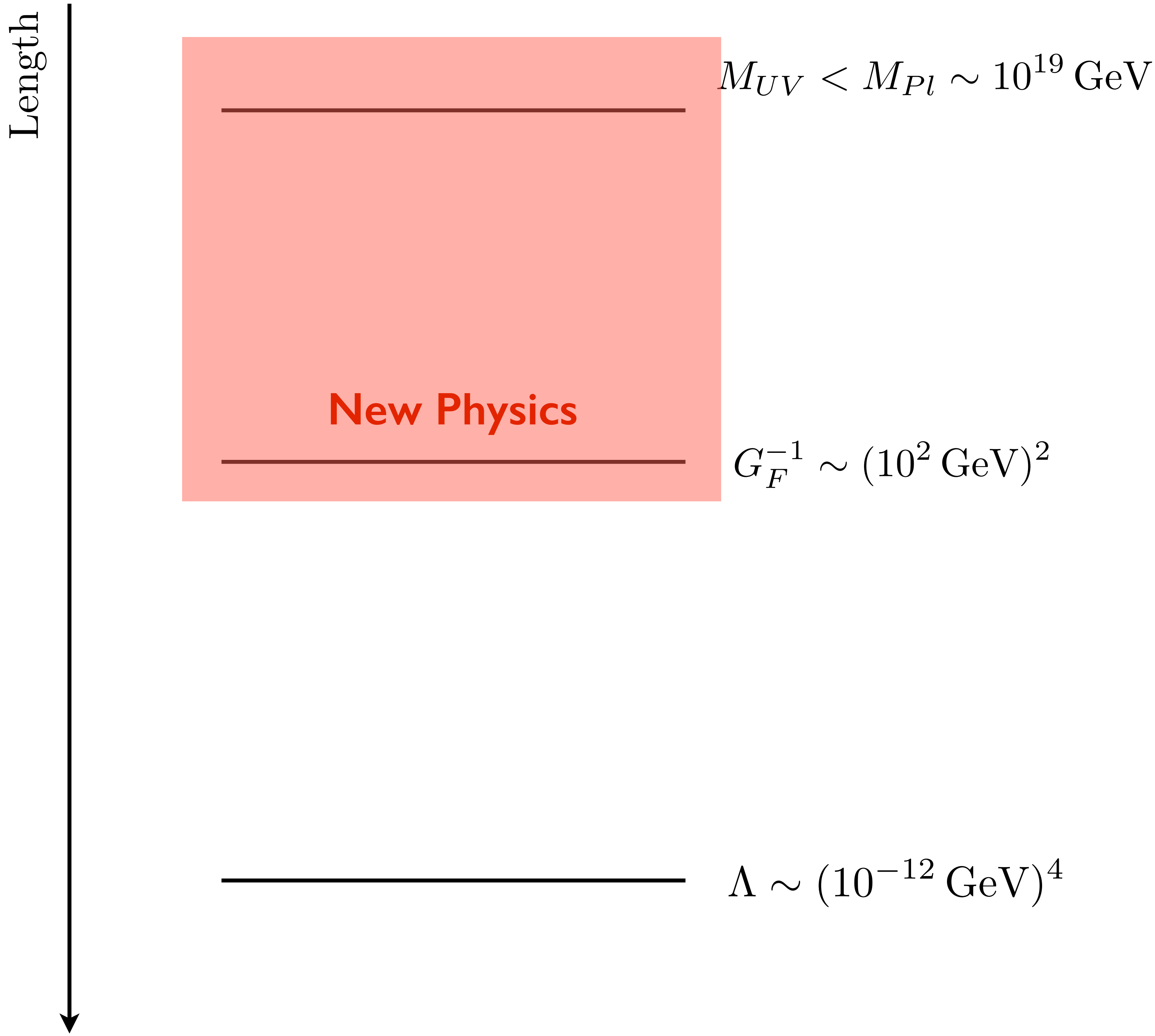
Dark matter and the Multiverse: WIMPs and axions

Duccio Pappadopulo - UC Berkeley



Length
$M_{UV} < M_{Pl} \sim 10^{19}\,\mathrm{GeV}$
New Physics
$G_F^{-1} \sim (10^2\,\mathrm{GeV})^2$
$\Lambda \sim (10^{-12}\,\mathrm{GeV})^4$

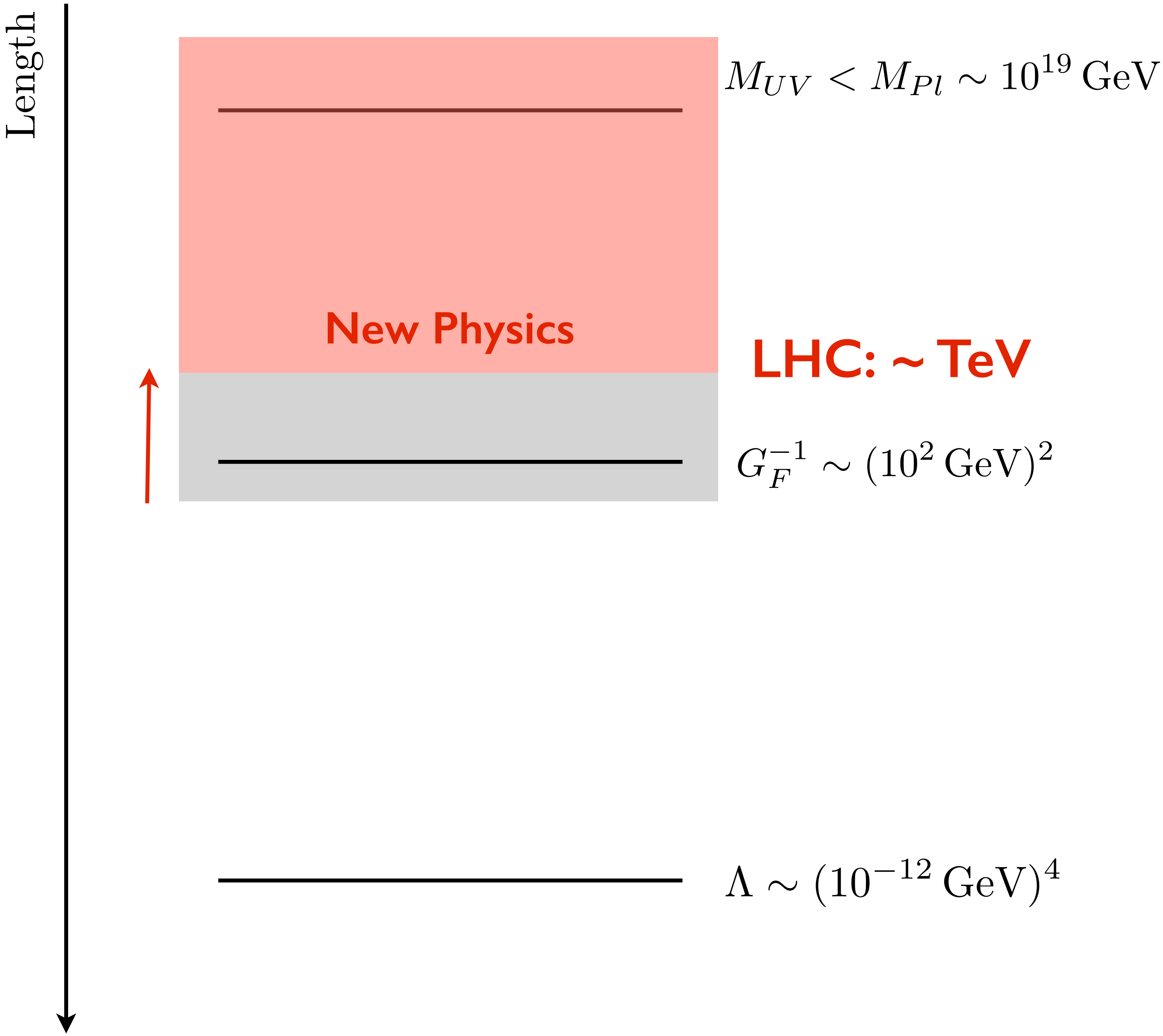
$$\Delta v^2 \sim M_{UV}^2$$

Known dynamical mechanisms to soften the quadratic sensitivity of the weak scale to heavy field theory thresholds.

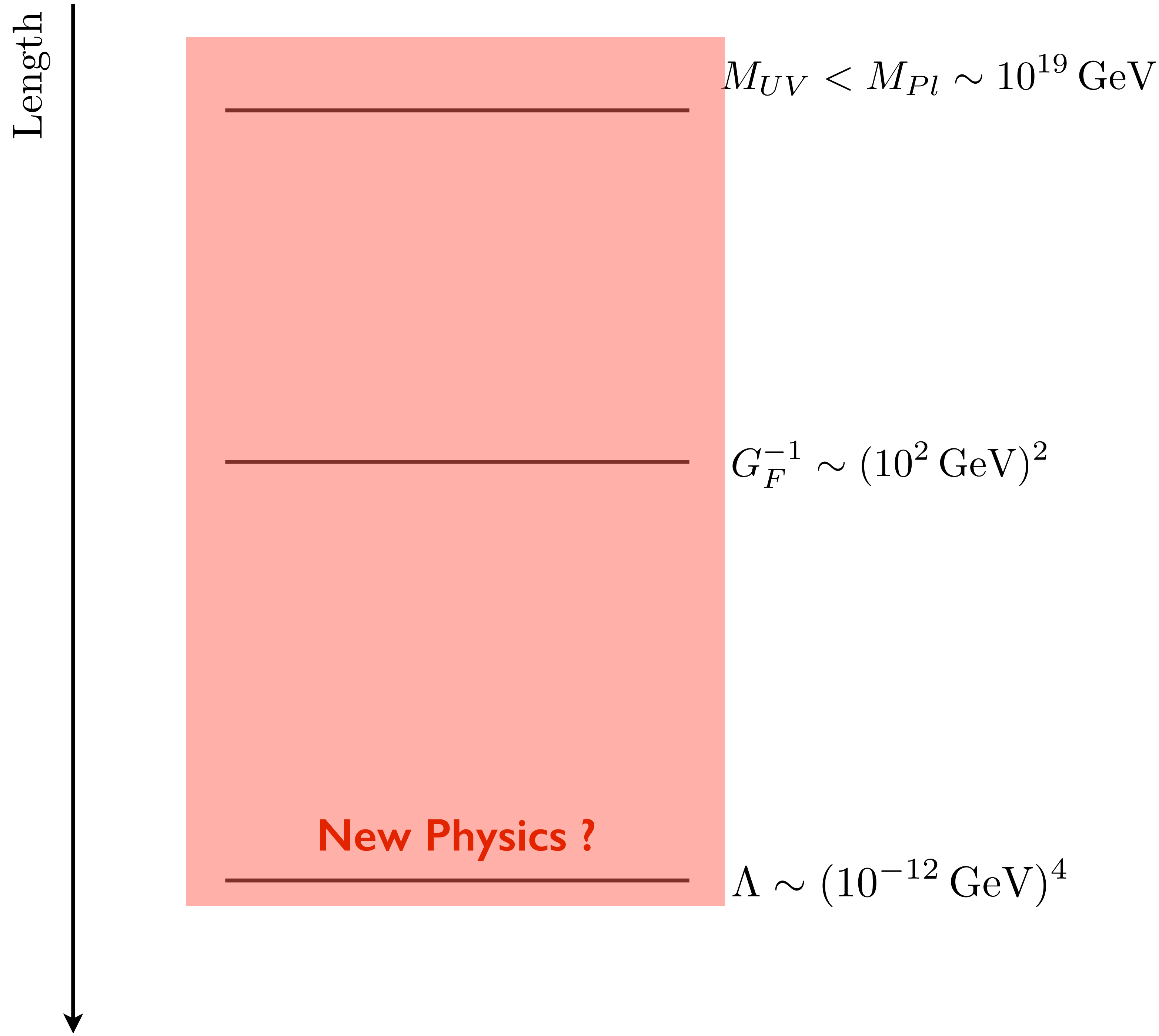
Supersymmetry

Compositeness

$$\Delta v^2 \sim m_{NP}^2 \log M_{UV}^2$$



Length
$M_{UV} < M_{Pl} \sim 10^{19}\,\mathrm{GeV}$
New Physics
LHC: ~ TeV
$G_F^{-1} \sim (10^2\,\mathrm{GeV})^2$
$\Lambda \sim (10^{-12}\,\mathrm{GeV})^4$


Length
$M_{UV} < M_{Pl} \sim 10^{19}\,\mathrm{GeV}$
$G_F^{-1} \sim (10^2\,\mathrm{GeV})^2$
New Physics ?
$\Lambda \sim (10^{-12}\,\mathrm{GeV})^4$

$$\Delta\Lambda \sim M_{UV}^4$$

NO compelling dynamical mechanisms to soften this quartic dependence.

The Weinberg prediction for the Cosmological Constant

Weinberg ('87)

Structures below horizon mass at equality grow by the same amount during matter domination

$$\delta \sim \frac{T_{eq}}{T_\Lambda} G(M)\delta_0 \sim \frac{\xi_m}{(\Lambda/\xi_m)^{1/3}} G(M)\delta_0 > 1$$

$$\xi_m \equiv \frac{\rho_m}{n_\gamma}$$

$$\delta_0 \approx 10^{-5}$$

$$M_{eq} = 10^{16} M_\odot \propto \xi_m^{-2}$$

In practice matter should dominate at the redshift where structures start to form

$$\left.\frac{\Lambda}{\rho_m}\right|_{\text{today}} \sim 2 \qquad \text{but} \qquad 1 + z_{SF} \lesssim 10$$

This result suggests strongly that if it is the anthropic principle that accounts for the smallness of the cosmological constant, then we would expect a vacuum energy density $\rho_V \sim (10-100)\rho_{M_0}$, because there is no anthropic reason for it to be any smaller.

