ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

# Quantum scale invariance and naturalness

**Mikhail Shaposhnikov**

**Topical Workshop in Frascati: Rethinking Naturalness**

**19 December 2014**

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- J. García-Bellido, J. Rubio, M.S., D. Zenhäusern, Phys. Rev. D84 (2011) 123504
- J. García-Bellido, J. Rubio, M.S., Phys. Lett. (2012)
- F. Bezrukov, G. K. Karananas, J. Rubio and M.S., Phys. Rev. D **87**, 096001 (2013)
- R. Armillis, A. Monin and M.S., JHEP **1310**, 030 (2013)
- A. Monin and M. S., Phys. Rev. D **88**, 067701 (2013)
- F. Bezrukov, J. Rubio and M.S., ArXiv:1412.3811

# Outline

- Classical scale invariance ?
- Exact quantum scale invariance
- Naturalness and quantum scale invariance
- The minimal model - scale invariant $\nu$MSM
- Cosmology and phenomenology of the minimal model
- Conclusions

# Classical scale invariance?

# Why scale invariance?

If the mass of the Higgs boson is put to zero in the SM, the Lagrangian has a wider symmetry: it is scale and conformally invariant:

Dilatations - global scale transformations ($\sigma = const$)

$$\Psi(x) \;\to\; \sigma^n \Psi(\sigma x)\;,$$

$n = 1$ for scalars and vectors and $n = 3/2$ for fermions.

It is tempting to use this symmetry for solution of the hierarchy problem

# Different approaches

- Lagrangian is invariant at the classical level, and scale symmetry is broken by quantum corrections a'la Coleman-Weinberg

Meissner, Nicolai; Iso, Okada, Orikasa; Boyle, Farnsworth, Fitzgerald, Schade; Salvio, Strumia

Simplest theory realising this idea, without gravity Iso, Okada, Orikasa : $SU(3) \times SU(2) \times U(1) \times U(1)_{B-L}$: Standard Model, one extra vector field - gauging of $B-L$, one extra complex scalar field $\chi$ with lepton number 2, 3 right-handed neutrinos.

Attractive features:

- Coleman-Weinberg mechanism works (it does not work in the SM)
- Quantisation of electric charges comes from requirement of cancellation of anomalies
- Phenomenology is OK: neutrino masses are generated, dark matter candidate, baryogenesis, possibilities for inflation

Main problem: inclusion of gravity in a scale-invariant way, to be consistent with the starting idea Iso, Kengo, MS

Simplest possibility – non-minimal coupling of $\chi$ to Ricci scalar, $|\chi|^2 R$ requires $\langle\chi\rangle \sim M_P$, and thus the small couplings to the SM fields, and thus to cosmological moduli problem: light scalar that have no time to setup in the Coleman-Weinberg ground state

More complicated theory, Salvio, Strumia - gravity with higher derivatives.

Attractive features:

- Renormalisable theory
- Coleman-Weinberg mechanism works (it does not work in the SM)
- Inflation

Main problem: ghost in the gravitational sector,

$$\boldsymbol{M_{gh} \simeq 3 \times 10^{10}\ \mathrm{GeV} \ll M_P}$$

the theory is unstable

# Exact quantum scale invariance

# Quantum scale invariance

# Quantum scale invariance

Common lore: quantum scale invariance does not exist, divergence of dilatation current is not-zero due to quantum corrections:

$$\partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\; a} \; ,$$

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Known exceptions - not realistic theories like N=4 SYM

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Known exceptions - not realistic theories like N=4 SYM

Does not make any sense to talk about it?

# Toy model

Classically scale-invariant Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu \chi)^2 - V(\varphi, \chi)$$

Potential ( $\chi$ - “dilaton”, $\varphi$ - “Higgs”):

$$V(\varphi, \chi) = \frac{\lambda}{4}\left(h^2 - \frac{\alpha}{\lambda}\chi^2\right)^2 + \beta\chi^4,$$

$\beta < 0$ : vacuum is unstable

$\beta = 0$ : flat direction, $h^2 = \frac{\alpha}{\lambda}\chi^2$. Choice of parameters:

$\alpha \sim \left(\frac{M_W}{M_P}\right)^2 \sim 10^{-32}$, to get the Higgs-Planck hierarchy correctly.

# Standard reasoning

Dimensional regularisation $d = 4 - 2\epsilon$, $\overline{MS}$ subtraction scheme:

mass dimension of the scalar fields: $1 - \epsilon$,

mass dimension of the coupling constant: $2\epsilon$

Counter-terms:

$$\lambda = \mu^{2\epsilon} \left[ \lambda_R + \sum_{k=1}^{\infty} \frac{a_n}{\epsilon^n} \right] ,$$

$\mu$ is a dimensionful parameter!!

One-loop effective potential along the flat direction:

$$V_1(\chi) = \frac{m_H^4(\chi)}{64\pi^2} \left[ \log \frac{m_H^2(\chi)}{\mu^2} - \frac{3}{2} \right] ,$$

Result: explicit breaking of the dilatation symmetry. Dilaton acquires a nonzero mass due to radiative corrections.

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Idea: Replace $\mu^{2\epsilon}$ by combinations of fields $\chi$ and $h$, which have the correct mass dimension:

$$\mu^{2\epsilon} \to \chi^{\frac{2\epsilon}{1-\epsilon}} F_\epsilon(x) \ ,$$

where $x = h/\chi$. $F_\epsilon(x)$ is a function depending on the parameter $\epsilon$ with the property $F_0(x) = 1$.

Zenhäusern, M.S

Englert, Truffin, Gastmans, 1976

# Example of computation

"Natural choice", to be explained below:

$$\mu^{2\epsilon} \to \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \;, (\xi_\chi \chi^2 + \xi_h h^2) \equiv \omega^2$$

Potential:

$$U = \frac{\lambda_R}{4} \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \left[h^2 - \zeta_R^2 \chi^2\right]^2 \;,$$

Counter-terms

$$U_{cc} = \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \left[A h^2 \chi^2 \left(\frac{1}{\bar{\epsilon}} + a\right) + B \chi^4 \left(\frac{1}{\bar{\epsilon}} + b\right) + C h^4 \left(\frac{1}{\bar{\epsilon}} + c\right)\right],$$

To be fixed from conditions of absence of divergences and presence of spontaneous breaking of scale-invariance

$$
\begin{aligned}
U_1 = \quad & \frac{m^4(h)}{64\pi^2}\left[\log\frac{m^2(h)}{v^2} + \mathcal{O}\left(\zeta_R^2\right)\right] \\
+ \quad & \frac{\lambda_R^2}{64\pi^2}\left[C_0 v^4 + C_2 v^2 h^2 + C_4 h^4\right] + \mathcal{O}\left(\frac{h^6}{\chi^2}\right),
\end{aligned}
$$

where $m^2(h) = \lambda_R(3h^2 - v^2)$ and

$$
\begin{aligned}
C_0 \quad &= \frac{3}{2}\left[2a - 1 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) + \frac{4}{3}\log 2\lambda_R + O(\zeta_R^2)\right] \;, \\
C_2 \quad &= -3\left[2a - 3 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) + O(\zeta_R^2)\right] \;, \\
C_4 \quad &= \frac{3}{2}\left[2a - 5 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) - 4\log 2\lambda_R + O(\zeta_R^2)\right] \;.
\end{aligned}
$$

# Origin of $\Lambda_{QCD}$

Consider the high energy ($\sqrt{s} \gg v$ but $\sqrt{s} \ll \chi_0$) behaviour of scattering amplitudes on the example of Higgs-Higgs scattering (assuming, that $\zeta_R \ll 1$). In one-loop approximation

$$\Gamma_4 = \lambda_R + \frac{9\lambda_R^2}{64\pi^2}\left[\log\left(\frac{s}{\xi_\chi \chi_0^2}\right) + \text{const}\right] + \mathcal{O}\left(\zeta_R^2\right) \ .$$

This implies that at $v \ll \sqrt{s} \ll \chi_0$ the effective Higgs self-coupling runs in a way prescribed by the ordinary renormalization group!

For QCD:

$$\Lambda_{QCD} = \chi_0 e^{-\frac{1}{2b_0\alpha_s}}, \quad \beta(\alpha_s) = b_0\alpha_s^2$$

Almost trivial statement - by construction: Quantum effective action is scale invariant in all orders of perturbation theory.

Less trivial statement Gretsch, Monin: Quantum effective action is conformally invariant in all orders of perturbation theory.

The main problem with this construction: theory is not renormalisable, one needs to add infinite number of counter-terms.

However:

- For $\alpha \ll 1$ all counter-terms are suppressed by the dimensionful parameter $\langle\chi\rangle$
- We get an effective field theory valid up to the energy scale fixed by $\langle\chi\rangle$
- Gravity is non-renormalisable anyway, and making $\langle\chi\rangle \sim M_P$ does not make a theory worse

# Hierarchy problem

For $\alpha = \beta = 0$ the classical Lagrangian has an extra symmetry : $\chi \to \chi + const$. Therefore, there are no large perturbative corrections to the Higgs mass: those proportional to $\chi$ contain necessarily $\alpha$ or $\beta$, those proportional to $\lambda$ contain only logs of $\chi$. This construction leads to "natural" hierarchy $\chi \gg h$. However, no explanation of why $\alpha \ll 1$.

