SUSY “naturalness” and LHC-14

Rethinking Naturalness

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Alberto Casas

No se puede mostrar la imagen. Puede que su equipo no tenga suficiente memoria para abrir la imagen o que ésta esté dañada. Reinicie el equipo y, a continuación, abra el archivo de nuevo. Si sigue apareciendo la x roja, puede que tenga que borrar la imagen e insertarla de nuevo.

First run of the LHC (7-8 TeV)

One historical success:
the discovery of the Higgs boson

Second run of the LHC (13-14 TeV)

Hopefully another historical success: the discovery of BSM physics

??

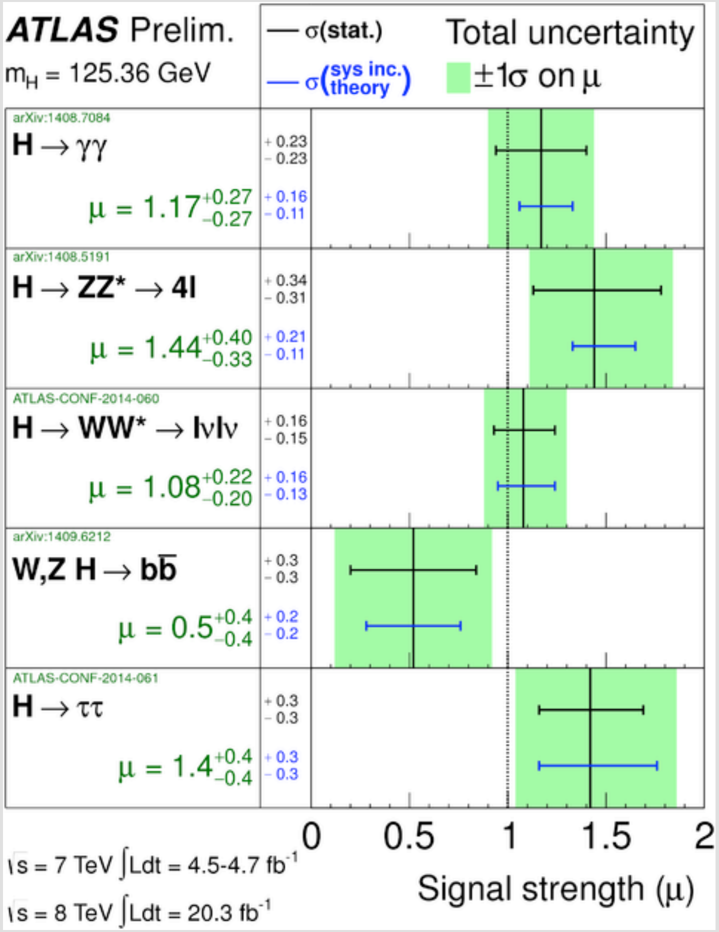
The RUN I of LHC (7/8 TeV) has not produced any real hint of BSM

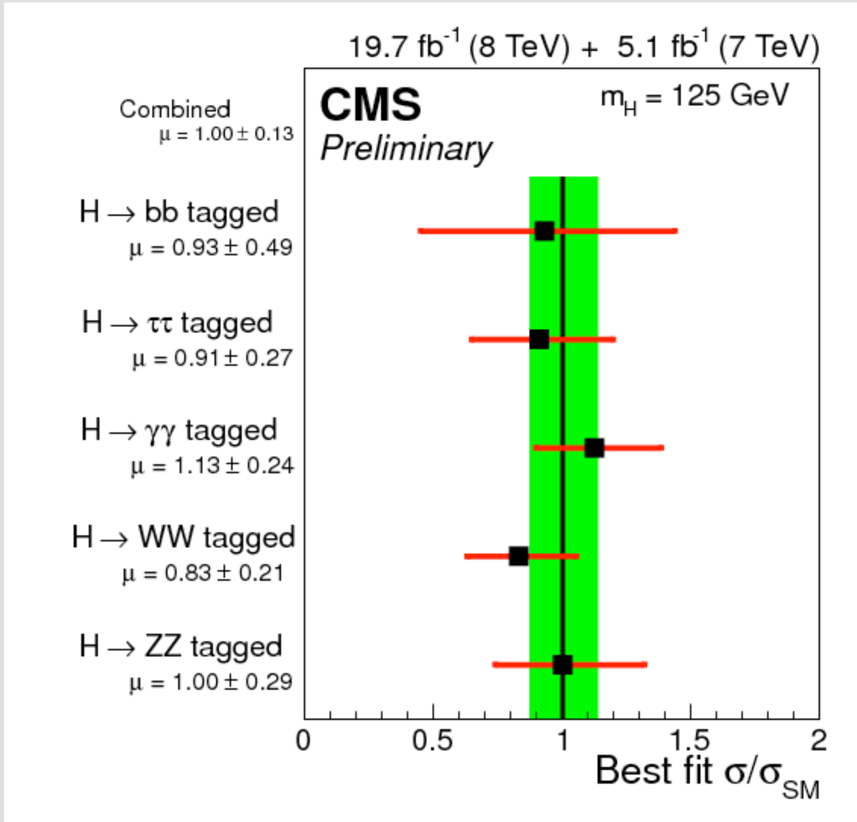
There was a reasonable hope to see signals of BSM in Higgs physics:

- The Higgs represents a new (the last) sector of the SM: “terra incognita”
- Main arguments to expect BSM at LHC rely on the Naturalness of the EWSB (Hierarchy Problem)