Weinberg

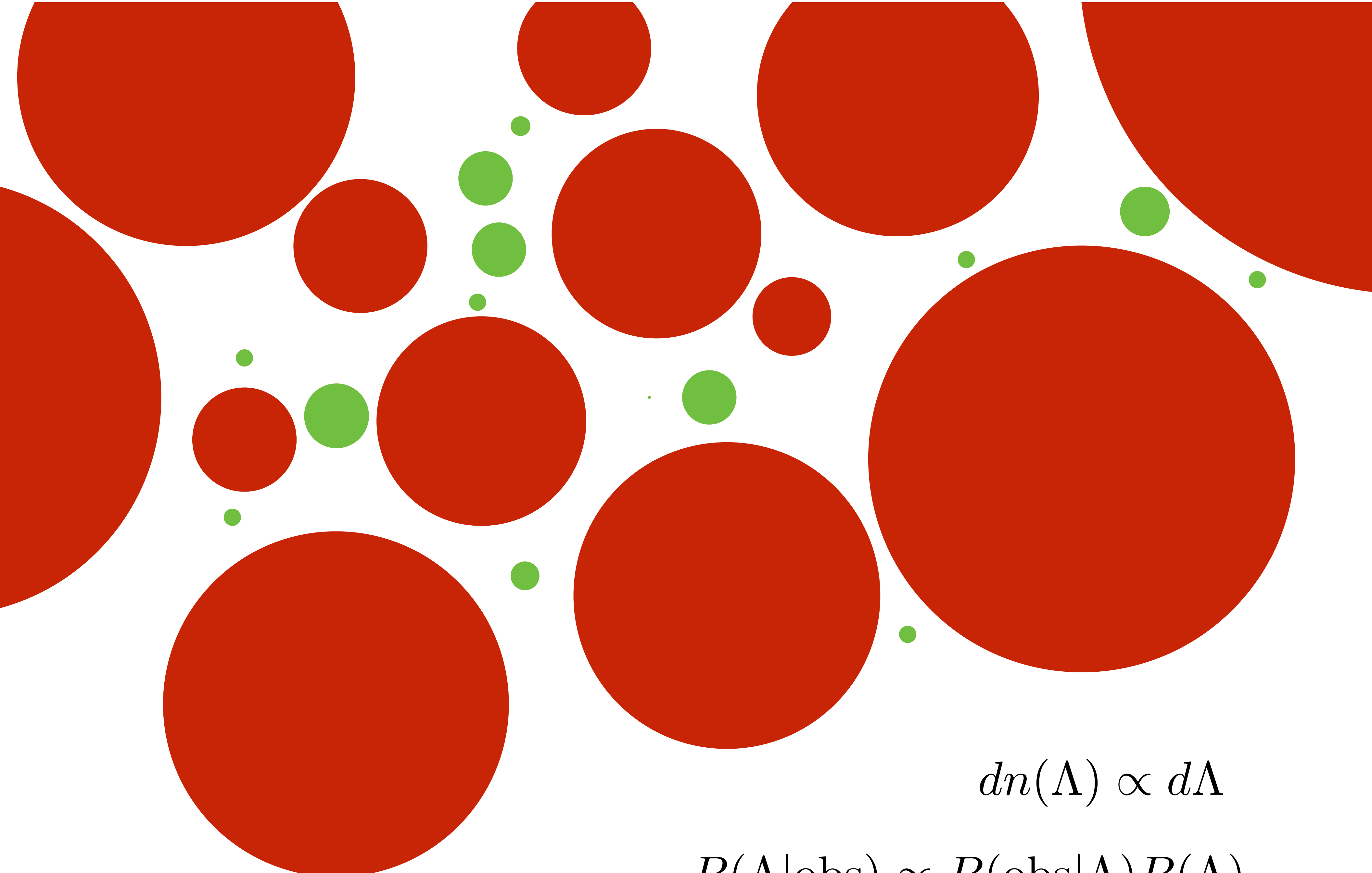
What's the physical meaning of changing the CC

Eternally inflating Landscape

Runaways if the primordial contrast or the DM density are changed

Not a quantitatively accurate prediction

$$dn(\Lambda) \propto d\Lambda$$



$$dn(\Lambda) \propto d\Lambda$$

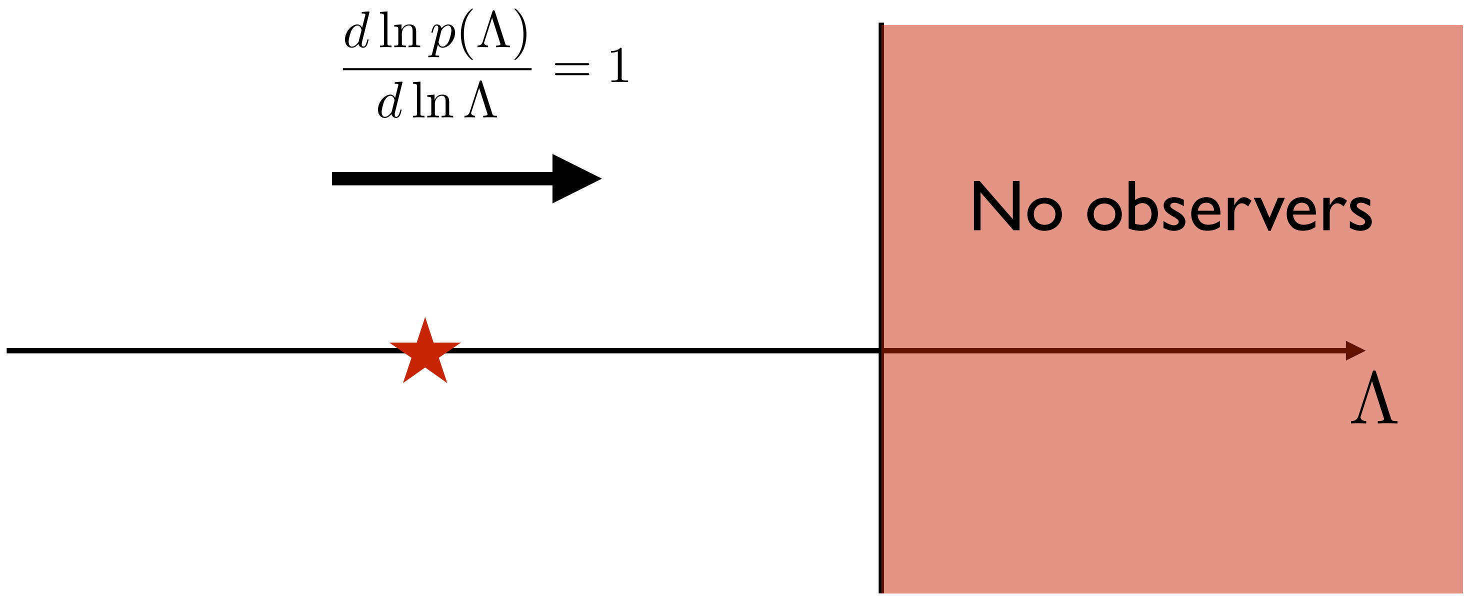
$$P(\Lambda|\text{obs}) \propto P(\text{obs}|\Lambda)P(\Lambda)$$

$$dP(\Lambda|\text{obs}) = f(\Lambda)\, n(\Lambda) d\ln\Lambda$$

Prior probability distribution: depends on UV details (string theory, eternal inflation) and is affected by the measure problem.

Anthropic factor: depends on IR physics, but it's definition cannot be precise. It is also affected by the measure problem.

$$\begin{aligned} dP(\Lambda|\text{obs}) &\sim f_{\text{eff}}(\Lambda)\theta(\Lambda_{\text{obs}} - \Lambda) d\ln\Lambda \\ &\sim \Lambda^p\, \theta(\Lambda_{\text{obs}} - \Lambda) d\ln\Lambda \end{aligned}$$



$\frac{d \ln p(\Lambda)}{d \ln \Lambda} = 1$
No observers
Λ

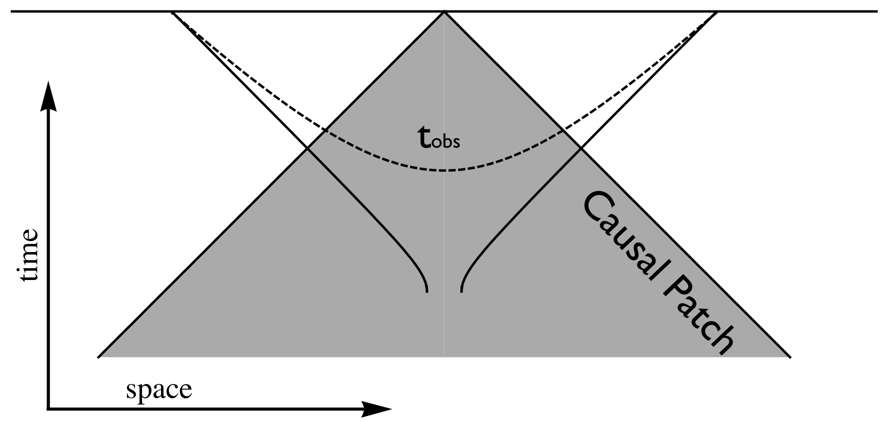
What's the physical meaning of changing the CC

Eternally inflating Landscape

Runaways if the primordial contrast or the DM density are changed

Not a quantitatively accurate prediction

Bousso, Freivogel, Leichenauer, Rosenhaus ('07)



$$t_\Lambda \equiv \frac{1}{\sqrt{G_N \Lambda}}$$

$$\frac{dp}{d\log t_\Lambda} = \frac{d\tilde{p}}{d\log t_\Lambda} \times n_{\rm obs}(t_{\rm obs}, t_\Lambda) \qquad \begin{cases} t_\Lambda^{-1} & \text{for} \quad t_{\rm obs} < t_\Lambda \\ t_\Lambda^{-1} e^{-3t_{\rm obs}/t_\Lambda} & \text{for} \quad t_{\rm obs} < t_\Lambda \end{cases}$$

$$\frac{d\tilde{p}}{d\log t_\Lambda} \longrightarrow \sim t_\Lambda^{-2} \qquad n_{\rm obs}(t_{\rm obs}, t_\Lambda) \longrightarrow \sim M_{CP}$$

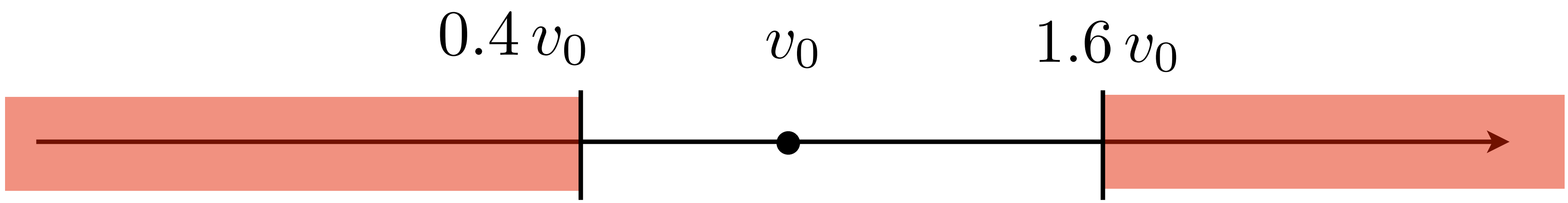
Bousso, Freivogel, Leichenauer, Rosenhaus ('07)

⟶ $t_\Lambda \sim t_{\rm obs}$ **Easy**: observers diluted exponentially after CC domination

⟶ $t_\Lambda \sim t_{\rm obs} \sim e^{\mathcal{N}^{\frac{1}{2}}}$ relates the size of the CC to the size of the landscape

Scanning the weak scale (and just the weak scale) in the multiverse

The EW vev is subject to the anthtopic requirement of the existence of complex chemistry.



No stable hydrogen

$H \to n\bar{\nu}_e$

No heavy elements

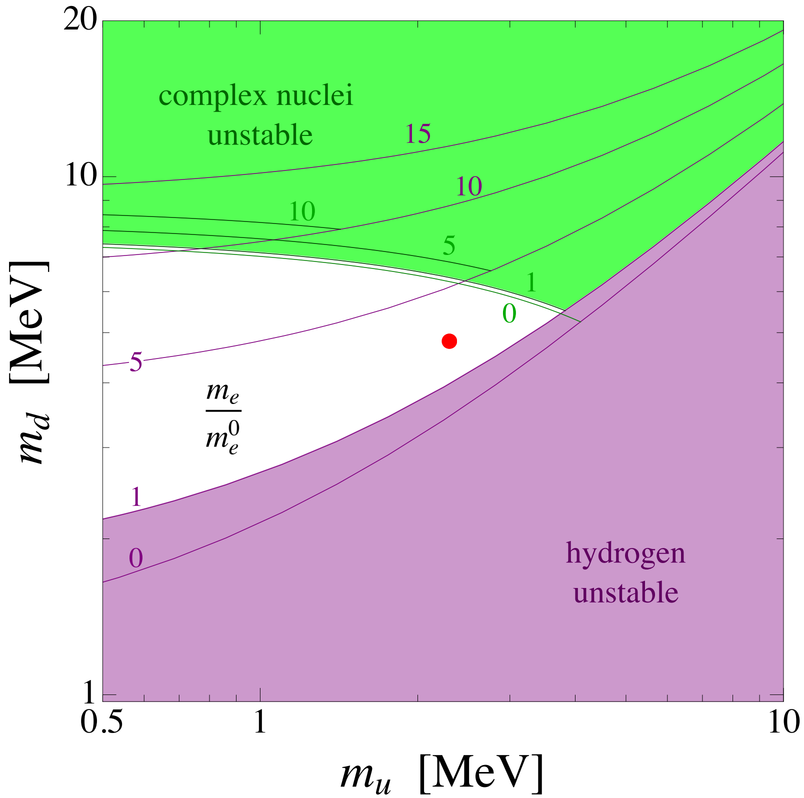
$|B/A| < m_n - m_p - m_e$

Notice that these are constraints on the fermion masses NOT on v

Weakless universe?