Important ingredient for naturalness: almost exact shift symmetry.

Requirement of the shift symmetry $\equiv$ requirement of absence of heavy particles with sufficiently strong interaction with the Higgs field and the dilaton, e.g.

$$\lambda_h h^2\phi^2 + \lambda_\chi \chi^2\phi^2$$

$\lambda_h \sim \lambda_\chi \sim 1$ spoils the argument!

Conjecture: natural theory should not have heavy particles between the Fermi and Planck scales

# Inclusion of gravity

Planck scale: through non-minimal coupling of the dilaton to the Ricci scalar,

Gravity part

$$\mathcal{L}_G = -\left(\xi_\chi \chi^2 + \xi_h h^2\right)\frac{R}{2}\,,$$

This term, for $\xi_\chi \sim 1$, does break the shift symmetry. However, this is a coefficient in front of graviton kinetic term. Since the graviton stays massless in any constant scalar background, the perturbative computations of gravitational corrections to the Higgs mass in scale-invariant regularisation are suppressed by $M_P$. There are no corrections proportional to $M_P$!

# Consequences

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- Theory is “natural” in perturbative sense: Higgs mass is stable against radiative corrections
- The dilaton is massless in all orders of perturbation theory
- Since it is a Goldstone boson of spontaneously broken symmetry it has only derivative couplings to matter (inclusion of gravity is essential!)
- Fifth force or Brans-Dicke constraints are not applicable to it

# Problems

- What happens beyond perturbation theory?
- What leads to selection of parameter $\beta = \mathbf{0} \equiv$ existence of flat direction $\equiv$ absence of the cosmological constant ?
- Unitarity and high-energy behaviour: What is the high-energy behaviour $(E > M_{Pl})$ of the scattering amplitudes? Is the theory unitary? Can it have a scale-invariant UV completion?

# The minimal model - scale invariant $\nu$MSM

# Particle content

Particles of the SM

+

graviton

+

dilaton

+

3 Majorana leptons

Scale-invariant Lagrangian

$$\mathcal{L}_{\nu\mathrm{MSM}} = \mathcal{L}_{\mathrm{SM}[\mathrm{M}\to 0]} + \mathcal{L}_G + \frac{1}{2}(\partial_\mu\chi)^2 - V(\varphi,\chi)$$
$$+ \left(\bar{N}_I i\gamma^\mu\partial_\mu N_I - h_{\alpha I}\,\bar{L}_\alpha N_I\tilde{\varphi} - f_I \bar{N}_I{}^c N_I\chi + \mathrm{h.c.}\right)\ ,$$

Potential ( $\chi$ - dilaton, $\varphi$ - Higgs, $\varphi^\dagger\varphi = 2h^2$):

$$V(\varphi,\chi) = \lambda\left(\varphi^\dagger\varphi - \frac{\alpha}{2\lambda}\chi^2\right)^2 + \beta\chi^4,$$

Gravity part

$$\mathcal{L}_G = -\left(\xi_\chi\chi^2 + 2\xi_h\varphi^\dagger\varphi\right)\frac{R}{2}\ ,$$

# Roles of different particles

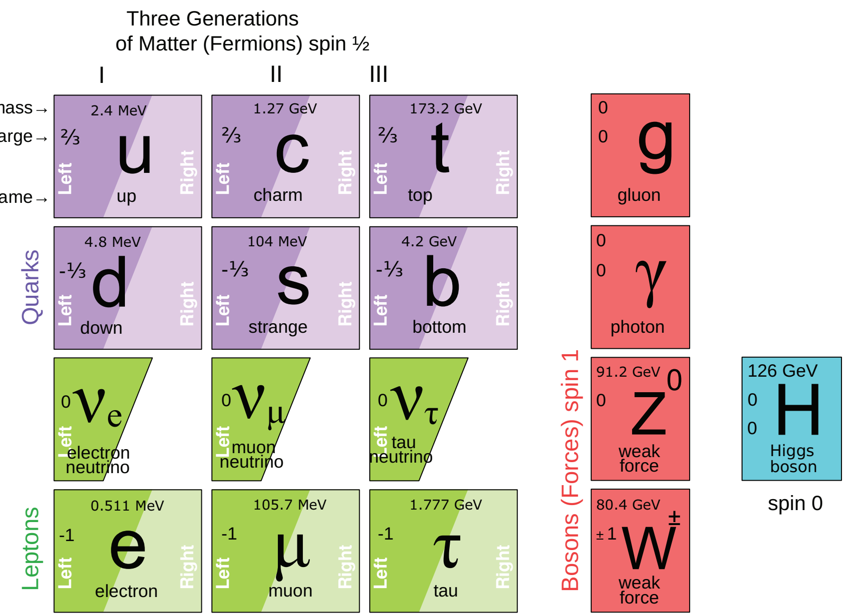
The roles of dilaton:

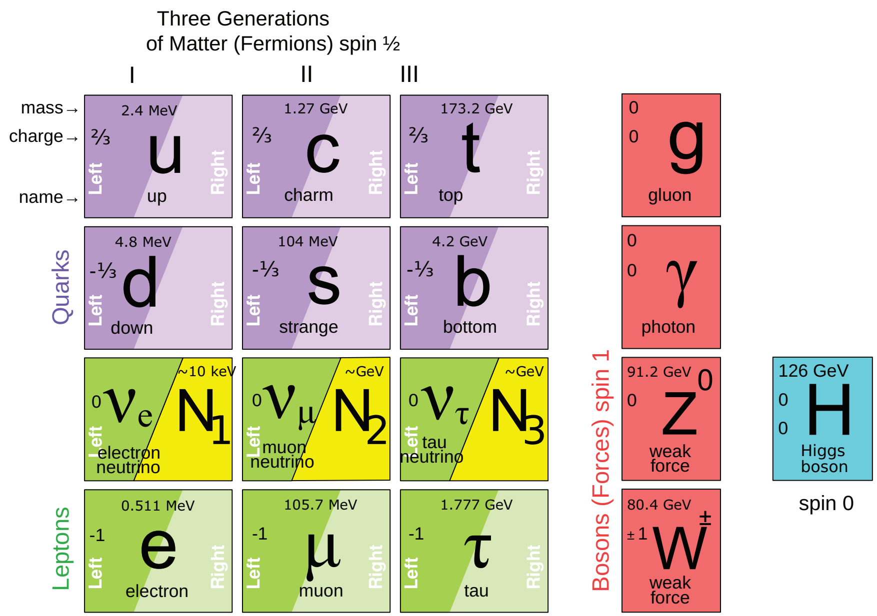
- determine the Planck mass
- give mass to the Higgs
- give masses to 3 Majorana leptons
- may lead to dynamical dark energy

Roles of the Higgs boson:

- give masses to fermions and vector bosons of the SM
- provide inflation

# New physics below the Fermi scale: the $\nu$MSM




Role of $N_1$ with mass in keV region: dark matter. Search - with the use of X-ray telescopes. Already found? Bulbul et al., Boyarsky et al

Role of $N_2,\ N_3$ with mass in 100 MeV – GeV region: “give” masses to neutrinos and produce baryon asymmetry of the Universe. Search - intensity and precision frontier, SHiP at CERN.

# The couplings of the $\nu$MSM

Particle physics part, accessible to low energy experiments: the $\nu$MSM. Mass scales of the $\nu$MSM:

$M_I < M_W$ (No see-saw)

Consequence: small Yukawa couplings,

$$F_{\alpha I} \sim \frac{\sqrt{m_{atm} M_I}}{v} \sim (10^{-6} - 10^{-13}),$$

here $v \simeq 174$ GeV is the VEV of the Higgs field,

$m_{atm} \simeq 0.05$ eV is the atmospheric neutrino mass difference.

Small Yukawas are also necessary for stability of dark matter and baryogenesis (out of equilibrium at the EW temperature).