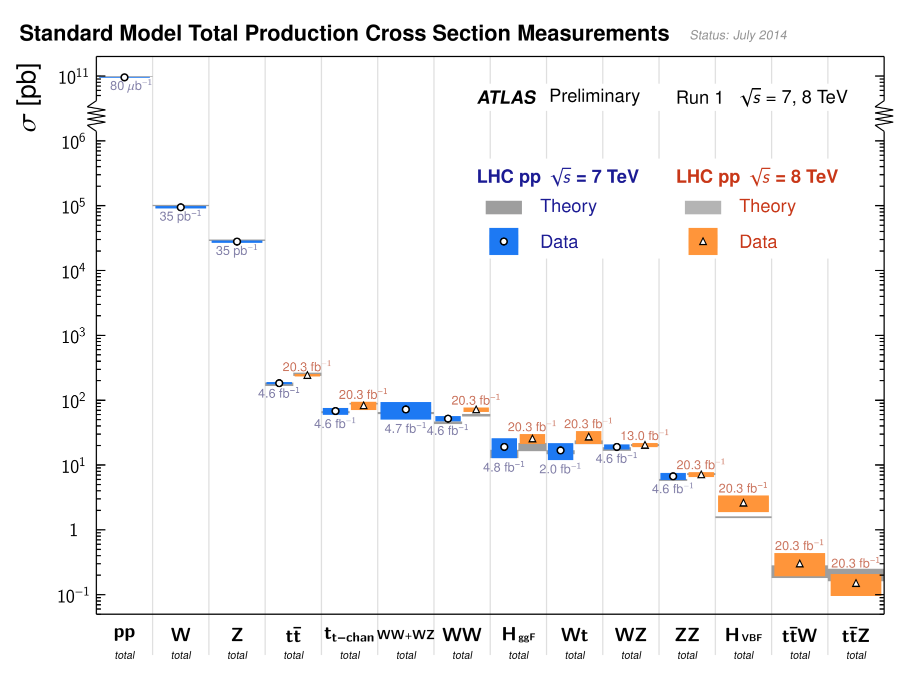
EWSB sector is a natural arena to find BSM

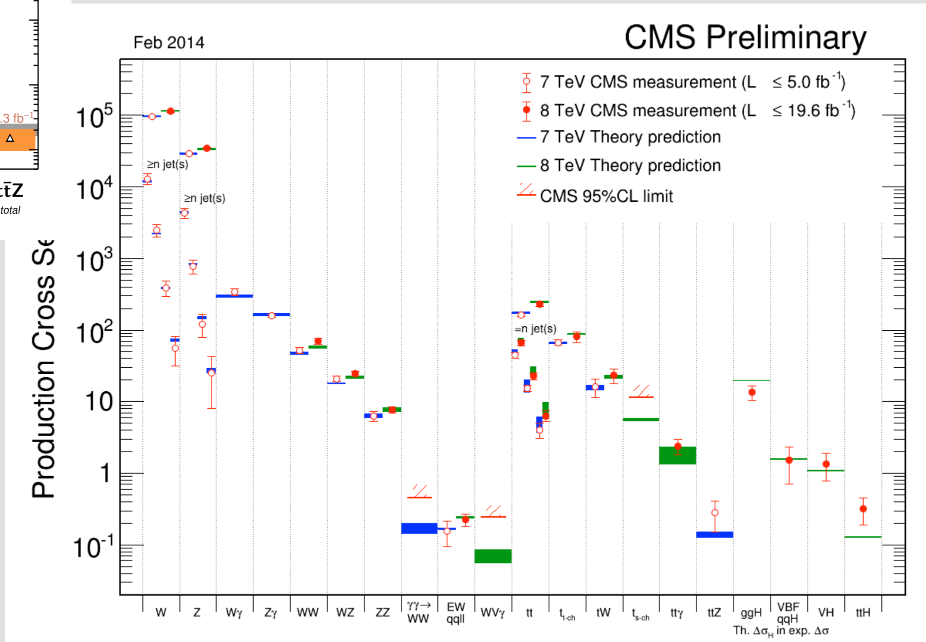
If BSM physics is related to Higgs properties, it must live within the error bars!





Apart from Higgs physics, impressive agreement with SM predictions:





Absence of hints of BSM from Run I

Tension... (?)

Naturalness arguments
(which are behind the Hierarchy Problem)

Recall (once more) the Hierarchy Problem....

$$\delta m^2 = \frac{\Lambda^2}{4\pi^2 v^2}\left(-3m_t^2 + \cdots\right)$$

$$\left|\frac{\delta m^2}{m^2}\right| \leq 10 \qquad \Rightarrow \qquad \Lambda \lesssim 1.5 \text{ TeV}$$

This is still the main reason to expect BSM at the reach of the LHC

Admittedly, Naturalness criterion (associated to the Hierarchy Problem) is

- quite imprecise
- maybe too naive
- maybe a misconception

But, to which extent the LHC results are in tension with it?

Notice:

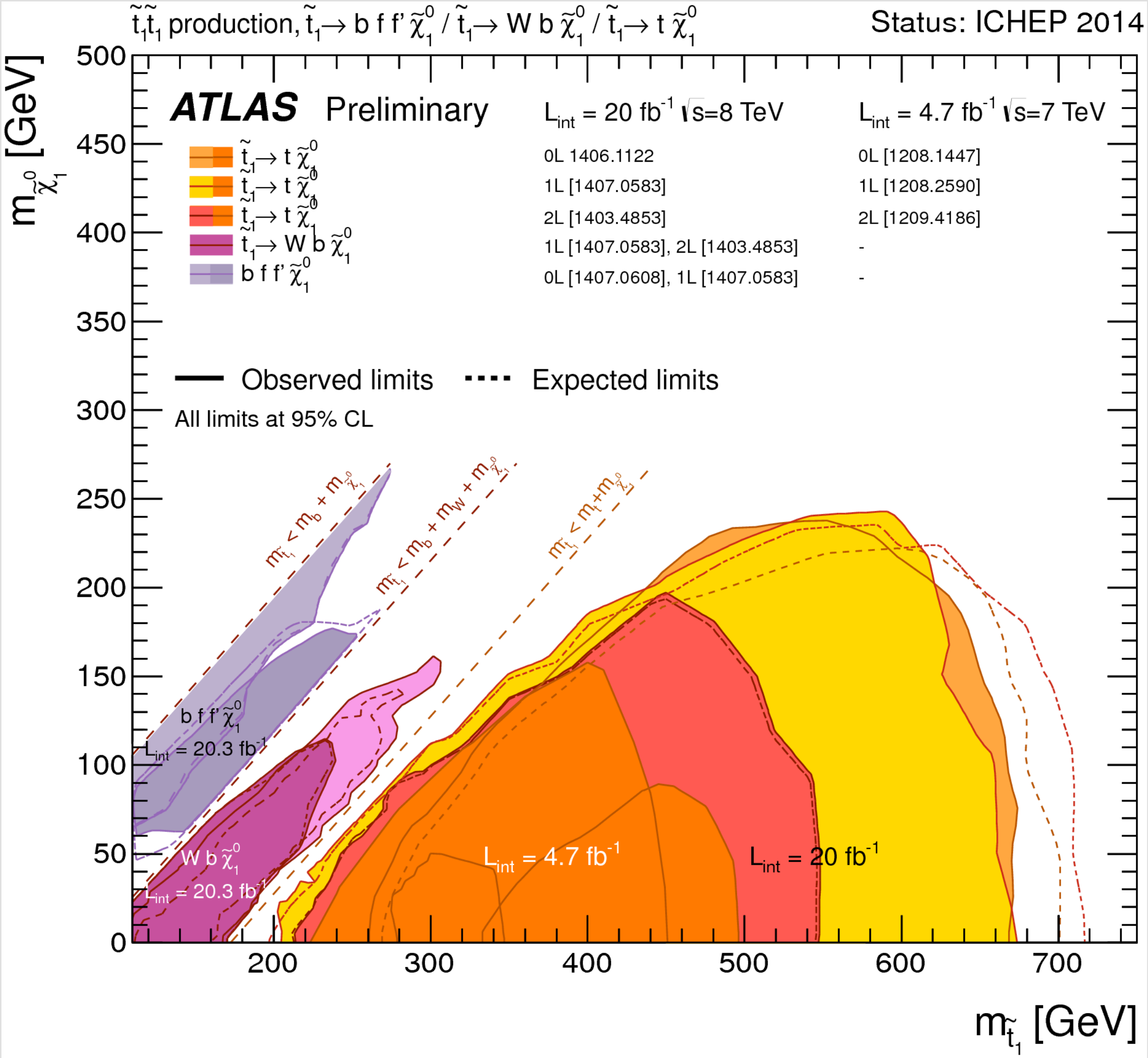
- ☆ The LHC has explored a lot of physics up to ~ 1.5 TeV, but not all (at all)

- ☆ The H.P. (naturalness) bound applies to the BSM physics associated to the top

  But, even if indeed there are top partners (of any kind) at ~1 TeV, they could have easily escaped the LHC (Run I)

E.g. if BSM $\equiv$ SUSY, top partners $\equiv$ stops

$\Longrightarrow$ $m_{\tilde{t}} = 700$ GeV (or smaller) is (could be) OK



So maybe it is a bit too soon to give up

It could even happen that the naturalness criterion is sound, but the BSM is just above the Run II reach (hope not)

On the other hand,

Naturalness bounds on BSM physics based on the crude form of the H.P. argument are too simplistic.

Naturalness bounds are more precise when they are analyzed for concrete BSM scenarios, evaluating the UV-physics contributions to m^2 in terms of the initial parameters of the theory.

E.g.

For the MSSM, the dominant contribution to m^2
(at 1-loop LL, DR-squeme):

$$\delta m^2 \sim -\frac{3m_t^2}{4\pi^2 v^2}\, m_{\tilde t}^2 \underbrace{\log\frac{\Lambda^2}{m_{\tilde t}^2}}_{\text{factor of enhancement}}$$

(naive) Naturalness doesn't apply

- Misconceptions about H.P.
- Landscape
- Other alternatives ("Agravity", ...)

(naive) Naturalness applies

- New Physics at the ~ TeV scale
- Possibly at the LHC reach
- SUSY, Composite, Extra Dim. ...

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- SUSY, Technicolor, Extra Dim. ...

We still don't know which way has been chosen by NATURE

Let us assume that naturalness arguments apply

So we expect BSM physics, hopefully at the reach of LHC-II. Which kind of BSM?

- ★ SUSY
- ★ Composite/NGB Higgs...
- ★ Warped Extra-Dimensions...

⋮

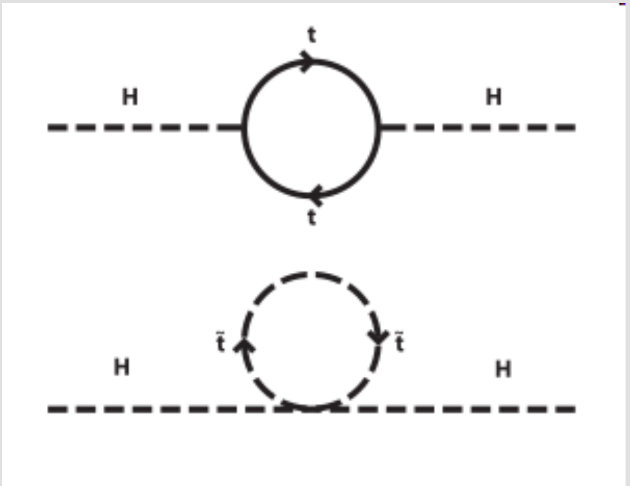
When you go to the details they all look uglier.

Which one you prefer is a matter of taste

SUSY

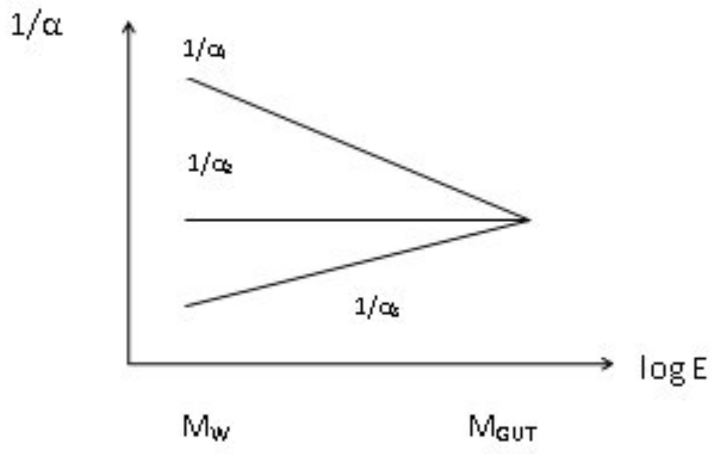
Motivations:

- Beautiful symmetry, strongly suggested by string theories
- Elegant solution to the Hierarchy Problem



SUSY is still one of the preferred candidated for BSM physics:

- Higgs looks fundamental, and $m_h < 135 \text{ GeV}$
- Radiative EW breaking
- Good DM (WIMPs) candidates
- Gauge Unification



BUT

- $m_h \simeq 125\ \mathrm{GeV}$ a bit too heavy for naive SUSY expectations

- No signal of SUSY from LHC-8 TeV

These two facts imply $m_{\text{SUSY particles}} \gtrsim 1\ \mathrm{TeV}$

fine-tuning to get the correct EW scale

(as all BSM scenarios)

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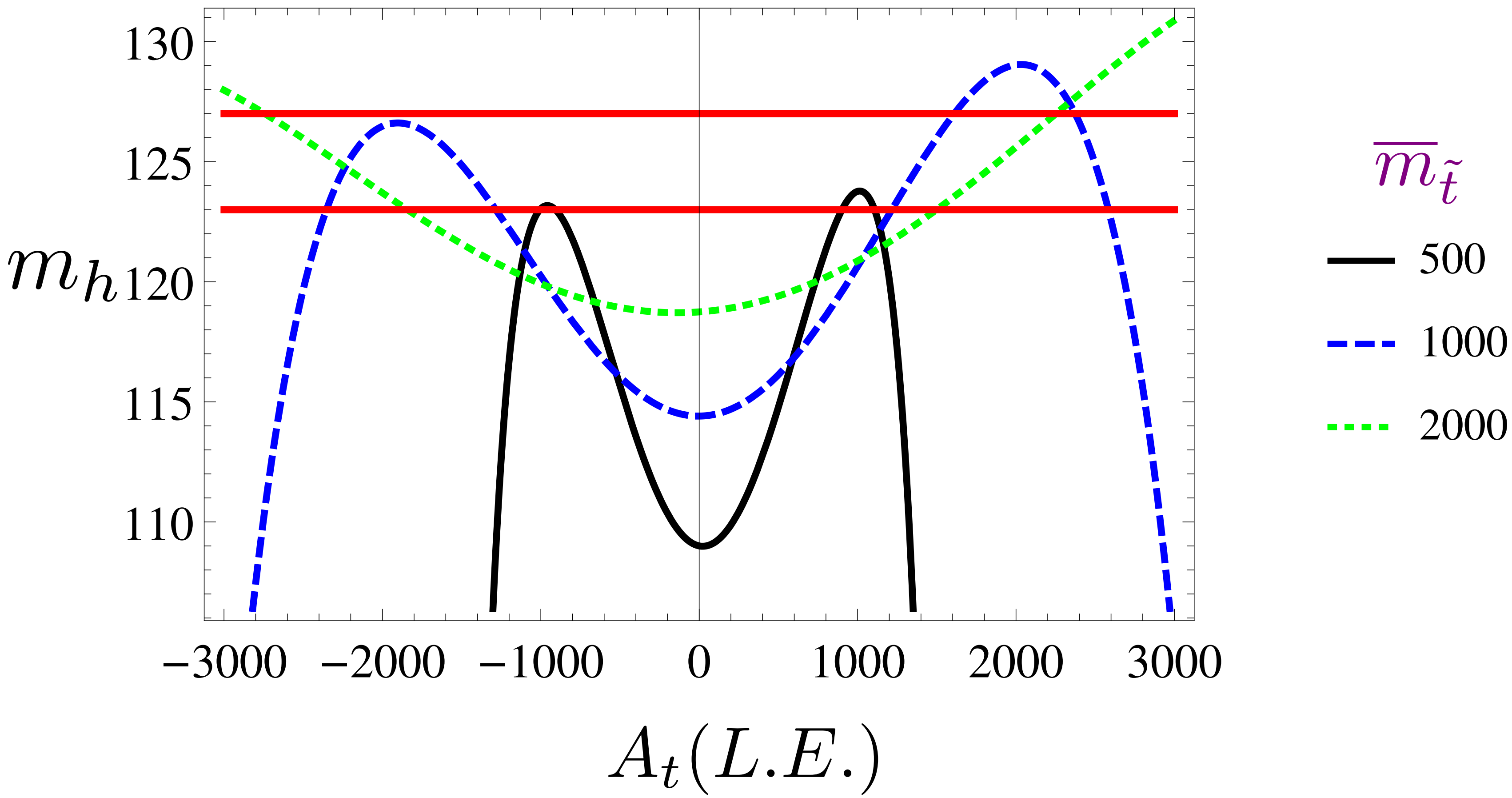
Recall (for MSSM) :

$\sim m_{\tilde{t}}$

$$m_h^2 \simeq \underbrace{M_Z^2 \cos^2 2\beta}_{\substack{\text{tree-level contrib.} \\ (\leq M_Z^2)}} + \frac{3}{2\pi^2}\frac{m_t^4}{v^2}\left[\underbrace{\log\frac{M_{\mathrm{SUSY}}}{M_t}}_{\text{rad. contrib.}} + \underbrace{f(A_t)}_{\text{threshold corr.}}\right] + \cdots$$

$m_h \simeq 125\ \mathrm{GeV}$ typically implies $m_{\tilde{t}} \gtrsim 1\ \mathrm{TeV}$



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It is not straightforward to translate LHC results into bounds on SUSY (MSSM)

MSSM has ~ 100 independent parameters !