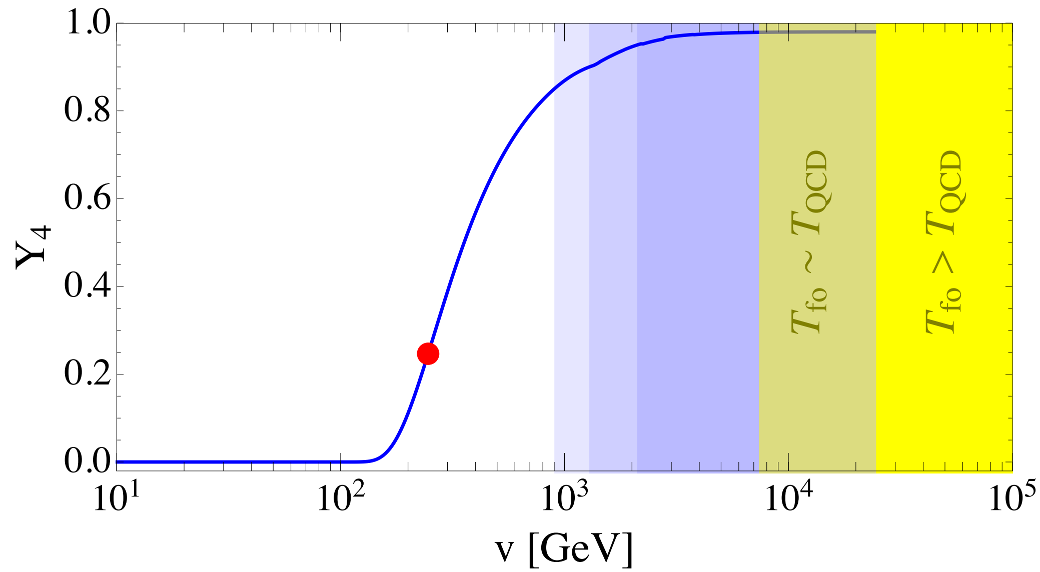
Agrawal, Barr, Donoghue, Seckel ('97)

Damour, Donoghue ('07)



Hall, Pinner, Ruderman ('14)

Is it possible to constrain BOTH light quark masses and v?



Hall, Pinner, Ruderman ('14)

$$\ln \frac{n}{p}\bigg|_{FO} \sim -\frac{(m_n - m_p) M_{Pl}^{1/3}}{v^{4/3}}$$

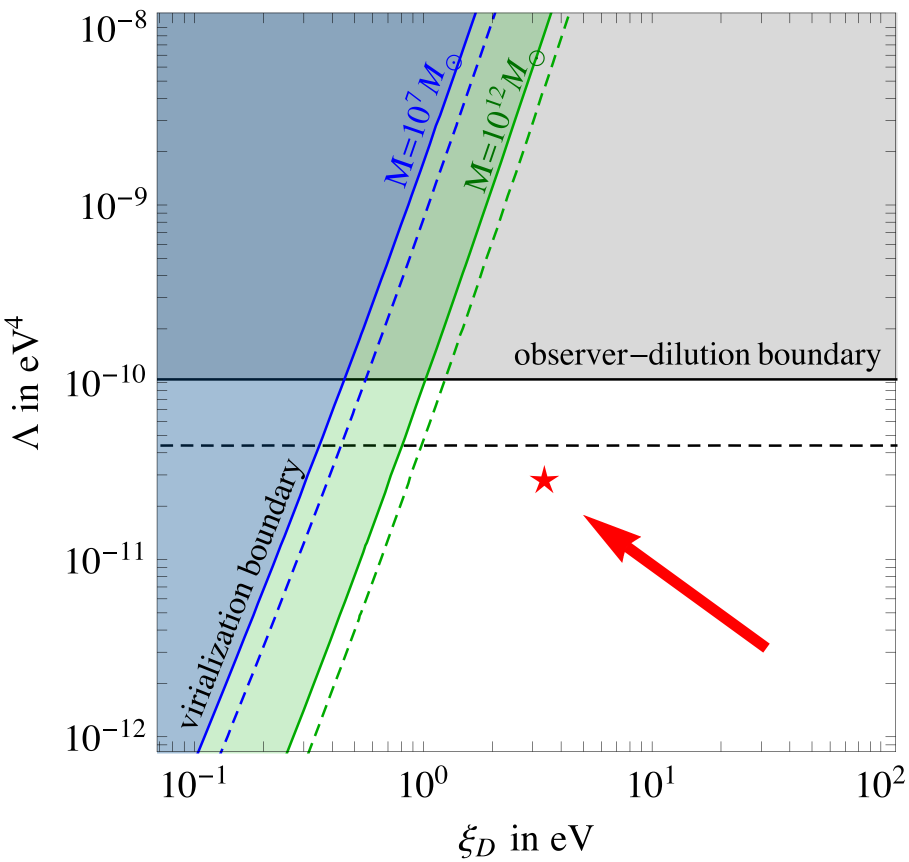
As v gets large the universe become He dominated

Likely (?) catastrophic

DM density from anthropic selection

$$\delta \sim \frac{\xi_m^{4/3}}{\Lambda^{1/3}} G(M)\delta_0$$

If the DM density is reduced by a sufficient amount structure will not go non-linear before CC domination



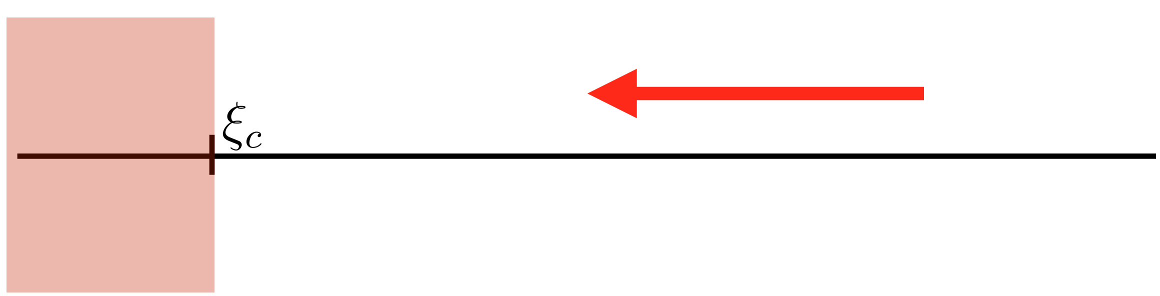
In the following: $\Lambda < \Lambda_c;$ $\quad \xi_D > \xi_c \left(\frac{\Lambda}{\Lambda_c}\right)^{1/4}$ $\quad (\xi_c, \Lambda_c) \sim (0.5\,\xi_{D0}, \Lambda_0)$

Probability distribution in the xi Lambda plane

$$dP \;=\; p(\xi_D)\,\frac{\xi_{B0}}{\xi_{B0}+\xi_D}^{\,*}\,d\ln\xi_D\,d\Lambda$$

Marginalizing over the CC

$$dP \;=\; p(\xi_D)\,\frac{\xi_{B0}}{\xi_{B0}+\xi_D}\,d\ln\xi_D\,\begin{cases}\Lambda_c & \xi_D > \xi_c\\ \Lambda_c(\xi_D/\xi_c)^4 & \xi_D < \xi_c\end{cases}$$



Freivogel ('08)

*

Assume observers are made of baryons

Assume they arise after equality

If the baryon to photon ratio is fixed then the total mass in the causal patch is independent of the DM to baryon ratio, then

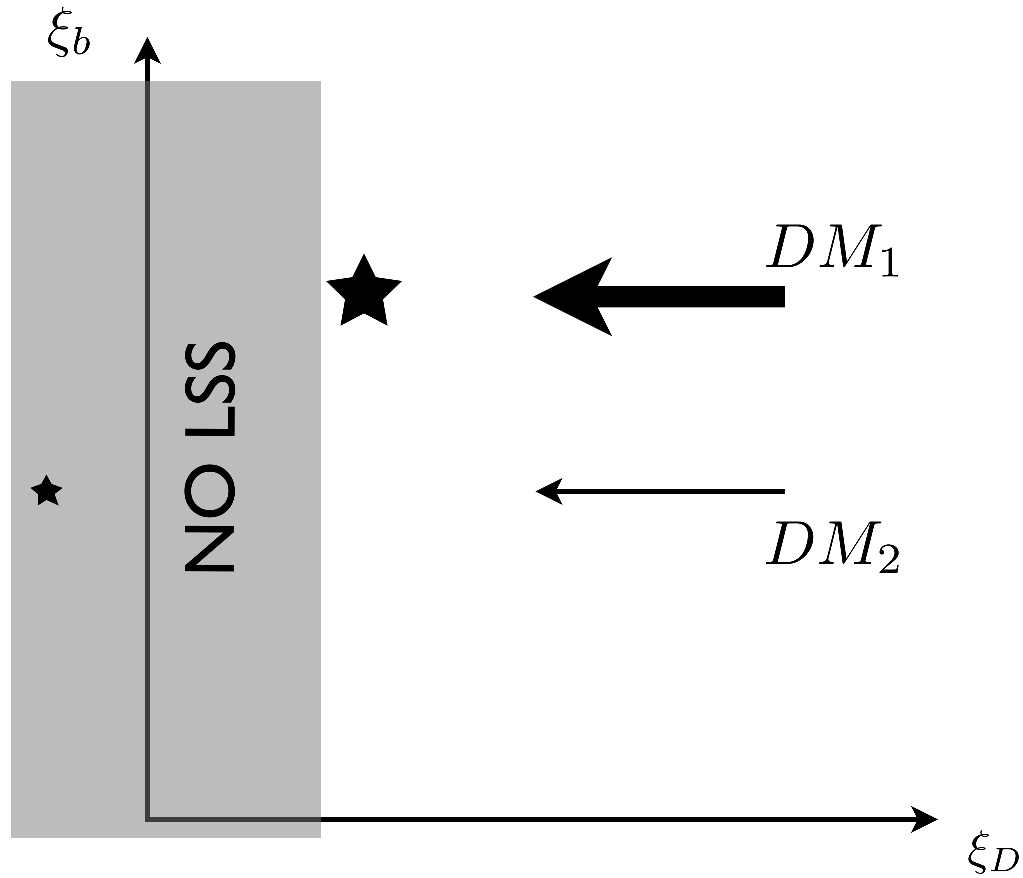
$$N_b \propto \boxed{\frac{\xi_b}{\xi_b + \xi_m}}$$

Assumption:

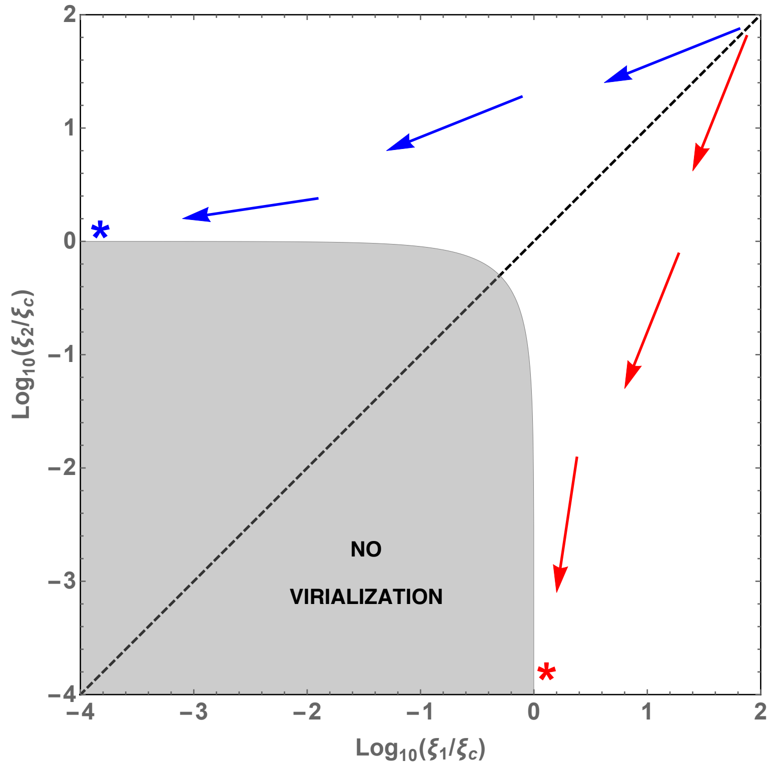
the vicinity of the LSS boundary is determined by environmental selection

(Requires a posterior pdf preferring small DM density)

