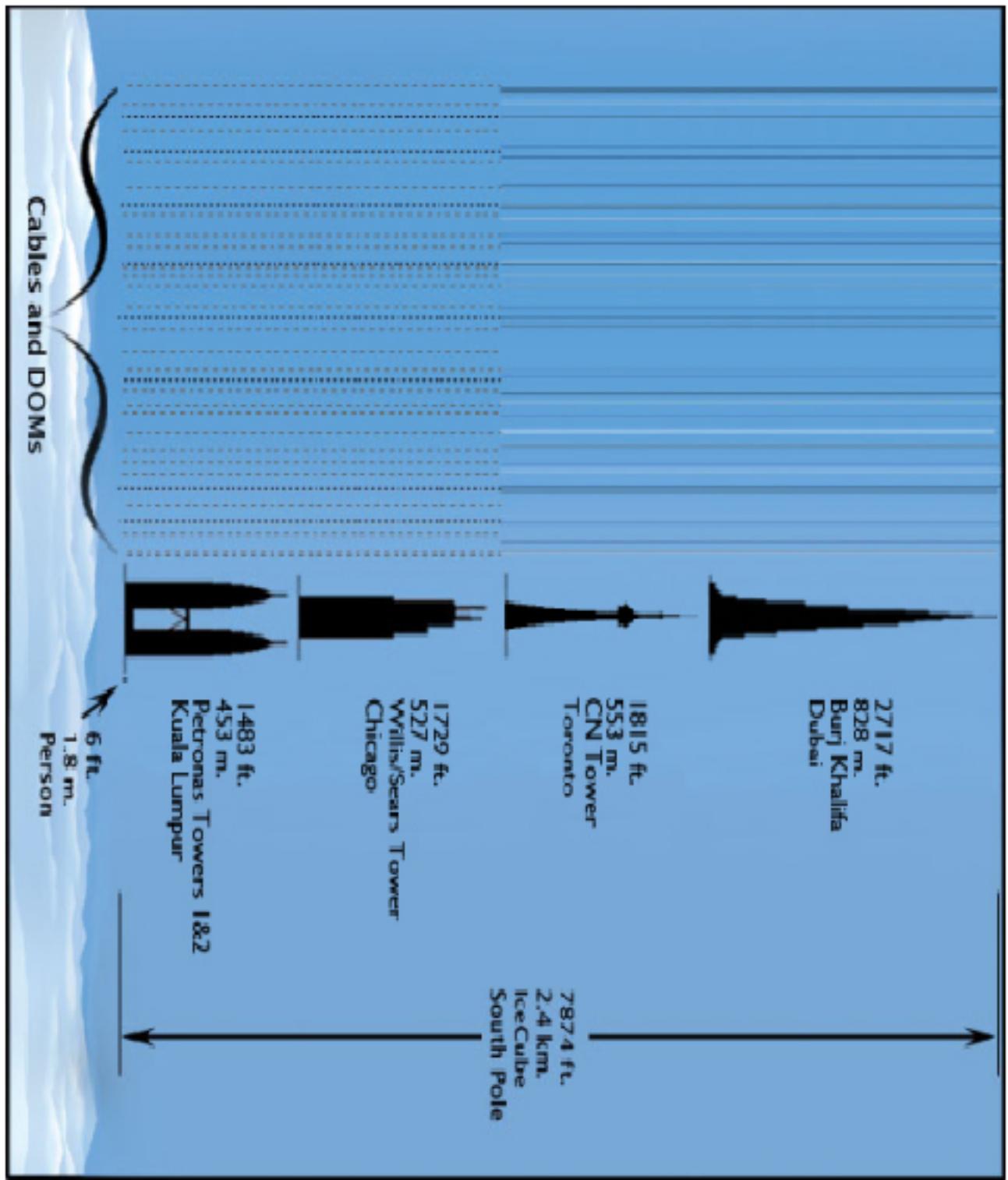
Neutrino Oscillation Source

- Oscillation
- IceCube-DeepCore Physics
- PingU
- Beyond

- Northern Hemisphere ν_μ oscillating over one earth radii produces ν_μ (ν_τ) oscillation minimum(maximum) at ~ 25 GeV
- Covers all possible terrestrial baselines
 - "Beam" is free and never turns off





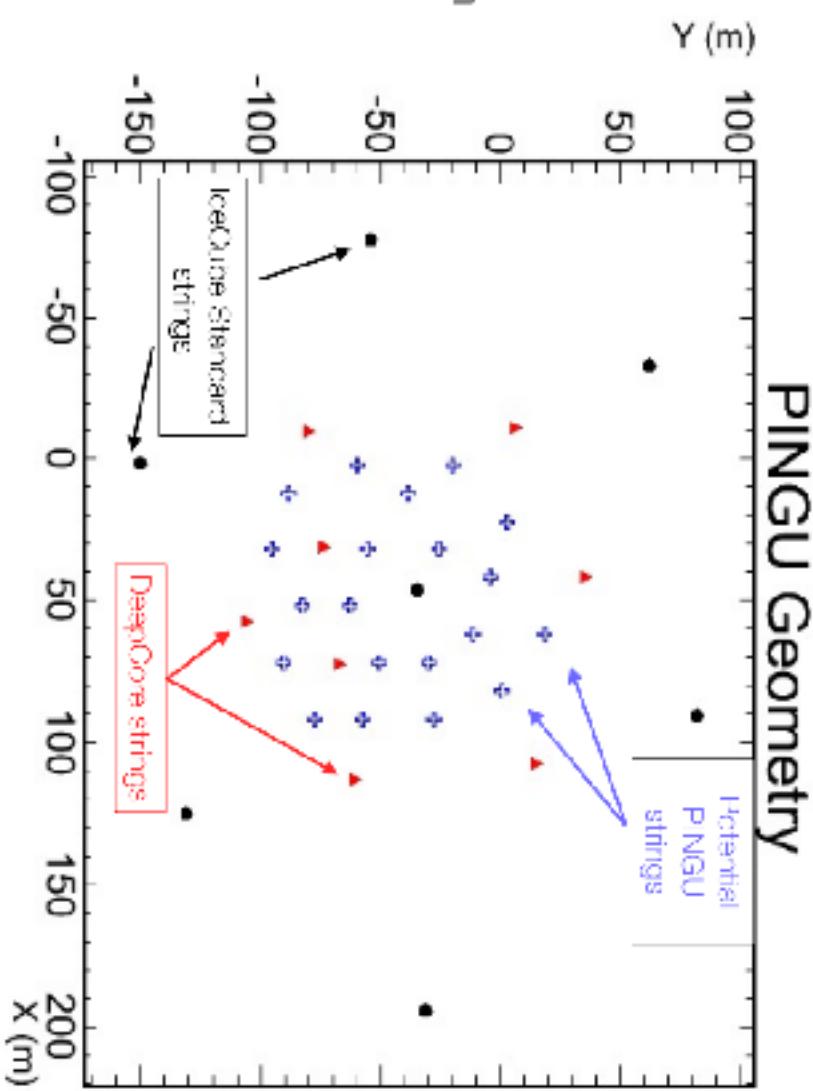


Cables and DOMs

PINGU: Possible Geometry

- Oscillation
- IceCube-DeepCore Physics
- PINGU
- Beyond

- ~20 strings within DeepCore volume w/ short string-string spacing
 - IC-IC: 125m
 - DC-DC: ~80m
 - PINGU-PINGU: <= 26m
- Shorter DOM-DOM spacing
 - IC-IC: 17m
 - DC-DC: 7m
 - PINGU-PINGU: <= 5m
- R & D for future water/ice cerenkov

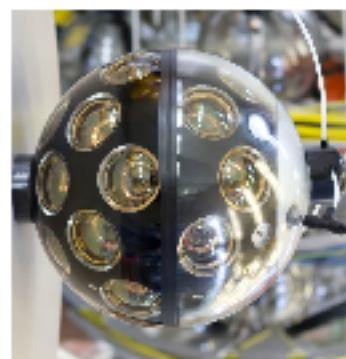


The ORCA proposed detector

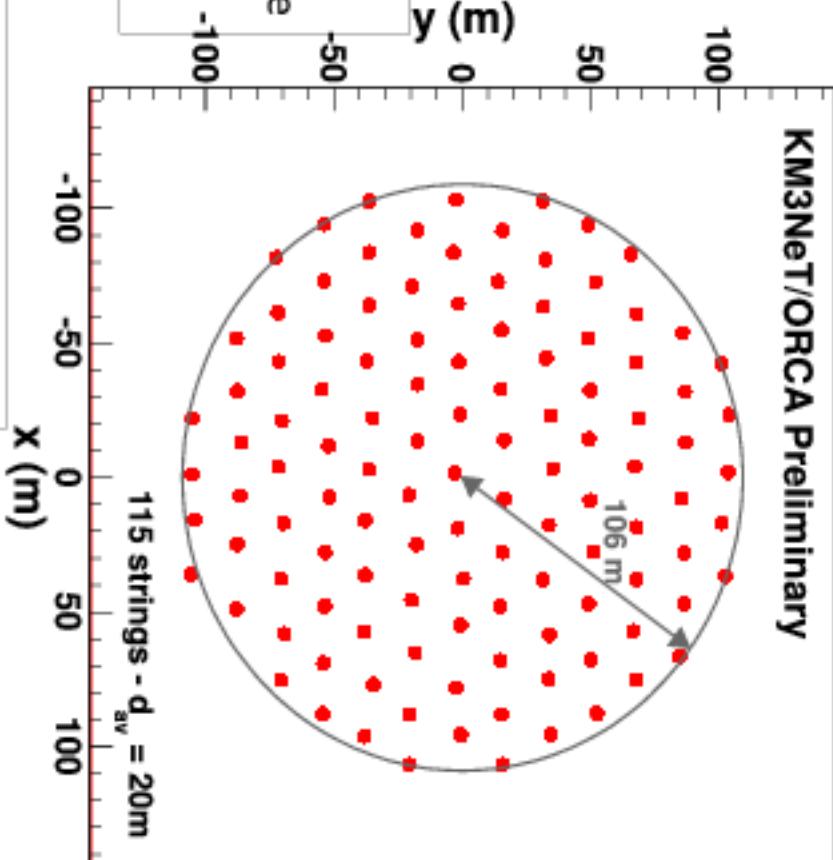
KM3NeT



TORCA



KM3NeT/TORCA Preliminary



Multi-PMT DOM
31 small PMTs
Almost uniform coverage
Photon counting
Direction of photon
All electronics inside

