Theoretical implications of present LHC unobservations

0) **What was found**
1) **Finite naturalness**
2) **A new principle**
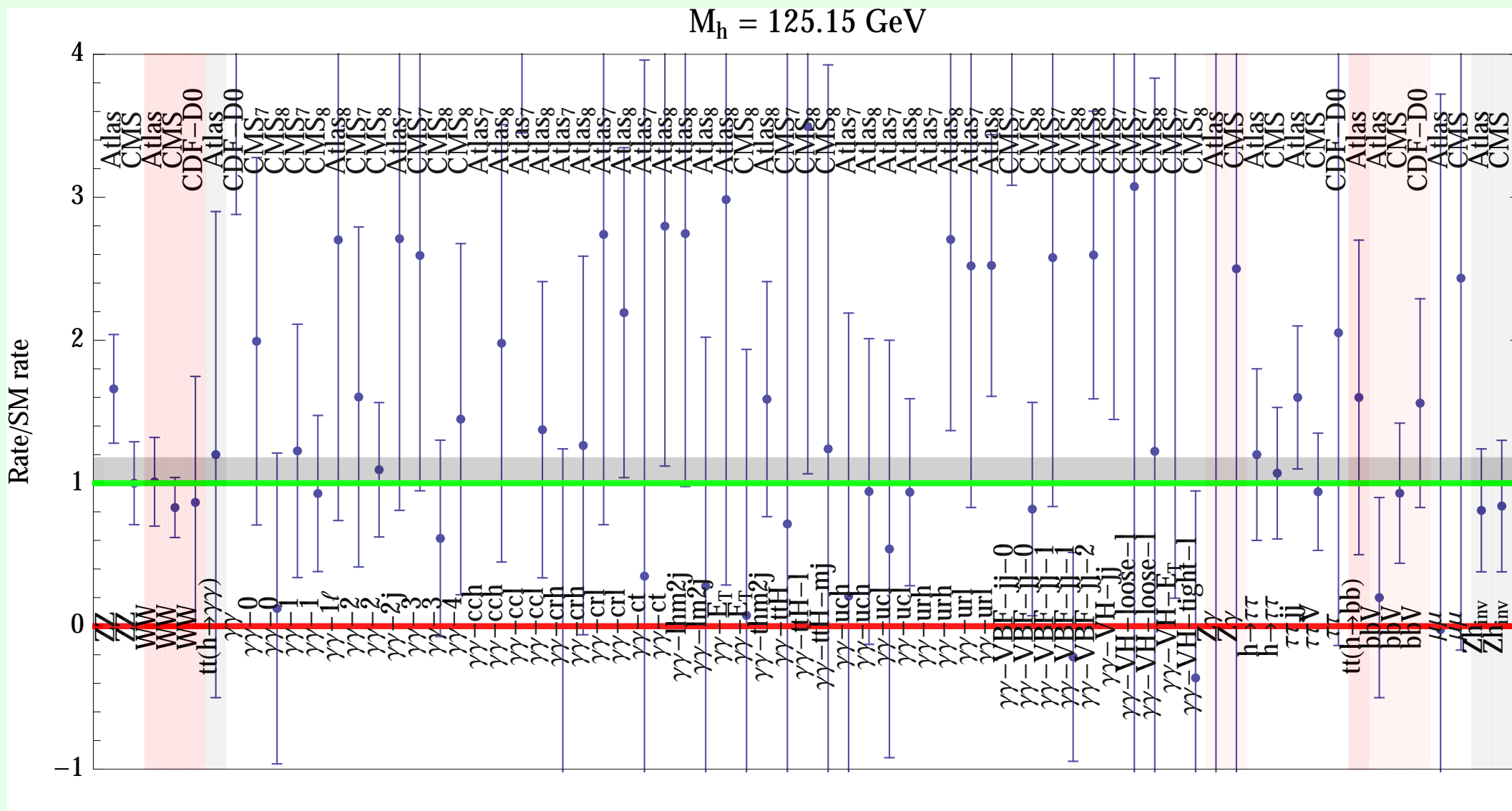3) **Agravity**
4) **Landau poles**

Alessandro Strumia
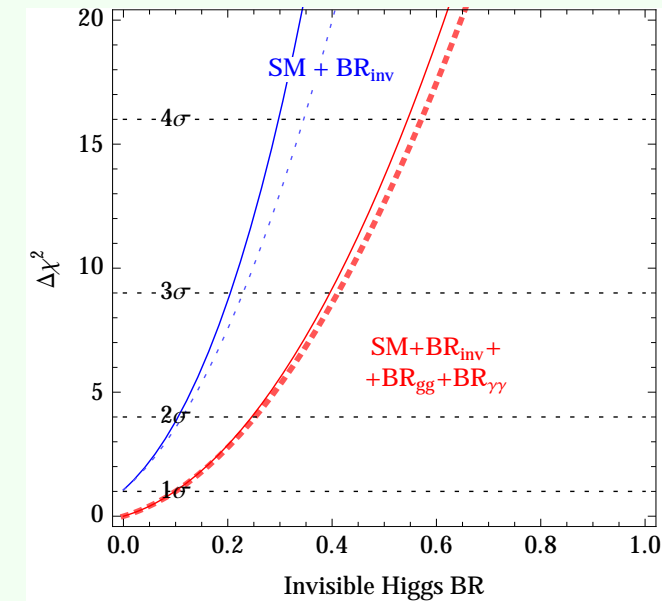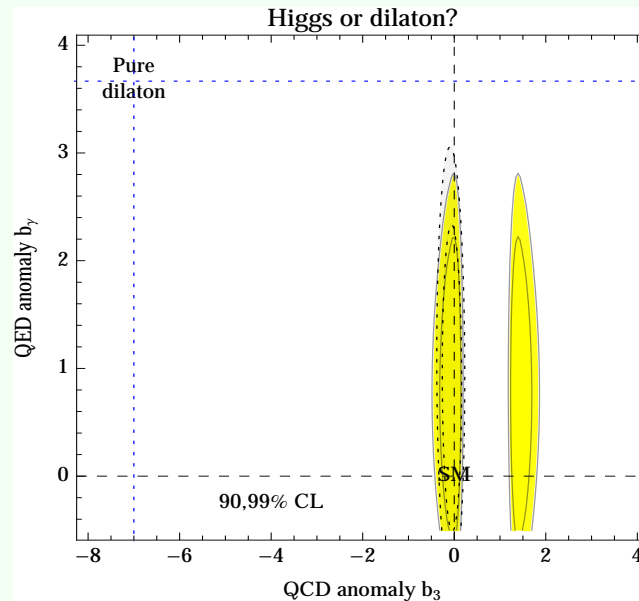
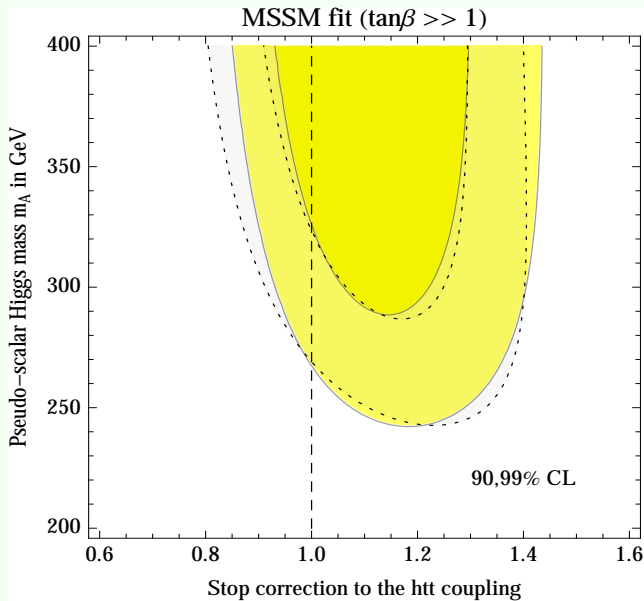Talk at 'Rethinking Naturalness', Frascati, December 17, 2014

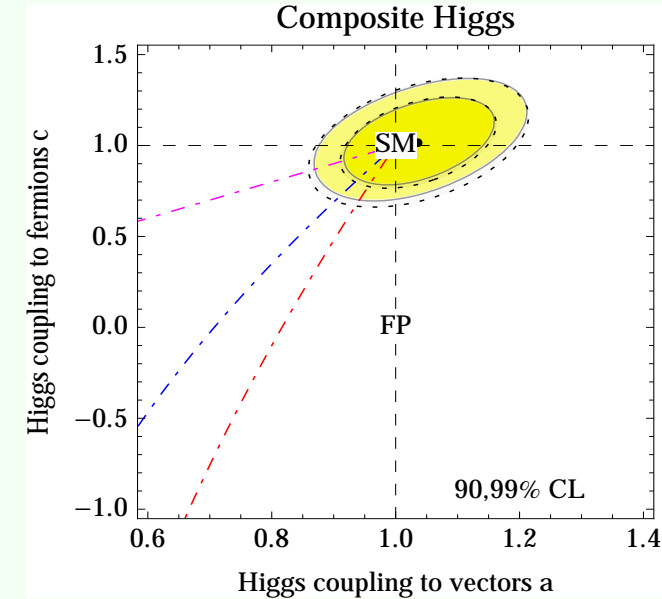# 0) What was found

But should not have been found

Only the Higgs

$M_h = 125.15$ GeV

The SM Higgs

Fit to Higgs couplings

New physics only in loops

Composite Higgs

MSSM fit ($\tan\beta \gg 1$)

Higgs or dilaton?
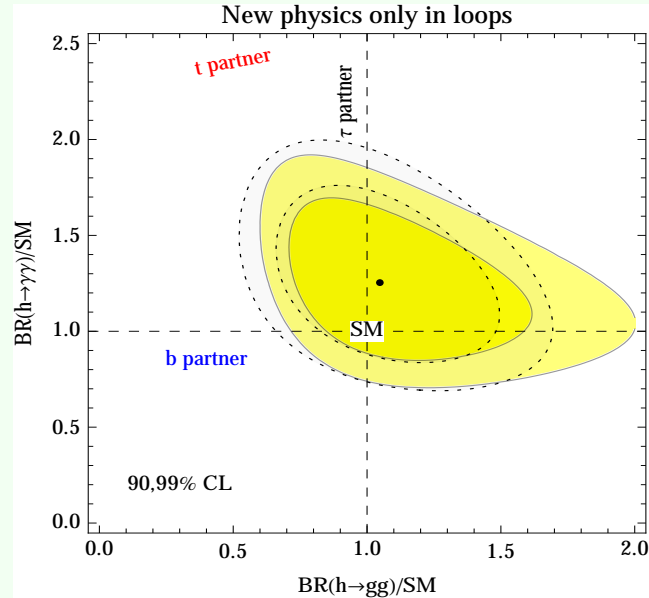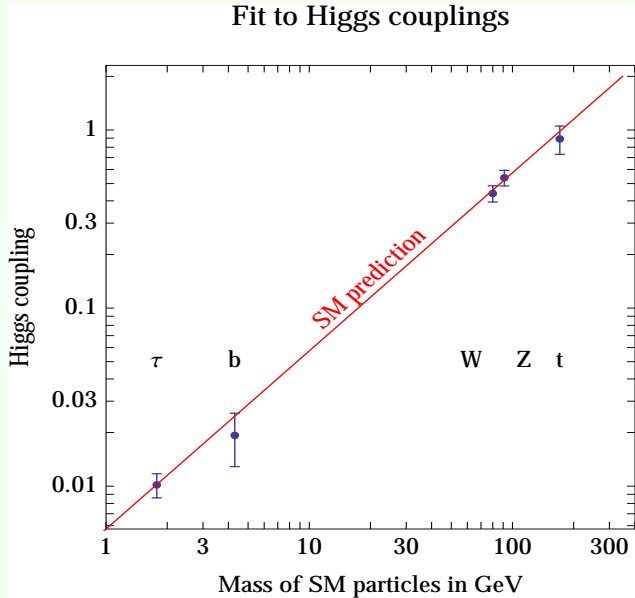
[Giardino, Kannike, Masina, Raidal Strumia, 1303.3570 + updates]

And nothing else

Maybe up to the Planck scale

For the measured M_h, M_t the SM can be extrapolated up to M_{Pl}.
And is close to vacuum meta-stability.