(most of them related to the unknown mechanism of SUSY and transmission to the observable sector):

$$\left\{m_{ij}^2, M_a, A_{ij}, B, \mu\right\}$$

Two main strategies:

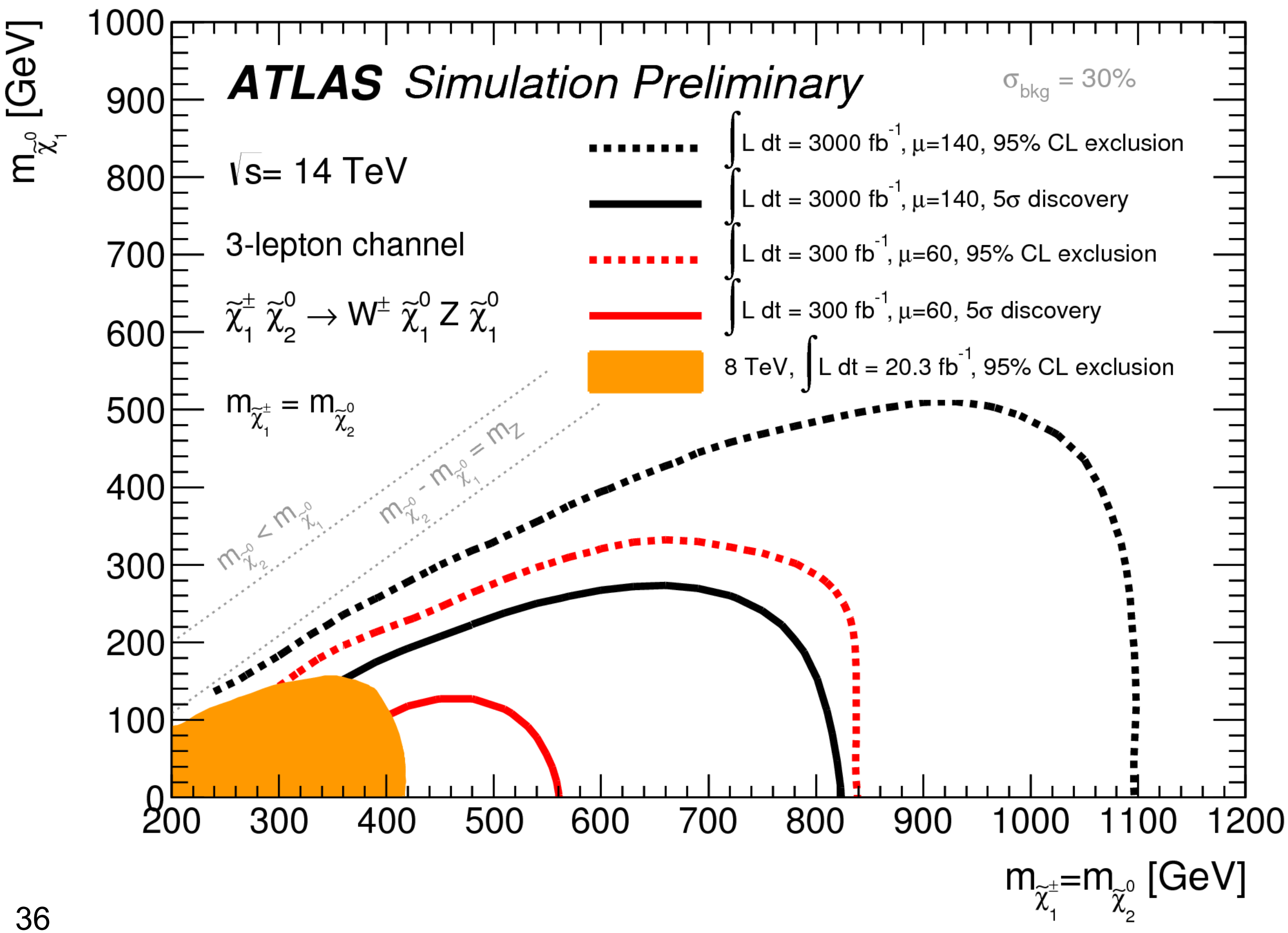
- Use simplified models to express the bounds

- Translate the LHC results into constraints on representative SUSY models:

  CMSSM, NUHM, NUGM, ...

Simplified models are very useful, but one has to be careful interpreting the results.

E.g. limits on electroweakinos from tri-lepton signal



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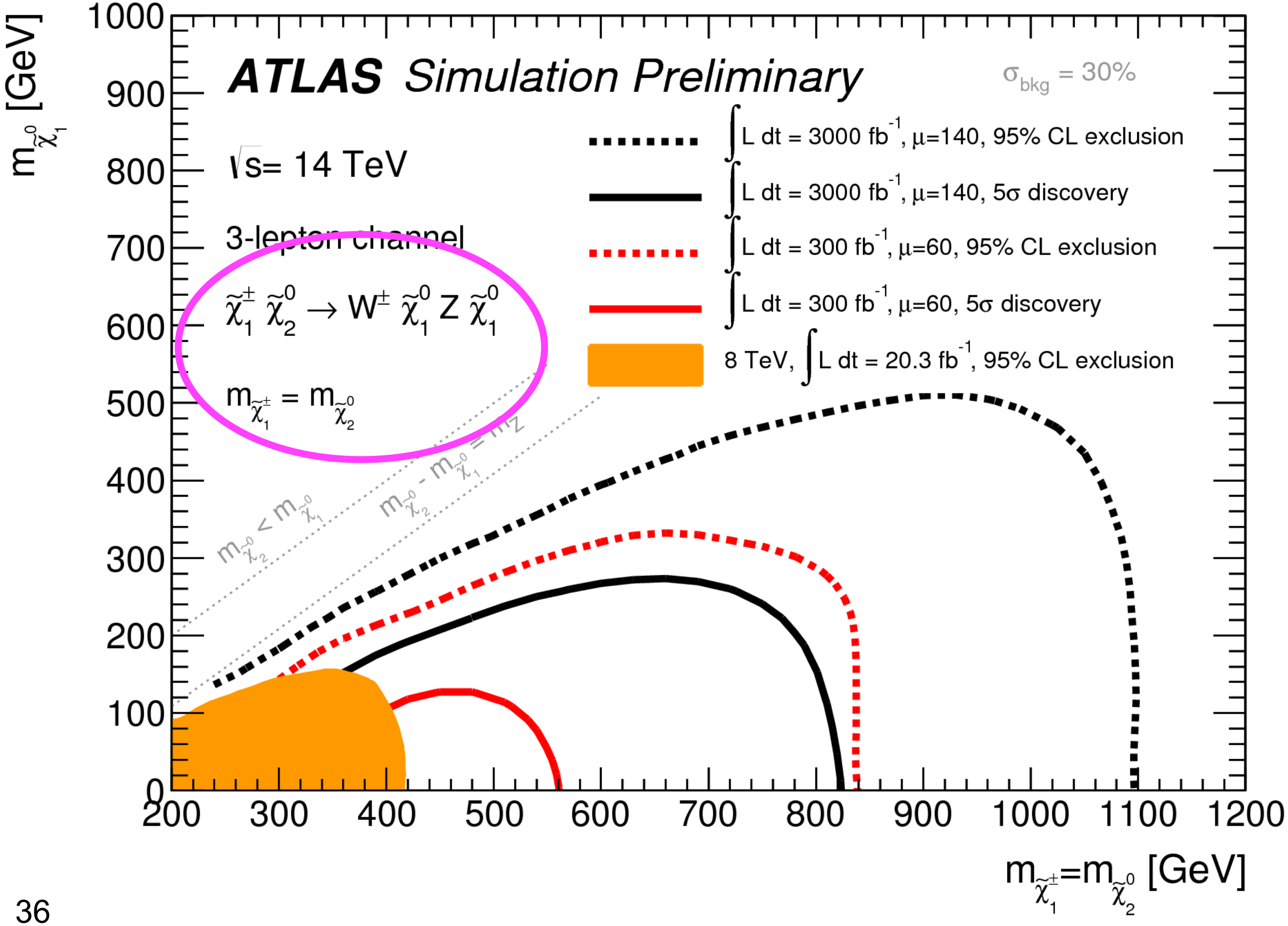