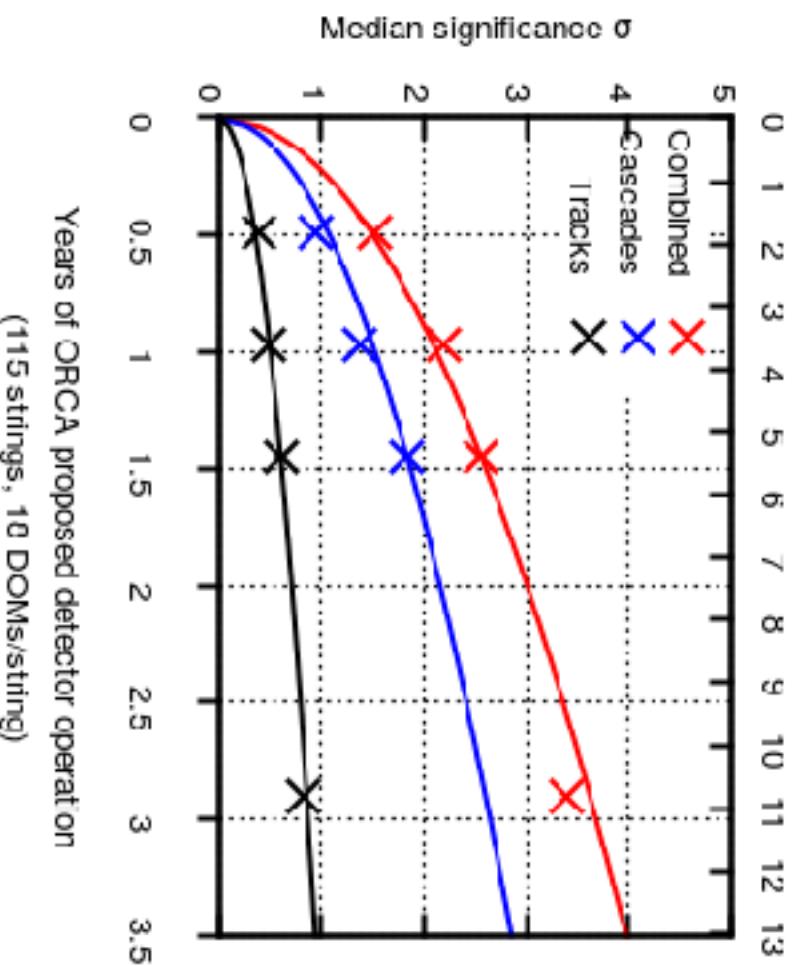
depth
water

115 strings (building block) of 18 DOMs each
Estimated cost 40M€

Sensitivity to the NMH

ORCA sensitivity (PRELIMINARY)

Mton × years



Years of ORCA proposed detector operation
(115 strings, 10 DOMs/string)

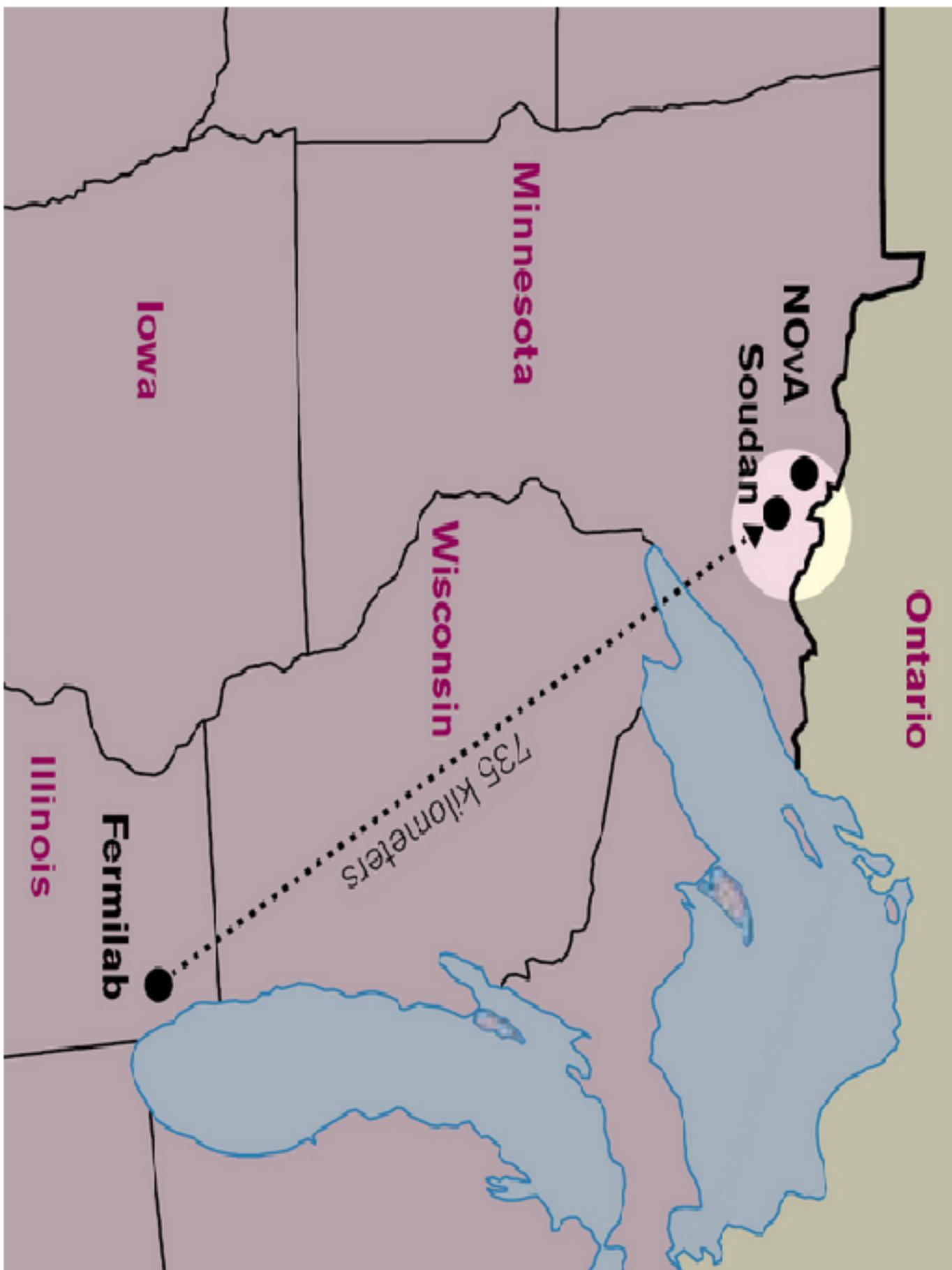
Future LBL Neutrino Oscillation Experiments on
 $\text{sgn}(\Delta m_{31}^2)$ (the Hierarchy) and CP Violation

LBL Oscillation Experiments NO ν A, LBNE, LBNO

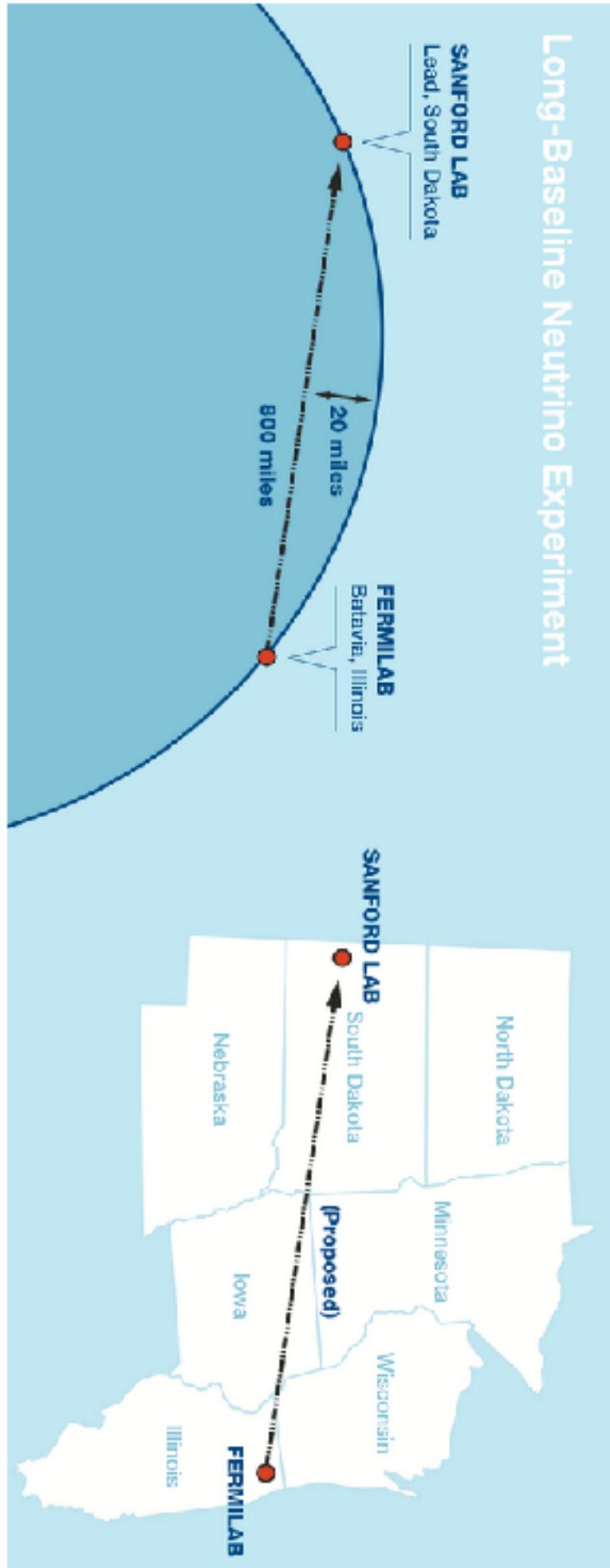
NO ν A: Fermilab - site in Minnesota; off-axis ν beam,
 $E = 2$ GeV, $L \cong 810$ km, 14 kt liquid scintillator; 2014.

LBNE: Fermilab-DUSEL, $L = 1290$ km, 700 kW wide
band ν beam (first and second osc. maxima at $E = 2.4$
GeV and 0.8 GeV); 2 or 3 100 kt Water Cherenkov with
15% to 30% PMT coverage, or multiple 17 kt fiducial
volume LAr detectors; plans to run 5 years with ν_μ and 5
years with $\bar{\nu}_\mu$; 2025 (?)

LBNO: CERN-Pyhasalmi, $L = 2290$ km, wide band ν_μ
1.6 MW super beam (first and second osc. maxima at
 $E \cong 4$ GeV and 1.5 GeV); 440 kt Water Cherenkov, or
100 kt LAr, or 50 kt liquid scintillator detector; 2023 (?)