For the measured masses even **the β-function of λ ~vanishes** around M_{Pl}

$$\lambda = \beta_\lambda = 0 \text{ at } M_{Pl}$$

The SM parameters at NNLO

SM parameters extracted with data at 2 loop accuracy: at $\bar{\mu} = M_t$

$$g_2 = 0.64822 + 0.00004 \left(\frac{M_t}{\text{GeV}} - 173.10 \right) + 0.00011 \frac{M_W - 80.384\,\text{GeV}}{0.014\,\text{GeV}}$$

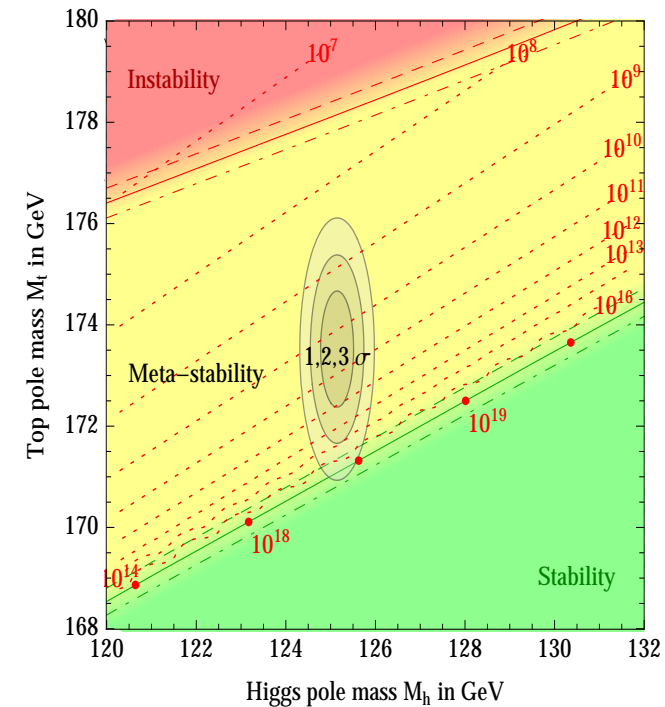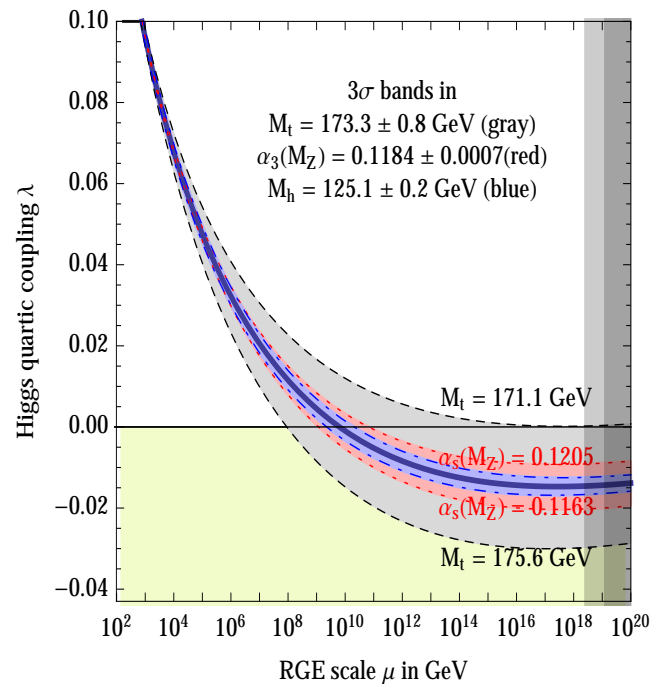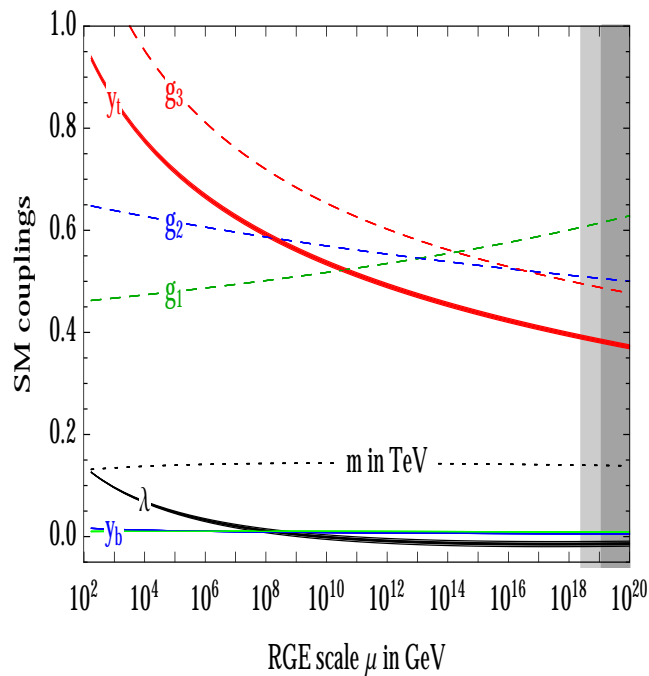$$g_Y = 0.35761 + 0.00011 \left(\frac{M_t}{\text{GeV}} - 173.10 \right) - 0.00021 \frac{M_W - 80.384\,\text{GeV}}{0.014\,\text{GeV}}$$

$$y_t = 0.9356 + 0.0055 \left(\frac{M_t}{\text{GeV}} - 173.10 \right) - 0.0004 \frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.0005_{\text{th}}$$

$$\lambda = 0.1271 + 0.0021 \left(\frac{M_h}{\text{GeV}} - 125.66 \right) - 0.00004 \left(\frac{M_t}{\text{GeV}} - 173.10 \right) \pm 0.0003_{\text{th}}$$

$$\frac{m}{\text{GeV}} = 132.03 + 0.94 \left(\frac{M_h}{\text{GeV}} - 125.66 \right) + 0.17 \left(\frac{M_t}{\text{GeV}} - 173.10 \right) \pm 0.15_{\text{th}}.$$

Renormalization to large energies is done with 3 loop RGE.

[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, 1307.3536]

What is the big message?

After LHC run I the mystery behind the Higgs got deeper

At some point somebody will understand

What is this talk about?

In the past decades, theory was driven by the naturalness principle:

"light fundamental scalars cannot exist, unless they are accompanied by new physics that protects their mass from quadratically divergent corrections".

Theorists proposed a beautiful plausible scenario with beautiful LHC signals:
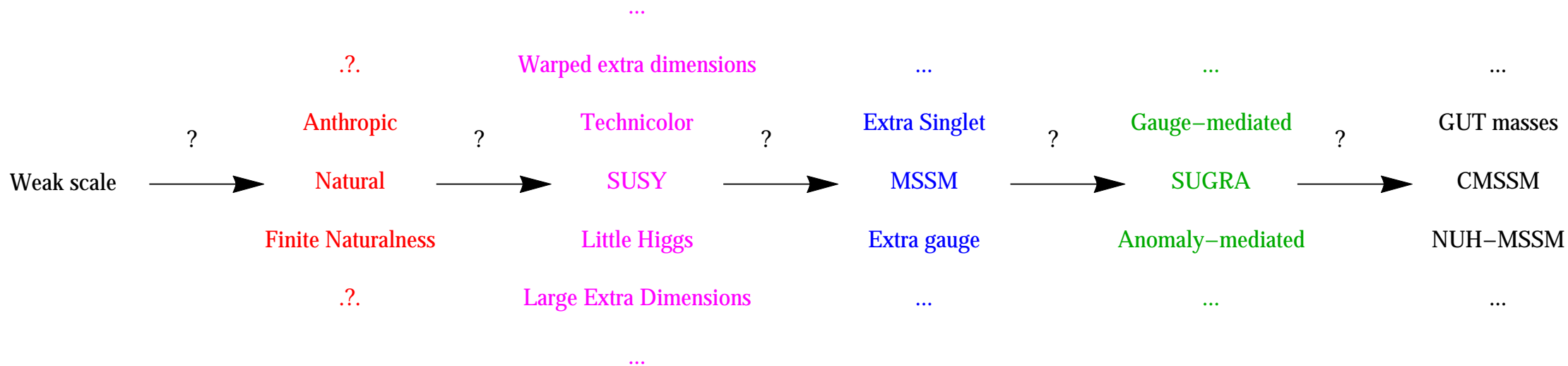
But LHC found the higgs and nothing else so far.

I assume that this will be the final outcome and reconsider the basic question.

The goal of this talk is presenting an alternative: a renormalizable theory valid above M_{Pl} such that M_h is naturally smaller than M_{Pl} without new physics at the weak scale. It naturally gives inflation and an anti-graviton ghost-like.

1) Finite Naturalness

[Pappadopulo Farina, Strumia, 1303.7244]

The good, the bad, the ugly

The **good possibility** is naturalness.
But it is in trouble.

The **bad possibility** is that the Higgs is light because of ant**pic selection.
But why nature should have preferred an unlikely fine-tuning to an existing natural solution, or to a natural SM-like, or to a smaller M_{Pl}?

The **ugly possibility** is that
quadratic divergences vanish and a modified Finite Naturalness applies.

Power divergences are unphysical, nobody knows if they vanish or not. The answer is chosen by the ultimate unknown physical cut-off. Surely it is not a Lorentz-breaking lattice. Maybe it behaves like dimensional regularization.

[Caution: this is when rotten tomatoes start to fly]

Ipse undixt

Wilson proposed the usual naturalness attributing a physical meaning to momentum shells of power-divergent loop integrals, used in the 'averaged action'.
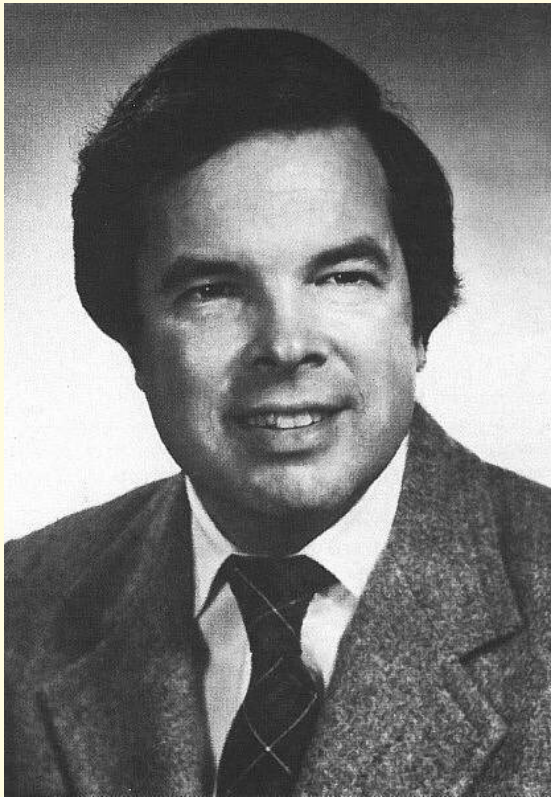
"The final blunder was a claim that scalar elementary particles were unlikely to occur in elementary particle physics at currently measurable energies unless they were associated with some kind of broken symmetry. The claim was that, otherwise, their masses were likely to be far higher than could be detected. The claim was that it would be unnatural for such particles to have masses small enough to be detectable soon.

But this claim makes no sense"

Kenneth G. Wilson

The SM satisfies Finite Naturalness

Quantum corrections to the dimensionful parameter $m^2 \simeq M_h^2$ in the SM Lagrangian $\frac{1}{2}m^2|H|^2 - \lambda|H|^4$ are small for the <u>measured</u> values of the parameters

Finite natualness in the SM
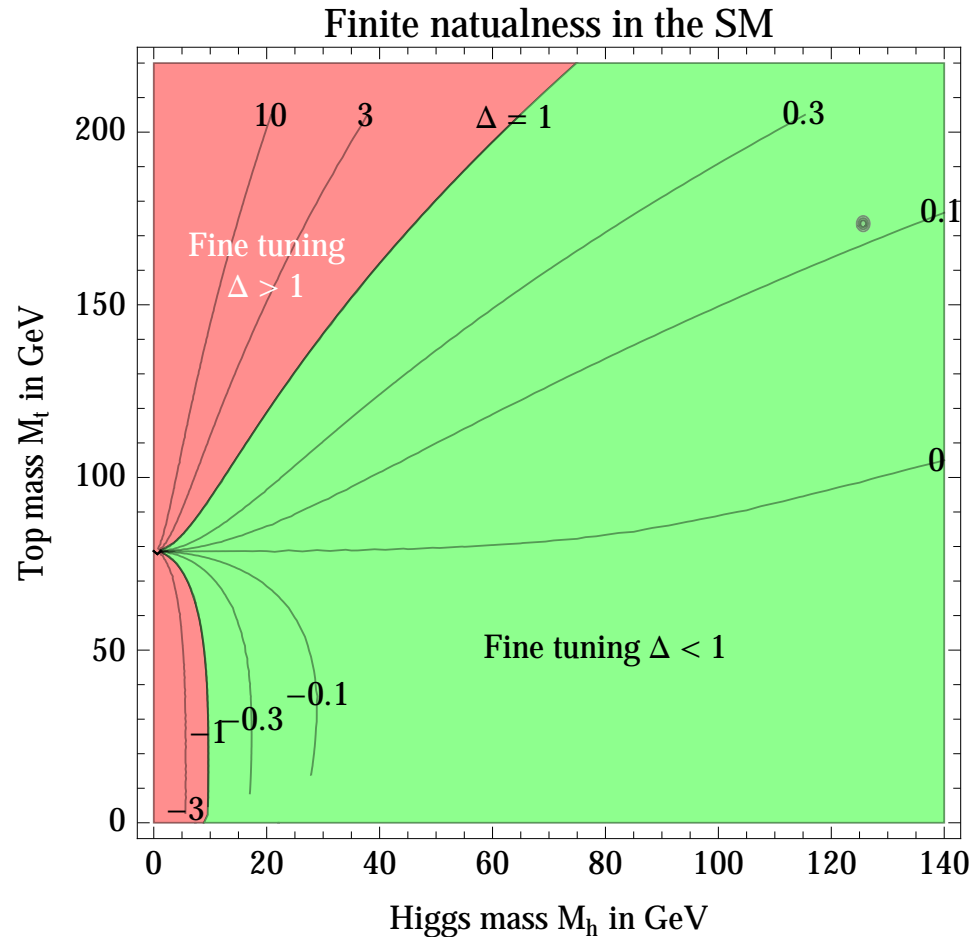
From arXiv:1303.7244

$$M_h = 125.6\,\text{GeV} \;\Rightarrow\; m(\bar{\mu} = M_t) = 132.7\,\text{GeV} \;\Rightarrow\; m(\bar{\mu} = M_{\text{Pl}}) = 140.9\,\text{GeV}$$

Finite Naturalness and new physics

FN would be ruined by new heavy particles too coupled to the SM.
Unlike in the other scenarios, high-scale model building is very constrained.
Imagine there is no GUT. No flavour models too. Above us only sky.

FN holds if the top really is the top — if the weak scale is the highest scale.

Data demand some new physics: DM, neutrino masses, maybe axions...

FN still holds if such new physics lies not much above the weak scale.

Is this possible? If yes what are the signals?

Finite Naturalness and new physics

Neutrino mass models add extra particles with mass M
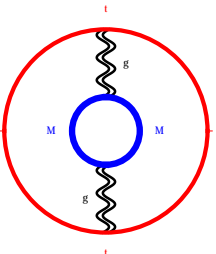
$$M \lesssim \begin{cases} 0.7 \ 10^7 \ \text{GeV} \times \sqrt[3]{\Delta} & \text{type I see-saw model,} \\ 200 \ \text{GeV} \times \sqrt{\Delta} & \text{type II see-saw model,} \\ 940 \ \text{GeV} \times \sqrt{\Delta} & \text{type III see-saw model.} \end{cases}$$

Leptogenesis is compatible with FN only in type I.