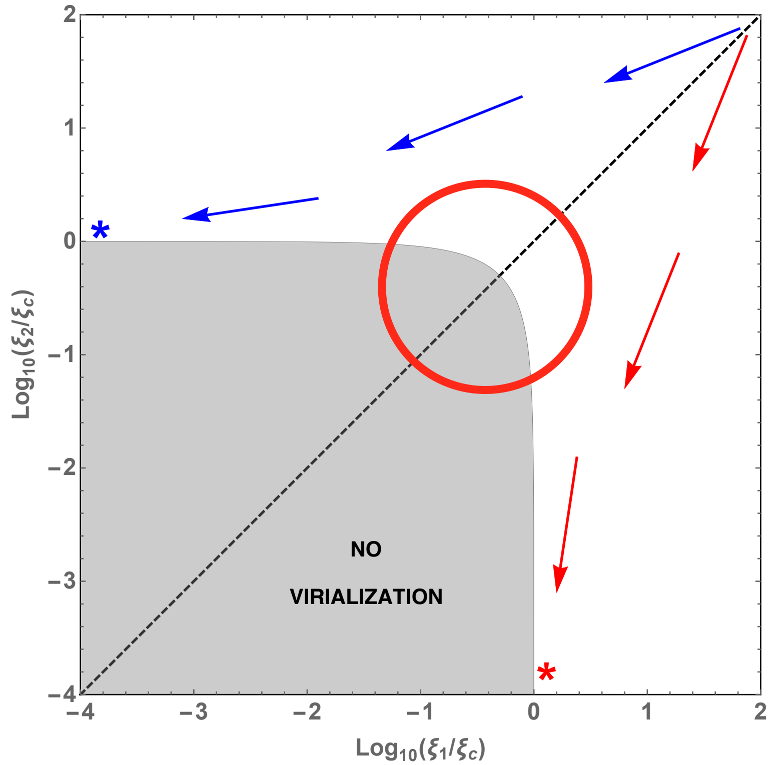
LSS boundary and multi-component DM



The LSS boundary predicts the DM to be single component

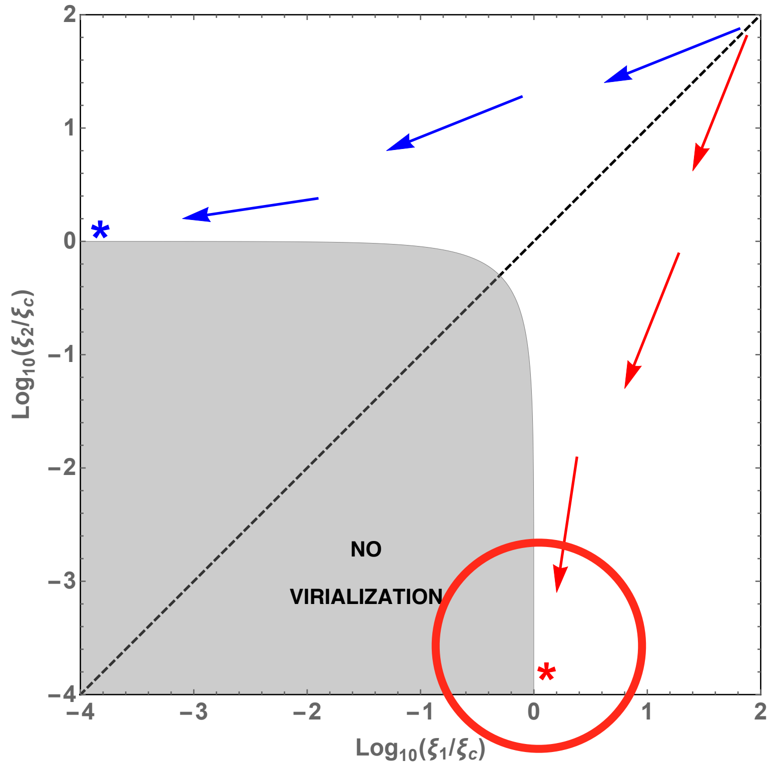


$$\frac{dP}{d\log\xi_1\, d\log\xi_2} = C\,\theta\left(\xi_1+\xi_2-\xi_c\right)\frac{1}{1+\frac{\xi_1+\xi_2}{\xi_B}}\,\xi_1^{n_1}\,\xi_2^{n_2}$$



$$\xi_1, \xi_2 \sim \xi_c$$
$$\Delta\xi_i \sim \xi_i$$

$$P_{\text{multi}} \sim \xi_c^{-|n_1|} \xi_c^{-|n_2|}$$



$$\xi_1 \sim \xi_c$$
$$\xi_2 \sim \xi_{2\,\mathrm{min}}$$
$$\Delta\xi_i \sim \xi_i$$

$$P_{\mathrm{single}} \sim \xi_c^{-|n_1|}\,\xi_{2\,\mathrm{min}}^{-|n_2|}$$

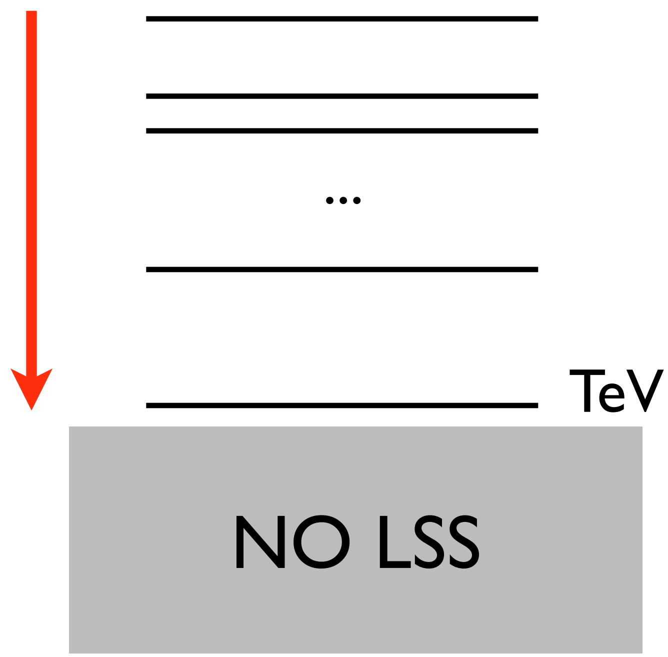
$$\longrightarrow \quad \frac{P_{\mathrm{multi}}}{P_{\mathrm{single}\,1}} \sim \left(\frac{\xi_{\mathrm{min}\,2}}{\xi_c}\right)^{|n_2|}$$

Single component DM:
WIMPs

Consider a SUSY spectrum defined by a fundamental SUSY breaking parameter m, in which the ratios between sparticle masses are roughly fixed

$$dP(\tilde{m}) \sim \frac{\tilde{m}^n}{1+\tilde{m}^2/v^2}\, d\ln\tilde{m} \qquad \tilde{m} > v: \quad \xi_D \sim \frac{\tilde{m}^2}{M_{Pl}}$$

EW fine tuning



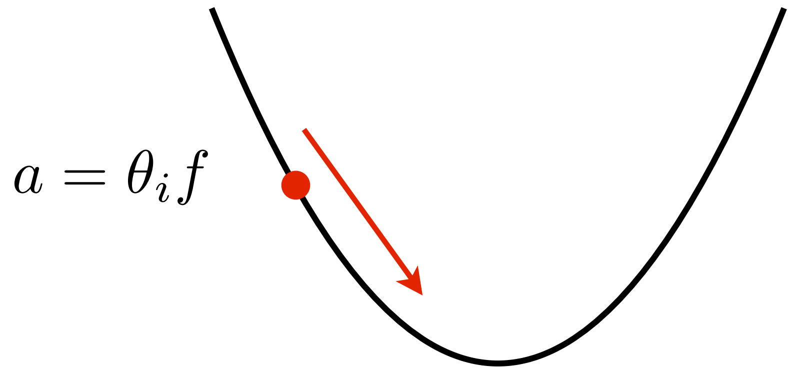
Predicts a little SUSY hierarchy

see also Nomura, Shirai ('14)

Single component DM:
QCD axion

Axion DM: misalignment mechanism

At high temperature (T < f) the axion field is stuck at some location in its potential



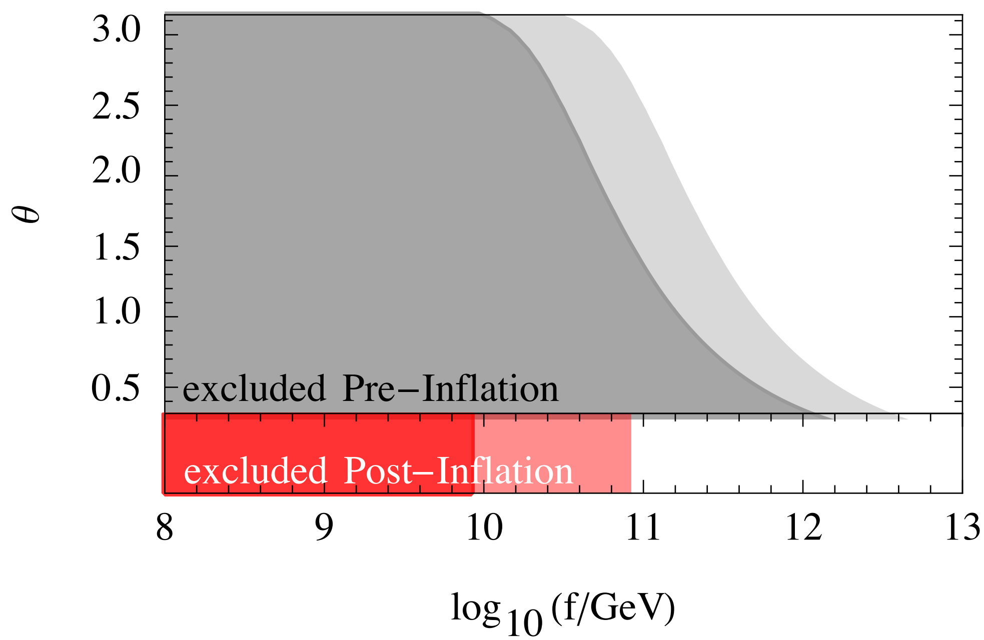
At $m_a(T_{\rm osc}) \sim H(T_{\rm osc})$ the axion starts to oscillate around its minimum and the energy density in the oscillations redshift like non-relativistic matter

$$m_a(T) \sim m_a(\Lambda_{QCD}/T)^{5.5} \;\Rightarrow\; T_{\rm osc} \sim 1\,{\rm GeV}$$

$$\xi_a = \frac{m_a}{m_a(T_{\rm osc})}\frac{\rho_a(T_{\rm osc})}{s(T_{\rm osc})} \approx 1.7\,\xi_{D0}\,\theta^2\left(\frac{f}{10^2\,{\rm GeV}}\right)^{1.18}$$

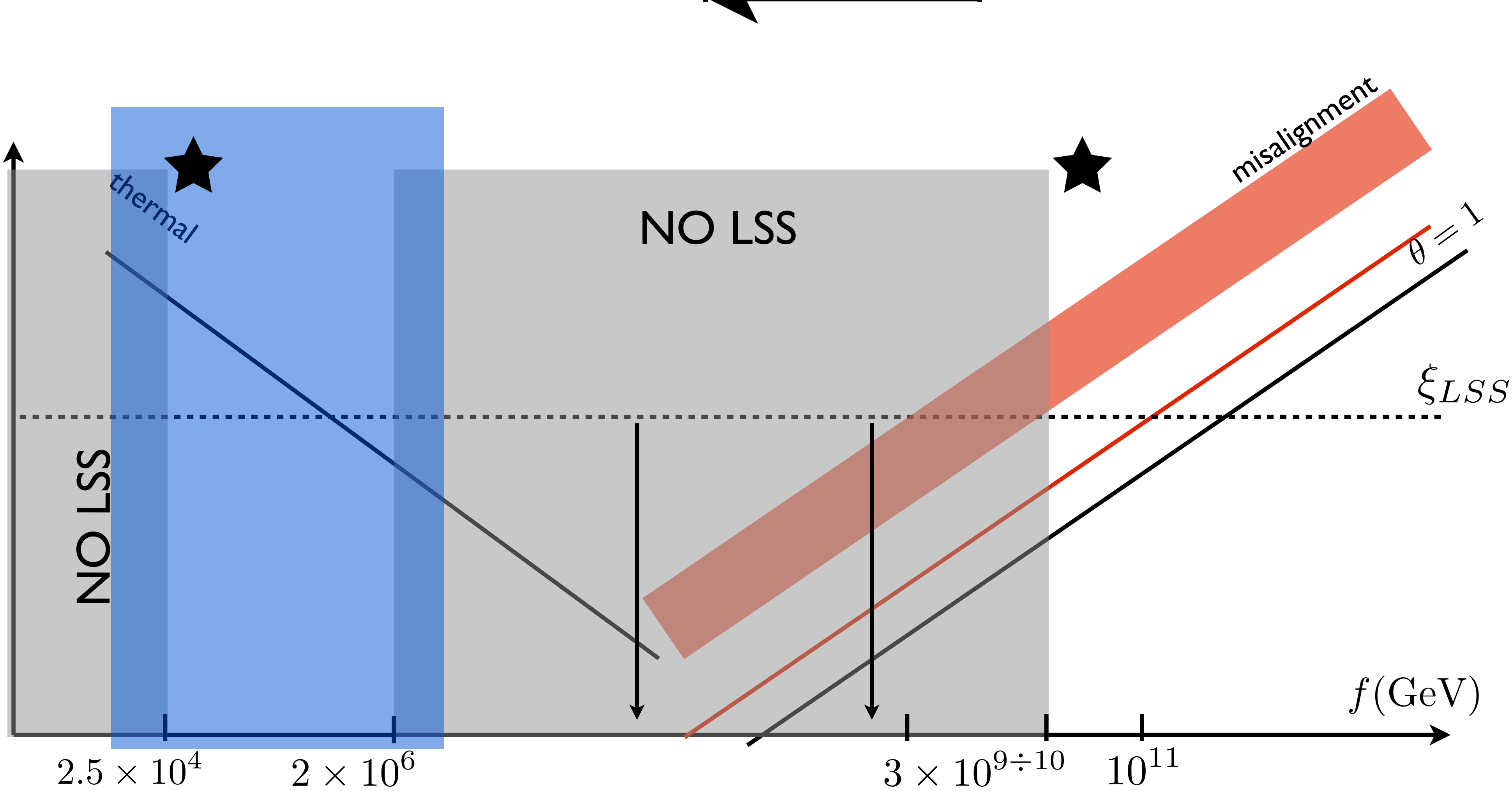
If the PQ is broken during inflation and is not restored after reheating θ takes a random value in our Hubble patch between 0 and π. On the other hand θ is averaged over the patch and $\theta_{\rm eff} = \pi/\sqrt{3}$

Kim et al ('08)



Axion production from decay of topological defects should be included. This can lower f by an order of magnitude.