Long-Baseline Neutrino Experiment





Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \ll 1$ and $\sin^2 \theta_{13} \ll 1$:

$$P_m^{3\nu\ man}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = -\alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

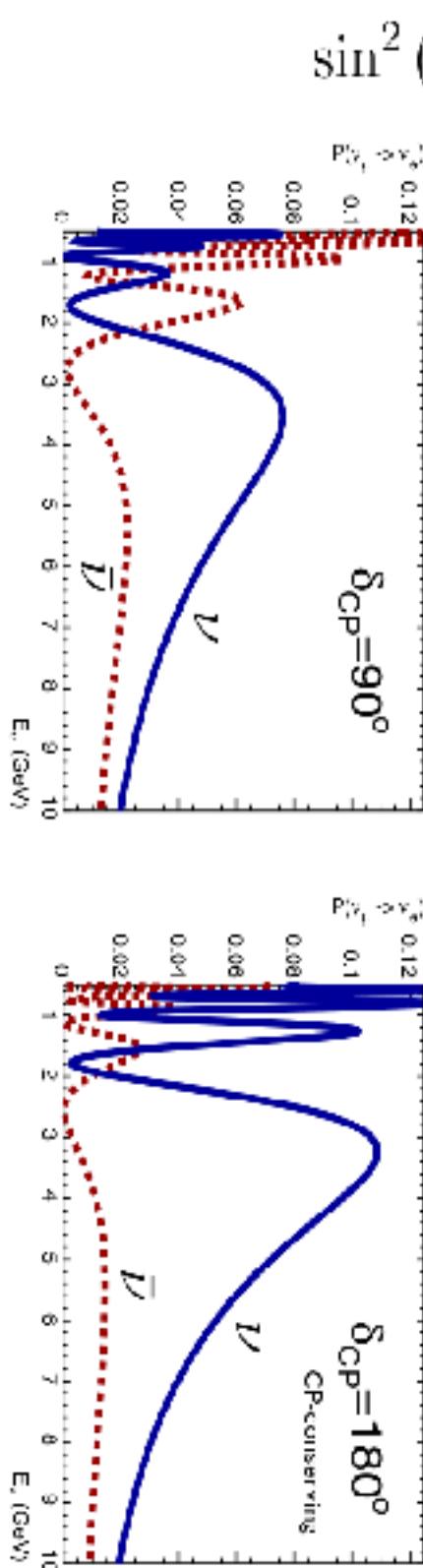
$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{man} \frac{2E}{\Delta m_{31}^2}.$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$

CERN-Pyhäsalmi: CP-effect $\nu_\mu \rightarrow \nu_e$

★ Normal mass hierarchy

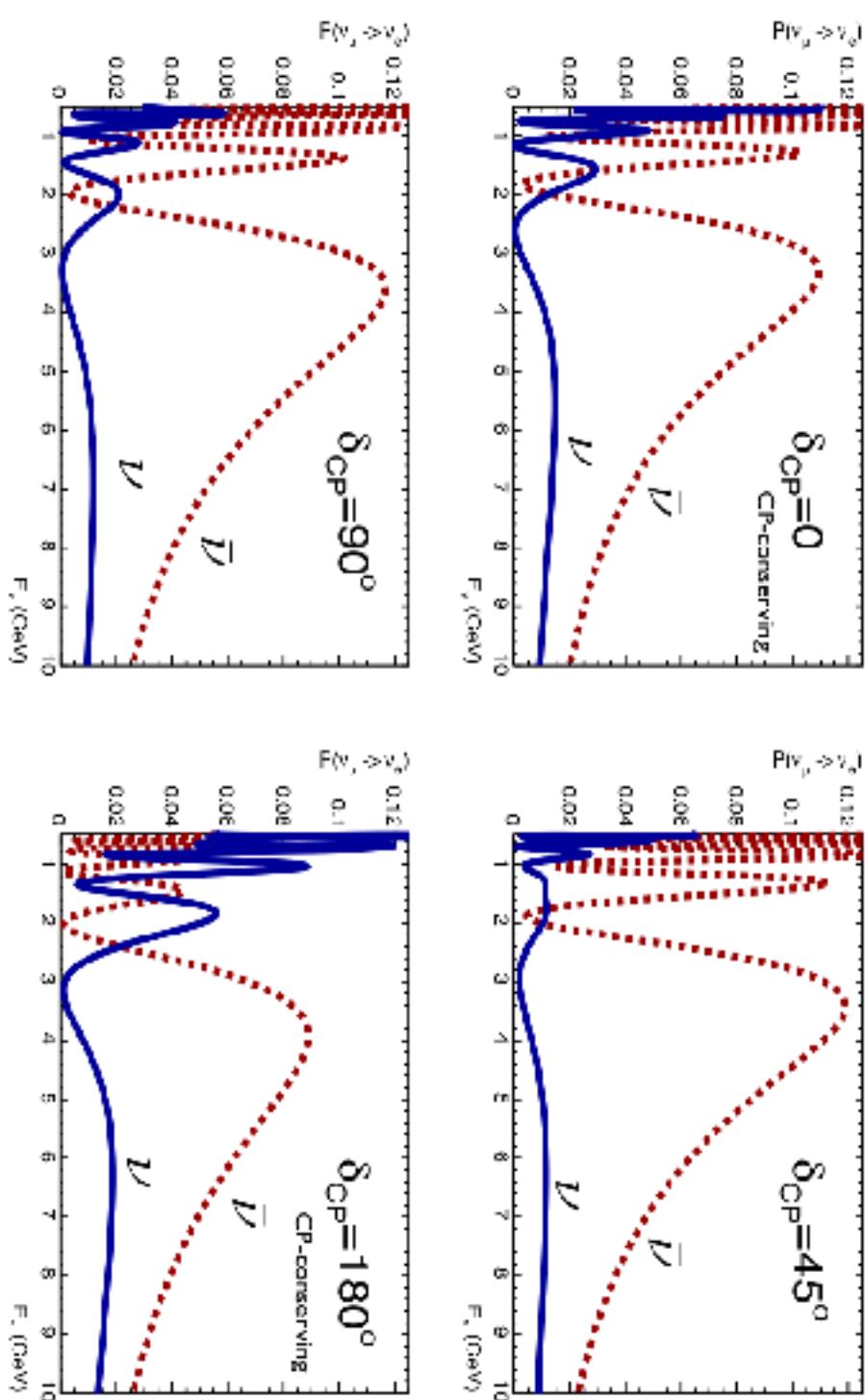
L=2300 km



CERN-Pyhäsalmi: CP-effect $\nu_\mu \rightarrow \nu_e$

★ Inverted mass hierarchy

L=2300 km



LBNE, for example, could achieve the determination of the mass hierarchy at 3σ in less than a year.

LBNE could also have very good sensitivity to CP-violation with a 60% coverage at 3σ in the allowed range of values of $\sin^2 2\theta_{13}$, for a 200 kton Water Cherenkov or 34 kton LAr detectors (assuming it will run for 5 years in neutrinos and 5 years in antineutrinos).

Determining the Nature of Massive Neutrinos

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}, \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP α_{21}, α_{31}

S.M. Bilenky, J. Hosek, S.T.P., 1980;
P. Langacker et al., 1987

$$A(\nu_l \leftrightarrow \nu_{l'}) = \sum_j U_{l'j} e^{-i(E_j t - p_j x)} U_{jl}^\dagger$$

$$U = V P : P_j e^{-i(E_j t - p_j x)} P_j^* = e^{-i(E_j t - p_j x)}$$

P - diagonal matrix of Majorana phases.

The result is valid also in the case of oscillations in matter: ν_l oscillations are not sensitive to the nature of ν_j .

ν_j – Dirac or Majorana particles, fundamental problem

ν_j – Dirac: conserved lepton charge exists, $L = L_e + L_\mu + L_\tau$, $\nu_j \neq \bar{\nu}_j$

ν_j – Majorana: no lepton charge is exactly conserved, $\nu_j \equiv \bar{\nu}_j$

The observed patterns of ν –mixing and of Δm_{atm}^2 and Δm_{\odot}^2 can be related to Majorana ν_j and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

See-saw mechanism: ν_j – Majorana

S. T.P., 1982

Establishing that ν_j are Majorana particles would be as important as the discovery of ν – oscillations.

If ν_j – Majorana particles, U_{PMNS} contains (3- ν mixing)

δ -Dirac, α_{21} , α_{31} - Majorana physical CPV phases

ν -oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau,$

• are not sensitive to the nature of ν_j ,

S.M. Bilenky et al., 1980;
P. Langacker et al., 1987

• provide information on $\Delta m_{jk}^2 = m_j^2 - m_k^2$, but not on the absolute values of ν_j masses.

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:



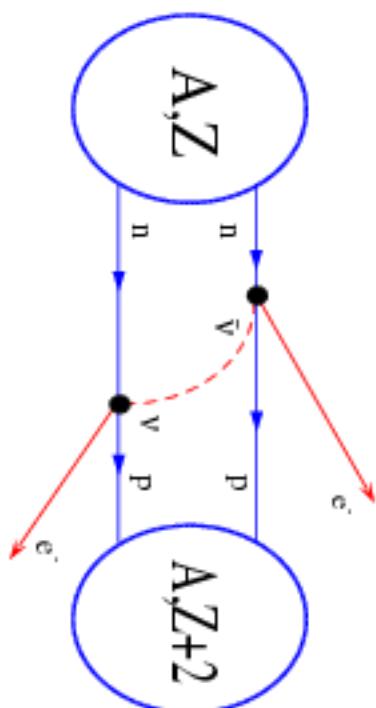
The process most sensitive to the possible Majorana nature of ν_j – $(\beta\beta)_{0\nu^-}$ decay



of even-even nuclei, ^{48}Ca , ^{76}Ge , ^{82}Se , ^{100}Mo , ^{116}Cd , ^{130}Te , ^{136}Xe , ^{150}Nd .

$2n$ from (Λ, Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of $(\Lambda, Z+2)$ and two free e^- .

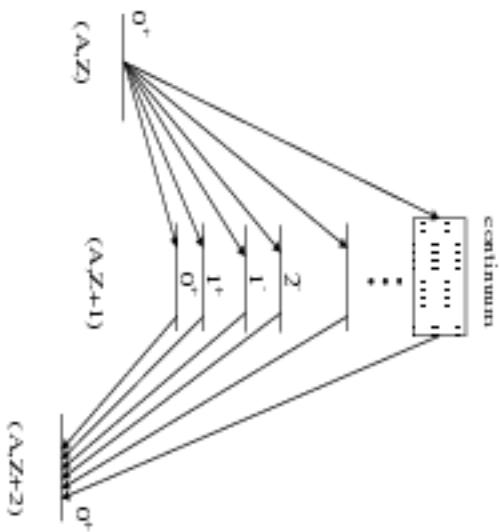
Nuclear $0\nu\beta\beta$ -decay



strong in-medium modification of the basic process

$$dd \rightarrow uu e^- e^- (\bar{\nu}_e \bar{\nu}_e)$$

virtual excitation
of states of all multipolarities
in $(A, Z+1)$ nucleus



$(\beta\beta)_{0\nu}$ -Decay Experiments:

- Majorana nature of ν_j
- Type of ν -mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

^3H β -decay , cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

$$A(\beta\beta)_{0\nu} \sim \langle m \rangle_{M(A,Z)}, \quad M(A,Z) - NME,$$

$$\begin{aligned} |\langle m \rangle| &= |m_1| |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}}| \\ &= |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i\alpha_{31}}|, \quad \theta_{12} \equiv \theta_\odot, \quad \theta_{13} - \text{CHOOZ} \end{aligned}$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix.