# Cosmology and phenomenology of a minimal model

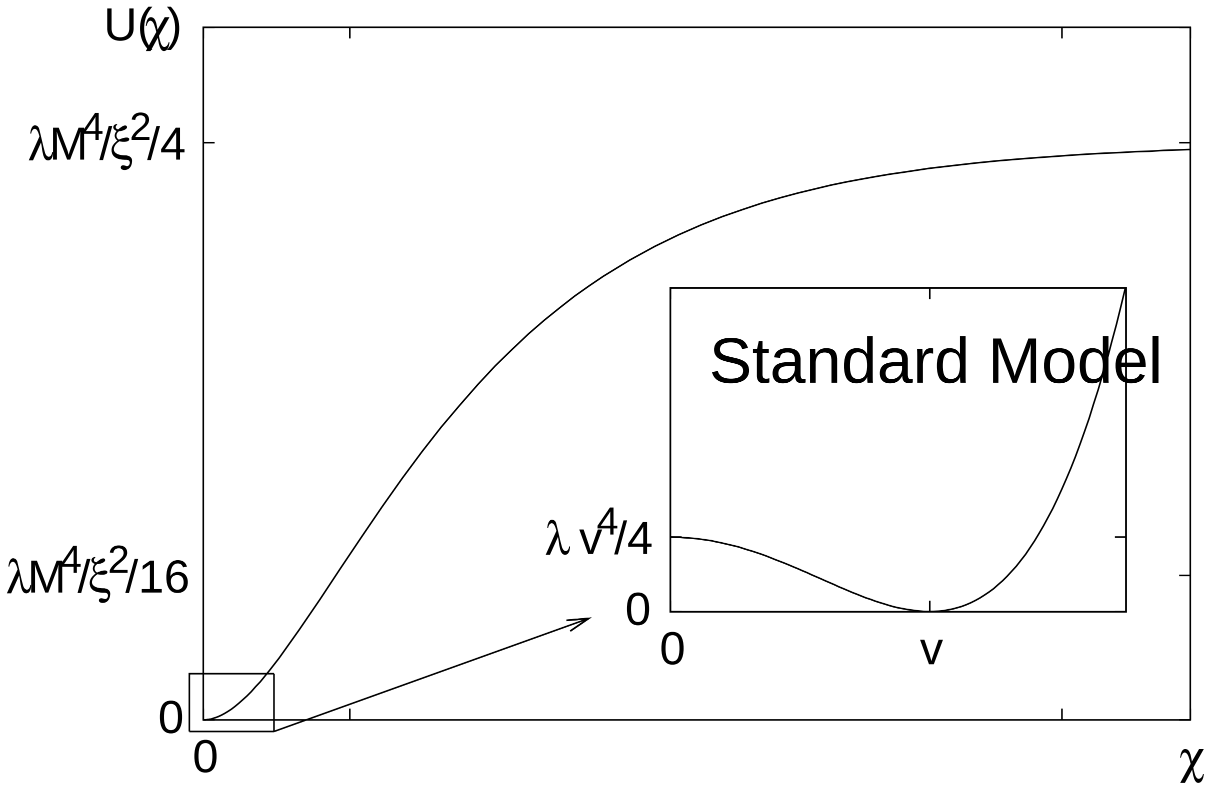
# Inflation

Chaotic initial condition: fields $\chi$ and $h$ are away from their equilibrium values.

Choice of parameters: $\xi_h \gg 1, \;\; \xi_\chi \ll 1$

Then - dynamics of the Higgs field is more essential $\chi \simeq const$ and is frozen $\Rightarrow$ Higgs inflation. Denote $\xi_\chi \chi^2 = M_P^2$.

# Potential in Einstein frame




$\chi$ - canonically normalized scalar field in Einstein frame.

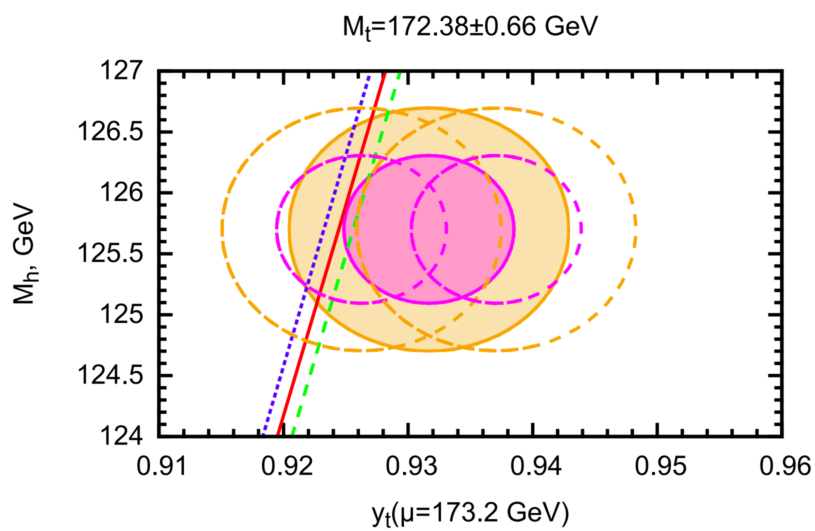
This form of the potential is universal for (Bezrukov, MS) $y_t(173.2) < y_t^{\text{crit}}$:

$$y_t^{\text{crit}} = 0.9223 + 0.00118\left(\frac{\alpha_s - 0.1184}{0.0007}\right) + 0.00085\left(\frac{M_H - 125.03}{0.3}\right) + 0.0023\left(\frac{\log\xi}{6.9}\right)$$

$y_t(173.2)$ - top Yukawa coupling in $\overline{\text{MS}}$- scheme at $\mu = 173.2$ GeV, $\alpha_s(M_Z)$ - strong coupling

theoretical uncertainty: $\delta y_t / y_t \simeq 2 \times 10^{-4}$ equivalent to changing of $M_H$ by $\sim 70$ MeV, or $m_t$ by $\sim 35$ MeV Buttazzo et al

Numerically for $\xi = 1$, $y_t^{\text{crit}}$ coincides with the metastability bound on the top Yukawa coupling



Potential for the Higgs field may be flat at large values of $h$: Linde chaotic inflation

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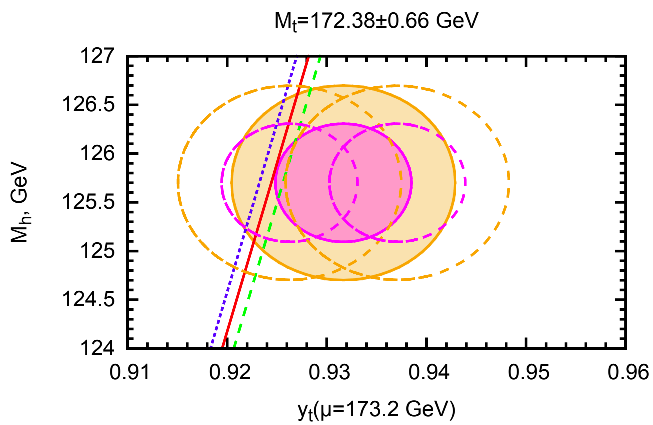
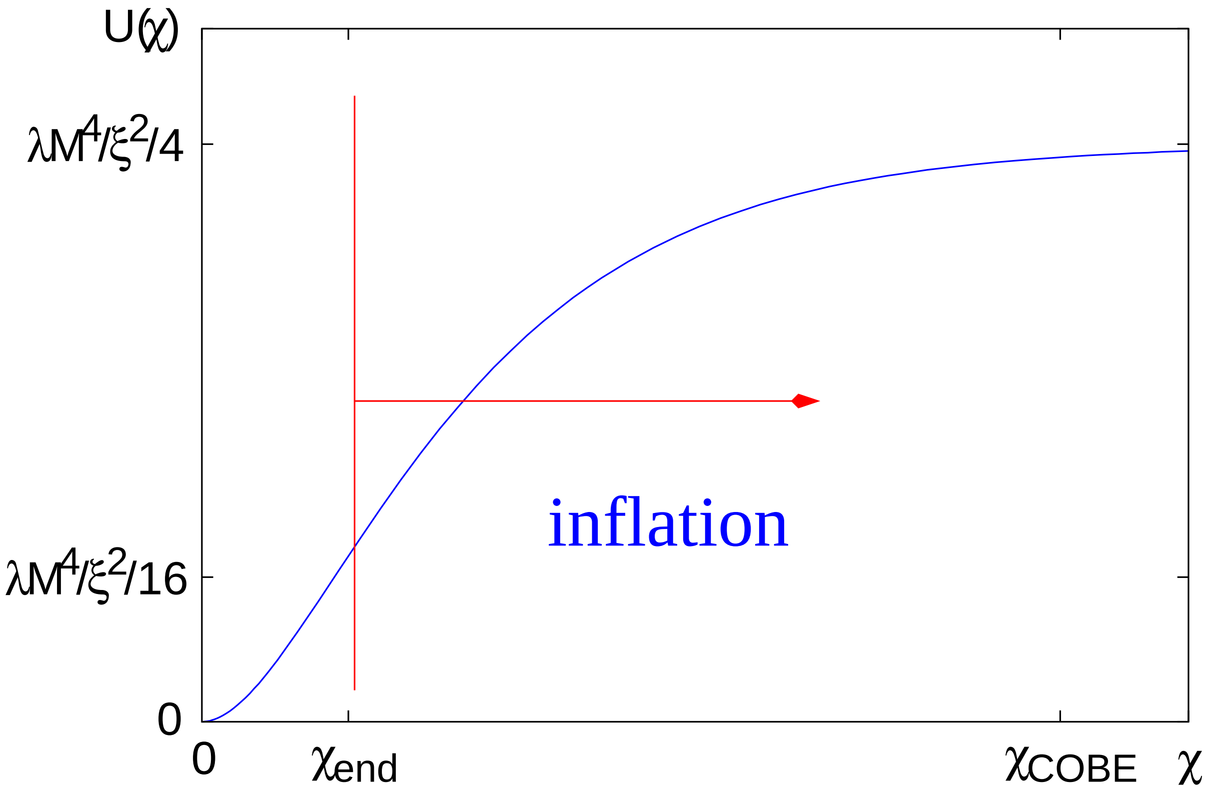
Inflation, Big Bang - all in the framework of the Standard Model!