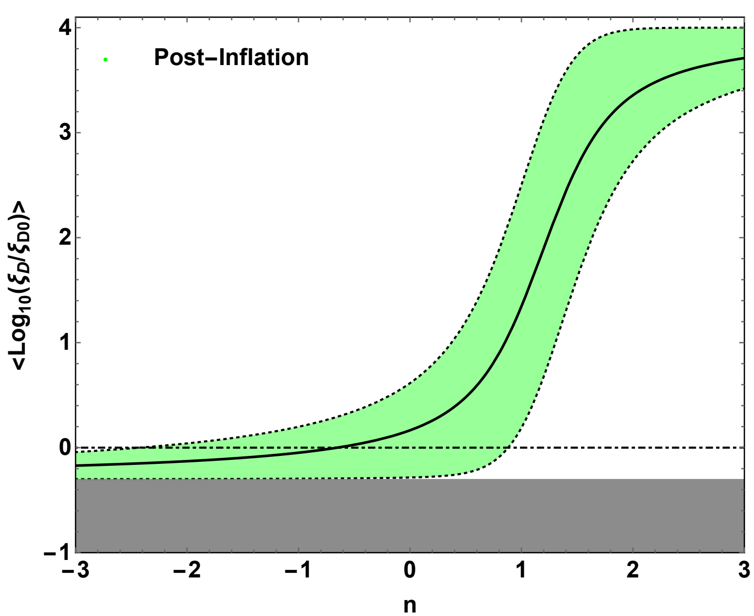
Axion DM parameter space

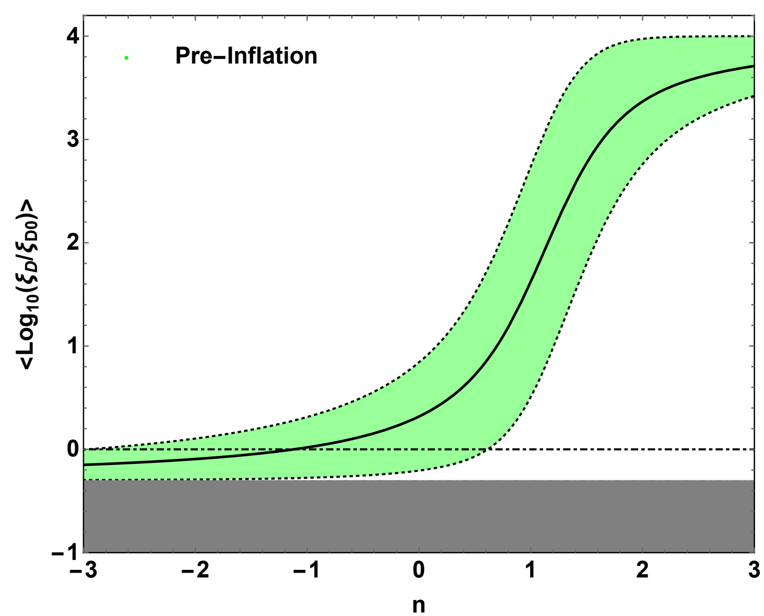


$$dP(f) \propto f^n \, d\ln f, \quad n < 0$$

The thermal axion window is likely not allowed by a variety of constraints: free-streaming of LSS by axion hot dark matter, large rate of axion emission by stars.

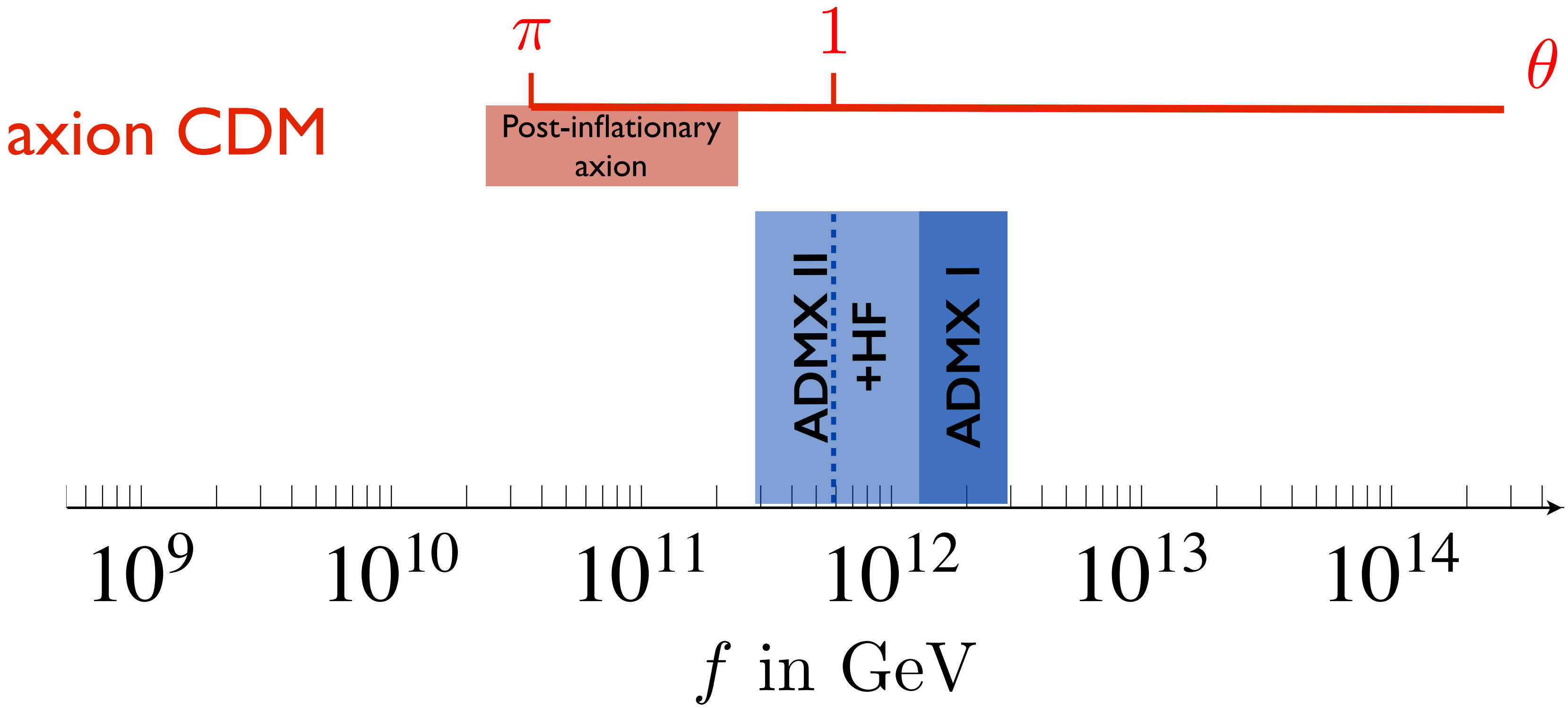
$$dP \propto \theta(\xi_D - \xi_c)\frac{1}{1+\xi_D/\xi_{b0}} f^n d\log f\ (d\theta) \qquad \xi_c = 0.5\,\xi_{D0}$$





$$\theta_{\min} = 10^{-2} < \theta < \pi$$

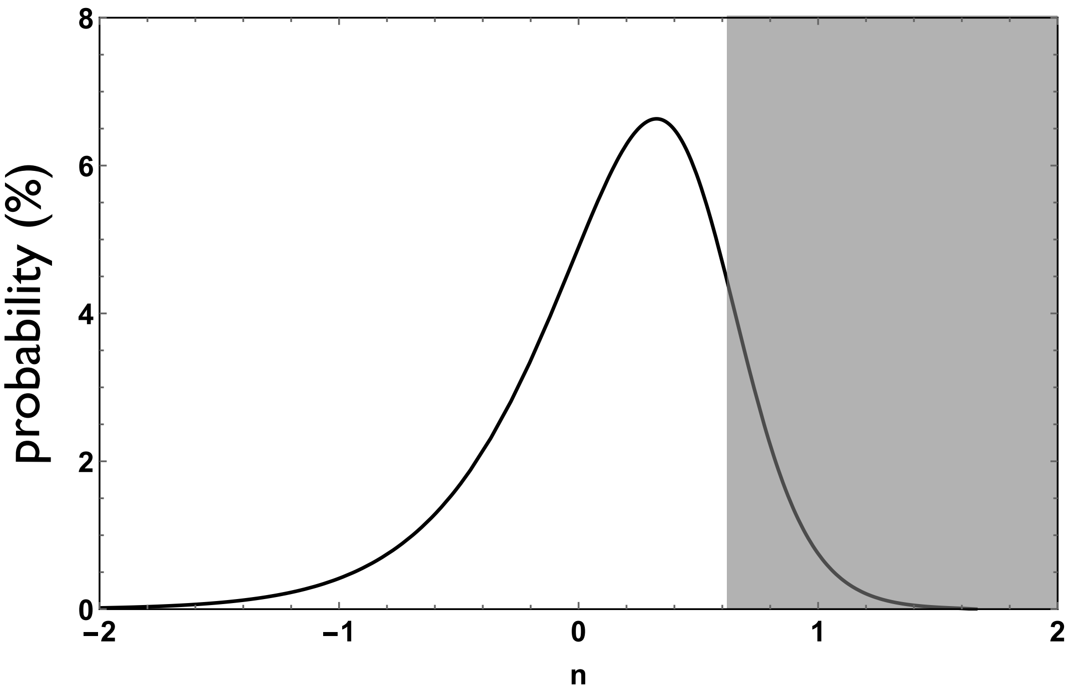
$$\xi_c < \xi_D < \xi_{\max} = 10^4$$



π
1
θ
axion CDM
Post-inflationary
axion
ADMX II
+HF
ADMX I
10^9
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
f in GeV

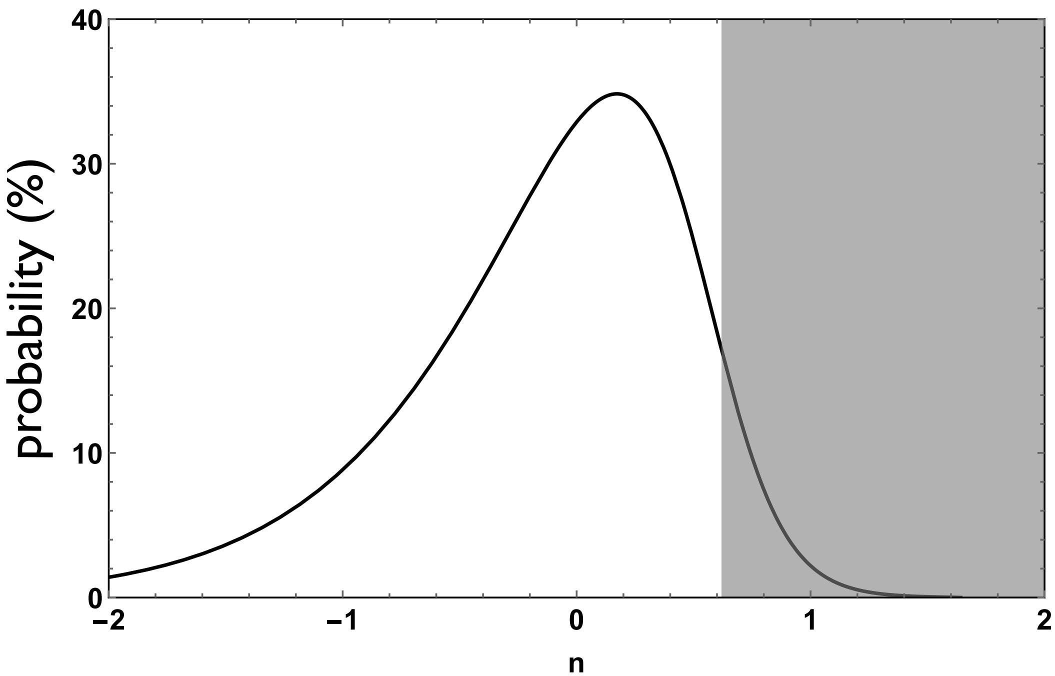
Pre-inflationary axion

ADMX I



Pre-inflationary axion

ADMX II



New ideas are needed to probe low f axions!

see for instance Arvanitaki, Geraci ('14)

A model & the Higgs mass

Ingredients

Axion

The multiverse motivates the existence of a solution to the strong CP problem. If we live close to the LSS boundary for a dynamical reason then the axion is likely to be DM.

Supersymmetry

Ameliorate the fine tuning of both the EW and CC hierarchies.