CP-invariance: $\alpha_{21} = 0, \pm\pi, \alpha_{31} = 0, \pm\pi$;

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2 , and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$$A(\beta\beta)_{0\nu} \sim \langle m \rangle_{\text{M(A,Z)}},$$

$$\text{M(A,Z)} - \text{NME},$$

$$|\langle m \rangle| \cong \left| \sqrt{\Delta m^2_\odot} \sin^2 \theta_{12} e^{i\alpha} + \sqrt{\Delta m^2_{31}} \sin^2 \theta_{13} e^{i\beta_M} \right|, \quad m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$|\langle m \rangle| \cong \sqrt{m_3^2 + \Delta m^2_{13}} \left| \cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12} \right|, \quad m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$|\langle m \rangle| \cong m \left| \cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12} \right|, \quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV (QD)},$$

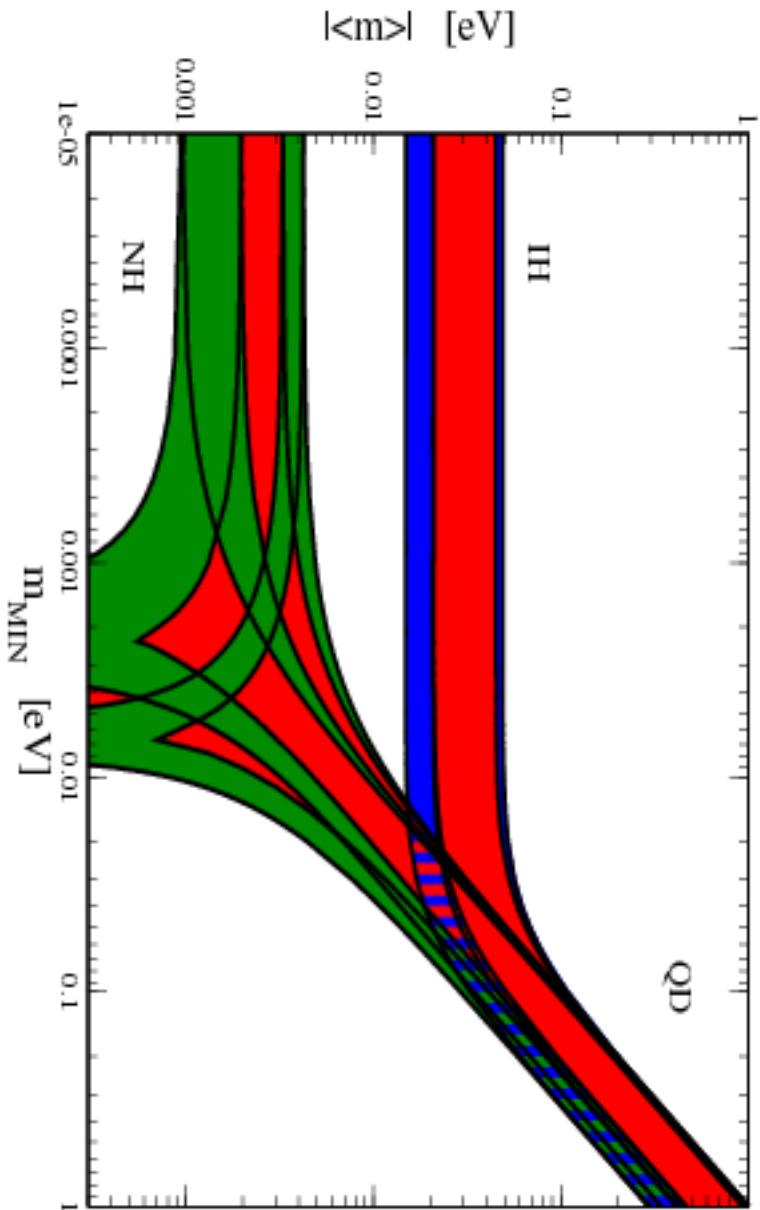
$$\theta_{12} \equiv \theta_\odot, \, \theta_{13}\text{-CHOOZ}; \, \alpha \equiv \alpha_{21}, \, \beta_M \equiv \alpha_{31}.$$

$$\textbf{CP-invariance: } \alpha = 0, \pm \pi, \, \beta_M = 0, \pm \pi,$$

$$|\langle m \rangle| \lesssim 5 \times 10^{-3} \text{ eV, NH};$$

$$\sqrt{\Delta m^2_{13}} \cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055 \text{ eV, IH};$$

$$m \cos 2\theta_{12} \lesssim |\langle m \rangle| \lesssim m, \, m \gtrsim 0.10 \text{ eV, QD}.$$



S. Pascoli, PDG, 2012

$$\sin^2 \theta_{13} = 0.0236 \pm 0.0042; \delta = 0.$$

$$1\sigma(\Delta m_{21}^2) = 2.6\%, \quad 1\sigma(\sin^2 \theta_{12}) = 5.4\%, \quad 1\sigma(|\Delta m_{31(23)}^2|) = 3\%.$$

From G.L. Fogli et al., arXiv:1205.5254v3

$2\sigma(|\langle m \rangle|)$ used.

Best sensitivity: GERDA (^{76}Ge), EXO (^{136}Xe), KamLAND-ZEN (^{136}Xe).

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|\langle m \rangle| = (0.1 - 0.9) \text{ eV} \text{ (99.73% C.L.)}$; b.f.v.: $|\langle m \rangle| = 0.33 \text{ eV}$.

IGEX ^{76}Ge : $|\langle m \rangle| < (0.33 - 1.35) \text{ eV}$ (90% C.L.).

Recent data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|\langle m \rangle| < (0.45 - 0.96) \text{ eV}$, $|\langle m \rangle| < (0.18 - 0.64) \text{ eV}$ (90% C.L.).

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$$\tau(^{76}\text{Ge}) = 2.23_{-0.31}^{+0.44} \times 10^{25} \text{ yr at 90\% C.L.}$$

Results from 2012-2013:

$$\tau(^{136}\text{Xe}) > 1.6 \times 10^{25} \text{ yr at 90\% C.L., EXO}$$

$\tau(^{136}\text{Xe}) > 1.9 \times 10^{25} \text{ yr at 90\% C.L., KamLAND - Zen}$

$\tau(^{76}\text{Ge}) > 2.1 \times 10^{25} \text{ yr at 90\% C.L., GERDA.}$

$\tau(^{76}\text{Ge}) > 3.0 \times 10^{25} \text{ yr at 90\% C.L., GERDA + IGEX + HdM.}$

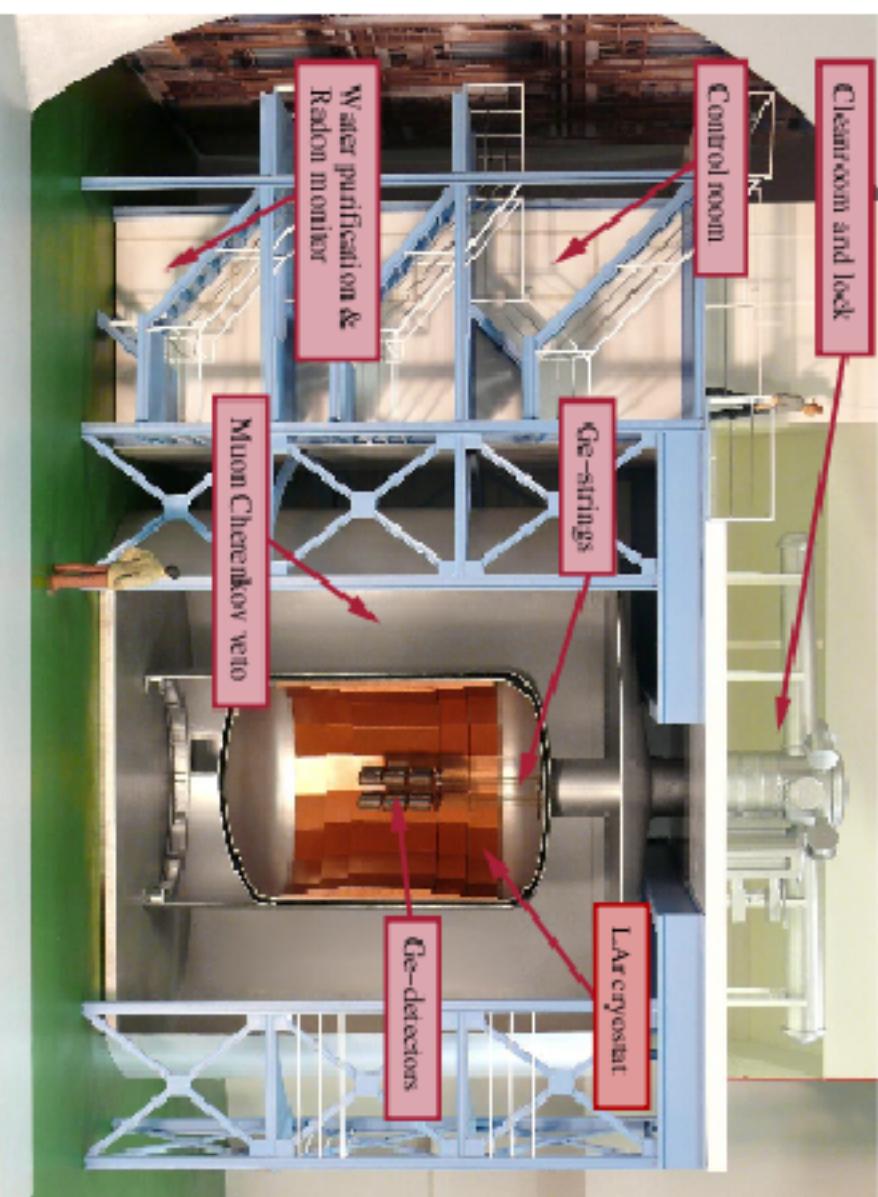
Large number of experiments: $|\langle m \rangle| \sim (0.01\text{-}0.05)$ eV