# Higgs inflation: $y_t < y_t^{\text{crit}}$



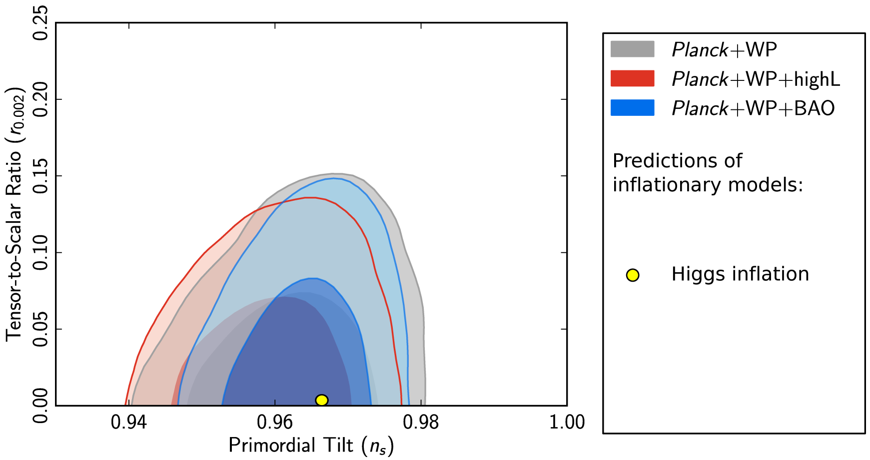
# Stage 1: Higgs inflation, $h > \frac{M_P}{\sqrt{\xi}}$, slow roll of the Higgs field

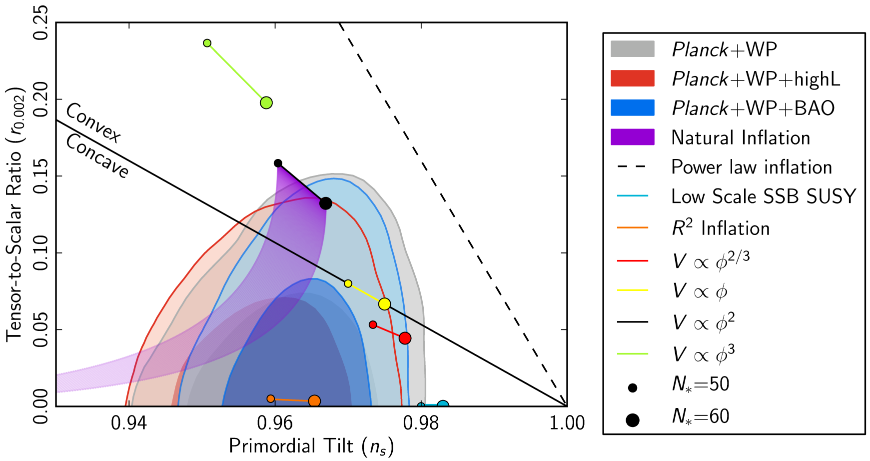



- Makes the Universe flat, homogeneous and isotropic
- Produces fluctuations leading to structure formation: clusters of galaxies, etc

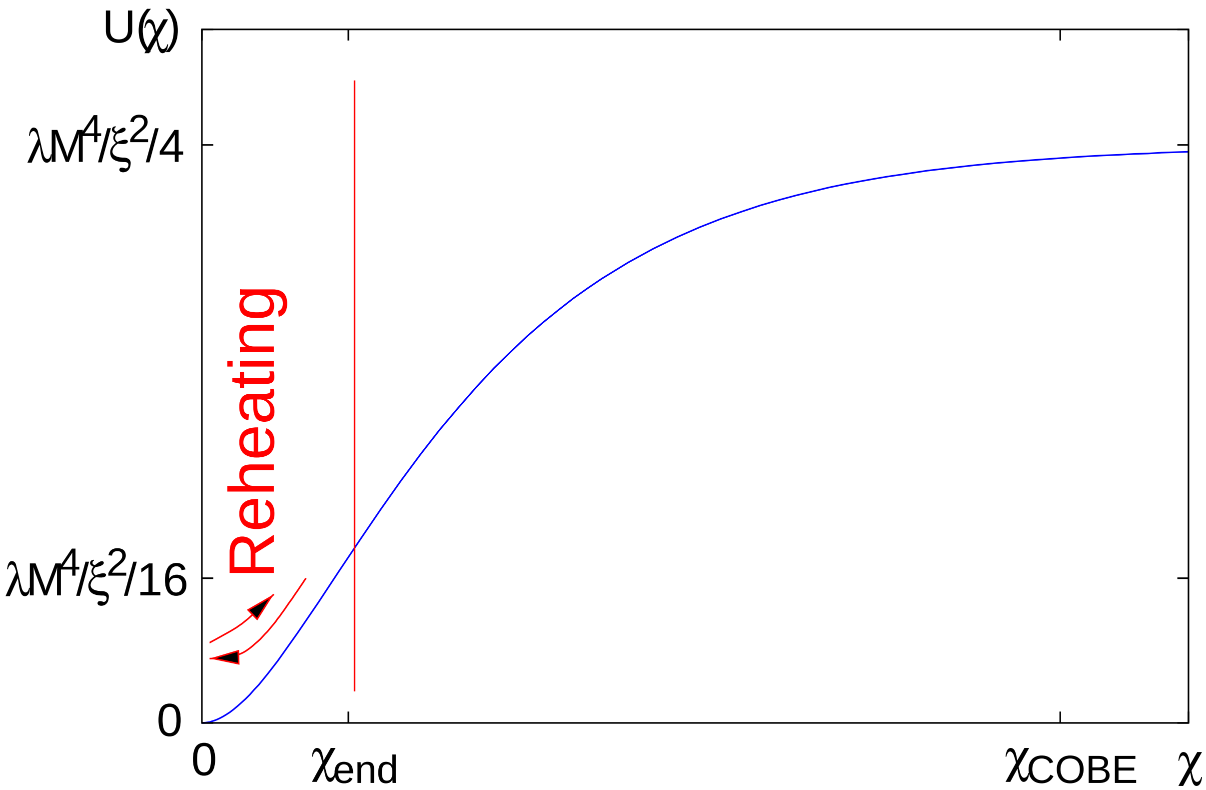
# CMB parameters - spectrum and tensor modes, $\xi \gtrsim 1000$

$$n_s = 0.97, \quad r = 0.003$$



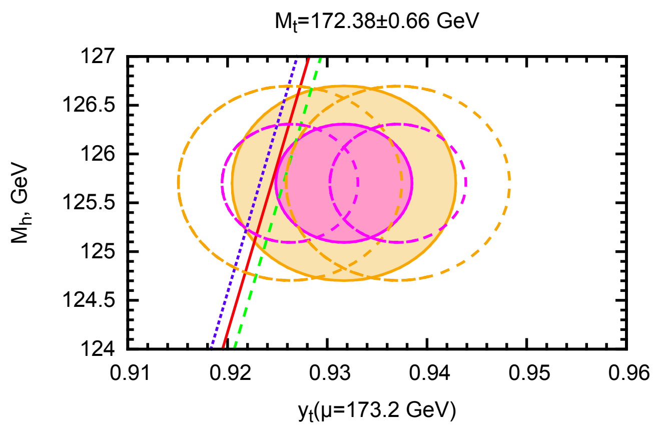



# Stage 2: Big Bang, $\frac{M_P}{\xi} < h < \frac{M_P}{\sqrt{\xi}}$, Higgs field oscillations




- All particles of the Standard Model are produced
- Coherent Higgs field disappears
- The Universe is heated up to $T \propto M_P/\xi \sim 10^{14}$ GeV