Provides a zeroth-order understanding of why the Higgs quartic coupling is small.

aMSSM

To the matter content of the MSSM add a singlet chiral superfield, coupled through

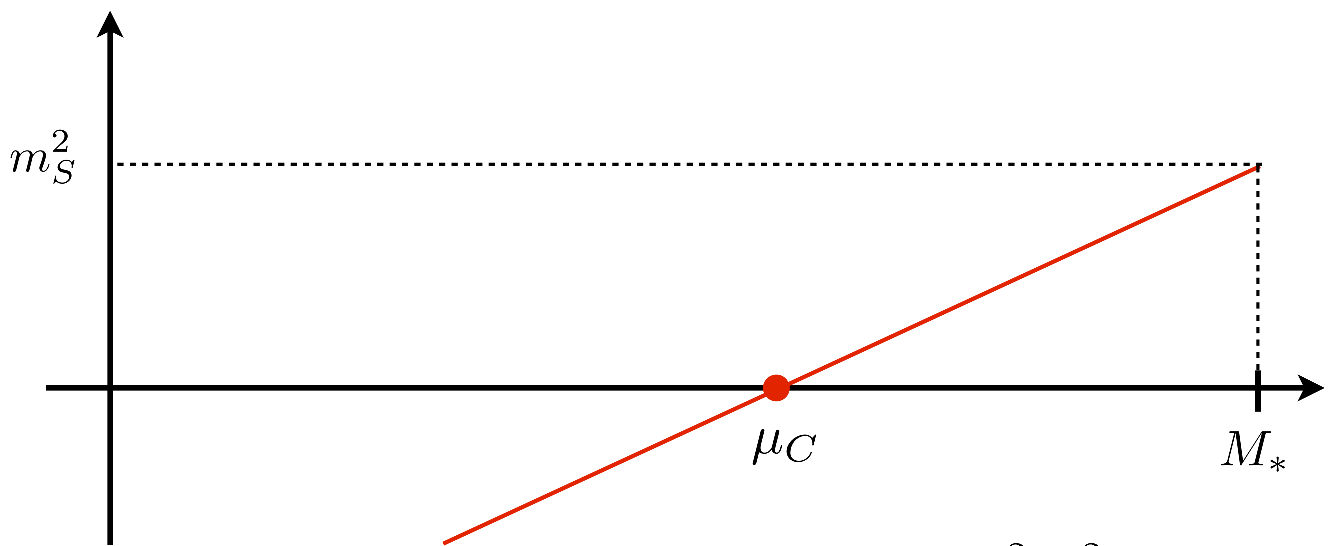
$$W = \xi S H_1 H_2 \qquad\qquad V_{\rm soft} = \xi A_\xi S H_1 H_2 + m_S^2 |S|^2$$

$$m_S^2 > 0$$

The model has an exact global PQ symmetry which has a color anomaly.
Simplest supersymmetric DFSZ model. The model has domain wall number 3.

At tree level both the PQ and EW symmetry are unbroken.

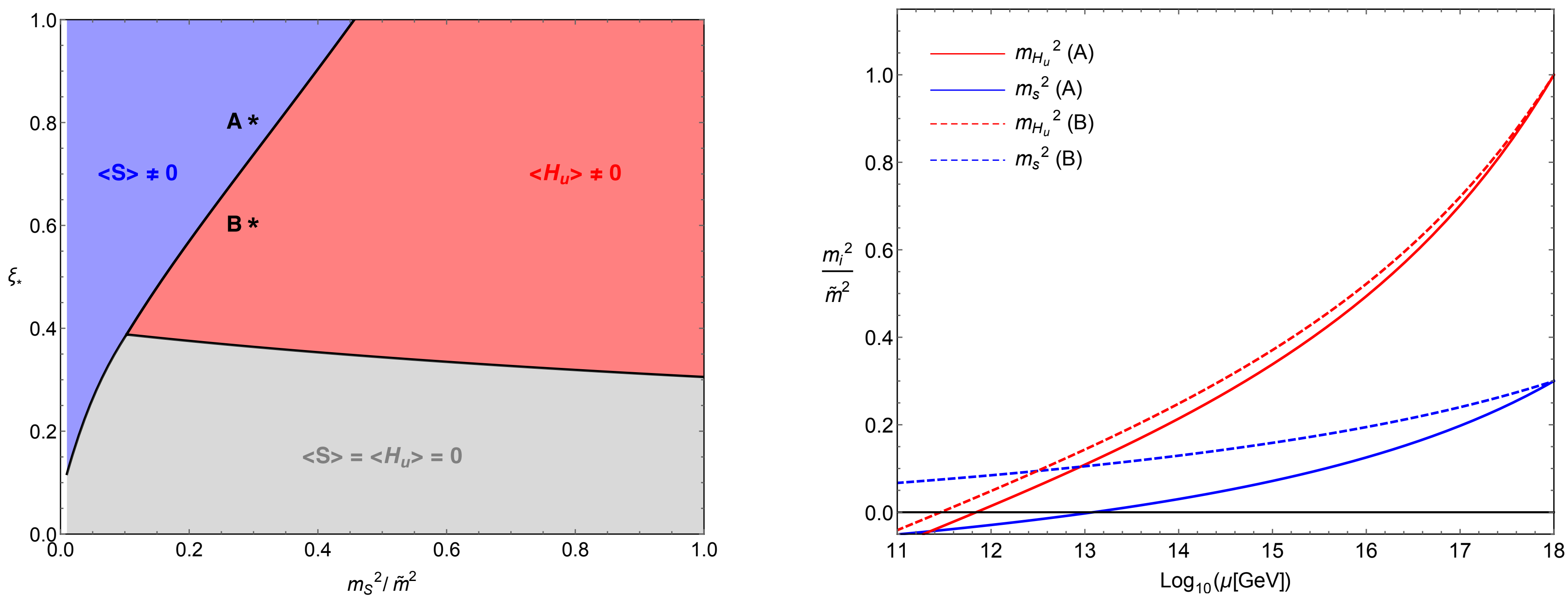
$$
\begin{aligned}
16\pi^2 \frac{d\xi}{dt} &= 4\xi^3 + O(\xi y_t^2) \\
8\pi^2 \frac{dm_{1,2}^2}{dt} &= \xi^2(m_1^2 + m_2^2 + m_S^2 + A_\xi^2) + O(y_t^2) \\
8\pi^2 \frac{dm_S^2}{dt} &= 2\xi^2(m_1^2 + m_2^2 + m_S^2 + A_\xi^2) \\
8\pi^2 \frac{dA_\xi}{dt} &= 4\xi^2 A_\xi + O(y_t^2)
\end{aligned}
$$



$$\mu_C \sim \xi\langle S\rangle \sim M_* e^{-4\pi^2/\xi^2}$$

PQ is broken spontaneously and radiatively. The dynamically generated scale is **a priori** independent of the absolute normalization of the soft masses.

LSS boundary: $\xi = O(1) \Rightarrow \mu_C \gg v$



A *
B *
<S> ≠ 0
<Hu> ≠ 0
<S> = <Hu> = 0
ξ*
mS2/ m̃2
mHu2 (A)
ms2 (A)
mHu2 (B)
ms2 (B)
mi2 / m̃2
Log10(μ[GeV])

$$\mathcal{M}_H^2 \approx \begin{pmatrix} \mu_C^2 + m_2^2 & A_\xi \mu_C \\ A_\xi \mu_C & \mu_C^2 + m_1^2 \end{pmatrix}$$

EWSB with high scale SUSY: $\det \mathcal{M}_H^2 \sim -m_Z^2 \tilde{m}^2$

$\mu_C \gg \tilde{m}$: $\det \mathcal{M}_H^2 \sim \mu_C^4$ NO EWSB

$\mu_C \ll \tilde{m}$: $\det \mathcal{M}_H^2 \sim \pm \tilde{m}^4$ NO EWSB

EWSB forces: $\mu_C \sim \tilde{m}$

A very concrete manifestation of the μ problem in this setup.
It has an anthropic solution.

$$V(S) = \Lambda(\mu) + m_S^2|S(\mu)|^2 + V^{(1)}(S;\mu) + \ldots \qquad \text{1-loop}$$

Expand around the point μ_C where the S soft mass vanishes. The leading log expansion of V then works fine.

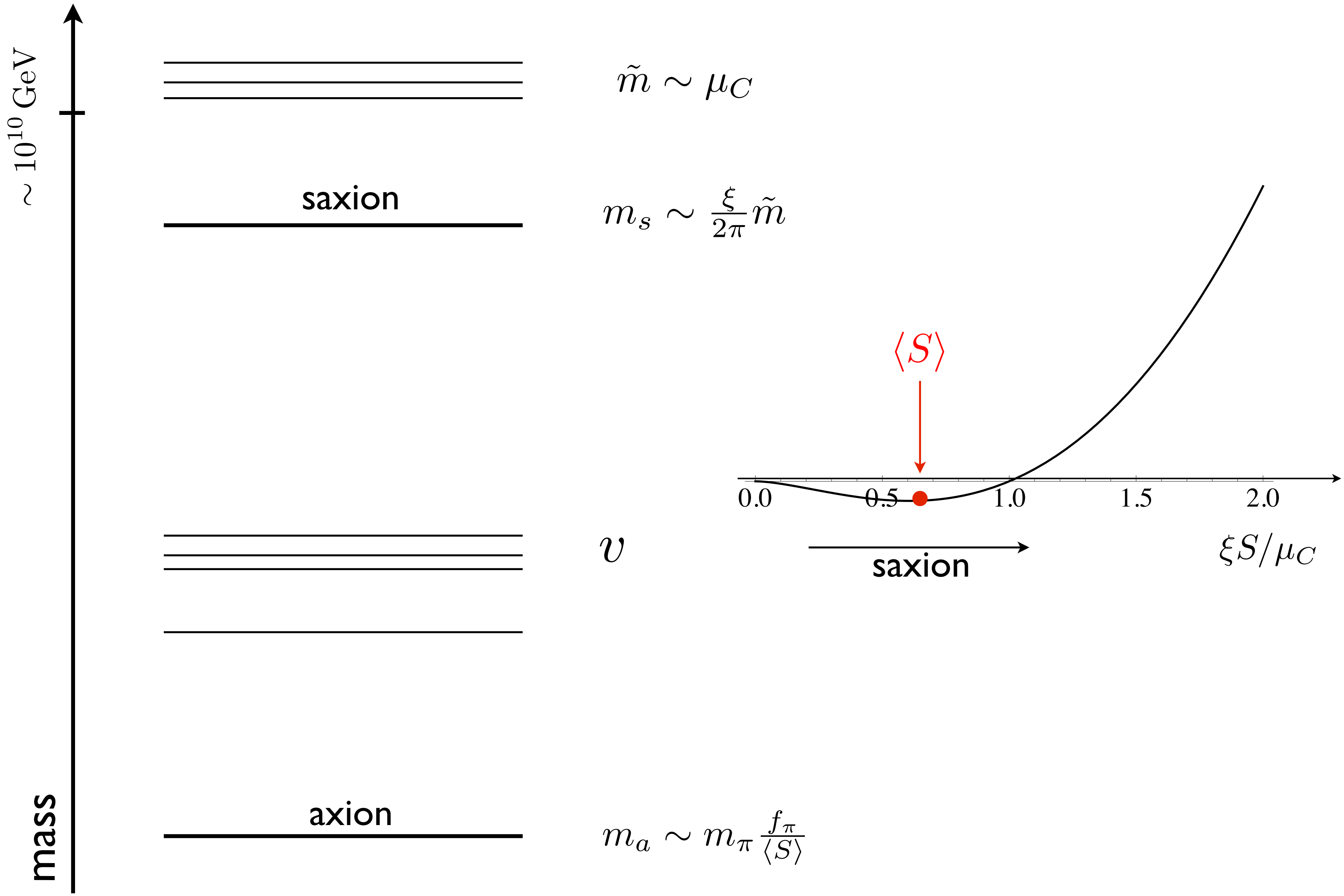
$$V(S) = \frac{1}{64\pi^2}\left[4m_H^4\left(\log\frac{m_H^2}{\mu_C^2} - \frac{3}{2}\right) + 4m_h^4\left(\log\frac{m_h^2}{\mu_C^2} - \frac{3}{2}\right) - 8m_F^4\left(\log\frac{m_F^2}{\mu_C^2} - \frac{3}{2}\right)\right]$$

$$\begin{aligned} m_{H,h}^2 &= \frac{m_1^2 + m_2^2}{2} + \xi^2|S|^2 \pm \sqrt{\frac{(m_1^2 - m_2^2)^2}{2} + \xi^2 A_\xi^2 |S|^2} \\ m_F &= \xi|S| \end{aligned}$$