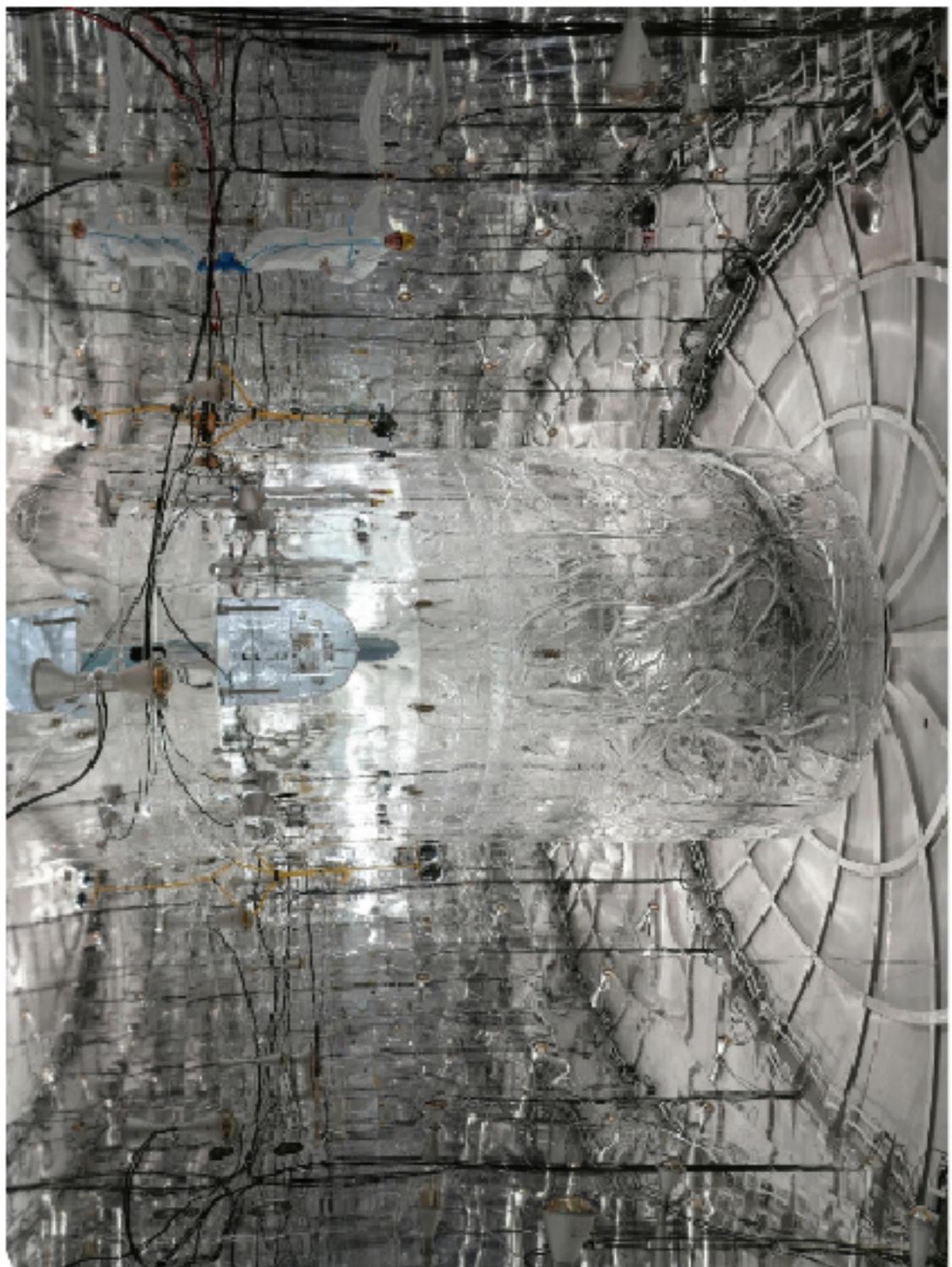
CUORE - ^{130}Te ,
GERDA - ^{76}Ge ,
KamLAND-ZEN - ^{136}Xe ;
EXO - ^{136}Xe ;
SNO+ - ^{130}Te ;
AMoRE - ^{100}Mo (S. Korea);
CANDLES - ^{48}Ca ;
SuperNEMO - ^{82}Se ,...;
MAJORANA - ^{76}Ge ;
COBRA - ^{116}Cd ;
MOON - ^{100}Mo .



GERDA: Experimental Setup

GERDA





Majorana CPV Phases and $|\langle m \rangle|$

CPV can be established provided

- $|\langle m \rangle|$ measured with $\Delta \lesssim 15\%$;
- Δm_{atm}^2 (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;
- $\xi \lesssim 1.5$;
- α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;
- $\tan^2 \theta_\odot \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

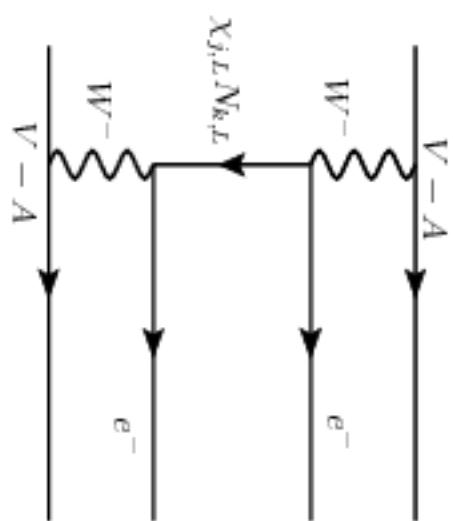
S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$ -decay"

V. Barger *et al.*, 2002

Different Mechanisms of $(\beta\beta)_{0\nu}$ -Decay



Light Majorana Neutrino Exchange

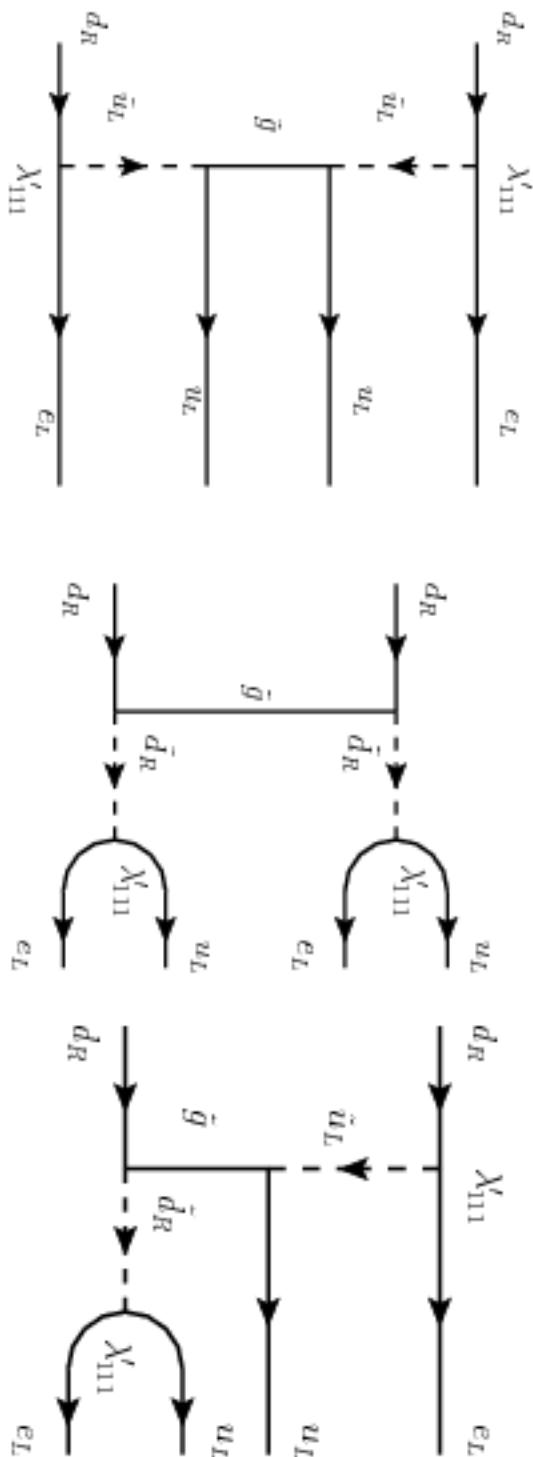
$$\eta_\nu = \frac{\langle m \rangle}{m_e}.$$

Heavy Majorana Neutrino Exchange Mechanisms

(V-A) Weak Interaction, LH N_k , $M_k \gtrsim 10$ GeV:

$$\eta_N^L = \sum_k^{heavy} U_{ek}^2 \frac{m_p}{M_k}, \text{ } m_p \text{ - proton mass, } U_{ek} \text{ - CPV.}$$

SUSY Models with R-Parity Non-conservation



$$\begin{aligned} \mathcal{L}_{R_p} = & \lambda'_{111} \left[(\bar{u}_L \bar{d}_L) \begin{pmatrix} e_R^c \\ -\nu_{eR}^c \end{pmatrix} \tilde{d}_R + (\bar{e}_L \bar{\nu}_{eL}) d_R \begin{pmatrix} \tilde{u}_L^* \\ -\tilde{d}_L^* \end{pmatrix} \right] + h.c. \end{aligned}$$

The problem of distinguishing between different sets of multiple (e.g., two) mechanisms being operative in $(\beta\beta)_{0\nu}$ -decay was studied in

1. A. Faessler, A. Meroni, S.T.P., F. Simkovic and J. Vergados, "Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$ -Decay", arXiv:1103.2434, Phys. Rev. D83 (2011) 113003.

2. A. Meroni, S.T.P. and F. Simkovic, "Multiple CP Non-conserving Mechanisms of bb0nu-Decay and Nu-nuclei with Largely Different Nuclear Matrix Elements", (arXiv:1212.1331, JHEP 1302 (2013) 025.

Earlier studies include:

- A. Halprin, S.T.P., S.P. Rosen, "Effects of Mixing of Light and Heavy Majorana Neutrinos in Neutrinoless Double Beta Decay", Phys. Lett. 125B (1983) 335.

Absolute Neutrino Mass Measurements

Troitzk, Mainz experiments on ${}^3\text{H} \rightarrow {}^3\text{He} + \text{e}^- + \bar{\nu}_e$:

$$m_{\nu_e} < 2.2 \text{ eV} \quad (95\% \text{ C.L.})$$

We have $m_{\nu_e} \cong m_{1,2,3}$ in the case of QD spectrum. The upcoming KATRIN experiment is planned to reach sensitivity

$$\text{KATRIN: } m_{\nu_e} \sim 0.2 \text{ eV}$$

i.e., it will probe the region of the QD spectrum.

Improved β energy resolution requires a **BIG** β spectrometer.