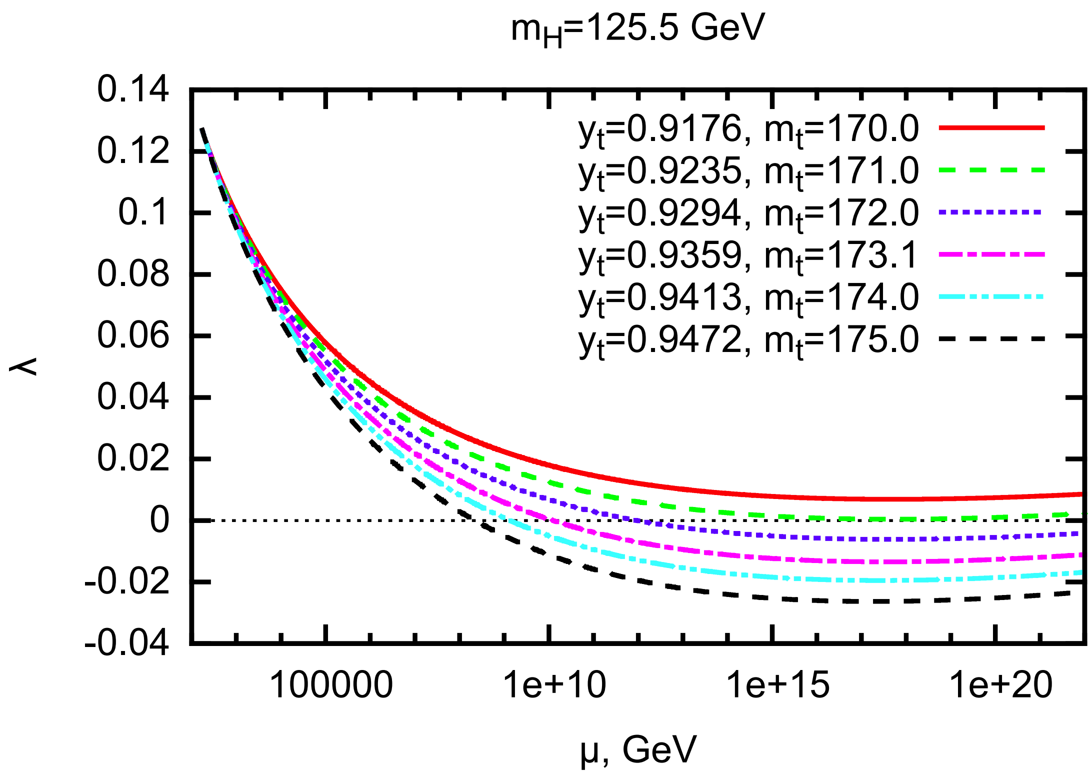
# Critical Higgs inflation: $y_t \approx y_t^{\text{crit}}$



Bezrukov, MS

For $y_t$ very close to $y_t^{\mathbf{crit}}$ : critical Higgs inflation - tensor-to-scalar ratio can be large, $\xi \sim 10$

Behaviour of $\lambda$:



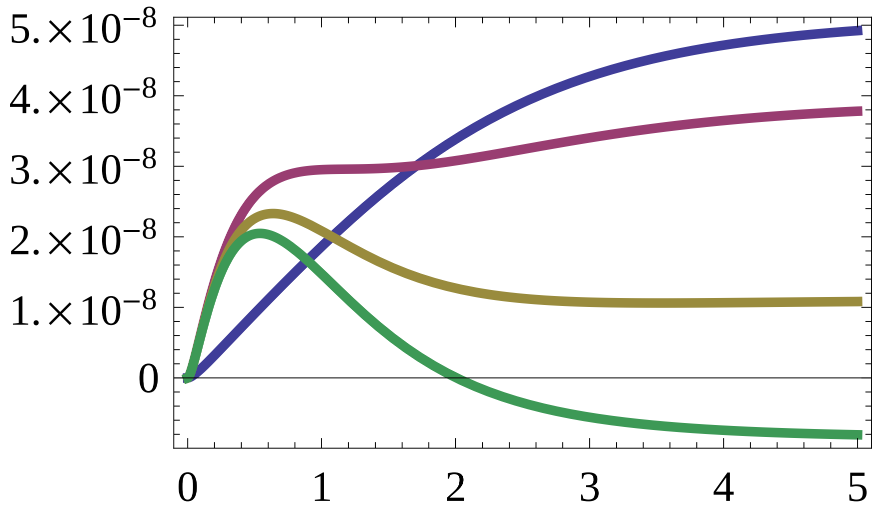
# Effective potential

$$U(\chi) \simeq \frac{\lambda(z')}{4\xi^2}\bar{\mu}^4\,, \;\; z' = \frac{\bar{\mu}}{\kappa M_P}, \;\; \bar{\mu}^2 = M_P^2\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)$$

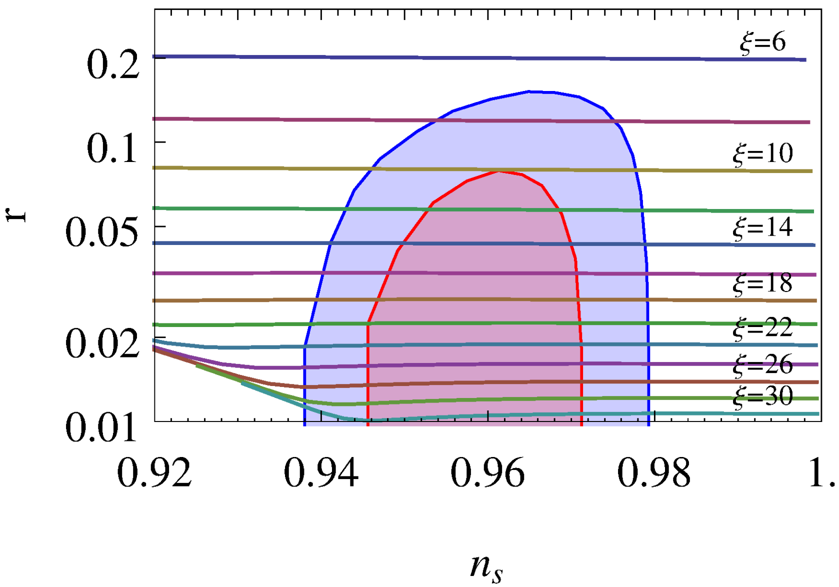
The parameter $\mu$ that optimises the convergence of the perturbation theory is related to $\bar{\mu}$ as

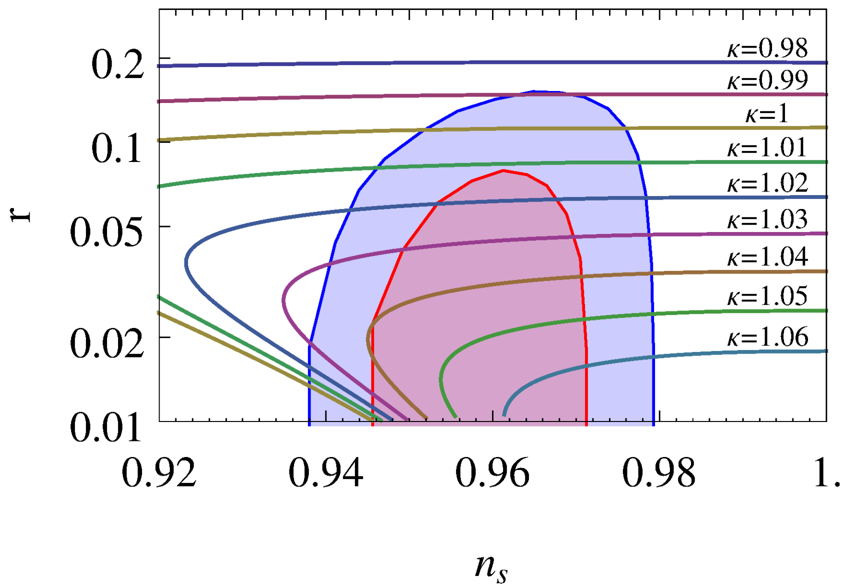
$$\mu^2 = \alpha^2\frac{y_t(\mu)^2}{2}\frac{\bar{\mu}^2}{\xi(\mu)}\,, \;\; \alpha \simeq 0.6$$

Behaviour of effective potential for $\lambda_0 \simeq b/16$:



# The inflationary indexes



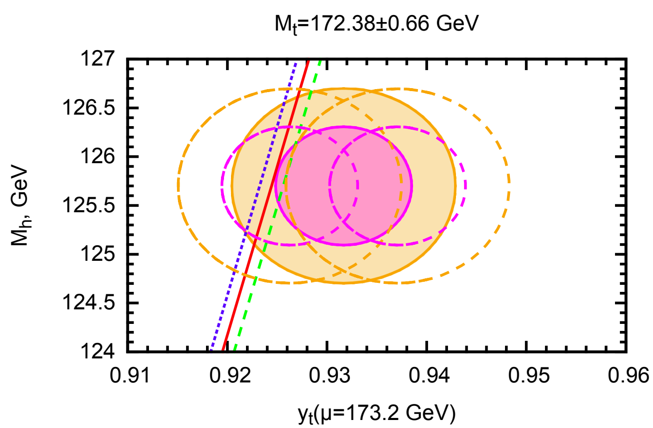




$r$ can be large! BICEP 2?

see also Hamada, Kawai, Oda and Park

Critical Higgs inflation only works if both Higgs and top quark masses are close to their experimental values.