Axion and LHC usually are like fish and bicycle because $f_a \gtrsim 10^9$ GeV. Axion models can satisfy FN, e.g. KSVZ models employ heavy quarks with mass M

$$M \lesssim \sqrt{\Delta} \times \begin{cases} 0.74 \ \text{TeV} & \text{if } \Psi = Q \oplus \bar{Q} \\ 4.5 \ \text{TeV} & \text{if } \Psi = U \oplus \bar{U} \\ 9.1 \ \text{TeV} & \text{if } \Psi = D \oplus \bar{D} \end{cases}$$

Inflation: flatness implies small couplings. Einstein gravity gives an upper bound on H_I and on any mass [Arvinataki, Dimopoulos..]

$$\delta m^2 \sim \quad \text{(diagram)} \quad \sim \frac{y_t^2 M^6}{M_{\text{Pl}}^4 (4\pi)^6} \qquad \text{so} \qquad M \lesssim \Delta^{1/6} \times 10^{14} \ \text{GeV}$$

Dark Matter: extra scalars/fermions with/without weak gauge interactions.

DM with EW gauge interactions

Consider a Minimal Dark Matter n-plet. 2-loop quantum corrections to M_h^2:

$$\delta m^2 = \frac{cnM^2}{(4\pi)^4}\left(\frac{n^2-1}{4}g_2^4 + Y^2 g_Y^4\right) \times \begin{cases} 6\ln\frac{M^2}{\Lambda^2} - 1 & \text{for a fermion} \\ \frac{3}{2}\ln^2\frac{M^2}{\Lambda\mu^2} + 2\ln\frac{M^2}{\Lambda^2} + \frac{7}{2} & \text{for a scalar} \end{cases}$$

Quantum numbers			DM could	DM mass	$m_{\mathrm{DM}^\pm} - m_{\mathrm{DM}}$	Finite naturalness	σ_{SI} in
SU(2)$_L$	U(1)$_Y$	Spin	decay into	in TeV	in MeV	bound in TeV, $\Lambda \sim M_{\mathrm{Pl}}$	10^{-46} cm^2
2	1/2	0	EL	0.54	350	$0.4 \times \sqrt{\Delta}$	$(2.3 \pm 0.3)\,10^{-2}$
2	1/2	1/2	EH	1.1	341	$1.9 \times \sqrt{\Delta}$	$(2.5 \pm 0.8)\,10^{-2}$
3	0	0	HH^*	2.5	166	$0.22 \times \sqrt{\Delta}$	0.60 ± 0.04
3	0	1/2	LH	2.7	166	$1.0 \times \sqrt{\Delta}$	0.60 ± 0.04
3	1	0	HH, LL	1.6+	540	$0.22 \times \sqrt{\Delta}$	0.06 ± 0.02
3	1	1/2	LH	1.9+	526	$1.0 \times \sqrt{\Delta}$	0.06 ± 0.02
4	1/2	0	HHH^*	2.4+	353	$0.14 \times \sqrt{\Delta}$	1.7 ± 0.1
4	1/2	1/2	(LHH^*)	2.4+	347	$0.6 \times \sqrt{\Delta}$	1.7 ± 0.1
4	3/2	0	HHH	2.9+	729	$0.14 \times \sqrt{\Delta}$	0.08 ± 0.04
4	3/2	1/2	(LHH)	2.6+	712	$0.6 \times \sqrt{\Delta}$	0.08 ± 0.04
5	0	0	(HHH^*H^*)	9.4	166	$0.10 \times \sqrt{\Delta}$	5.4 ± 0.4
5	0	1/2	stable	10	166	$0.4 \times \sqrt{\Delta}$	5.4 ± 0.4
7	0	0	stable	25	166	$0.06 \times \sqrt{\Delta}$	22 ± 2

DM without EW gauge interactions

DM coupling to the Higgs determines Ω_{DM}, σ_{SI} and Finite Naturalness δm^2

scalar DM singlet
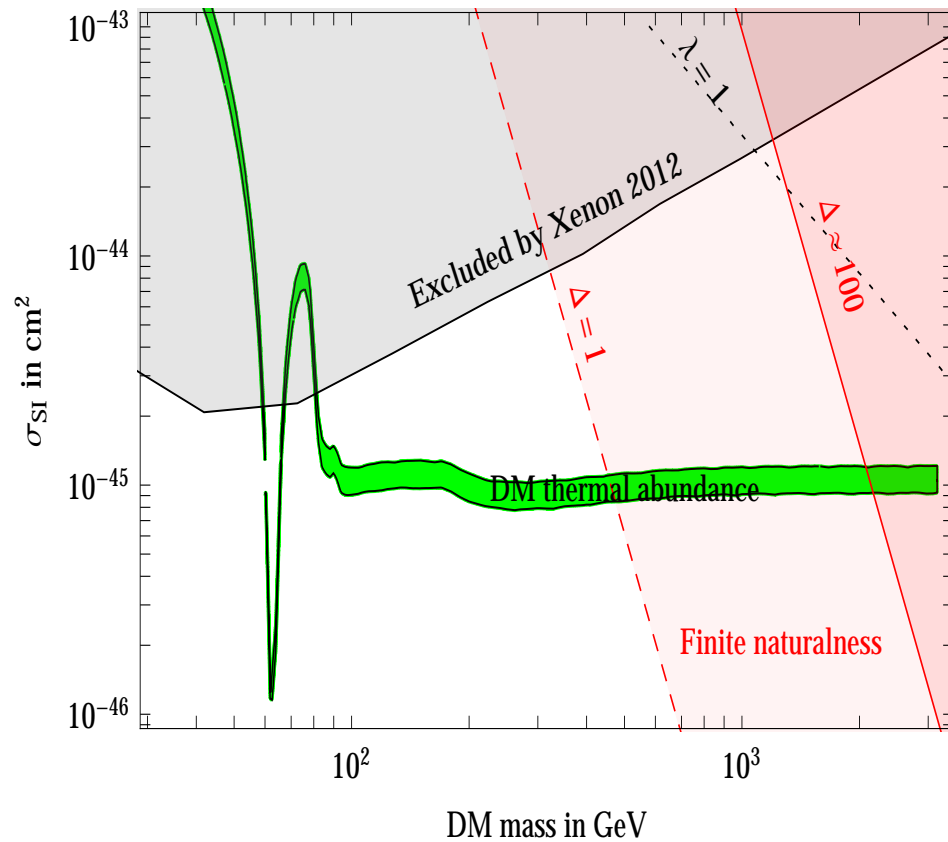
Fermion DM singlet ($m_S = 300$ GeV)
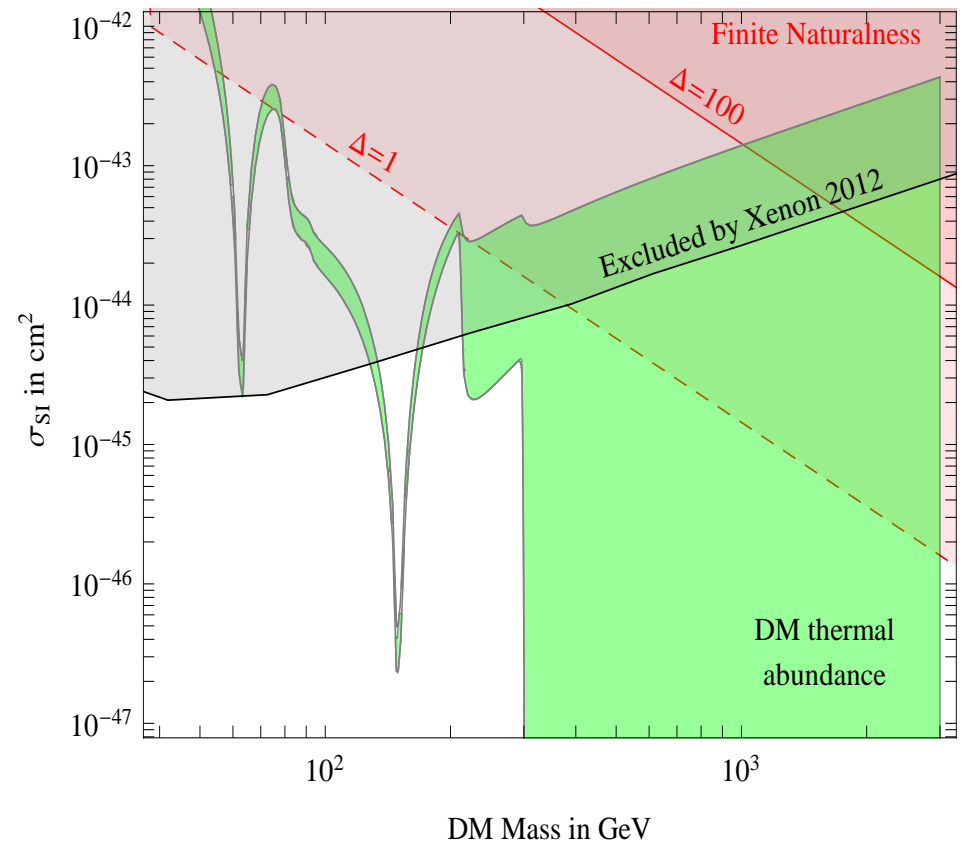
Observable DM satisfies Finite Naturalness if lighter than $\approx 1\,\mathrm{TeV}$

2) A new principle

Finite Naturalness is phenomenologically viable, what about its theory?

Nature has no scale

FN needs something different from the effective field theory ideology

$$\mathscr{L} \sim \Lambda^4 + \Lambda^2 |H|^2 + \mathscr{L}_4 + \frac{H^6}{\Lambda^2} + \cdots$$

that leads to the hierarchy problem. Nature is singling out $\mathscr{L}_4$. Why?

Principle: "**Nature has no fundamental scales** Λ".

Then, the fundamental QFT is described by $\mathscr{L}_4$: only a-dimensional couplings.

Power divergences vanish simply because they have mass dimension, and there are no masses. [Other authors assume scale or conformal invariance as quantum symmetries and argue that the regulator must respect them. I assume that scale invariance is just an accidental symmetry, like baryon number].

Quantum corrections break scale invariance and should generate M_h, M_{Pl}

Can this happen? I apply this principle first to matter and later to gravity.

What is the weak scale?

○ Could be the only scale of particle physics. Just so.

• Could be generated from nothing by heavier particles.

  - See-saw, axions, gravity...

• Could be generated from nothing by weak-scale dynamics.

  - Another gauge group might become strong around 1 TeV.

  - The quartic of another scalar might run negative around 1 TeV.

WEAKLY COUPLED MODELS

The Coleman-Weinberg mechanism can dynamically generate the weak scale

In the SM it predicts a too light M_h. Add an extra scalar S with a gauge interaction (U(1), SU(2), SU(2)$\otimes$U(1)...) such that the quartic $\lambda_S|S|^4$ runs negative. Then S develops a vev and $\lambda_{HS}|S|^2|H|^2$ becomes a Higgs mass.

Hambye, Strumia, 1306.2329 proposed a model that:
1) **Dynamically generates** the weak scale and weak scale DM
2) **Preserves** the successful automatic features of the SM: $B, L...$
3) **Gets DM stability** as one extra automatic feature.

Model:

$G_{\mathsf{SM}} \otimes \mathsf{SU}(2)_X$ with one extra scalar S, doublet under $\mathsf{SU}(2)_X$ and potential

$$V = \lambda_H|H|^4 - \lambda_{HS}|HS|^2 + \lambda_S|S|^4.$$

Weakly coupled SU(2) model

1) λ_S runs negative at low energy:

$$\lambda_S \simeq \beta_{\lambda_S} \ln \frac{s}{s_*} \qquad \text{with} \qquad \beta_{\lambda_S} \simeq \frac{9g_X^4}{8(4\pi)^2}$$

$$S(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ w + s(x) \end{pmatrix} \qquad w \simeq s_* e^{-1/4}$$
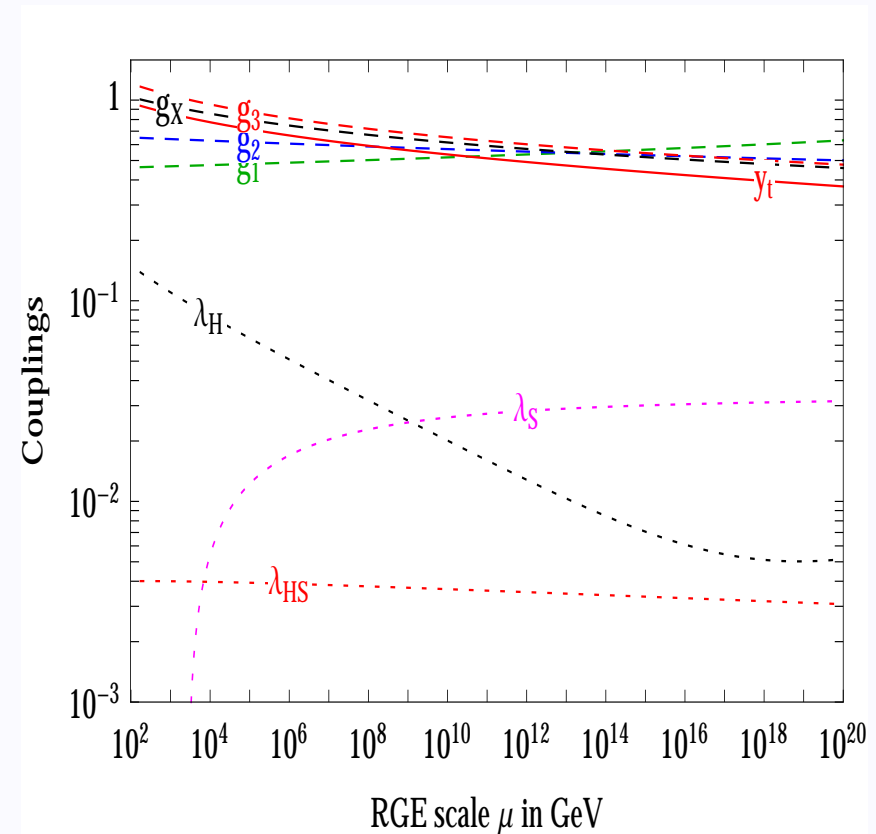
$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \qquad v \simeq w \sqrt{\frac{\lambda_{HS}}{2\lambda_H}}$$

Problem: vacuum energy must be negative???