KATRIN

5σ signal if $m_i > 0.35$ eV



Leopoldshafen, 25.11.06

KATRIN'S JOURNEY

Scale 1: 19,500,000

La Mart Conformal Conic Projection
standard parallels 40°N and 56°N

卷之三



Mass and Hierarchy from Cosmology

Cosmological and astrophysical data on $\sum_j m_j$: the Planck + WMAP (low $l \leq 25$) + ACT (large $l \geq 2500$) CMB data + Λ CDM (6 parameter) model + assuming 3 light massive neutrinos, implies

$$\sum_j m_j \equiv \Sigma < 0.66 \text{ eV} \quad (95\% \text{ C.L.})$$

Adding data on the baryon acoustic oscillations (BAO) leads to:

$$\sum_j m_j \equiv \Sigma < 0.23 \text{ eV} \quad (95\% \text{ C.L.})$$

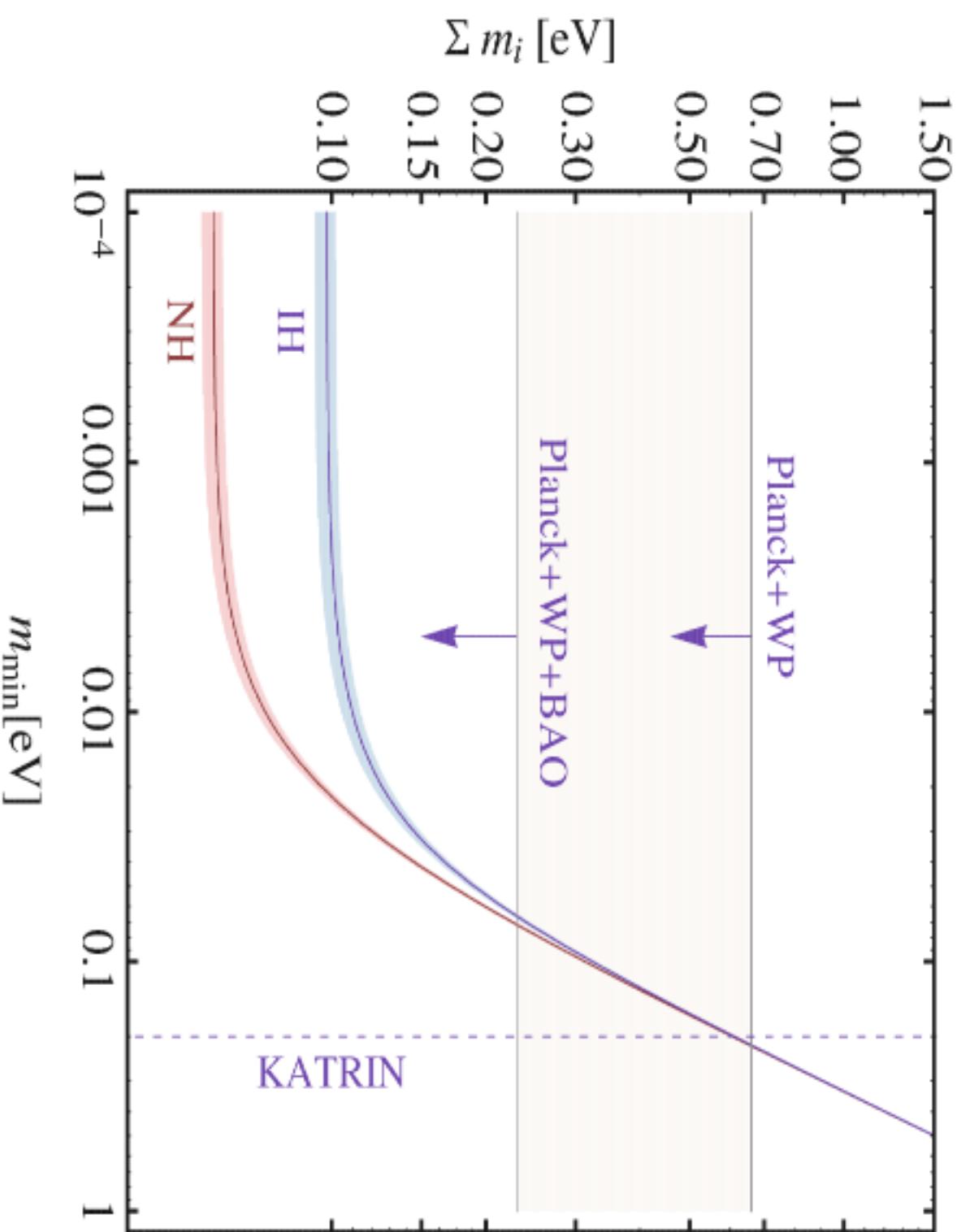
Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP, and Planck experiments, and/or data from future EUCLID experiment, might allow to determine

$$\sum_j m_j : \quad \delta \cong (0.01 - 0.04) \text{ eV.}$$

NH: $\sum_j m_j \leq 0.05 \text{ eV } (3\sigma);$

IH: $\sum_j m_j \geq 0.10 \text{ eV } (3\sigma).$

Mass and Hierarchy from Cosmology



These data imply that

$$m_{\nu_j} \ll m_{e,\mu,\tau}, m_q, q = u, c, t, d, s, b$$

For $m_{\nu_j} \lesssim 1$ eV: $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family: $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^2$

Instead of Conclusions

We are at the beginning of the Road...

The future of neutrino physics is bright.

Supporting Slides

The Nature of Massive Neutrinos III:

The Seesaw Mechanisms of Neutrino Mass Generation M_ν from the See-Saw Mechanism

P. Minkowski, 1977.

M. Gell-Mann, P. Ramond, R. Slansky, 1979;
T. Yanagida, 1979;

R. Mohapatra, G. Senjanovic, 1980.

- Explain the smallness of ν -masses.
- Through leptogenesis theory link the ν -mass generation to the generation of baryon asymmetry of the Universe.

S. Fukugita, T. Yanagida, 1986.

Three Types of Seesaw Mechanisms

Require the existence of new degrees of freedom (particles) beyond those present in the ST

Type I seesaw mechanism: $\nu_{lR} - RH \nu s'$ (heavy).

Type II seesaw mechanism: $H(x)$ - a triplet of H^0, H^-, H^{--} Higgs fields (HTM).

Type III seesaw mechanism: $\Gamma(x)$ - a triplet of fermion fields.

The scale of New Physics determined by the masses of the New Particles.

Massive neutrinos ν_j - Majorana particles.

All three types of seesaw mechanisms have TeV scale versions, predicting rich low-energy phenomenology ($(\beta\beta)^0\nu$ -decay, LFV processes, etc.) and New Physics at LHC.

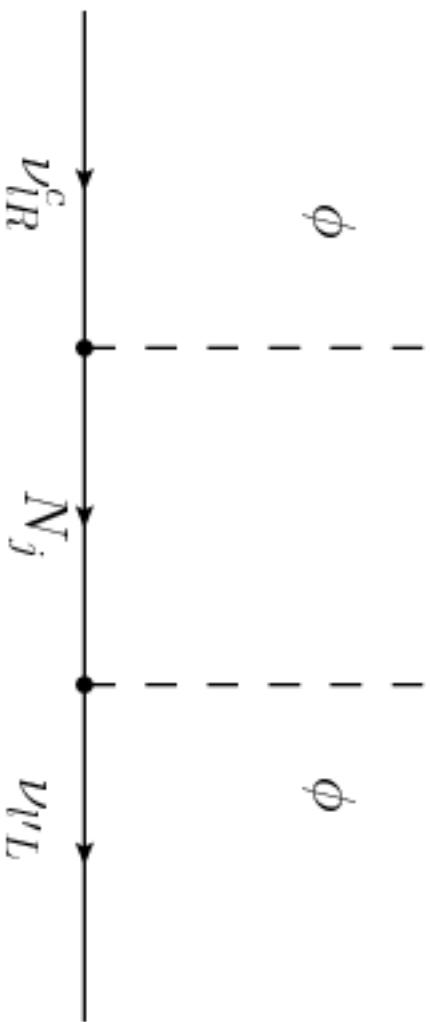
Type I Seesaw Mechanism

- Requires both $\nu_{lL}(x)$ and $\nu_{l'R}(x)$.
- Dirac+Majorana Mass Term: $M_{LL} = 0$, $|M_D| = v Y^\nu / \sqrt{2}$ $\ll |M^{RR}|$.
- Diagonalising M^{RR} : N_j - heavy Majorana neutrinos, $M_j \sim \text{TeV}$; or $(10^9 - 10^{13}) \text{ GeV}$ in GUTs.

For sufficiently large M_j , Majorana mass term for $\nu_{lL}(x)$:

$$M_\nu \cong v_u^2 (Y^\nu)^T M_j^{-1} Y^\nu = U_{\text{PMNS}}^* m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger .$$

$v_u Y^\nu = M_D$, $M_D \sim 1 \text{ GeV}$, $M_j = 10^{10} \text{ GeV}$: $M_\nu \sim 0.1 \text{ eV}$.



- $\nu_{lR}(x)$: Majorana mass term at "high scale" ($\sim \text{TeV}$; or $(10^9 - 10^{13}) \text{ GeV}$) in $SO(10) \text{ GUT}$

$$\mathcal{L}_M^\nu(x) = + \frac{1}{2} \nu_{lR}^T(x) C^{-1} (M^{RR})_{ll}^\dagger \nu_{lR}(x) + h.c. = - \frac{1}{2} \sum_j \bar{N}_j M_j N_j ,$$

- Yukawa type coupling of $\nu_{lL}(x)$ and $\nu_{lR}(x)$ involving $\Phi(x)$:

$$\begin{aligned} \mathcal{L}_Y(x) &= \bar{Y}_{ll}^\nu \overline{\nu_{lR}}(x) \Phi^T(x) (i\tau_2) \psi_{lL}(x) + h.c. , \\ &= Y_{jl}^\nu \overline{N_{jR}}(x) \Phi^T(x) (i\tau_2) \psi_{lL}(x) + h.c. , \\ M_D &= \frac{v}{\sqrt{2}} Y^\nu , \quad v = 246 \text{ GeV} . \end{aligned}$$

Type II Seesaw Mechanism

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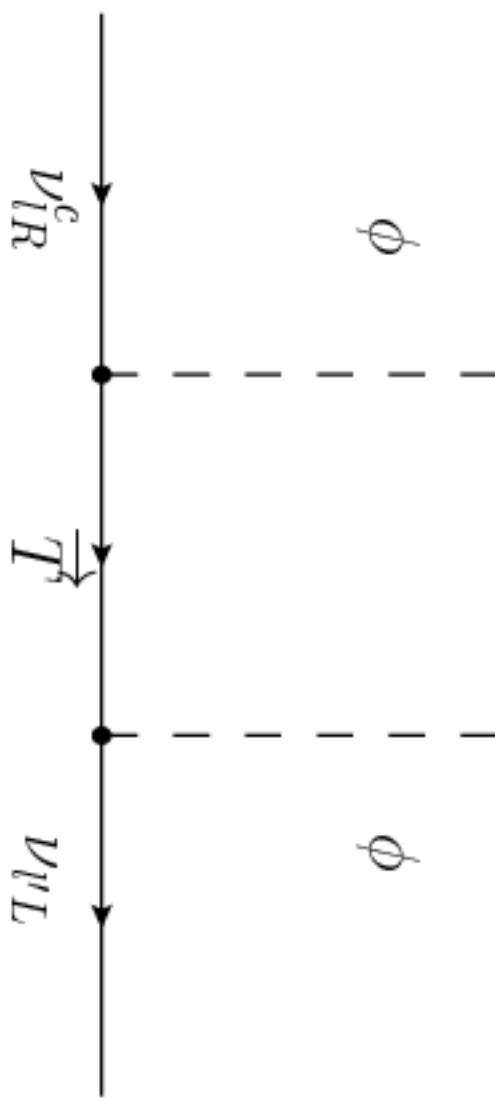


Due to I. Girardi

$$M_\nu \cong h v^2 M_H^{-1} = U_{\text{PMNS}}^* m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger .$$

$h \sim 10^{-2}$, $v = 246$ GeV, $M_H \sim 10^{12}$ GeV; $M_\nu \sim 0.6$ eV.