# Living beyond the edge: Higgs inflation and vacuum metastability, $y_t > y_t^{\rm crit}$



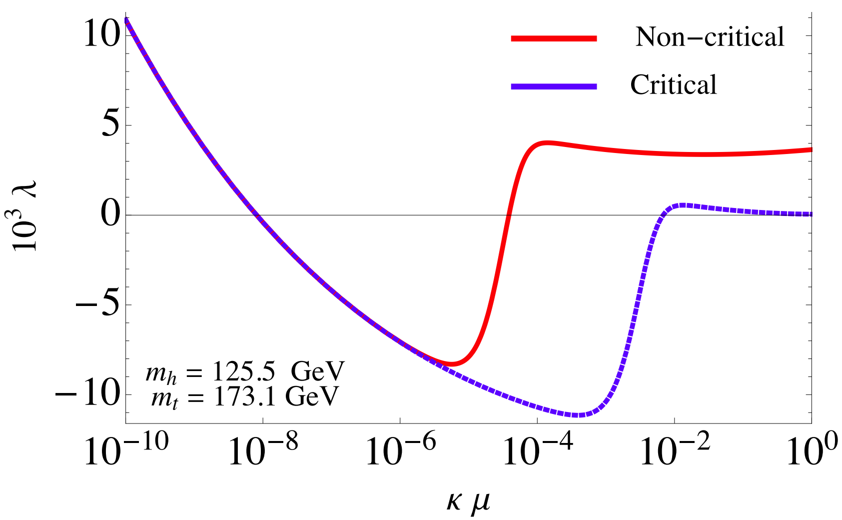
Bezrukov, Rubio, MS

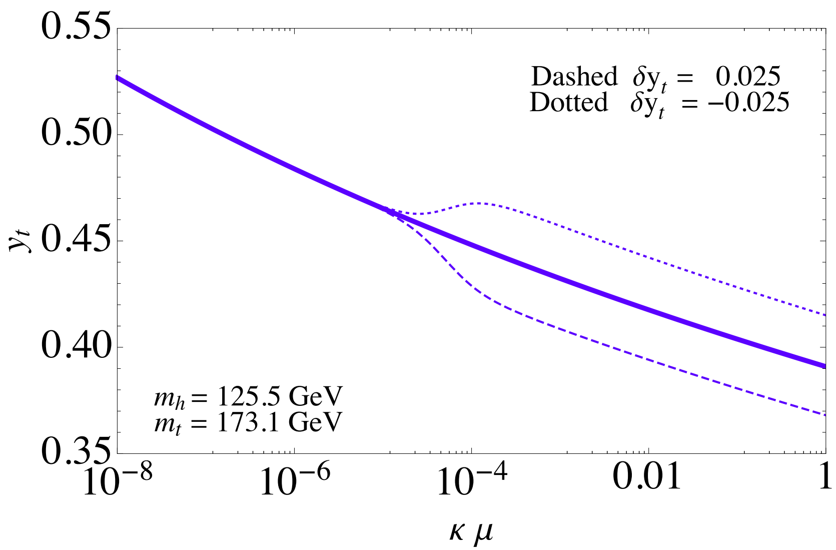
Renormalisation of the SM coupling constants at the scale $M_P/\xi$:

"jumps" of $\lambda$ and $y_t$ controlled by UV completion of the SM, which cannot be found from low-energy observables of the SM

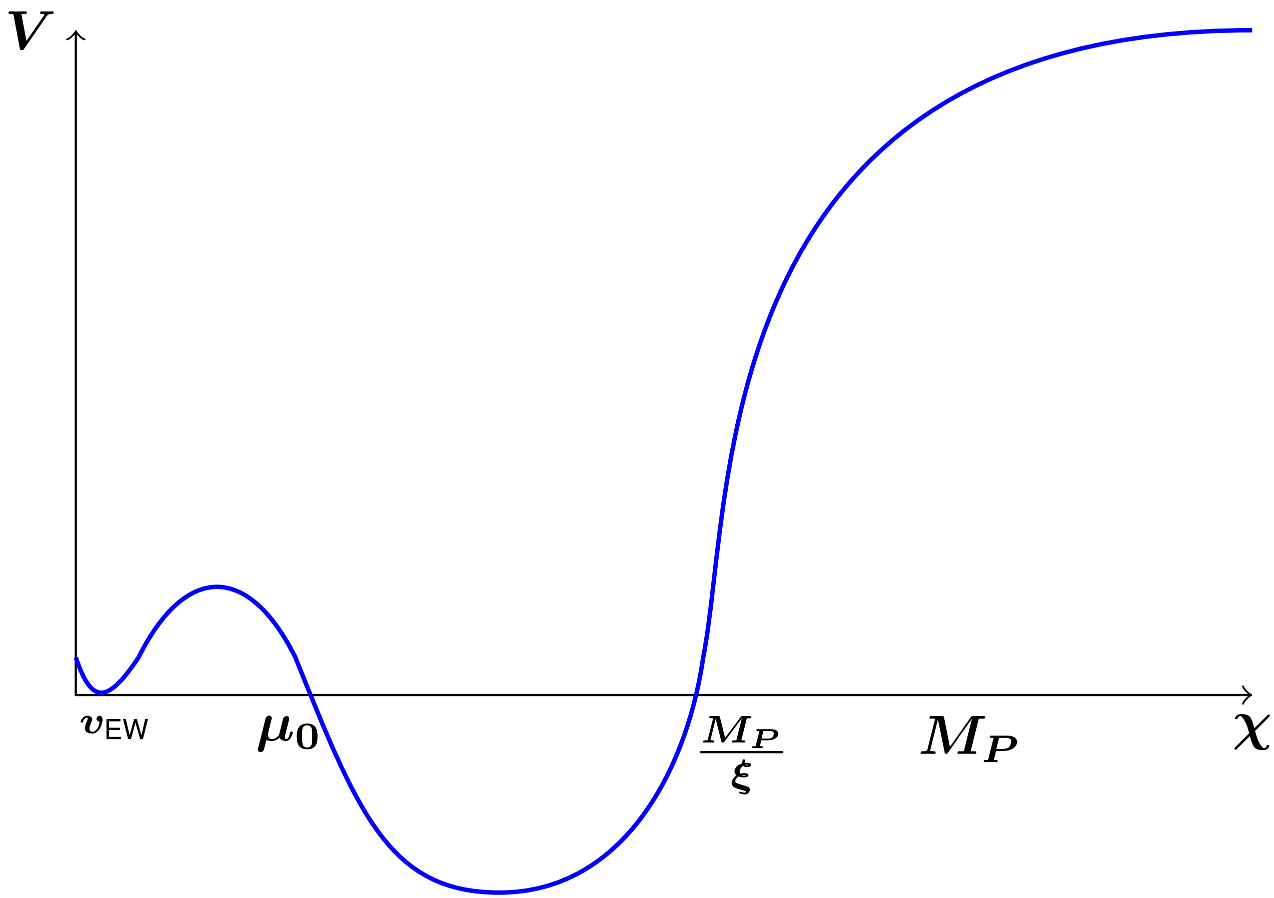
Bezrukov, Magnin, MS., Sibiryakov

$\lambda(M_P/\xi)$ is small due to cancellations between fermionic and bosonic loops: $\delta\lambda$ can be of the order of $\lambda$