Example of representative model: CMSSM

$$\left\{m_{ij}^2, M_a, A_{ij}, B, \mu\right\}$$

$$\{m, M, A, B, \mu\}$$ (at M_X)

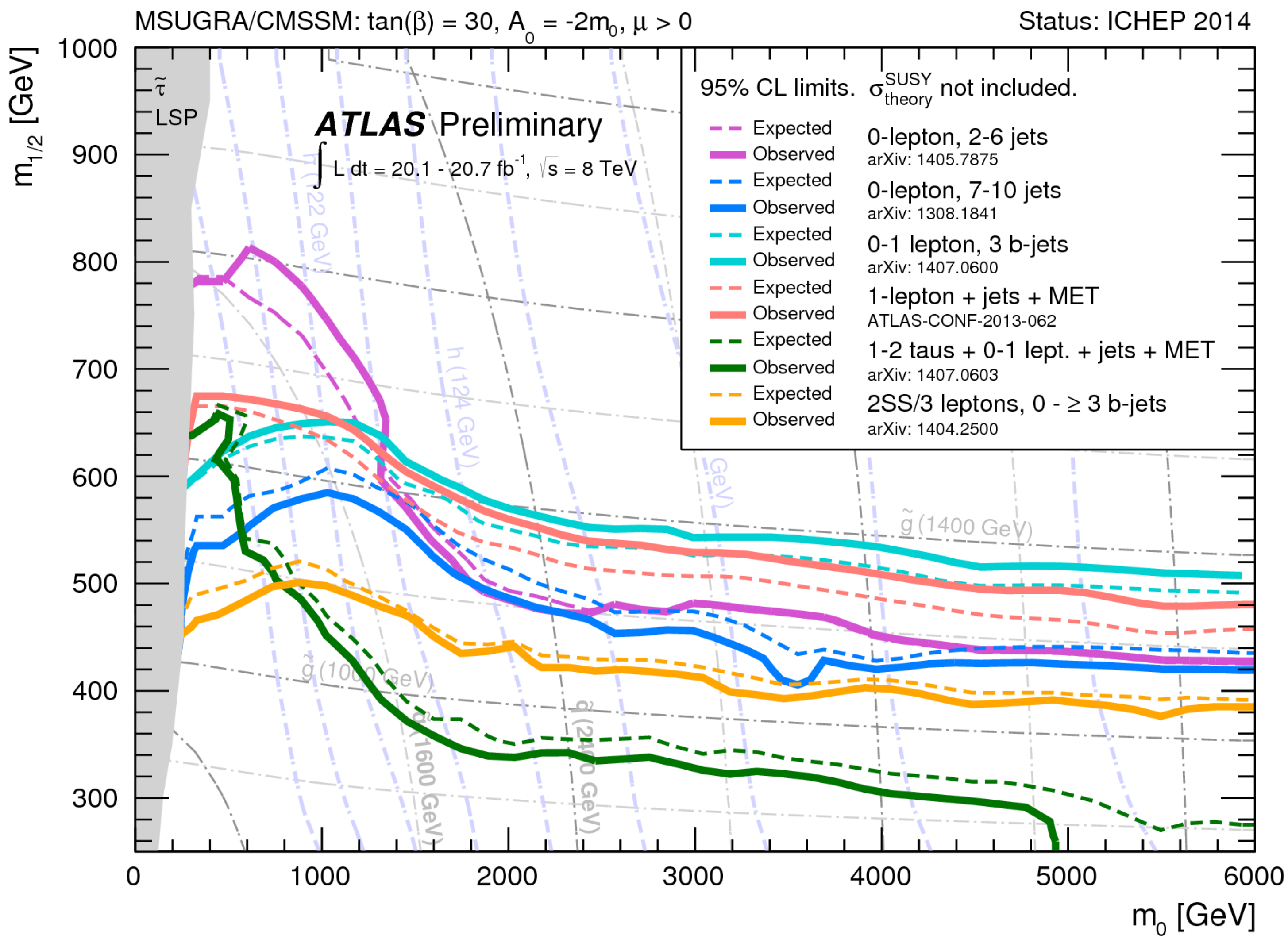
Typical Spectrum

$$M_{\tilde{g}} \sim m_{\tilde{q}} > m_{\tilde{l}}$$

$$M_{\tilde{g}} > M_{\chi^\pm} \gtrsim M_{\chi^0}$$

$$\chi_1^0 \equiv \mathrm{LSP}$$

LHC constraints on the CMSSM



Mostly from multijet + $\not{E}_T$

$$\tan\beta = 10, \;\; A = 0$$

Roughly speaking, for the CMSSM:

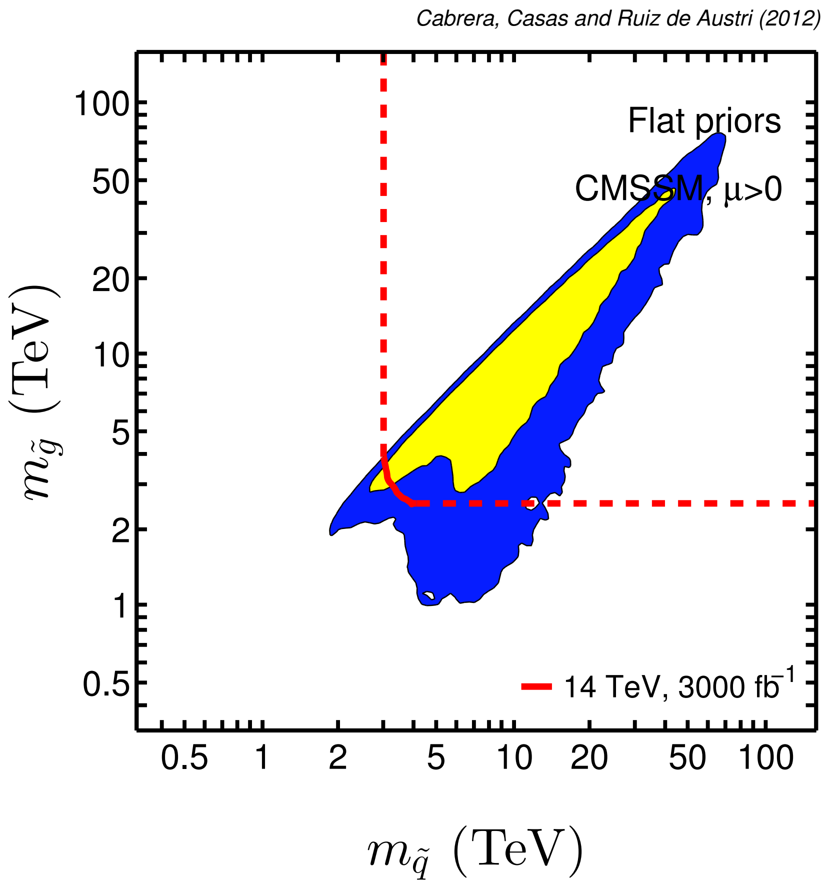
$$m_{\tilde{q}} \gtrsim 1.8 \text{ TeV}, \;\; M_{\tilde{g}} \gtrsim 1.4 \text{ TeV}$$

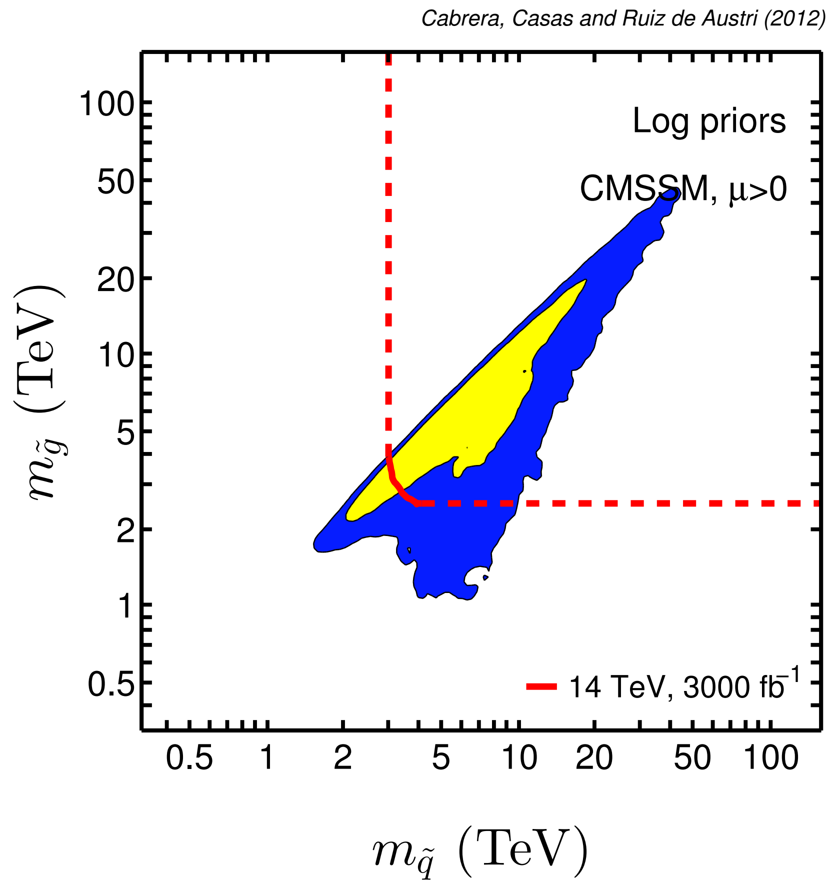
Besides, stops are typically not much lighter than squarks

$\Rightarrow$ CMSSM is in trouble

Not only the CMSSM is fine-tuned, but even if the model is true, the chances to be discovered at the LHC are decreasing dramatically.

Bayesian analysis of CMSSM





ME Cabrera, J.A.C., R. Ruiz de Austri 2012

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see also

C. Balazs, A. Buckley, D. Carter, B. Farmer and M. White

A. Fowlie, M. Kazana, K. Kowalska, S. Munir, L. Roszkowski et. al.;

S. Akula, P. Nath and G. Peim

O. Buchmueller, R. Cavanaugh, M. Citron, A. De Roeck, M. Dolan et al.

C. Strege, G. Bertone, F. Feroz, M. Fornasa, R. R. de Austri et. al., Certainly, the enormous universality of the CMSSM is not a theoretical or phenomenological requirement

(only partially, to avoid FV processes)

Going beyond CMSSM is very plausible

Still, there quite general LHC constraints for most MSSM models

- ★ Heavy (1st and 2nd gen.) squarks, $m_{\tilde{q}} \gtrsim 1.8\ \mathrm{TeV}$
- ★ Heavy gluino, $M_{\tilde{g}} \gtrsim 1.4\ \mathrm{TeV}$
- ★ At least one heavy stop (from m_h), $m_{\tilde{t}} \gtrsim 1\ \mathrm{TeV}$

  unless $A_t \simeq A_t^{\max}$

generic problems with fine-tuning for the MSSM

There are possible exceptions, if SUSY leaves in special corners of the parameter space,

e.g. if the SUSY spectrum is “compressed”, so that visible particles in the events have small p^T.