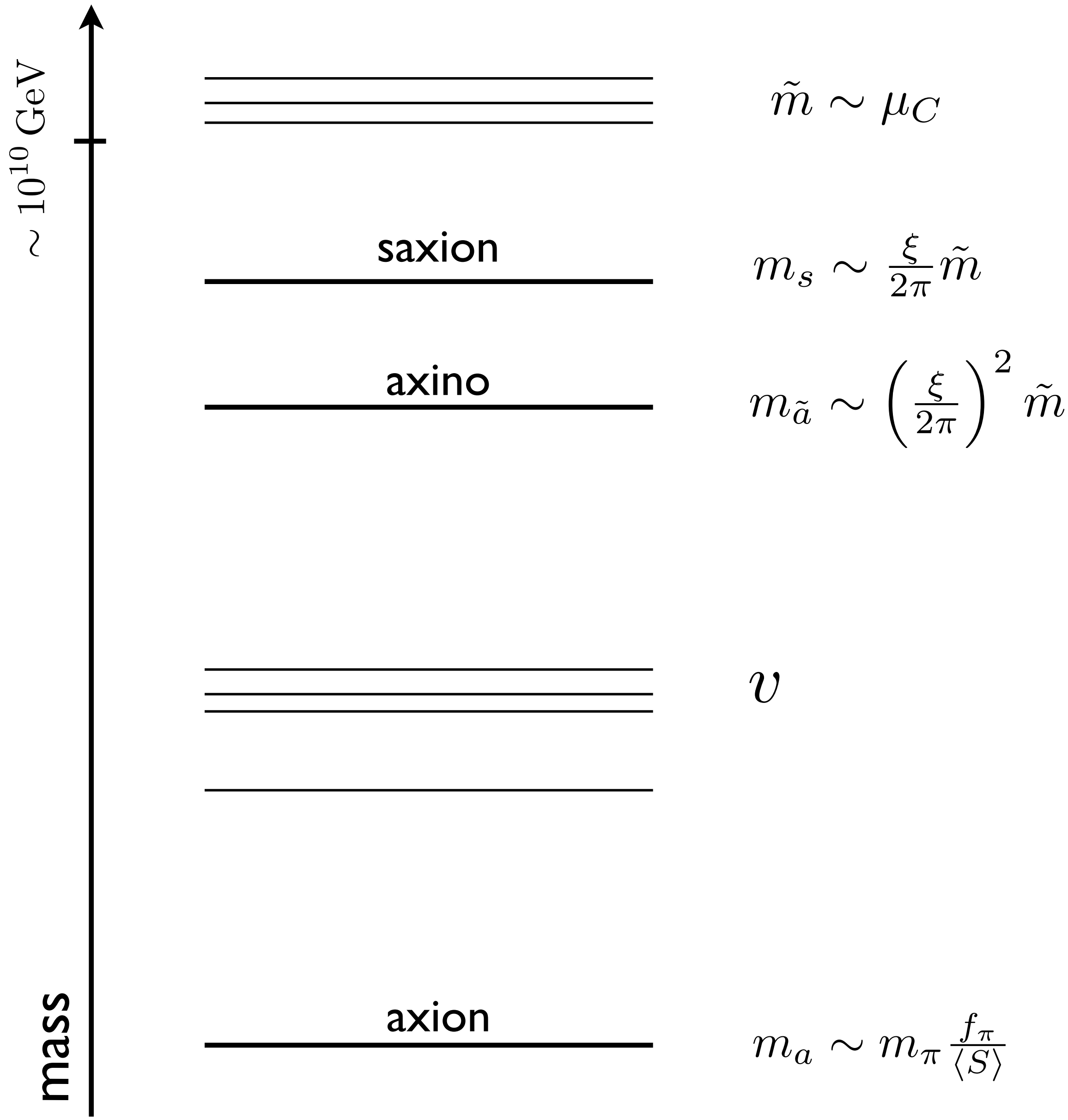
Inputs: $m_1, \quad m_2, \quad A_\xi, \quad \mu_C, \quad \xi$

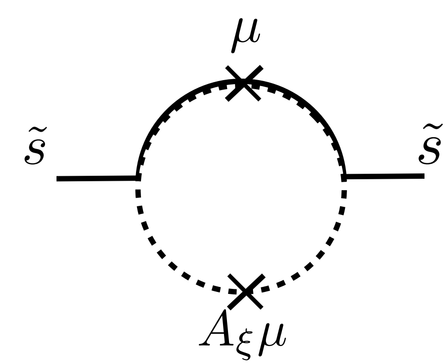
Outputs: $\det \mathcal{M}_H^2, \quad \tan\beta, \quad f, \quad A_{SHH}$

The spectrum around H=0



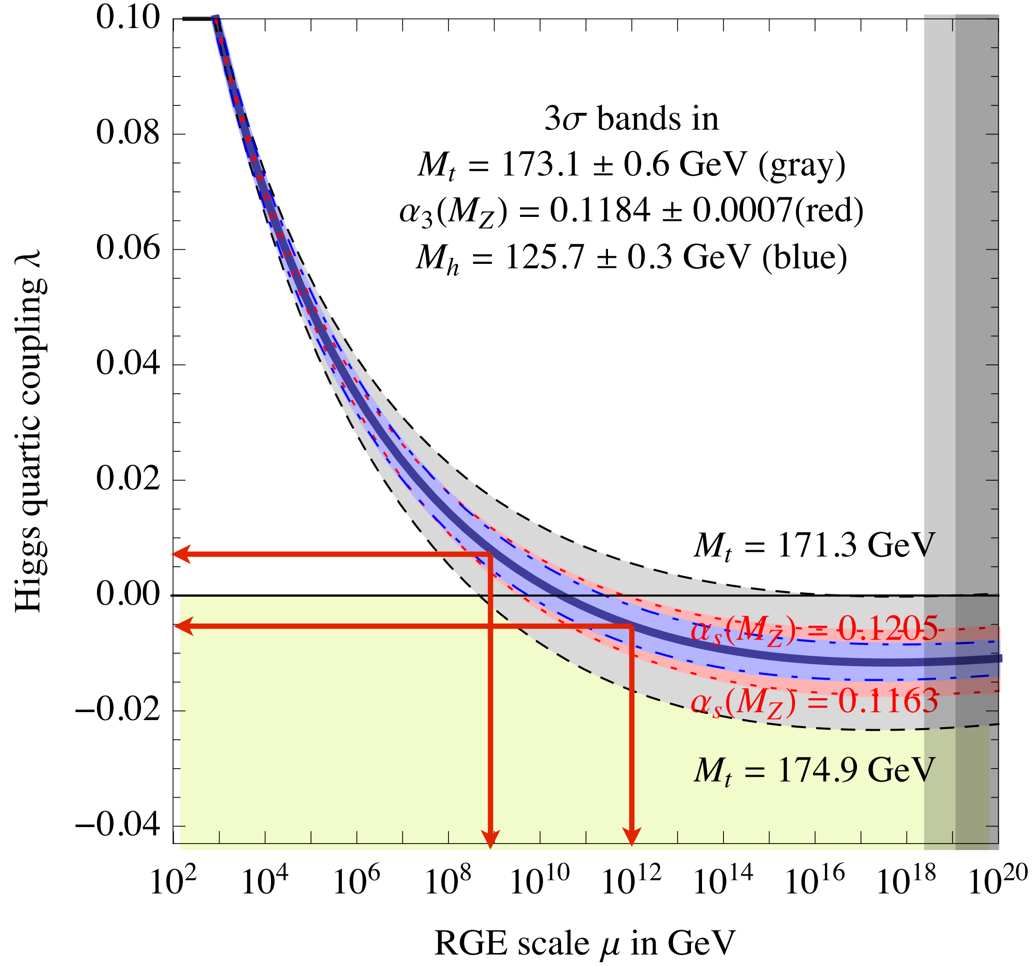
The spectrum around H=0





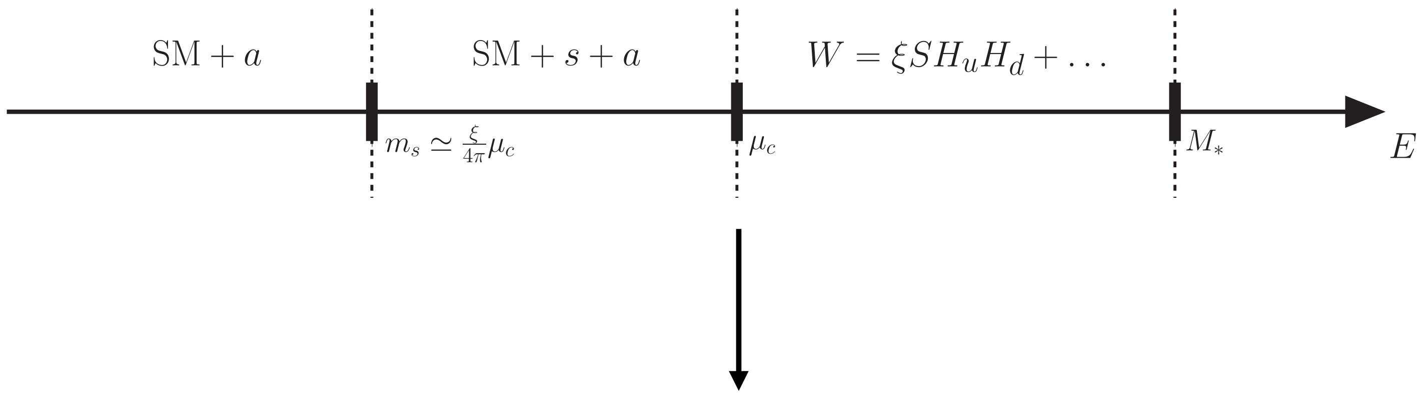
The Higgs quartic around H=0

$$V_{SM}(H) = \lambda_{SM}|H|^4$$



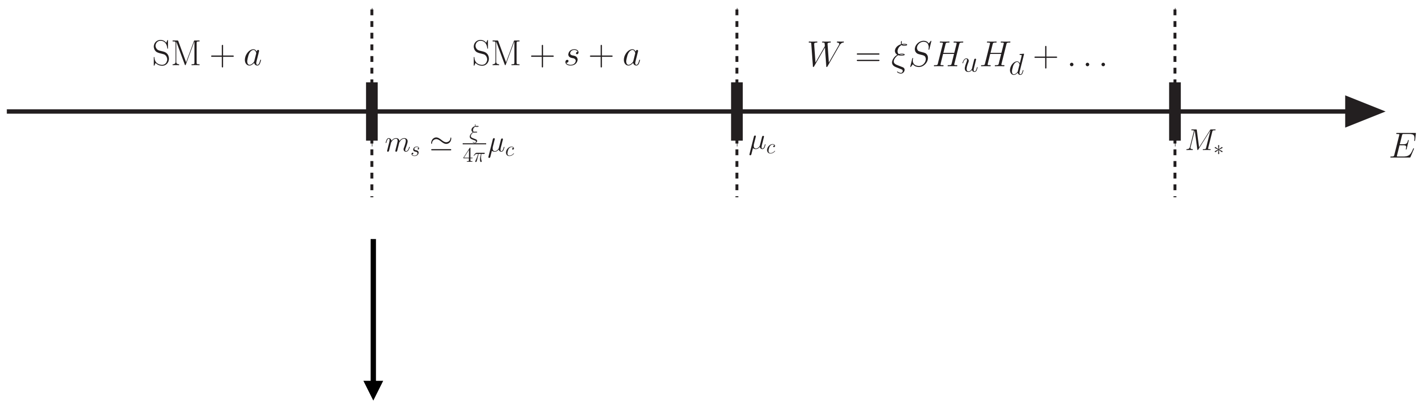
Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia ('13)

A postdiction for the Higgs mass?

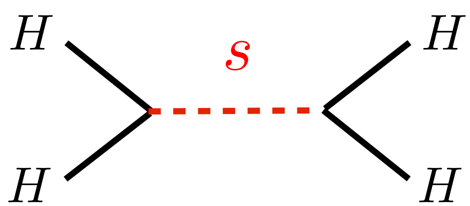


$$\lambda_h = \underbrace{\frac{g^2 + g'^2}{8}\cos^2 2\beta}_{\text{D-terms}} + \underbrace{\frac{\xi^2}{4}\sin^2 2\beta}_{\text{F-term}}$$

A postdiction for the Higgs mass?



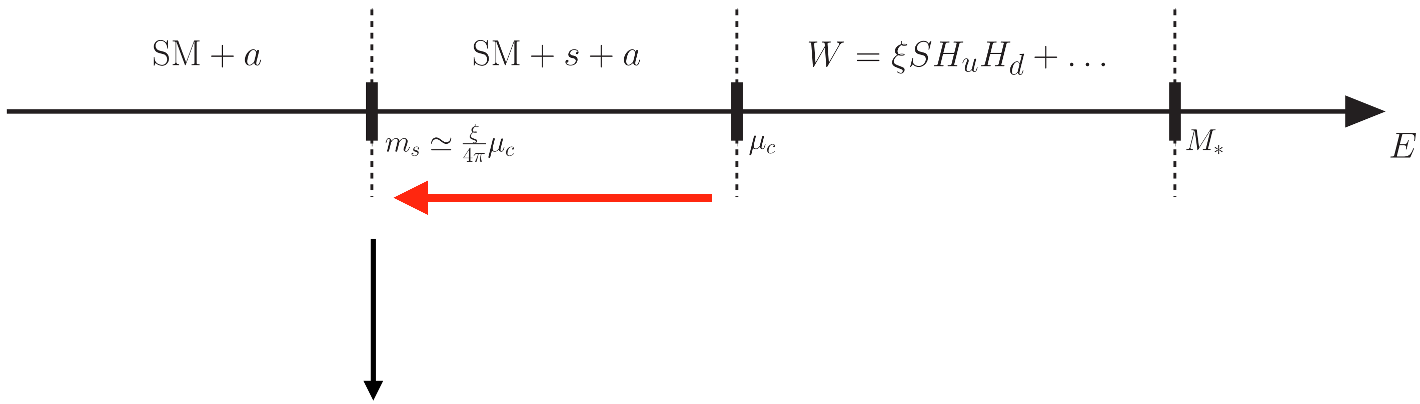
$$\lambda(m_s) = \lambda_h(m_s) - \frac{(A\xi)^2}{12\, m_s^2} \qquad A = \frac{1}{\sqrt{2}}\left(\mu - \frac{A_\xi}{2}\sin 2\beta\right)$$



The saxion mass is one-loop below the other sparticles. Integrating it out gives a large and negative contribution to the Higgs quartic coupling. The H=0 vacuum is unstable.

$$\Delta\lambda_{SM} \sim -16\pi^2$$

A postdiction for the Higgs mass?