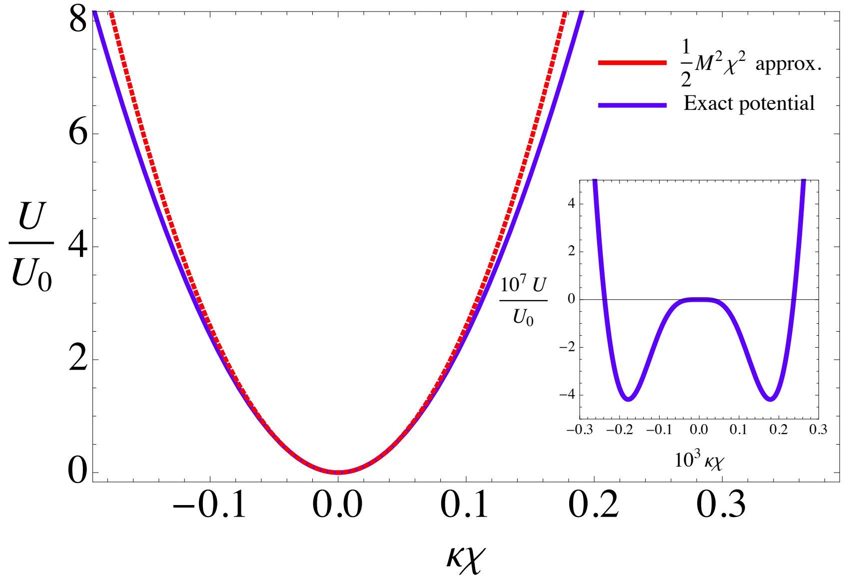


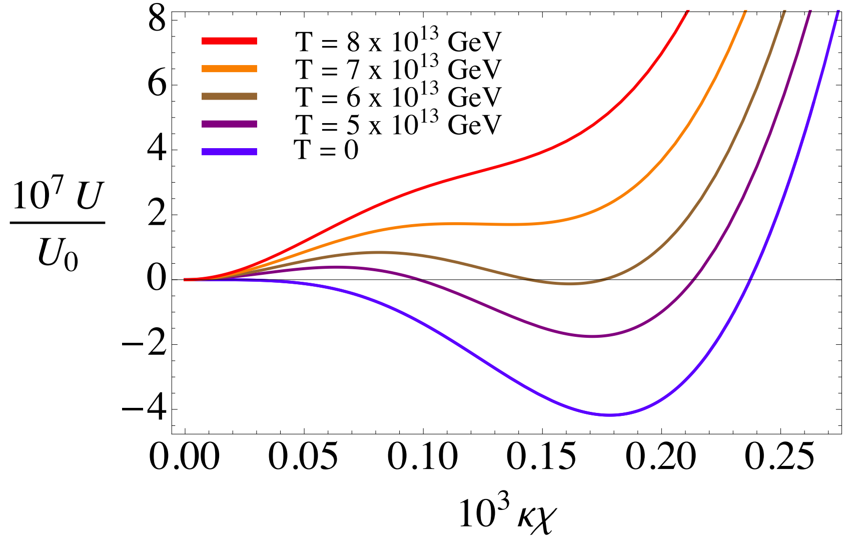



# Higgs potential



# Symmetry restoration

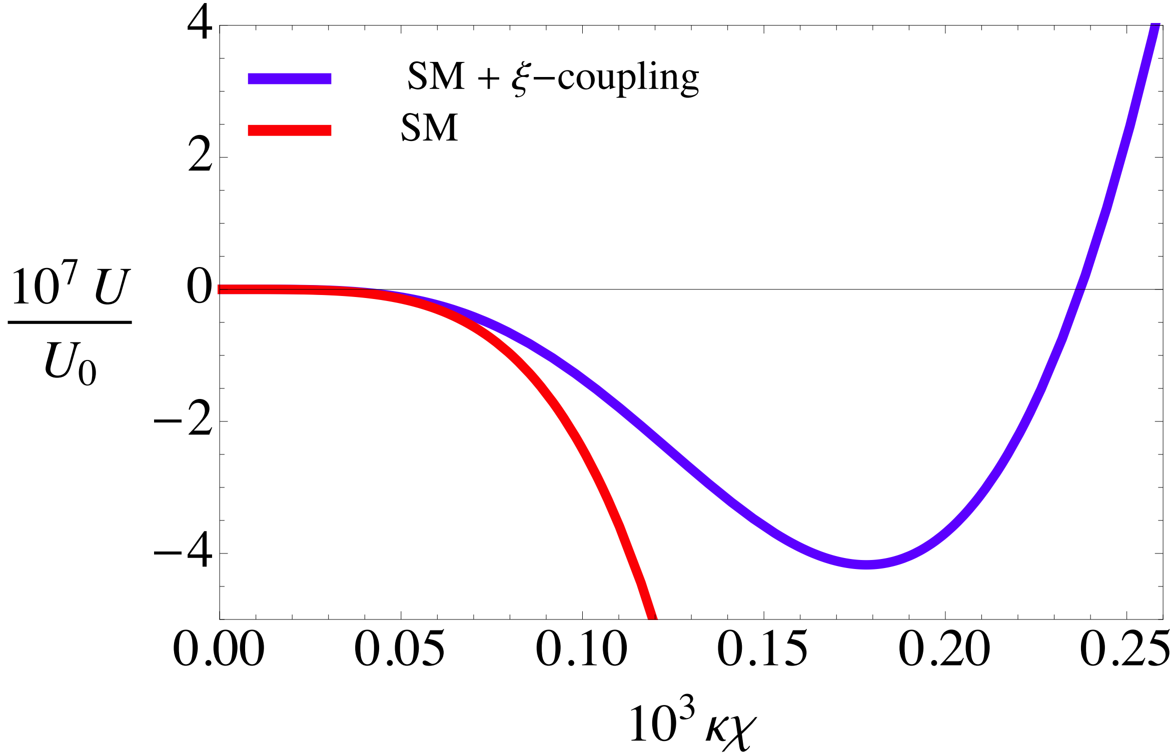



Reheating temperature $T_R \simeq 2 \times 10^{14}$ GeV $> T_+ \simeq 7 \times 10^{13}$ GeV, $T_c = 6 \times 10^{13}$ GeV

# (Meta) stability of false vacuum

Computation for SM: Espinosa, Giudice, Riotto



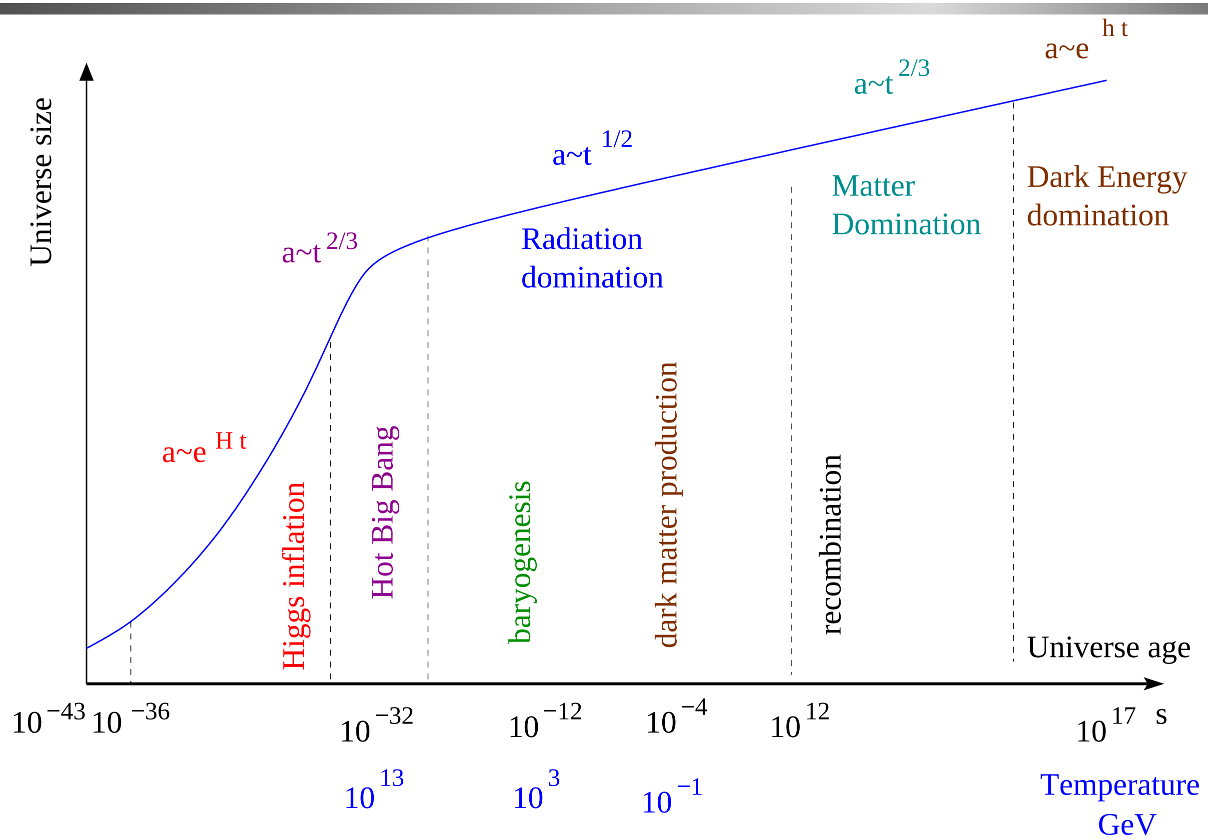
Predictions for critical indexes $n_s$ and $r$ are the same as for non-crtitical Higgs inflation

$$n_s = 0.97, \;\; r = 0.003$$

# Critical Higgs inflation at $y_t > y_t^{\rm crit}$?

Critical Higgs inflation : small $\xi \sim 10$ - the depth of the large Higgs value vacuum is comparable with the energy stored in the Higgs after inflation: the required reheating temperature is too large, $T_+ \simeq 10^{16}$ GeV and cannot be achieved.

# History of the Universe




The problems on neutrino masses, baryon asymmetry of the Universe, and of Dark Matter can all be solved by particles lighter than the Electroweak scale: 3 RH neutrinos

# Conclusions

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  - Unique source for all mass scales.
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  - Higgs mass is stable against radiative corrections (scale symmetry + approximate shift symmetry $\chi \to \chi + const$)
  - The massless sector of the theory contains dilaton, which has only derivative couplings to matter
  - All observational drawbacks of the SM can be solved by new physics below the Fermi scale

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- Find heavy neutral leptons $N_{2,3}$ - responsible for neutrino masses and baryogenesis: SHiP

# Back up slides

# Self-consistency of Higgs inflation

# Large $\xi$ - bad or good?

Sibiryakov, '08; Burgess, Lee, Trott, '09; Barbon and Espinosa, '09

Tree amplitudes of scattering of scalars above electroweak vacuum hit the tree unitarity bound at energies

$$E > \Lambda \sim \frac{M_P}{\xi}$$

The typical energy scale at inflation is

$$\frac{M_P}{\sqrt{\xi}} \gg \Lambda$$

Does it mean that the Higgs inflation is inconsistent?