2) No new Yukawas.

3) $SU(2)_X$ vectors get mass $M_X = \frac{1}{2} g_X w$ and are automatically stable.

4) Bonus: threshold effect stabilises $\lambda_H = \lambda + \lambda_{HS}^2 / \beta_{\lambda_S}$.

Experimental implications

1) New scalar s: like another h with suppressed couplings; $s \to hh$ if $M_s > 2M_h$.

2) Dark Matter coupled to s, h. Assuming that DM is a thermal relict

$$\sigma v_{\text{ann}} + \frac{1}{2}\sigma v_{\text{semi-ann}} = \frac{11g_X^2}{1728\pi w^2} + \frac{g_X^2}{64\pi w^2} \approx 2.2 \times 10^{26}\,\frac{\text{cm}^3}{\text{s}}$$

fixes $g_X = w/2\,\text{TeV}$, so all is predicted in terms of one parameter e.g. g_X:
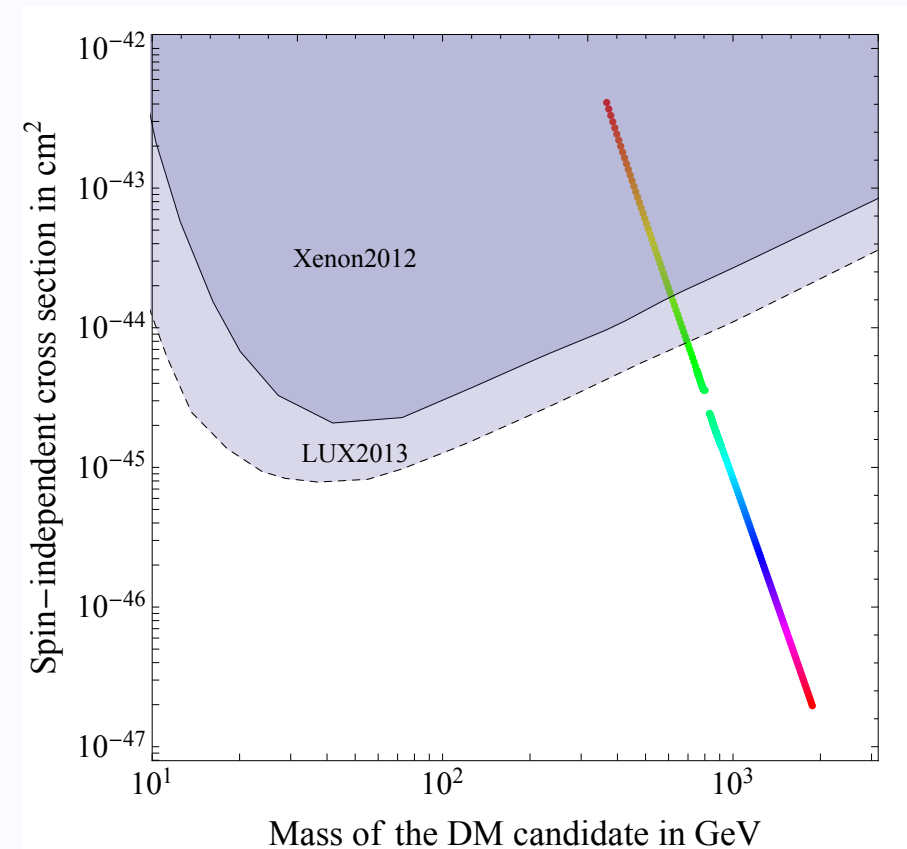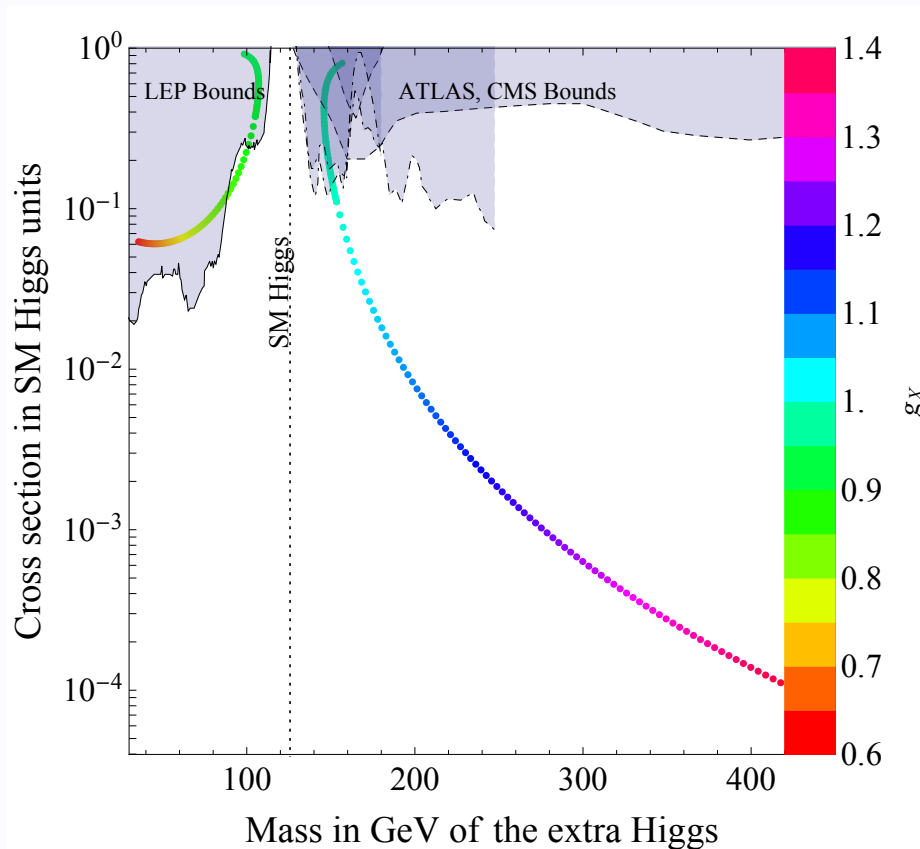
Dark/EW phase transition

The universe remains trapped at $s = 0$ until the potential energy ΔV is violently released via thermal tunnelling: $\Gamma \sim T^4 e^{-S/T}$ with $S \propto g_X^4$.
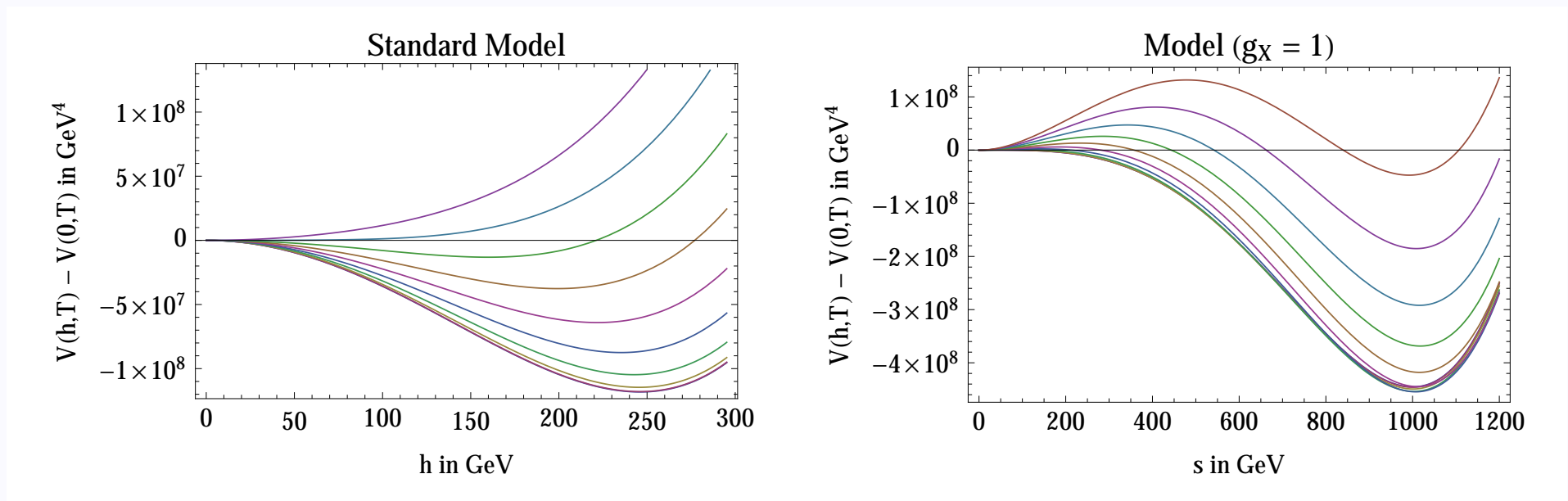
- For the critical value $g_X \approx 1.2$ one has $\Delta V \approx \rho$ such that

$$f_{\text{peak}} \approx 0.3 \, \text{mHz} \qquad \Omega_{\text{peak}} h^2 \approx 2 \, 10^{-11} \qquad \text{detectable at LISA}$$

- For $g_X > 1.2$ gravitational waves become weaker.
- For $g_X < 1.2$ the universe gets trapped in a (too long?) inflationary phase. Allows for EW baryogenesis from the QCD axion [Servant].

STRONGLY COUPLED MODELS

Techni-Color can dynamically generate a mass scale for an <u>elementary</u> Higgs

$\star$ Hur-Ko and Raidal et al. proposed models where a scalar S interacts with the Higgs and techni-quarks such that $\lambda_{HS}|S|^2|H|^2$ becomes a Higgs mass.

$\star\star$ Lindner et al.: models where a TC scalar S develops a condensate $\langle S^*S \rangle$. (Kubo, Lim, Lindner observe that even QCD alone could do the job, if S in a big SU(3)$_c$ representation. A 15$'$ condenses around 1 TeV, when $\frac{28}{3}\alpha_3 \sim 1$).

$\star\star\star$ Antipin, Redi, Strumia 1410.1817: the scalar S is not necessary.

Model:

$G_{\mathsf{SM}} \otimes \mathsf{SU}(N)_{\mathsf{TC}}$ with one extra fermion in the $(0_Y, 3_L, 1_c, N \oplus \bar{N})$. $V = \lambda_H|H|^4$

No extra scalars, no masses: as many parameters as the SM!

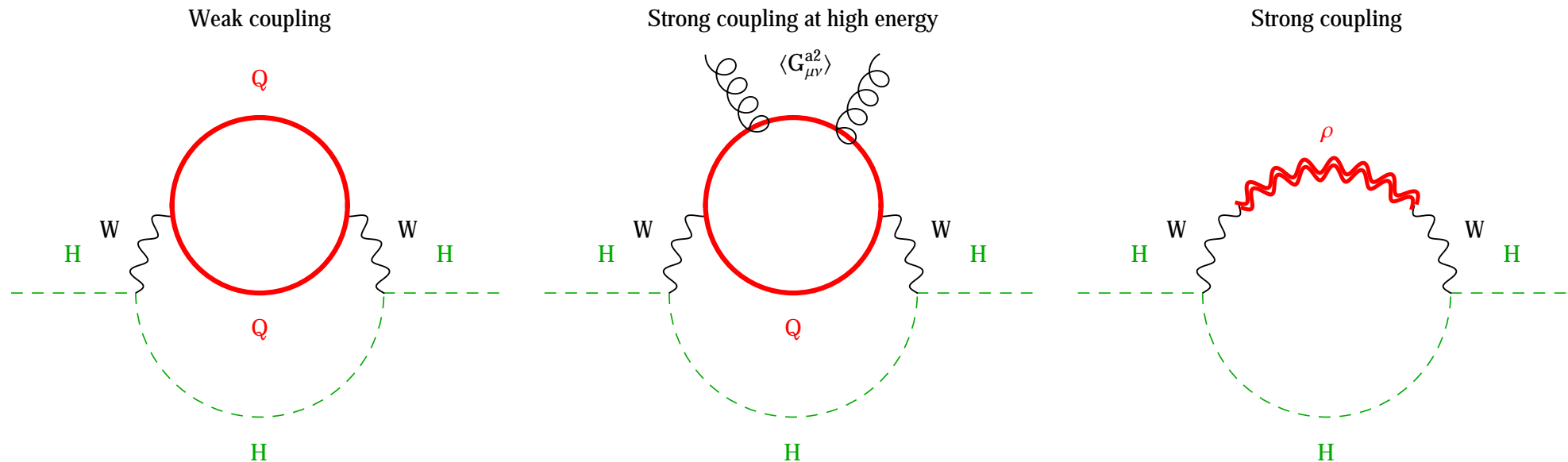# The weak scale from strong dynamics

The Higgs mass arises as

$$m^2 = -\frac{9g_2^4}{4i} \int \frac{d^4q}{(2\pi)^4} \frac{\Pi_{WW}(q^2)}{q^2} = \frac{9g_2^4}{4(4\pi)^2} \int dQ^2 \Pi_{WW}(-Q^2).$$

where strong dynamics is parameterised by Π_{WW}:

$$i \int d^4x\, e^{iq\cdot x} \langle 0| \mathsf{T} J_\mu^a(x) J_\nu^b(0) |0\rangle \equiv \delta^{ab}(q^2 g_{\mu\nu} - q_\mu q_\nu) \Pi_{WW}(q^2).$$

The weak scale from strong dynamics

- Dispersion relations show that $m^2 < 0$:

$$\frac{\partial \Pi_{VV}}{\partial \Lambda_{\mathsf{TC}}^2} = -\frac{q^2}{\Lambda_{\mathsf{TC}}^2} \frac{\partial \Pi_{VV}}{\partial q^2}, \qquad \frac{\partial \Pi_{WW}(q^2)}{\partial q^2} = \frac{1}{\pi} \int_0^\infty ds \, \frac{\overbrace{\mathrm{Im}\, \Pi_{WW}(s)}^{\sim -\sigma < 0}}{(s-q^2)^2} < 0$$

- Operator Product Expansions demand that m^2 is UV-finite

$$\Pi_{WW}(q^2) \overset{q^2 \gg \Lambda_{\mathsf{TC}}^2}{\simeq} \underbrace{c_1(q^2)}_{\text{dimensionless}} + \underbrace{c_3(q^2)}_{-C/q^4} \underbrace{\langle 0 | \frac{\alpha_{\mathsf{TC}}}{4\pi} \mathcal{G}_{\mu\nu}^{A2} | 0 \rangle}_{\text{positive}} + \cdots$$

- Vector Meson Dominance estimates

$$\Pi_{WW}(q^2) = \frac{m_\rho^2}{g_\rho^2(q^2 - m_\rho^2 + i\epsilon)} \qquad \Rightarrow \qquad m^2 \sim -\frac{g_2^4 m_\rho^2}{(4\pi)^2 g_\rho^2}$$

Natural result: $m_\rho \sim 20\,\mathrm{TeV}$, $m_B \sim 50\,\mathrm{TeV}$, $m_{\pi_n} \approx \frac{g_2 m_\rho}{4\pi} \sqrt{\frac{3}{4}(n^2-1)} \sim 2\,\mathrm{TeV}$.
Pions in the $3 \otimes 3 - 1 = 3 \oplus 5$ of $\mathsf{SU}(2)_L$. π_5 decays via the anomaly $\pi_5 \to WW$.