Type III Seesaw Mechanism



$$M_\nu \cong v^2 (Y_T)^T M_T^{-1} Y_T = U_{\text{PMNS}}^* m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger .$$

$v Y_T \sim 1 \text{ GeV}, M_T \sim 10^{10} \text{ GeV}; M_\nu \sim 0.1 \text{ eV}.$

TeV Scale Type I See-Saw Mechanism

Type I see-saw mechanism, heavy Majorana neutrinos N_j at the TeV scale:

$$m_\nu \simeq -M_D \hat{M}_N^{-1} M_D^T, \quad \hat{M} = \text{diag}(M_1, M_2, M_3), \quad M_j \sim (100 - 1000) \text{ GeV}.$$

$$\mathcal{L}_{CC}^N = -\frac{g}{2\sqrt{2}} \bar{\ell} \gamma_\alpha (RV)_{\ell k} (1 - \gamma_5) N_k W^\alpha + \text{h.c.}, \quad (RV)_{\ell k} \equiv U_{\ell 3+k},$$

$$\mathcal{L}_{NC}^N = -\frac{g}{2c_w} \bar{\nu}_{\ell L} \gamma_\alpha (RV)_{\ell k} N_{kL} Z^\alpha + \text{h.c.}$$

- All low-energy constraints can be satisfied in a scheme with two heavy Majorana neutrinos $N_{1,2}$, which form a pseudo-Dirac pair:
 $M_2 = M_1(1+z)$, $0 < z \ll 1$.
- only NH and IH ν mass spectra possible: $\min(m_j) = 0$.

- Requirements: $|{}_{\ell k}^{(RV)}|$ “sizable”
+ reproducing correctly the neutrino oscillation data:

$$\begin{aligned}|{}_{e1}^{(RV)}|^2 &= \frac{1}{2} \frac{y^2 v^2}{M_1^2 m_2 + m_3} \left| U_{e3} + i \sqrt{m_2/m_3} U_{e2} \right|^2, \quad \text{NH}, \\ |{}_{\mu 1}^{(RV)}|^2 &= \frac{1}{2} \frac{y^2 v^2}{M_1^2 m_1 + m_2} \left| U_{\ell 2} + i \sqrt{m_1/m_2} U_{\ell 1} \right|^2 \cong \frac{1}{4} \frac{y^2 v^2}{M_1^2} |U_{\ell 2} + i U_{\ell 1}|^2, \quad \text{IH},\end{aligned}$$

$${}_{\ell 2}^{(RV)} = \pm i {}_{\ell 1}^{(RV)} \sqrt{\frac{M_1}{M_2}}, \quad \ell = e, \mu, \tau,$$

y - the maximum eigenvalue of Y^ν , $v_u \simeq 174$ GeV.

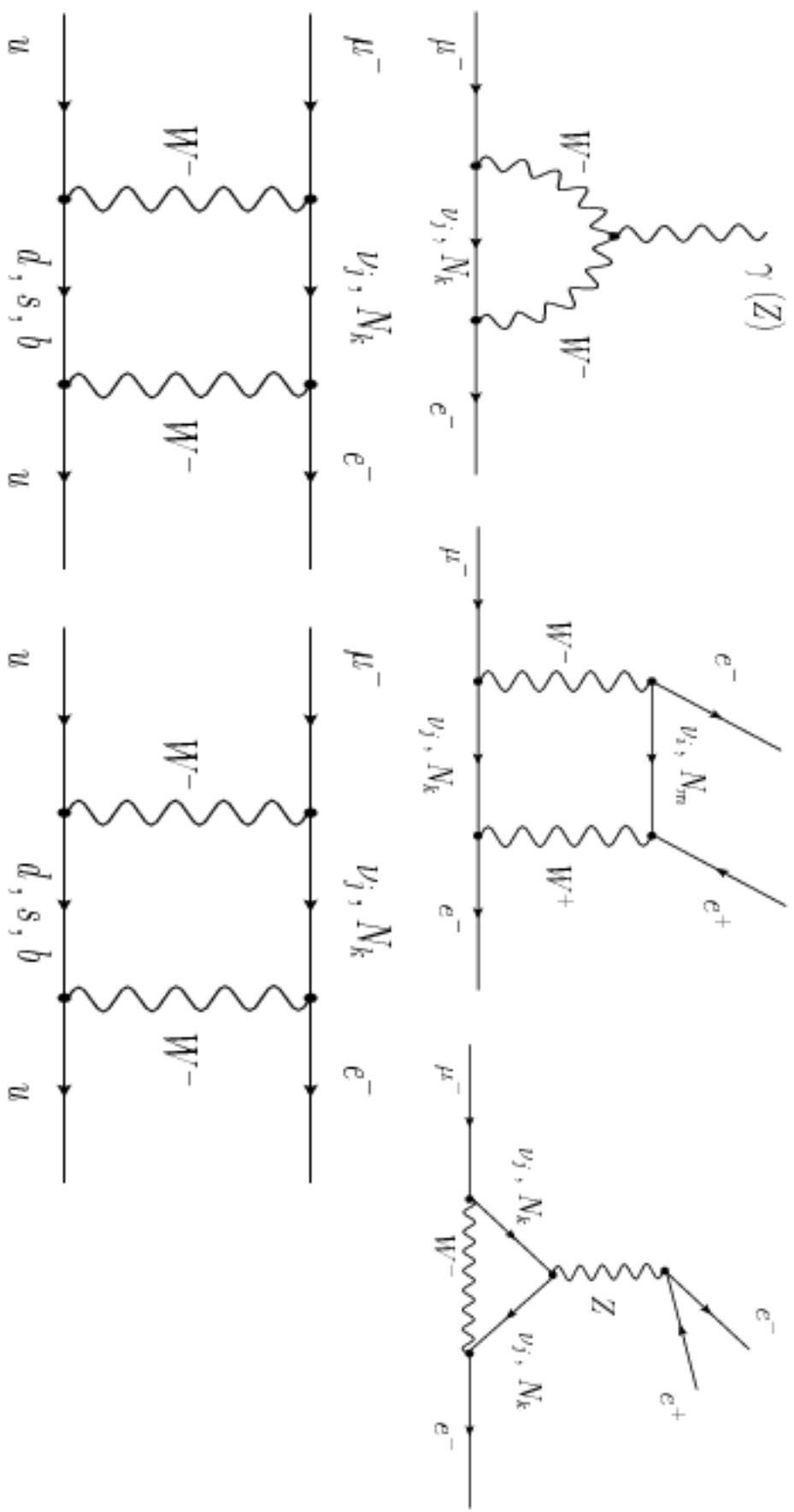
4 parameters: M , z , y and a phase ω . A. Ibarra, E. Molinaro, S.T.P., 2010 and 2011

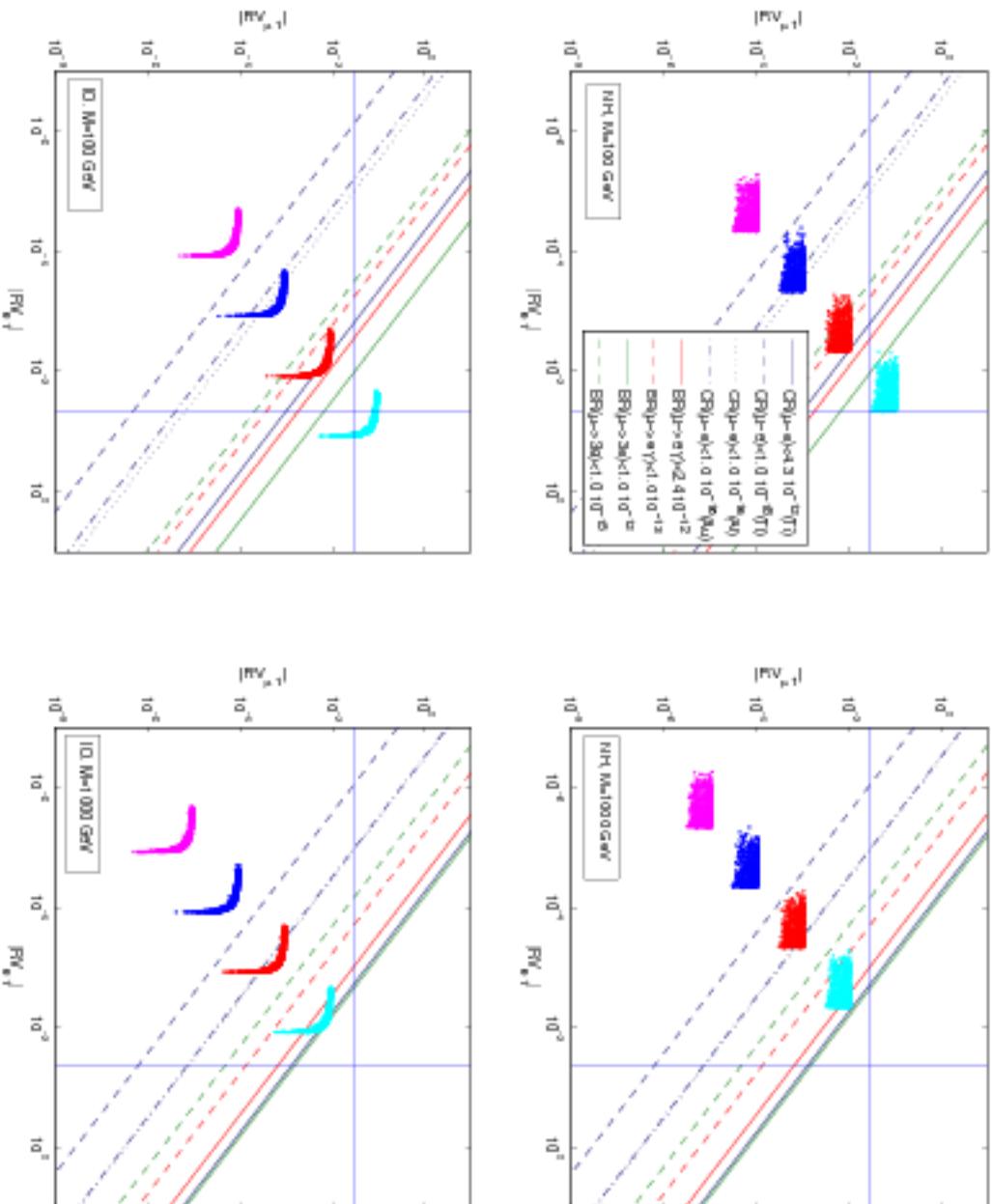
Low energy data:

$$\begin{aligned}|{}_{e1}^{(RV)}|^2 &\lesssim 2 \times 10^{-3}, \\ |{}_{\mu 1}^{(RV)}|^2 &\lesssim 0.8 \times 10^{-3}, \\ |{}_{\tau 1}^{(RV)}|^2 &\lesssim 2.6 \times 10^{-3}.\end{aligned}$$

Observation of $N_{1,2}$ at LHC - problematic.

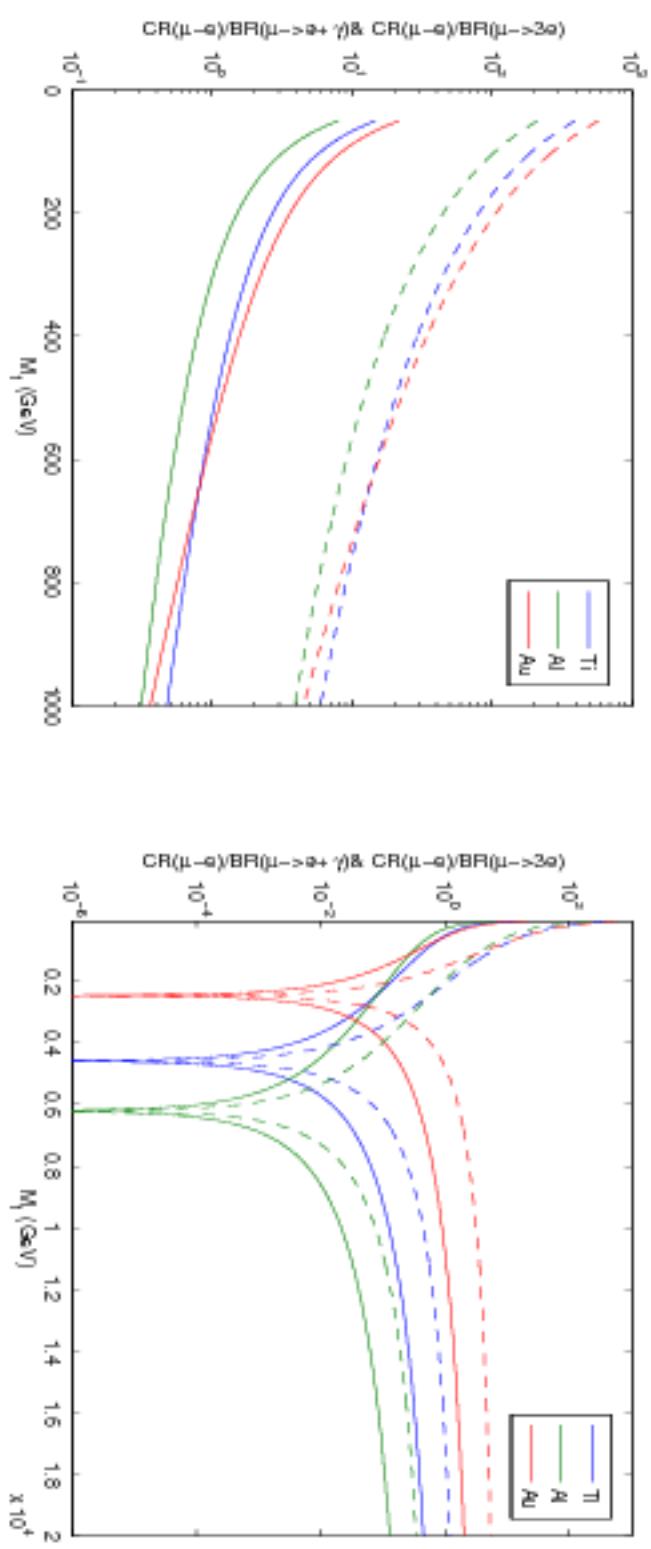
LFV processes: $\mu^- \rightarrow e^- + \gamma$, $\mu^- \rightarrow 3e^-$, $\mu^- + N_j \rightarrow e^- + N_j$: can proceed with exchange of virtual N_j :





Current limits and potential sensitivity to $|(\text{RV})_{e1}|$ and $|(\text{RV})_{\mu 1}|$ from data on LFV processes for NH (upper panels) and IH (lower panels) spectra, for $M_1 = 100$ (1000) GeV and, *i*) $y = 0.0001$ (magenta pts), *ii*) $y = 0.001$ (blue pts), *iii*) $y = 0.01$ (red pts) and *iv*) $y = 0.1$ (cyan pts).

The ratio of the $\mu - e$ relative conversion rate and the branching ratio of the I) $\mu \rightarrow e\gamma$ decay (solid lines), II) $\mu \rightarrow 3e$ decay (dashed lines), versus the type I see-saw mass scale M_1 , for three different nuclei: ^{48}Ti (blue lines), ^{27}Al (green lines) and ^{197}Au (red lines).



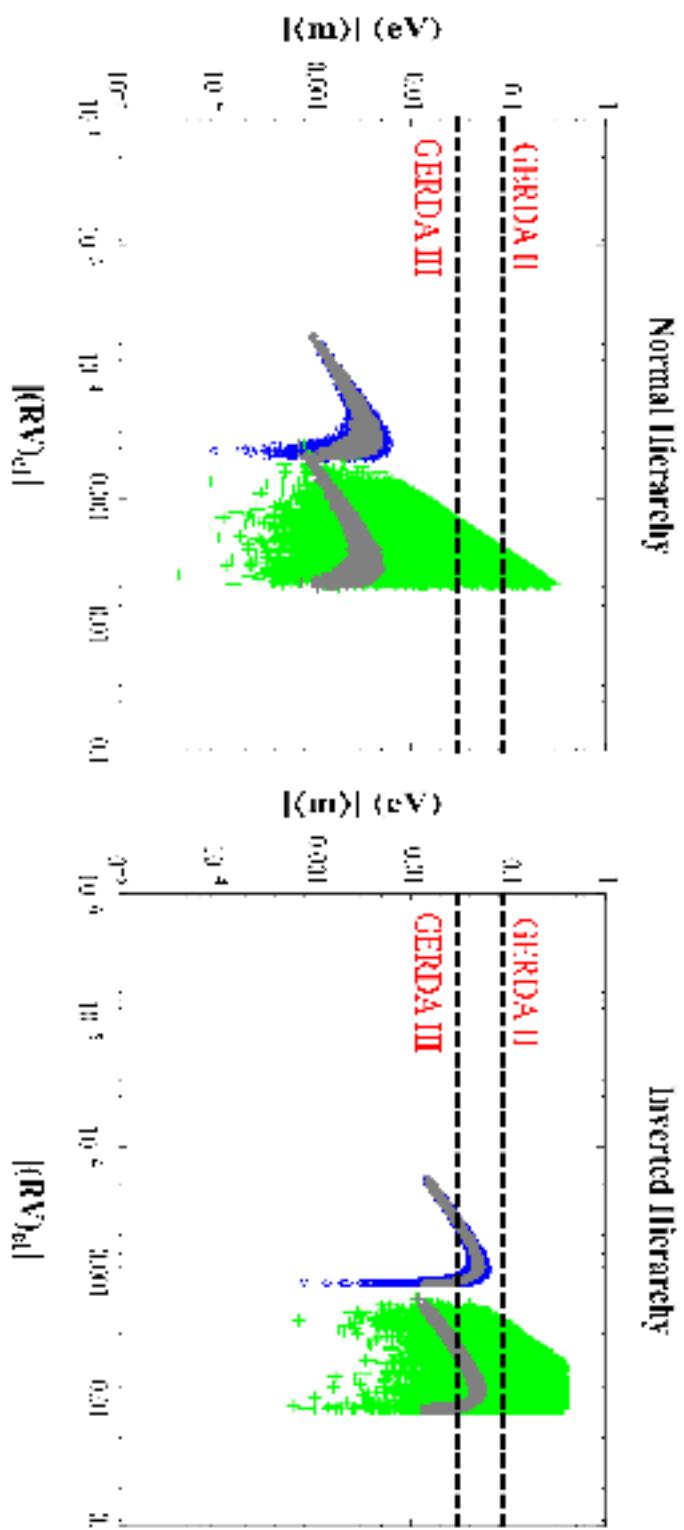
The exchange of virtual N_j gives a contribution to $|\langle m \rangle|$:

$$|\langle m \rangle| \cong \left| \sum_i (U_{PMNS})_{ei}^2 m_i - \sum_k f(A, M_k) (RV)_{ek}^2 \frac{(0.9 \text{ GeV})^2}{M_k} \right|,$$

$$f(A, M_k) \cong f(A).$$

For, e.g., ^{48}Ca , ^{76}Ge , ^{82}Se , ^{130}Te and ^{136}Xe , the function $f(A)$ takes the values $f(A) \cong 0.033$, 0.079 , 0.073 , 0.085 and 0.068 , respectively.

- **The Predictions for $|\langle m \rangle|$ can be modified considerably.**



$|\langle m \rangle|$ vs $|(RV)_{e1}|$ for ^{76}Ge in the cases of NH (left panel) and IH (right panel) light neutrino mass spectrum, for $M_1 = 100$ GeV and *i)* $y = 0.001$ (blue), *ii)* $y = 0.01$ (green). The gray markers correspond to $|\langle m \rangle^{\text{std}}| = |\sum_i (U_{PMNS})_{ei}^2 m_i|$.