# Self-consistency of Higgs inflation

A way to proceed:

Bezrukov, Magnin, M.S., Sibiryakov; Ferrara, Kallosh, Linde, A. Marrani, Van Proeyen

- Take the non-zero Higgs field background, and define the region of applicability of perturbation theory. This region is background dependent.
- Compare the background dependent "cutoff" with the different energy scales important for inflation and for subsequent evolution of the Universe.

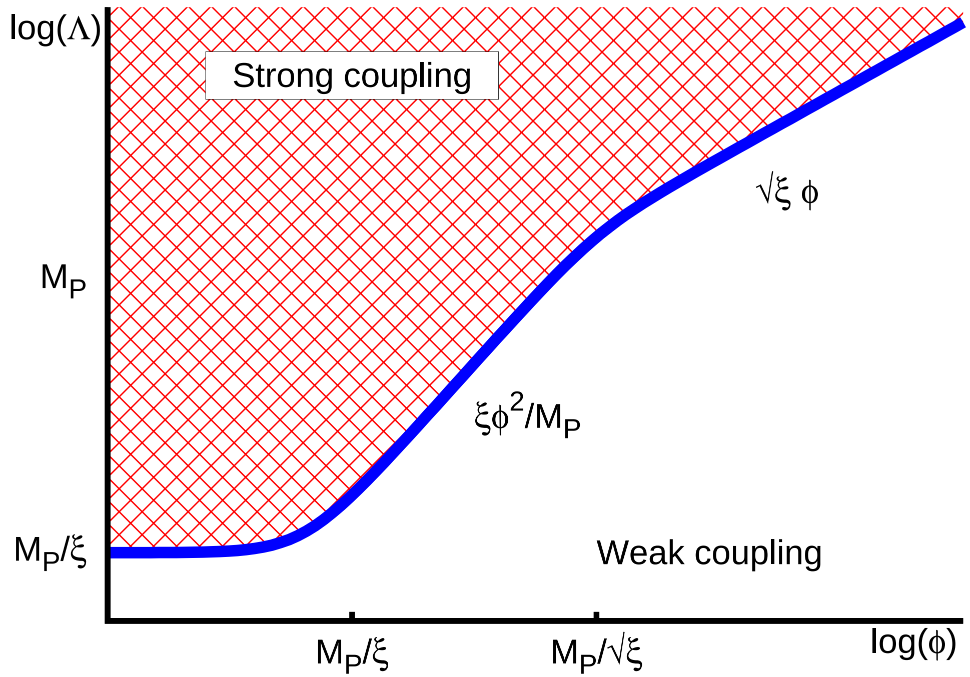
The background dependent "cutoff" - nothing unusual: Fermi interaction - "cutoff" does depend on the Higgs field.

# Dynamical cutoff

Computation for the Higgs-gravity part of the SM:

$$\Lambda(h) \simeq \begin{cases} \frac{M_P}{\xi} \; , & \text{for } h \lesssim \frac{M_P}{\xi} \; , \\ \frac{h^2\xi}{M_P} \; , & \text{for } \frac{M_P}{\xi} \lesssim h \lesssim \frac{M_P}{\sqrt{\xi}} \; , \\ \sqrt{\xi} h \; , & \text{for } h \gtrsim \frac{M_P}{\sqrt{\xi}} \; . \end{cases}$$

# Higgs-dependent cutoff




Cutoff is higher than the relevant dynamical scales throughout the whole history of the Universe, including the inflationary epoch and reheating!!

The Higgs-inflation is self-consistent.

# Higgs inflation: radiative corrections

# Main assumptions

- The SM is a valid effective theory up to the Planck scale, i.e. its Lagrangian can only be modified by higher dimensional operators suppressed by $M_P$
- The quadratic divergences are ignored (the valid procedure at small energies). Technically, this corresponds to the use of the minimal subtraction scheme.

# Effective theory of Higgs inflation

Theory is non-renormalizable (as any theory with gravity).

Let's add to all counter-terms necessary to make it finite with all possible constant parts having the same structure as counter-terms.

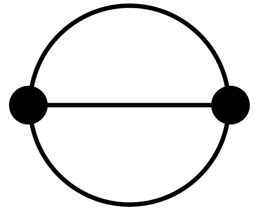
The procedure must respect the classical symmetries of the theory (scale invariance in Jordan frame = shift symmetry in Einstein frame). Technically - use dimensional regularisation and $\overline{\text{MS}}$ subtraction procedure.

Starting Lagrangian:

$$L = \frac{(\partial_\mu \chi)^2}{2} - U(\chi)$$

where $U(\chi)$ has at large fields the generic form

$$U(\chi) = U_0 \left( 1 + \sum_{n=1}^{\infty} u_n e^{-n\chi/M} \right)$$

$$\frac{U_0 u_n}{M^3} e^{-n\bar{\chi}/M}$$

$$\frac{U_0 u_m}{M^3} e^{-m\bar{\chi}/M}$$

$$\propto \frac{1}{\epsilon} \cdot \frac{U_0^2}{M^8} u_n u_m (\partial_\mu \bar{\chi})^2 e^{-(n+m)\bar{\chi}/M} \quad ,$$

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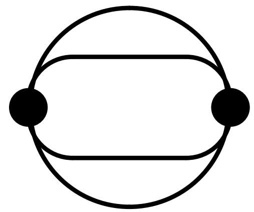

$$\frac{U_0 u_m}{M^4} e^{-m\bar{\chi}/M}$$

$$\propto \frac{1}{\epsilon} \cdot \frac{U_0^2}{M^8} u_n u_m \left( \frac{(\partial^2 \bar{\chi})^2}{M^2} + \frac{(\partial \bar{\chi})^4}{M^4} \right) e^{-(n+m)\bar{\chi}/M}$$

Effective action, incorporating radiative corrections:

$$L = f^{(1)}(\chi)\frac{(\partial_\mu \chi)^2}{2} - U(\chi) + f^{(2)}(\chi)\frac{(\partial^2 \chi)^2}{M^2} + f^{(3)}(\chi)\frac{(\partial \chi)^4}{M^4} + \cdots$$

where dots stand for terms with more derivatives. The coefficient functions are (formal) series in the exponent,

$$f^{(i)}(\chi) = \sum_{n=0}^{\infty} f_n^{(i)} e^{-n\chi/M}$$

Important: asymptotic shift symmetry $\chi \to \chi + const$ in Einstein frame (or scale invariance in the Jordan frame).

# Physics of the "jump" of the constants

Top quark Yukawa interaction in the Einstein frame:

$$L_t = \frac{y_t}{\sqrt{2}} \bar{t} t F(\chi) \,, F(\chi) = \frac{h}{\Omega} \,,$$

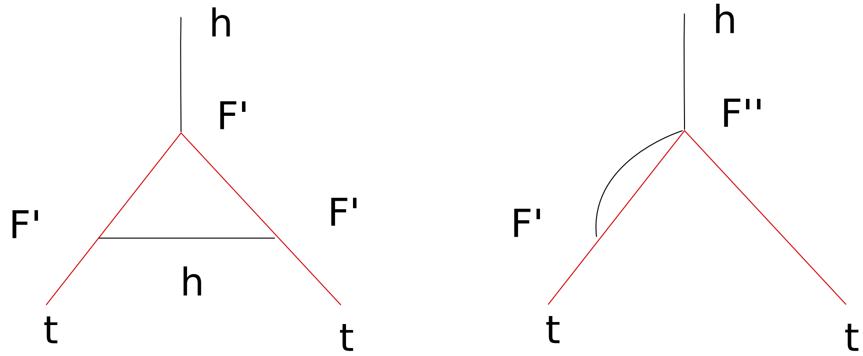
Here $\Omega^2 = 1 + \xi h^2/M_P^2$ is the conformal factor, and the canonically normalised field $\chi$ is related to the Higgs field $h$ via

$$\frac{dh}{d\chi} = \frac{\Omega^2}{\sqrt{\Omega^2 + \xi(6\xi + 1)h^2/M_P^2}} \,.$$

In the background field $\chi$ coupling of the top to $\chi$ is proportional to $dF/d\chi = F'$.

Counter-term:

$$y_t \to y_t + \frac{y_t^3}{16\pi^2}\left(\frac{3}{\epsilon} + C_t\right) F'^2$$



$$F'(\chi) = \begin{cases} 1 & \text{for } h \lesssim M_P/2\sqrt{6}\xi \\ 0 & \text{for } h \gtrsim M_P/2\sqrt{6}\xi \end{cases}$$

For $h \lesssim h^*$ C is absorbed into definition of low energy coupling and is not observable:

$y_t^{\text{phys}} = y_t + \frac{y_t^3}{16\pi^2} C_t$ .

For $h \gtrsim h^*$ contribution from $F'$ disappears: determines a jump of the coupling around $h^*$

Finite parts of counter-terms:

Top Yukawa

$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1\right] \ ,$$

Higgs self-coupling

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \frac{1}{3}F''F\right)^2 - 1\right],$$