Dark Matter from strong dynamics

The model has **two** DM candidates:

- **The lightest techni-baryon**, presumably subdominant: its thermal abundance reproduces the DM abundance for $m_B \sim 200\,\text{TeV}$ (estimate: $\sigma v \sim g_{\text{TC}}^4/4\pi M^2$ e.g. $\sigma_{p\bar{p}}v \sim 100/m_p^2$). Its magnetic dipole can give a detectable signal with a characteristic recoil energy dependence.

- **The techni-pion** π_3, stable because of accidental G-parity $\mathcal{Q} \to \exp(i\pi T^2)\mathcal{Q}^c$. It has a negligible quartic behaving as Minimal Dark Matter: $m_{\pi_3} = 2.5\,\text{TeV}$, $\sigma_{\text{SI}} \approx 0.12\ 10^{-46}\,\text{cm}^2$.
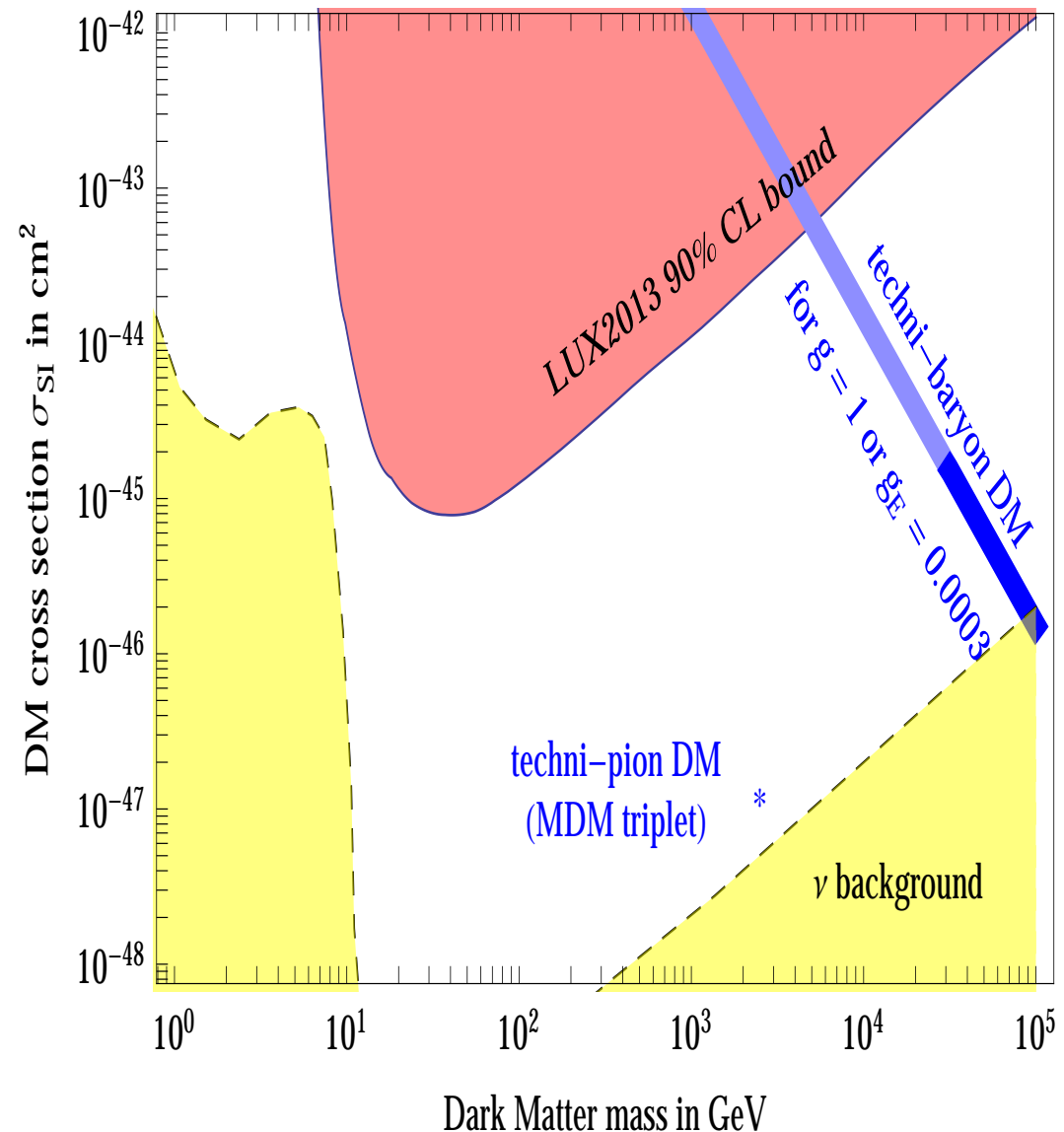
More strong models

Other assignments of massless $\mathcal{Q}$ lead to good DM:

number of techni-flavors	Yukawa	$N=3$ TCb	TCπ	$N=4$ TCb	TCπ	
$N_F = 2$		2	3	1	3	under TC-flavor SU(2)
model 1: $\mathcal{Q} = 2_{Y=0}$	0	charged	3	1	3	DM, under SU(2)$_L$
$N_F = 3$		8	8	$\bar{6}$	8	under TC-flavor SU(3)
model 1: $\mathcal{Q} = 1_Y + 2_{Y'}$	1	1	no	1	no	DM, under SU(2)$_L$
model 2: $\mathcal{Q} = 3_{Y=0}$	0	3	3	1	3	DM, under SU(2)$_L$
$N_F = 4$		$\overline{20}$	15	$20'$	15	under TC-flavor SU(4)
model 1: $\mathcal{Q} = 4_{Y=0}$	0	charged	3	1	3	DM, under SU(2)$_L$
$N_F = 5$		$\overline{40}$	24	$\overline{50}$	24	under TC-flavor SU(5)
model 1: $\mathcal{Q} = 2_Y + 3_{Y'}$	1	1	no	charged	no	DM, under SU(2)$_L$
model 2: $\mathcal{Q} = 5_{Y=0}$	0	3	3	1	3	DM, under SU(2)$_L$

Multiple rep.s give $m_{\pi_1} = 0$ (excluded) unless Yukawas $yH\mathcal{Q}_L^1\mathcal{Q}_R^2 + y'H^\dagger\mathcal{Q}_R^1\mathcal{Q}_L^2$ with the elementary H are possible such that

- $m_{\pi_1} \sim |y|vm_\rho/m_{\pi_2}$.
- π_2/H mixing: $\Delta m^2 \sim -y^2 m_\rho^2/m_{\pi_2}^2$.
- π_3 becomes unstable.

Weak coupling

Strong coupling
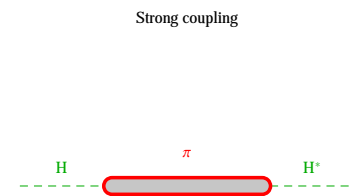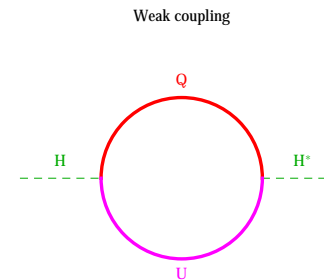
$\mathcal{Q}$ with color interactions give $m^2 > 0$ at 3 loops; gravity gives $\Lambda < 0$.

3) Agravity

What about gravity?

Does quantum gravity give $\delta M_h^2 \sim M_{\text{Pl}}^2$ ruining Finite Naturalness?

Maybe M_{Pl}^{-1} is just a small coupling and there are no new particles around M_{Pl}.

Quantum gravity would be very different from what strings suggest...

[Salvio, Strumia, 1403.4226]

Adimensional gravity

Applying the adimensional principle to the SM plus gravity and a scalar S gives:

$$\mathscr{S} = \int d^4x \; \sqrt{|\det g|} \, \mathscr{L}$$

$$\mathscr{L} = \mathscr{L}_{\mathsf{SM}} + \frac{R^2}{3f_0^2} + \frac{R^2 - 3R_{\mu\nu}^2}{3f_2^2} + |D_\mu S|^2 - \xi_S |S|^2 R - \lambda_S |S|^4 + \lambda_{HS}|HS|^2$$

where f_0, f_2 are the adimensional 'gauge couplings' of gravity and $R \sim \partial_\mu \partial_\nu g_{\mu\nu}$.

<u>Of course</u> the theory is renormalizable, and indeed the graviton propagator is:

$$\frac{-i}{k^4}\left[2f_2^2 P_{\mu\nu\rho\sigma}^{(\mathsf{spin}\ 2)} - f_0^2 P_{\mu\nu\rho\sigma}^{(\mathsf{spin}\ 0)} + \mathsf{gauge\text{-}fixing}\right].$$

The Planck scale should be generated dynamically as $\xi_S \langle S \rangle^2 = \bar{M}_{\mathsf{Pl}}^2/2$.

Then, the spin-0 part of $g_{\mu\nu}$ gets a mass $M_0 \sim f_0 M_{\mathsf{Pl}}$ and the spin 2 part splits into the usual graviton and an anti-graviton with mass $M_2 = f_2 \bar{M}_{\mathsf{Pl}}/\sqrt{2}$ that acts as a Pauli-Villars in view its negative kinetic term [Stelle, 1977].

A ghost?

Classically, higher derivatives are bad [Ostrogradski, 1850]:

$$\partial^4 \Rightarrow \text{unbounded negative kinetic energy} \Rightarrow \text{the theory is dead.}$$

The dispersion relation $P^4 = m^4$ has 4 solutions: $E = \pm m$ and $E = \pm im$.

In presence of masses, ∂^4 can be decomposed as 2 fields with 2 derivatives:

$$\frac{1}{k^4} \rightarrow \frac{1}{k^4 - M_2^2 k^2} = \frac{1}{M_2^2} \left[\frac{1}{k^2} - \frac{1}{k^2 - M_2^2} \right]$$

Quantistically, the state with negative kinetic term can be reinterpreted as **positive energy and negative norm** by swapping $a \leftrightarrow a^\dagger$.

This is the $i\epsilon$ choice that makes the theory renormalizable.

Lee, Wick, Cutkosky... claim that it gives a slightly acausal unitary S matrix.

Anti-particles teach us that sometimes we have the right equations before understanding what they mean. I ignore the ghost issue and compute.