$$\lambda(m_s) = \lambda_+ - \lambda_-$$

running

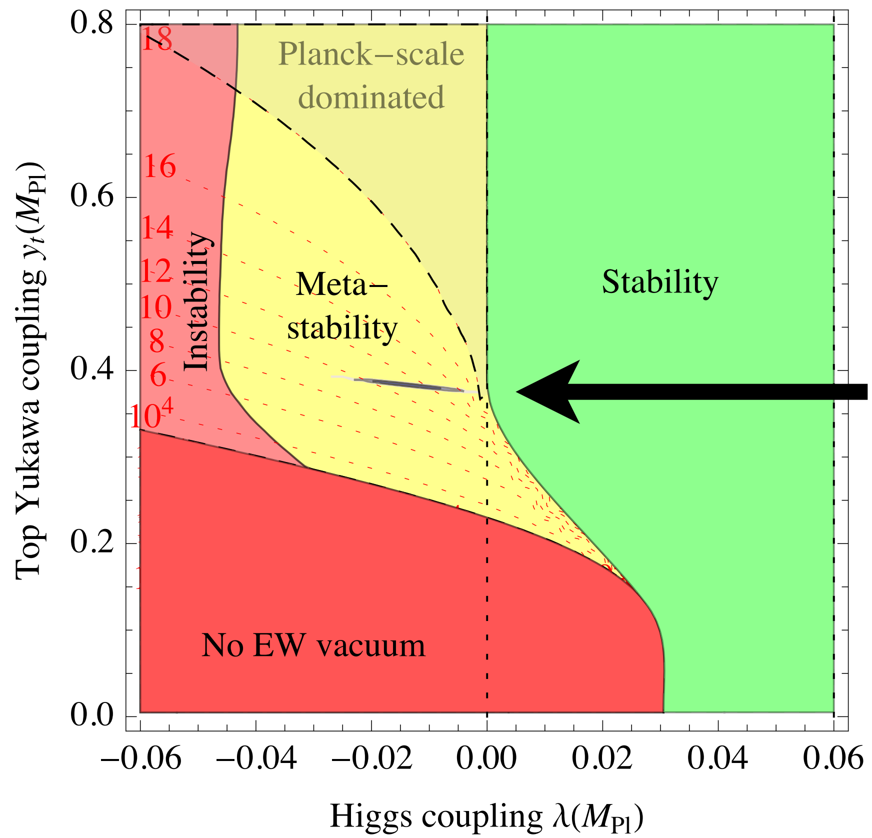
$$\lambda_+ = \left(\frac{g^2+g'^2}{8}\cos^2 2\beta + \frac{\xi^2}{4}\sin^2 2\beta + \boxed{\frac{3y_t^4}{8\pi^2}\ln(4\pi/\xi)} \right)_{\mu_c}$$

$$\lambda_- = \lambda_0\epsilon^2 \qquad \lambda_0 \sim (4\pi)^2 \qquad \epsilon \equiv A/\mu$$

$$dP(\epsilon) \propto d\epsilon \quad \longrightarrow \quad dP(\lambda_-) \propto \lambda_-^{-1/2}\, d\lambda_-$$

Solid anthropic lower bound on λ from vacuum stability

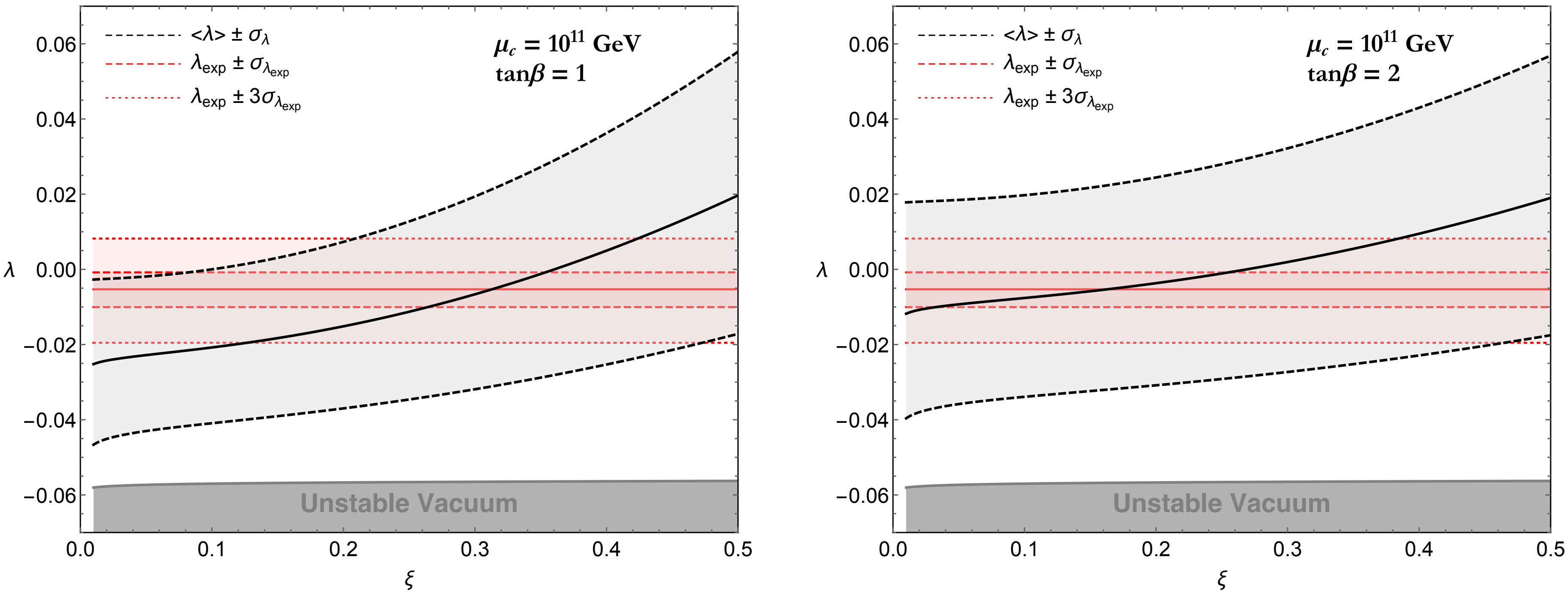
$$\lambda_{SM}(M_*) > \lambda_{\rm crit} \approx -0.04$$



Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia ('13)

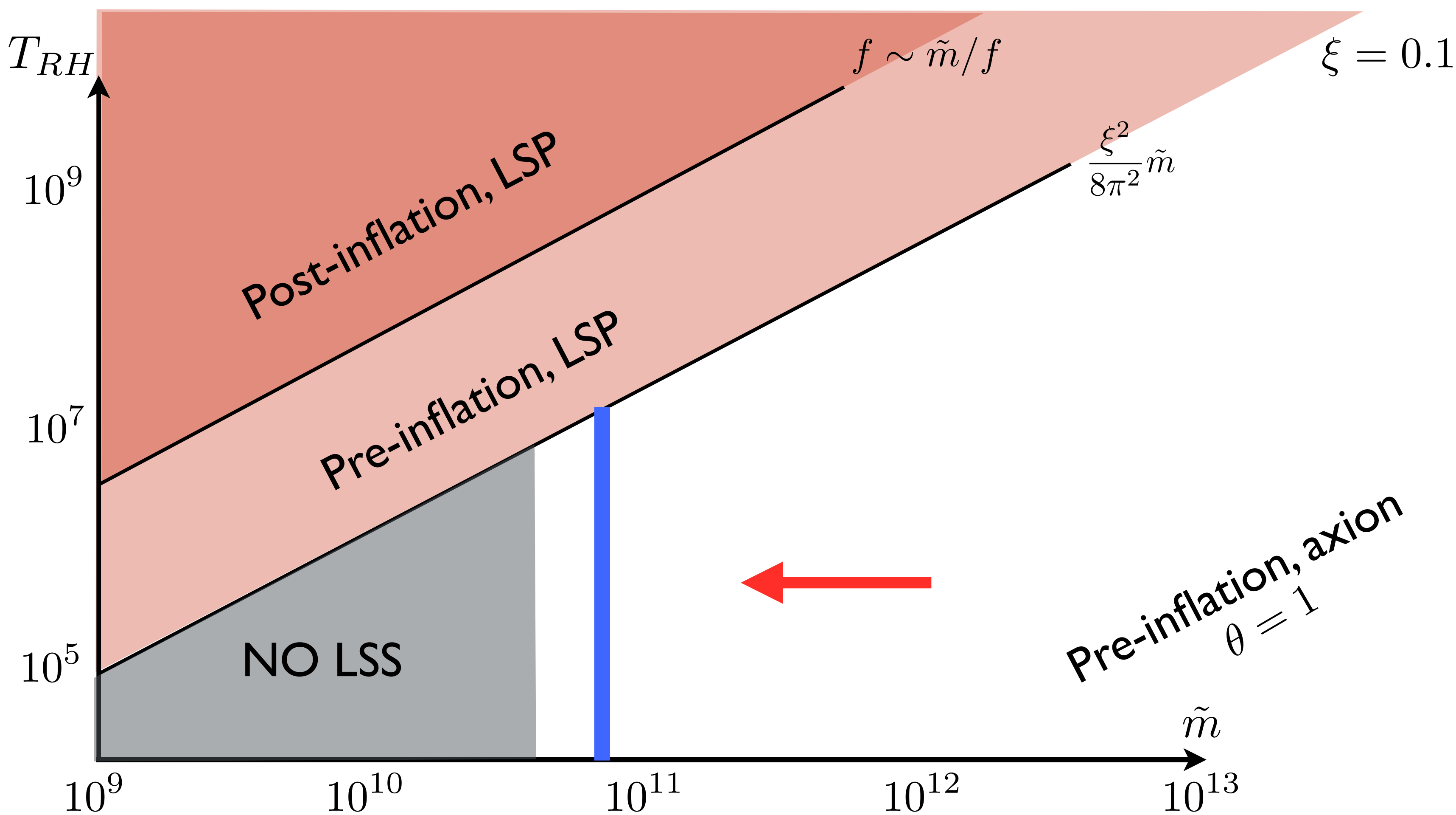
We are pushed against the instability boundary for λ.
Anthropic explanation for the existence of a heavy quark?

$$\lambda_- < \lambda_+ - \lambda_{\rm cr} \qquad\qquad dP(\lambda_-) = \frac{1}{2}\,\frac{\lambda_-^{-1/2}}{\sqrt{\lambda_+ - \lambda_{\rm cr}}}\,d\lambda_-$$



$$\langle\lambda\rangle = \frac{2}{3}\lambda_+ + \frac{1}{3}\lambda_{\rm cr}, \qquad \sigma_\lambda = \frac{2}{3\sqrt{5}}\,|\lambda_+ - \lambda_{\rm cr}|$$

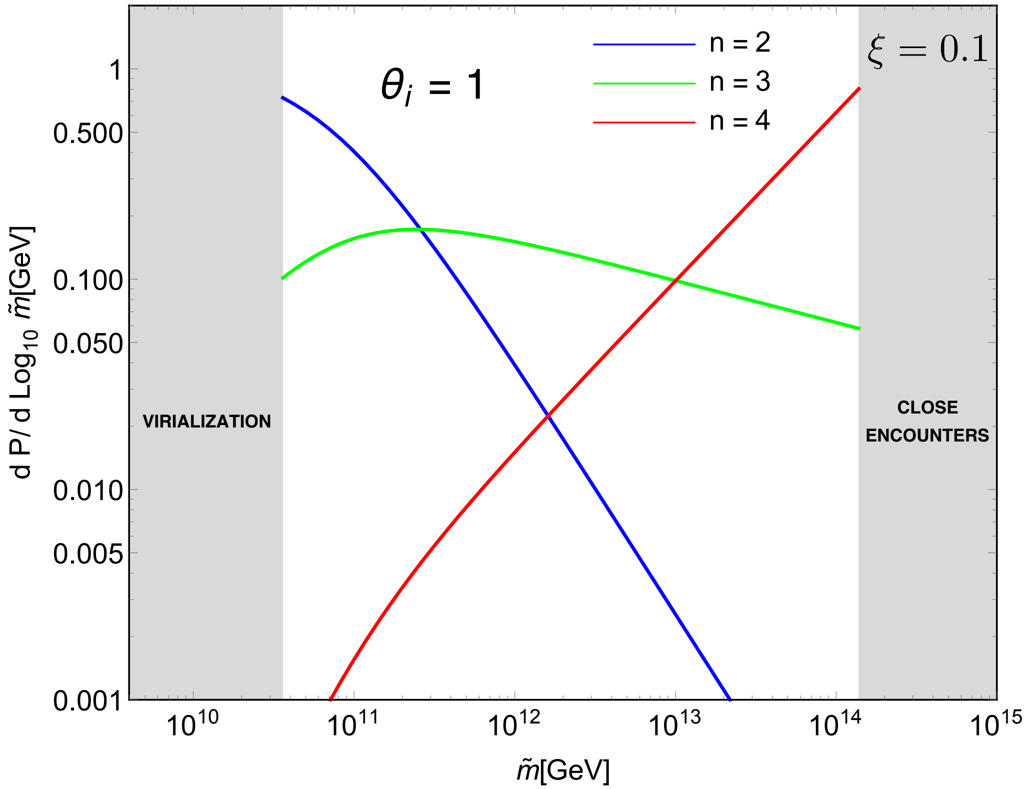
Assume instantaneous reheating



Virialization: $\rho/\rho_0 < 0.5$

Close encounters: $\rho/\rho_0 > 10^4$

$$\frac{dP}{d\log\tilde{m}\;d\theta} \;\propto\; \theta\left(\rho_a-\rho_a^{\rm min}\right)\theta\left(\rho_a^{\rm max}-\rho_a\right)\;\frac{v^2}{v^2+\tilde{m}^2}\;\frac{1}{1+\rho_a/\rho_B}\,\tilde{m}^n$$



For mSUSY ~ 100TeV axino DM is again allowed

A model with low scale SUSY?

To the matter content of the MSSM add a singlet chiral superfield, coupled through

$$W = \frac{1}{M_*} S^2 H_1 H_2$$

work in progress…