A ghost?
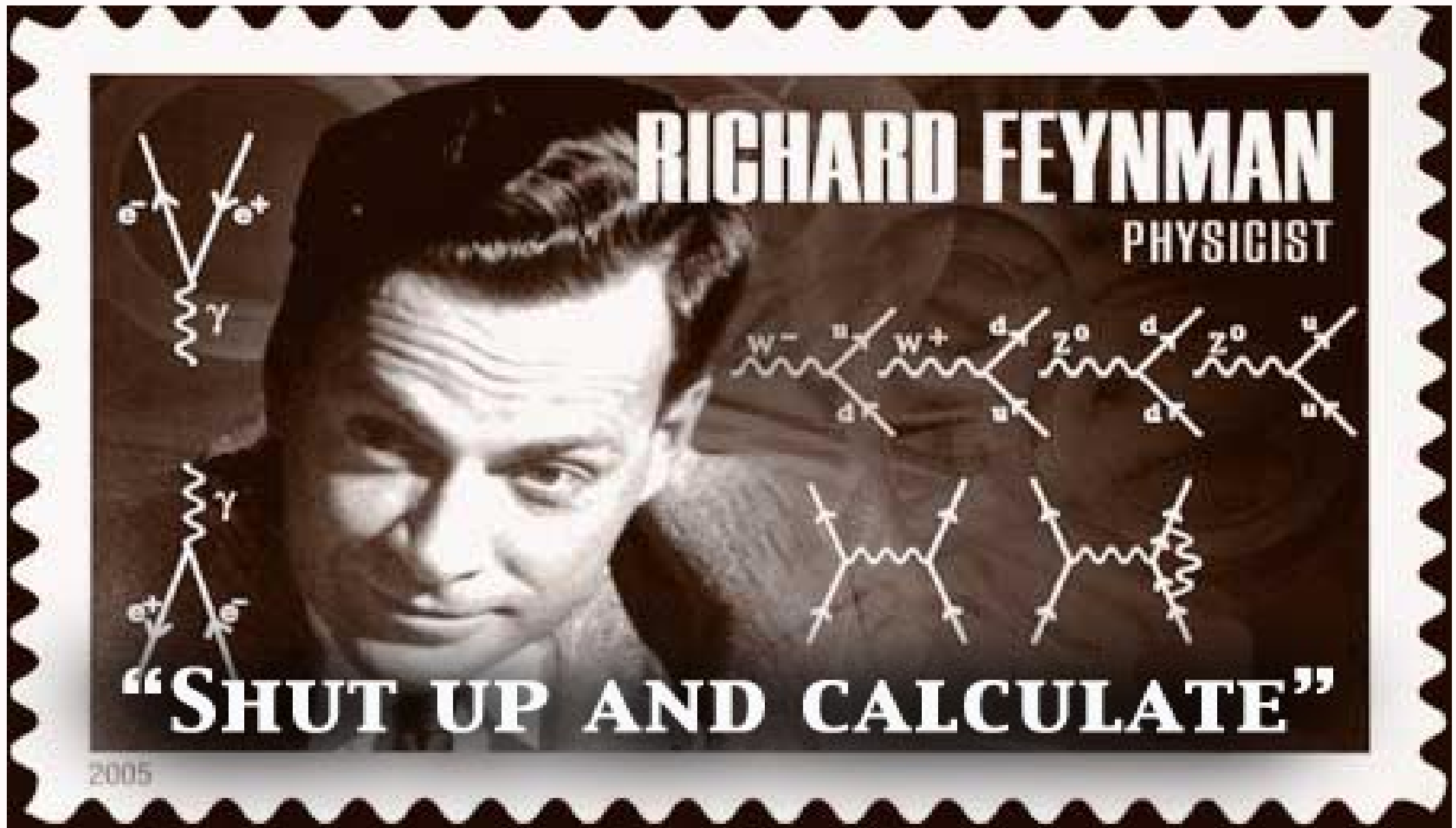
A ghost?

If we knew
what we were doing
it wouldn't be research

Albert Einstein

A ghost?

Me ne frego !

Quantum Agravity...

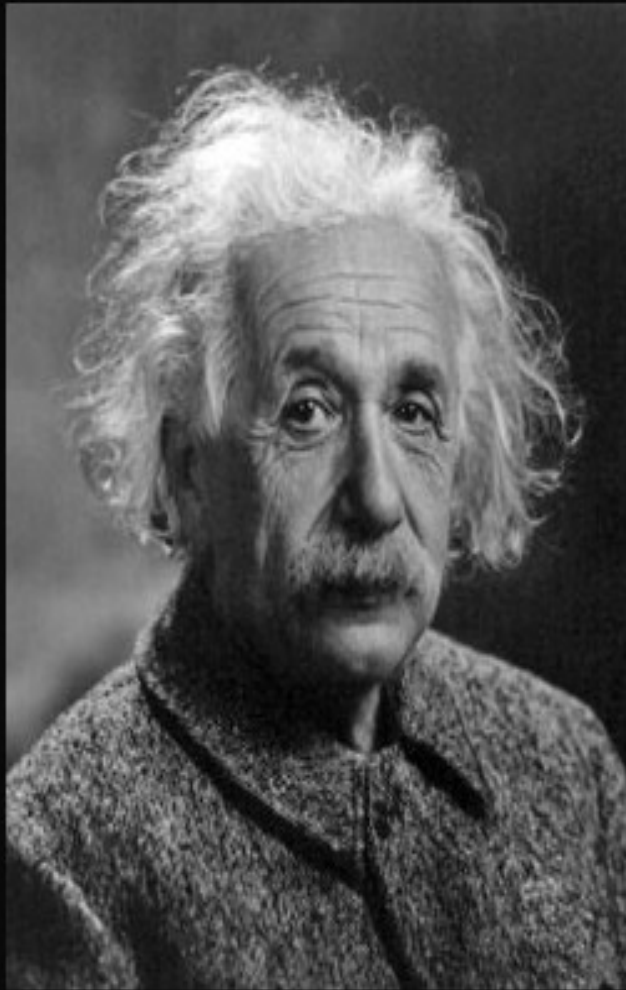The quantum behaviour of a renormalizable theory is encoded in its RGE.
The unusual $1/k^4$ makes easy to get signs wrong. Literature is contradictory.

- f_2 is asymptotically free:

$$(4\pi)^2 \frac{df_2^2}{d\ln\mu} = -f_2^4\left[\frac{133}{10} + \frac{N_V}{5} + \frac{N_f}{20} + \frac{N_s}{60}\right]$$

- Gravity does not affect running of gauge couplings: these two diagrams cancel
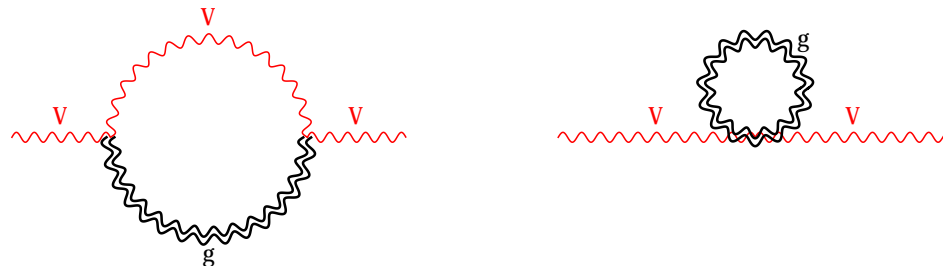
  presumably because abelian g is undefined without charged particles.

- f_0 is not asymptotically free unless $f_0^2 < 0$

$$(4\pi)^2 \frac{df_0^2}{d\ln\mu} = \frac{5}{3}f_2^4 + 5f_2^2f_0^2 + \frac{5}{6}f_0^4 + \frac{f_0^4}{12}\sum_s(1+6\xi_s)^2$$

...Quantum Agravity

- Yukawa couplings get an extra multiplicative RGE correction:

$$(4\pi)^2 \frac{dy_t}{d\ln\mu} = \frac{9}{2}y_t^3 - y_t\left(8g_3^2 - \frac{15}{8}f_2^2\right)$$

- RGE for ξ

$$(4\pi)^2 \frac{d\xi_H}{d\ln\mu} = -\frac{5}{3}\frac{f_2^4}{f_0^2}\xi_H + f_0^2\xi_H(6\xi_H + 1)\left(\xi_H + \frac{2}{3}\right) + (6\xi_H + 1)\left[2y_t^2 - \frac{3}{4}g_2^2 + \cdots\right]$$

- Agravity makes quartics small at low energy:

$$(4\pi)^2 \frac{d\lambda_H}{d\ln\mu} = \xi_H^2[5f_2^4 + f_0^4(1 + 6\xi_H)^2] - 6y_t^4 + \frac{9}{8}g_2^4 + \cdots$$

- Agravity creates a mixed quartic:

$$(4\pi)^2\frac{d\lambda_{HS}}{d\ln\mu} = \frac{\xi_H\xi_S}{2}[5f_2^4 + f_0^4(6\xi_S + 1)(6\xi_H + 1)] + \text{multiplicative}$$

Generation of the Planck scale

Some mechanisms can generate dynamically the Planck scale

a) λ_S runs negative below M_{Pl} or   b) f_2 or ξ_S run non-perturbative.

Focus on a): scalar Planckion. ξ_S makes the vacuum equations non-standard:

$$\frac{\partial V}{\partial S} - \frac{4V}{S} = 0 \qquad \text{i.e.} \qquad \frac{\partial V_E}{\partial S} = 0$$

where $V_E = V/(\xi S^2)^2 \sim \lambda_S(S)/\xi_S^2(S)$ is the Einstein-frame potential. The vev

$$\langle S \rangle = \bar{M}_{\text{Pl}}/\sqrt{2\xi_S}$$

needs a condition different from the usual Coleman-Weinberg:

$$\frac{\beta_{\lambda_S}(\bar{\mu} \sim \langle S \rangle)}{\lambda_S(\bar{\mu} \sim \langle S \rangle)} - 2\frac{\beta_{\xi_S}(\bar{\mu} \sim \langle S \rangle)}{\xi_S(\bar{\mu} \sim \langle S \rangle)} = 0$$

The cosmological constant vanishes if

$$\lambda_S(\bar{\mu} \sim \langle S \rangle) = 0$$

Then the minimum simplifies to

$$\beta_{\lambda_S}(\bar{\mu} \sim \langle S \rangle) = 0$$

Is this fine-tuned running possible?

This is how λ_H runs in the SM

RGE running of the $\overline{\text{MS}}$ quartic Higgs coupling in the SM

3σ bands in
$M_t = 173.3 \pm 0.8$ GeV (gray)
$\alpha_s(M_Z) = 0.1184 \pm 0.0007$ (red)
$M_h = 125.1 \pm 0.2$ GeV (blue)
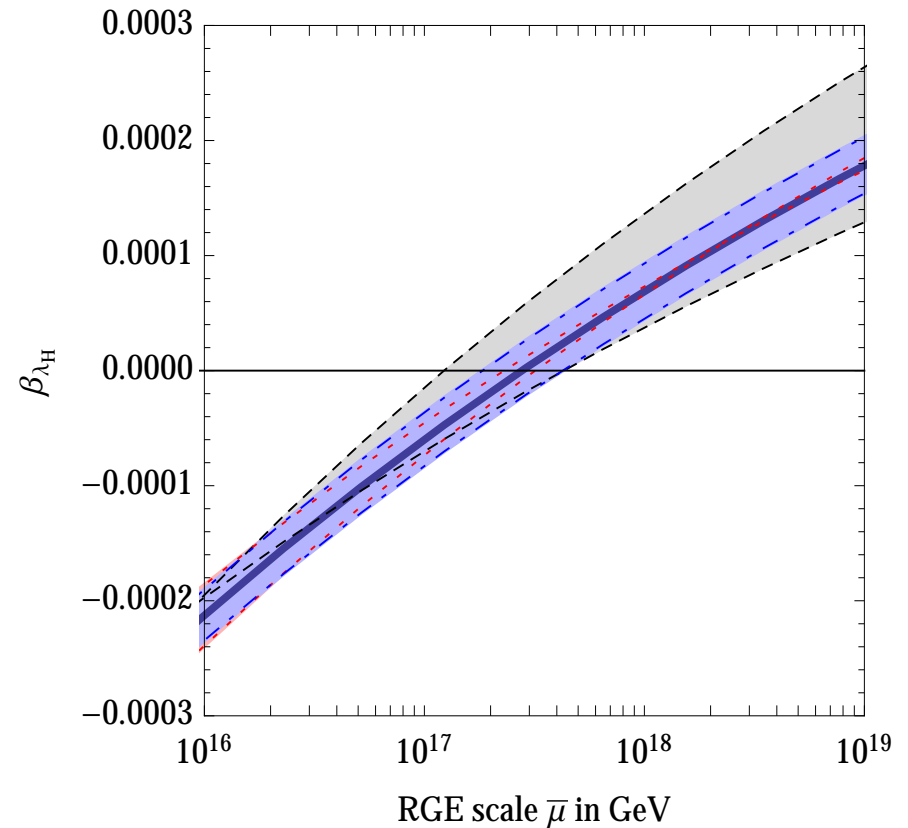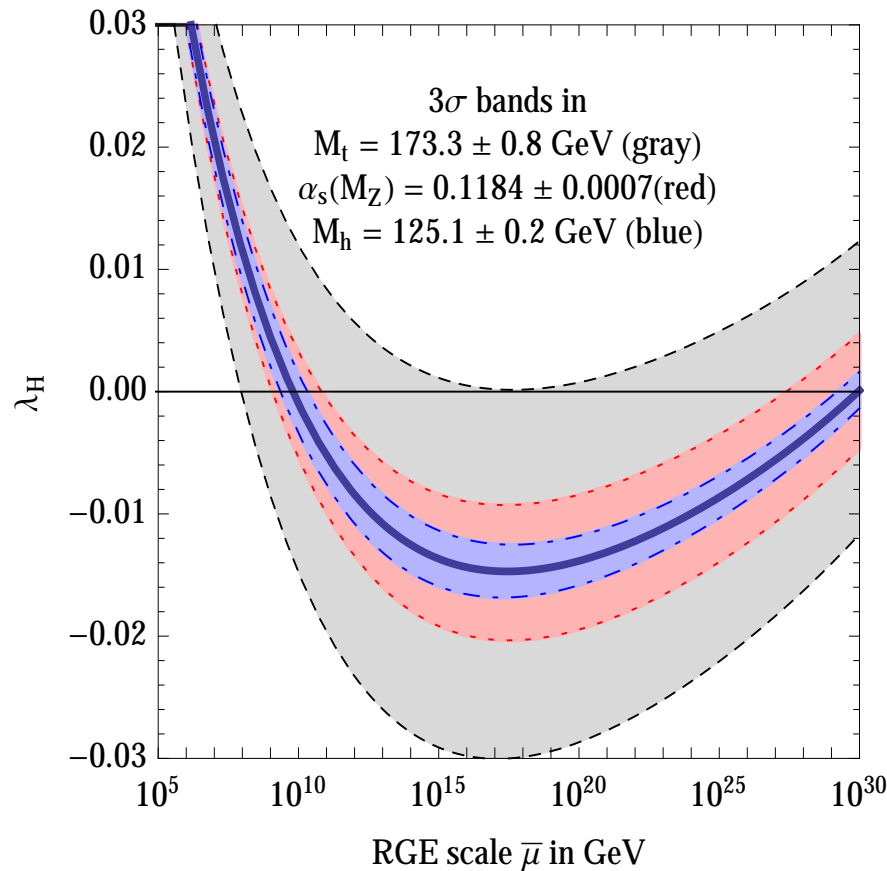
H cannot get a Planck-scale vev. Model: add a mirror copy of the SM, broken by the fact that S, the Higgs mirror, lies in the Planck minimum: $\xi_S \sim 10^{1 \div 2}$.

Inflation = perturbative agravity

A successful class of models is ξ-inflation: a scalar S with $-\frac{1}{2}f(S)R + V(S)$.
Redefine $g_{\mu\nu} = g_{\mu\nu}^E \times \bar{M}_{\mathsf{Pl}}^2/f$ to the Einstein frame to make the graviton canonical

$$\sqrt{\det g}\left[-\frac{f}{2}R + \frac{(\partial_\mu s)^2}{2} - V\right] = \sqrt{\det g_E}\left[-\frac{\bar{M}_{\mathsf{Pl}}^2}{2}R_E + \bar{M}_{\mathsf{Pl}}^2\left(\frac{1}{f} + \frac{3f'^2}{2f^2}\right)\frac{(\partial_\mu s)^2}{2} - V_E\right]$$

where $V_E = \bar{M}_{\mathsf{Pl}}^4 V/f^2$ is flat (good for inflation) if $V(S) \propto f^2(S)$ **above** M_{Pl}.

In general, this restriction is unmotivated and uncontrollable.

In quantum agravity $f(S) = \xi_S(\bar{\mu} \sim S)|S|^2$ and $V(S) = \lambda_S(\bar{\mu} \sim S)|S|^4$!

Inflation is a typical phenomenon in agravity: the slow-roll parameters are the β-functions, which are small if the theory is perturbative. In the Einstein frame

$$\epsilon \equiv \frac{\bar{M}_{\mathsf{Pl}}^2}{2}\left(\frac{1}{V_E}\frac{\partial V_E}{\partial s_E}\right)^2 = \frac{1}{2}\frac{\xi_S}{1 + 6\xi_S}\left[\frac{\beta_{\lambda_S}}{\lambda_S} - 2\frac{\beta_{\xi_S}}{\xi_S}\right]^2,$$

$$\eta \equiv \frac{\bar{M}_{\mathsf{Pl}}^2}{V_E}\frac{\partial^2 V_E}{\partial s_E^2} = \frac{\xi_S}{1 + 6\xi_S}\left[\frac{\beta(\beta_{\lambda_S})}{\lambda_S} - 2\frac{\beta(\beta_{\xi_S})}{\xi_S} + \frac{5 + 36\xi_S}{1 + 6\xi_S}\frac{\beta_{\xi_S}^2}{\xi_S^2} - \frac{7 + 48\xi_S}{1 + 6\xi_S}\frac{\beta_{\lambda_S}\beta_{\xi_S}}{2\lambda_S\xi_S}\right].$$

Inflationary predictions

Agravity predicts at least two scalars, the inflaton is the lighter one:

- The scalar that breaks scale invariance generating M_{Pl}; if it is the inflaton one has $n_s \approx 0.967$ and $r \approx 0.13$.

- The scalar component of the graviton; if it is the inflaton one has $n_s \approx 0.967$ and $r \approx 0.003$.

- Intermediate values of r are obtained if their masses are comparable.

WORK IN PROGRESS

Generation of the Weak scale

RGE running generates M_h from M_{Pl}. 3 regimes:

1) below $M_{0,2}$: ignore agravity, M_h runs logarithmically as in the SM

$$(4\pi)^2 \frac{dM_h^2}{d\ln\bar{\mu}} = \beta_{\text{SM}} M_h^2 \qquad \beta_{\text{SM}} = 12\lambda_H + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}$$

2) between $M_{0,2}$ and M_{Pl}: the apparent masses run:

$$(4\pi)^2 \frac{dM_h^2}{d\ln\bar{\mu}} = \left[\beta_{\text{SM}} + 5f_2^2 + \frac{5}{3}\frac{f_2^4}{f_0^2} + \cdots\right] M_h^2 - \xi_H\left[5f_2^4 + f_0^4(1 + 6\xi_H)\right]\bar{M}_{\text{Pl}}^2$$

3) above M_{Pl} couplings are adimensional: $\lambda_{HS}|H|^2|S|^2$ leads to $M_h^2 = \lambda_{HS}\langle s\rangle^2$:

$$(4\pi)^2 \frac{d\lambda_{HS}}{d\ln\bar{\mu}} = -\xi_H\xi_S[5f_2^4 + f_0^4(6\xi_S + 1)(6\xi_H + 1)] + \cdots$$

The weak scale arises if $f_{0,2} \sim \sqrt{M_h/M_{\text{Pl}}} \sim 10^{-8}$ i.e. $M_{0,2} \sim 10^{11}\,\text{GeV}$

All small parameters such as $f_{0,2}$ and $\lambda_{HS} \sim f_{0,2}^4$ are naturally small

The Planckion s can have any mass between M_h and M_{Pl}

Black holes

Non-perturbative quantum gravity (a black hole with mass M_{BH}) could give

$$\delta M_h^2 \sim M_{\mathsf{BH}}^2 e^{-M_{\mathsf{BH}}^2/M_{\mathsf{Pl}}^2}.$$

The black holes possibly dangerous for FN have mass $M_{\mathsf{BH}} \sim M_{\mathsf{Pl}}$.

Such black holes do not exist if the fundamental coupling of gravity is small.
The minimal mass of a black hole is $M_{\mathsf{BH}} > M_{\mathsf{Pl}}/f_{0,2}$ because of

$$V_{\mathsf{Newton}} = -\frac{Gm}{r}\left[1 - \frac{4}{3}e^{-M_2 r} + \frac{1}{3}e^{-M_0 r}\right]$$

Conclusion: non-perturbative QG corrections $\delta M_h^2 \propto e^{-1/f_{0,2}^2}$ can be neglected.

4) Landau poles

Landau poles in the SM + soft gravity

We have the RGE above M_Pl, **can the theory reach infinite energy?**
Problem: Landau poles for g_Y at 10^{43} GeV, possibly λ, y_t, y_b, y_τ.
We assume that Landau poles give $\delta M_h^2 \sim M_\mathsf{Landau}^2$ and must be avoided.

We view agravity as an example of the more generic scenario of **soft-gravity**: the gravitational coupling $g \sim E/M_\mathsf{Pl}$ stops growing at $E \lesssim 10^{11}$ GeV when it is still small enough that gravitational corrections to M_h^2 are naturally small. The RGE of soft-gravity are dominated by the bigger SM couplings like in agravity.

[Giudice, Isidori, Salvio, Strumia, 1412.2769]

TAFfing

This procedure allows to check if any QFT obeys Total Asymptotic Freedom:

Rewrite RGE in terms of $t = \ln \mu^2/(4\pi)^2$ and of $x_I = \{\tilde{g}_i, \tilde{y}_a, \tilde{\lambda}_m\}$ as

$$g_i^2(t) = \frac{\tilde{g}_i^2(t)}{t}, \qquad y_a^2(t) = \frac{\tilde{y}_a^2(t)}{t}, \qquad \lambda_m(t) = \frac{\tilde{\lambda}_m(t)}{t}.$$

Get

$$\frac{dx_I}{d\ln t} = V_I(x) = \begin{cases} \tilde{g}_i/2 + \beta_{g_i}(\tilde{g}), \\ \tilde{y}_a/2 + \beta_{y_a}(\tilde{g}, \tilde{y}), \\ \tilde{\lambda}_m + \beta_{\lambda_m}(\tilde{g}, \tilde{y}, \tilde{\lambda}). \end{cases}$$
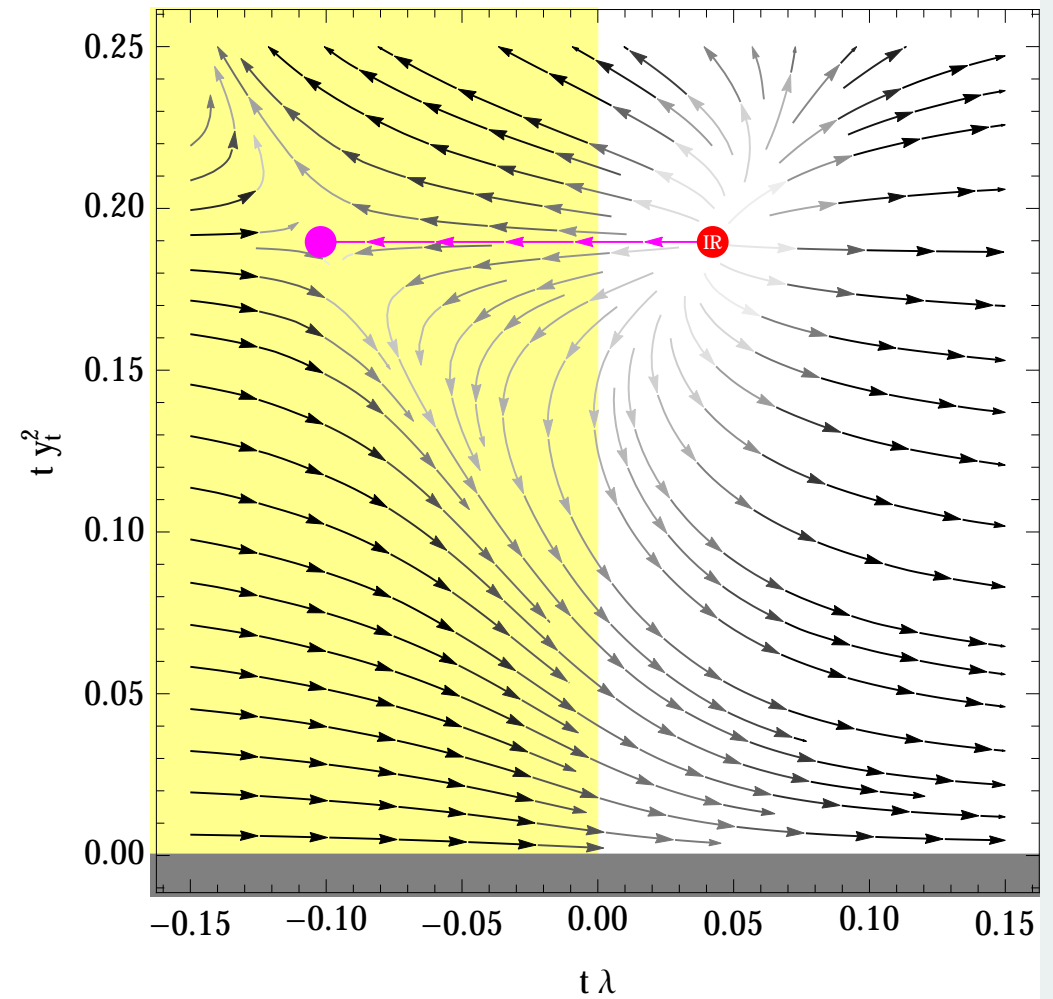
Fixed-points $x_I(t) = x_\infty$ are determined by the <u>algebraic</u> equation $V_I(x_\infty) = 0$.

Linearize around each fixed-point:

$$V_I(x) \simeq \sum_J M_{IJ}(x_J - x_{J\infty}) \qquad \text{where} \qquad M_{IJ} = \left.\frac{\partial V_I}{\partial x_J}\right|_{x=x_\infty}$$

Negative eigenvalues of M are UV-attractive. Each positive eigenvalue implies a UV-repulsive direction: to reach the FP a coupling is univocally predicted.

Flows in the SM with $g_1 = 0$
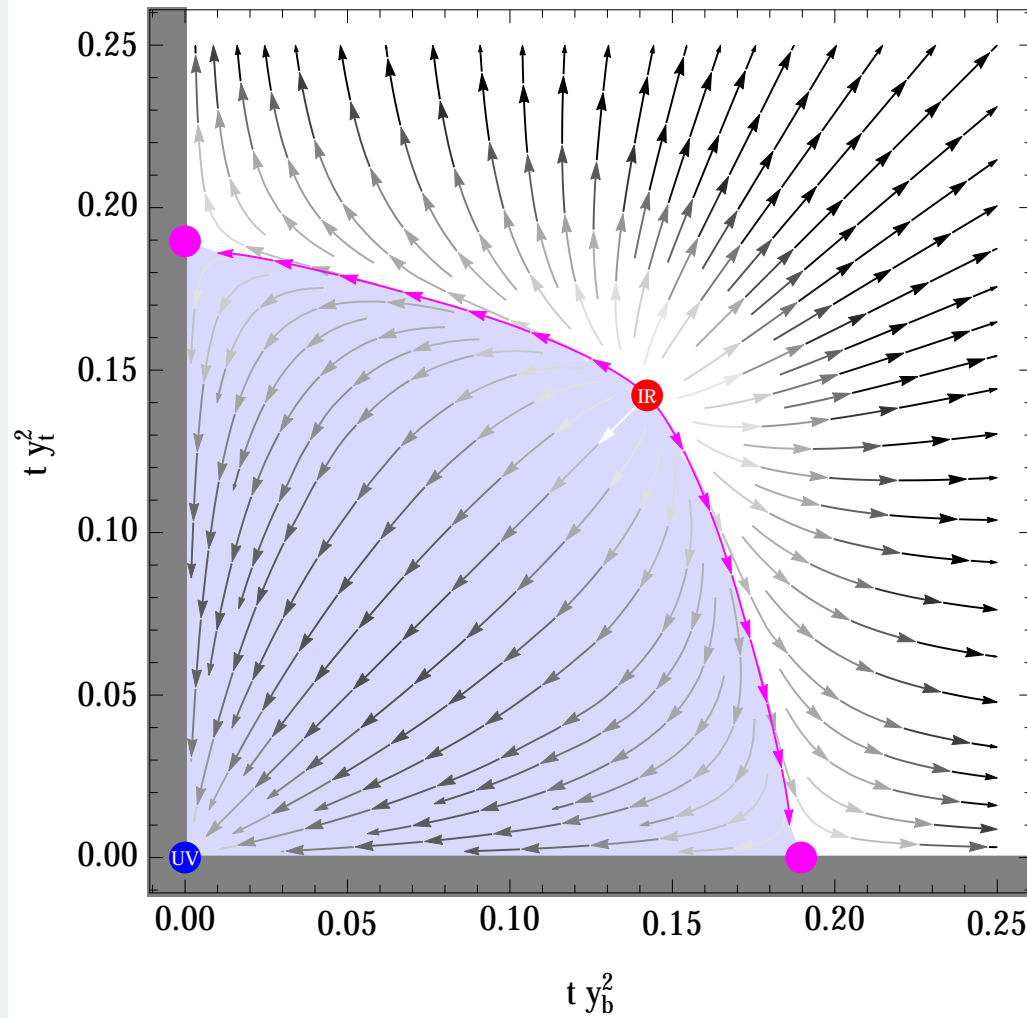
	$\tilde{y}_{t\infty}^2$	$\tilde{y}_{b\infty}^2$	$\tilde{y}_{\tau\infty}^2$	$\tilde{y}_{\nu\infty}^2$	Eigenvalues
Solution 1	227/1197	0	0	0	$+-++$
Solution 2	0	227/1197	0	0	$-+++$
Solution 3	227/1596	227/1596	0	0	$++++$
Solution 4	0	0	0	0	$--++$

	$\tilde{\lambda}_\infty$	Eig
Sol. 1	$\dfrac{-143 + \sqrt{119402}}{4788} \approx +0.0423$	$+$
Sol. 2	$\dfrac{-143 - \sqrt{119402}}{4788} \approx -0.1020$	$-$

SM up to infinite energy?

Predictions: 1) $g_Y = 0$; in this limit 2) $y_t^2 \simeq 227/1197t$ i.e. $M_t = 186\,\text{GeV}$; 3) $y_{\tau,\nu} = 0$; 4) $\lambda \simeq (-143 \pm \sqrt{119402})/4788t$ i.e. $M_h \leq 163\,\text{GeV}$. Equality avoids $\lambda < 0$ at large energy, and too fast vacuum decay $\lambda < -1/12t$.

SM for $g_1 = 0$ and $M_t = 185.6$ GeV
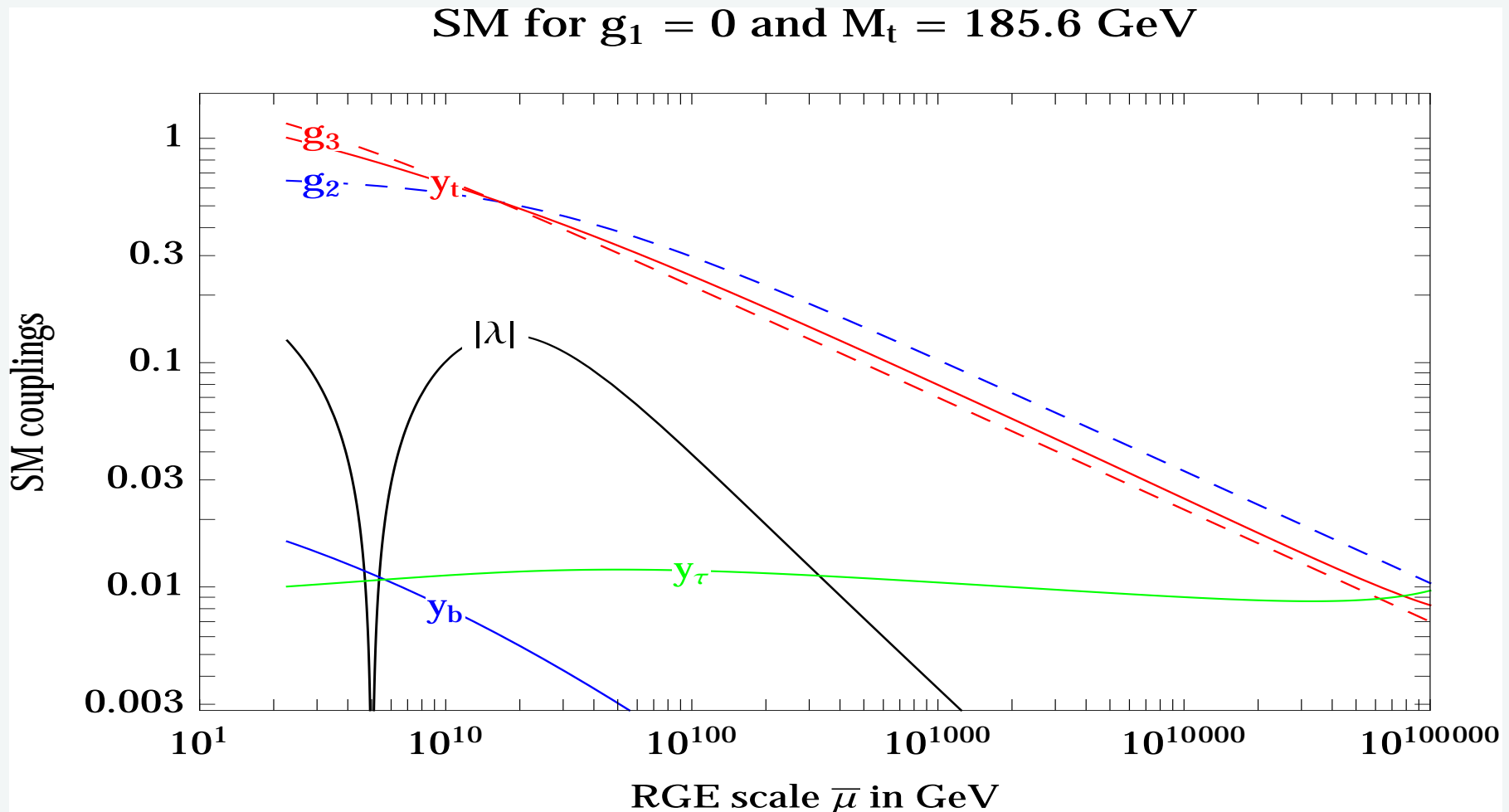
TAF extensions of the SM

Can the SM be extended into a theory valid up to infinite energy?
Avoid Landau poles by making hypercharge non abelian.

We found realistic SU(5) TAF models. But GUTs are not compatible with finite naturalness, that demands a TAF extension at the weak scale. Making sense of $Y = T_{3R} + (B - L)/2$ needs SU(2)$_R$. We see 2 possibilities:

$$\text{SU(4)}_c \otimes \text{SU(2)}_L \otimes \text{SU(2)}_R \qquad \text{and} \qquad \text{SU(3)}_c \otimes \text{SU(3)}_L \otimes \text{SU(3)}_R$$

TAF is tough

Too many experimental signals

- A W_R boson and a Z'_{B-L}: $M_{W_R} > 2.2\,\text{TeV}$, $M_{Z'_{B-L}} > 2.6_{333}, 3.8_{224}\,\text{TeV}$

$$\delta M_h^2 = -\frac{9g_R^2 M_{W_R}^2}{(4\pi)^2} \ln(\frac{M_{W_R}^2}{\bar{\mu}^2}) \approx M_h^2 \left(\frac{M_{W_R}}{2.5\,\text{TeV}}\right)^2$$

- The Higgs $(2_L, \bar{2}_R)$ contains 2 doublets coupled to u and d: new flavour violations controlled by a right-hand CKM matrix.

$$M_H > \begin{cases} 18\,\text{TeV} & \text{if } V_R = V_{\text{CKM}} \\ 3\,\text{TeV} & \text{if } V_R^{ij} = V_{\text{CKM}}^{ij} \times \min(m_i, m_j)/\max(m_i, m_j) \text{ (natural texture)} \end{cases}$$

- A lighter singlet that mixes with the higgs if $G_{\text{TAF}} \to G_{\text{SM}}$ dynamically.

- And we still have to find models where all Yukawas and quartics obey TAF

Pati-Salam

Fields	spin	generations	SU(2)$_L$	SU(2)$_R$	SU(4)$_{PS}$
$\psi_L = \begin{pmatrix} \nu_L & e_L \\ u_L & d_L \end{pmatrix}$	1/2	3	$\bar{2}$	1	4
$\psi_R = \begin{pmatrix} \nu_R & u_R \\ e_R & d_R \end{pmatrix}$	1/2	3	1	2	$\bar{4}$
ϕ_R	0	1	1	2	$\bar{4}$
$\phi = \begin{pmatrix} H_U^0 & H_D^+ \\ H_U^- & H_D^0 \end{pmatrix}$	0	2	2	$\bar{2}$	1
ψ	1/2	1, 2, 3	2	$\bar{2}$	1
Q_L	1/2	2	1	1	10
Q_R	1/2	2	1	1	$\overline{10}$
Σ	0	1	1	1	15

No extra chiral fermions. Two ways to get acceptable fermions masses:

Foot: add ψ and ϕ_L: $-\mathcal{L}_Y = Y_N\,\psi_L\psi\phi_R + Y_L\,\psi\psi_R\phi_L + Y_U\psi_R\psi_L\phi + Y_D\,\psi_R\psi_L\phi^c$.
Avoids ℓ_L/d_L unification so $M_{W'} > 8.8\,\text{TeV}$. No TAF found for the 24 quartics.

Vodkas: add $Q_{L,R}$ to mix d_R. Has ℓ_L/d_L unification so $M_{W'} > 100\,\text{TeV}$.

TAF Pati-Salam

Add Σ and 2 $Q_{L,R}$: TAF found, 5 Yukawas and $\sim \frac{1}{2} 19$ quartics predicted.

But we found no TAF model with ϕ_L: strong flavor bound $M_{W'} > 100 \, \text{TeV}$

$SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$

Matter fields	spin	$SU(3)_L$	$SU(3)_R$	$SU(3)_c$
$Q_R = \begin{pmatrix} u_R^1 & u_R^2 & u_R^3 \\ d_R^1 & d_R^2 & d_R^3 \\ d_R'^1 & d_R'^2 & d_R'^3 \end{pmatrix}$	$1/2$	1	3	$\bar{3}$
$Q_L = \begin{pmatrix} u_L^1 & d_L^1 & d_R'^1 \\ u_L^2 & d_L^2 & d_R'^1 \\ u_L^3 & d_L^3 & d_R'^3 \end{pmatrix}$	$1/2$	$\bar{3}$	1	3
$L = \begin{pmatrix} \nu_L' & e_L' & e_L \\ \bar{e}_L' & \nu_L' & \nu_L \\ e_R & \nu_R & \nu' \end{pmatrix}$	$1/2$	3	$\bar{3}$	1
H_1, H_2	0	3	$\bar{3}$	1

No bad vectors, but extra fermions chiral under $SU(3)^3$. Yukawas

$$-\mathscr{L}_Y = \sum_{i=1}^{2} \left(y_Q^i \, Q_L Q_R H_i + \frac{y_L^i}{2} LLH_i^* \right)$$

admit TAF solution. For $1H$: 2 quartics, TAF solutions. But $2H$ are needed: 20 quartics, no TAF solutions. TAF demands conservation of baryon number.

Conclusions
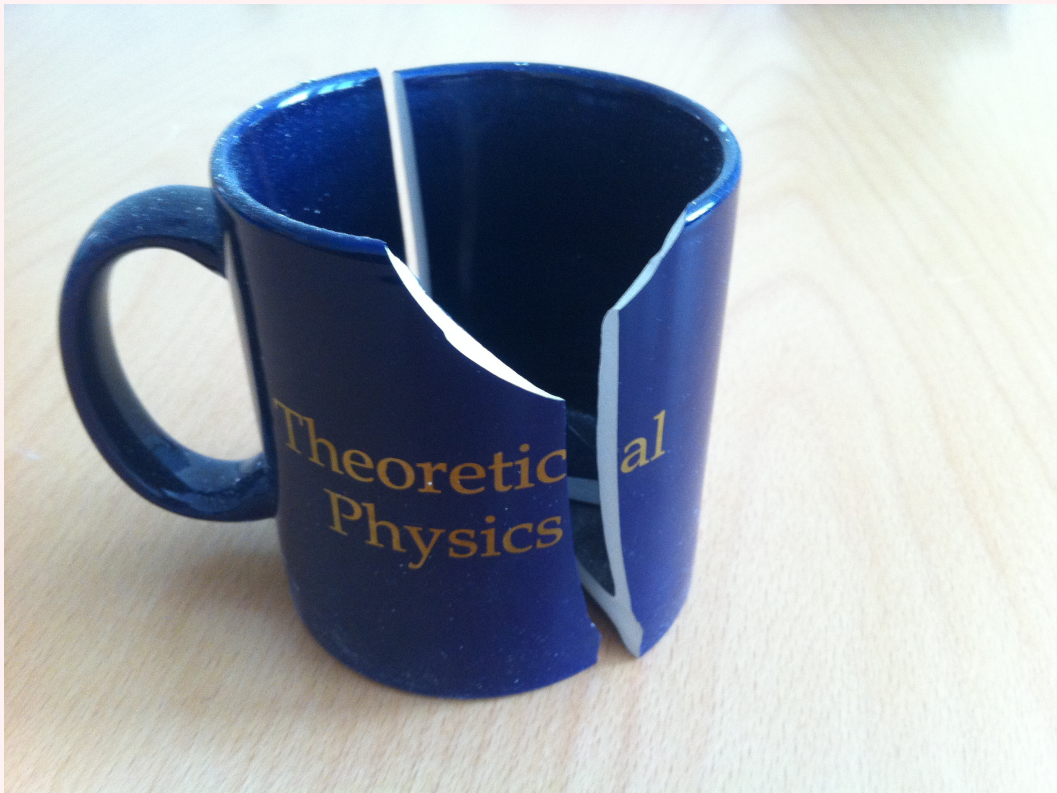
The exploration is still in progress.
The truth can be somewhere along this set of ideas.
Of course, going from Higgs and no SUSY to modified naturalness to an anti-graviton ghost at 10^{11} GeV is risky. Of course, it is much more reasonable to imagine ant***pic selection within a SUSY multiverse of branes wrapped on compactified 6 or 7 extra dimensions.