Such situation would fool the LHC to some extent. It is certainly possible, but it sounds artificial (a “trick” to save low-energy SUSY)

There are further possiblities going beyond the MSSM: *NMSSM, BMSSM, etc.*

In any case, we cannot just “forget” about the fine-tuning problem, since the main reason to consider Weak-Scale SUSY was to avoid the Hierarchy Problem (fine-tuning of EW breaking in the SM)

- ☆ To which extent is the CMSSM (or a generic MSSM) fine-tuned?

- ☆ Is there any MSSM scenario with little fine-tuning?

☆ To which extent is the CMSSM (or a generic MSSM) fine-tuned?

☆ Is there any MSSM scenario with little fine-tuning?

Since fine-tuning seems to be the main problem with SUSY, a reasonable guide to explore SUSY is to look for scenarios as little fine-tuned as possible

Natural SUSY $\equiv$ MSSM as natural (non-fine-tuned) as possible

These questions require a careful analysis of the fine-tuning issue, admittedly an slippery question

Studies of fine-tuning in SUSY have been done since long ago

Natural SUSY is not really a new idea!

J. R. Ellis, K. Enqvist, D. V. Nanopoulos and F. Zwirner '86; R. Barbieri and G. Giudice '88, de Carlos and J.A.C. '92, G. Kane, C. Kolda, L. Roszkowski and J. Wells '94, G. W. Anderson and D. J. Castano '95; J. L. Feng, K. T. Matchev and T. Moroi '00; J. A. C., , J. R. Espinosa and I. Hidalgo '04, ...

The idea was re-launched by Papucci, Ruderman & Weiler ‘11

and it has motivated a lot of work in recent times

J. E. Younkin and S. P. Martin '12; Arbey et al. ’12; S. Fichet '12; E. Hardy '13; K. Kowalska and E. M. Sessolo '13; C. Han, K. -i. Hikasa, L. Wu, J. M. Yang and Y. Zhang '13; E. Dudas, G. von Gersdorff, S. Pokorski and R. Ziegler '13; J. Fan and M. Reece '14; T. Gherghetta, B. von Harling, A. D. Medina and M. A. Schmidt '14; K. Kowalska, L. Roszkowski, E. M. Sessolo and S. Trojanowski '14; J. L. Feng '14; H. Baer, V. Barger, P. Huang, A. Mustafayev and X. Tata '14; S. P. Martin '14; Baer, Barger, Mickelson, Padeffe-Kirkland '14, ...

The "standard" Natural SUSY scenario

Papucci, Ruderman & Weiler '11

In the effective SM theory

$$V_{\rm SM} = m^2|H_{\rm SM}|^2 + \lambda|H_{\rm SM}|^4$$

$$m^2 \simeq |\mu|^2 + m_{H_u}^2 = \frac{m_h^2}{2}$$

1-loop LL corrections

$$\delta m_{H_u}^2|_{stop} = -\frac{3}{8\pi^2}y_t^2\left(m_{Q_3}^2 + m_{u_3}^2 + |A_t|^2\right)\log\left(\frac{\Lambda}{\rm TeV}\right)$$

$$\delta m_{H_u}^2|_{gluino} = -\frac{2}{\pi^2}y_t^2\left(\frac{\alpha_s}{\pi}\right)|M_3|^2\log^2\left(\frac{\Lambda}{\rm TeV}\right)$$

Demanding $\frac{\delta m^2}{m^2} \lesssim \Delta, \qquad (\Delta \sim 10)$

"popular" predictions of Natural SUSY

- ★ stops should be light (< 1 TeV)
- ★ gluino not too heavy (< 2 TeV)
- ★ very light Higgsinos (< 500 GeV)

(for $\Delta = 10$, i.e. ~10% fine-tuning)

However the “standard” arguments are too simplistic

- 1-loop LL is not accurate enough
- physical squark, gluino and Higgsino masses are not initial parameters
  - one should evaluate the required cancellation in terms of the initial parameters
  - there is not a one-to-one correspondence between initial parameters and physical masses

E.g. if the scalar masses are universal at H.E.

$$m_{H_u}^2 = m_{U_3}^2 = m_{Q_3}^2 = m_0^2$$

then

$$\begin{bmatrix}\delta m_{H_u}^2 \\ \delta m_{U_3}^2 \\ \delta m_{Q_3}^2\end{bmatrix} = \frac{\delta m_0^2}{2}\left\{\begin{bmatrix}3\\2\\1\end{bmatrix}\underbrace{\exp\left[\int_0^t \frac{6y_t^2}{8\pi^2}dt'\right]}_{\simeq 1/3} - \begin{bmatrix}1\\0\\-1\end{bmatrix}\right\}$$

(if you start the running from M_X)

$$\delta m_{H_u}^2 \simeq 1.6 M_3^2 - \underbrace{0.026}_{\ll 1}\, \delta m_0^2 + \cdots$$

Focus-point behaviour

Feng, Matchev, Moroi,'2000

$$m_{H_u}^2 \simeq 1.6 M_3^2 - \boxed{0.026}\, m_0^2 + \cdots$$

$$m_{\tilde{t}}^2 \simeq 2.97 M_3^2 + \boxed{0.5}\, m_0^2 + \cdots$$

If the stops are heavy because m_0 is large, then there is no fine-tuning price

This is a clear counter-example to Natural SUSY requiring light stops

Before, examining more closely the issue of the electroweak fine-tuning, it is worth noticing that

there are other potential fine-tunings, which are normally left aside in analyses

Fine-tunings left aside (in common analyses)

☆ If the stops are too light, there is an extra fine-tuning in A_t to get $m_h \simeq 125\ \mathrm{GeV}$

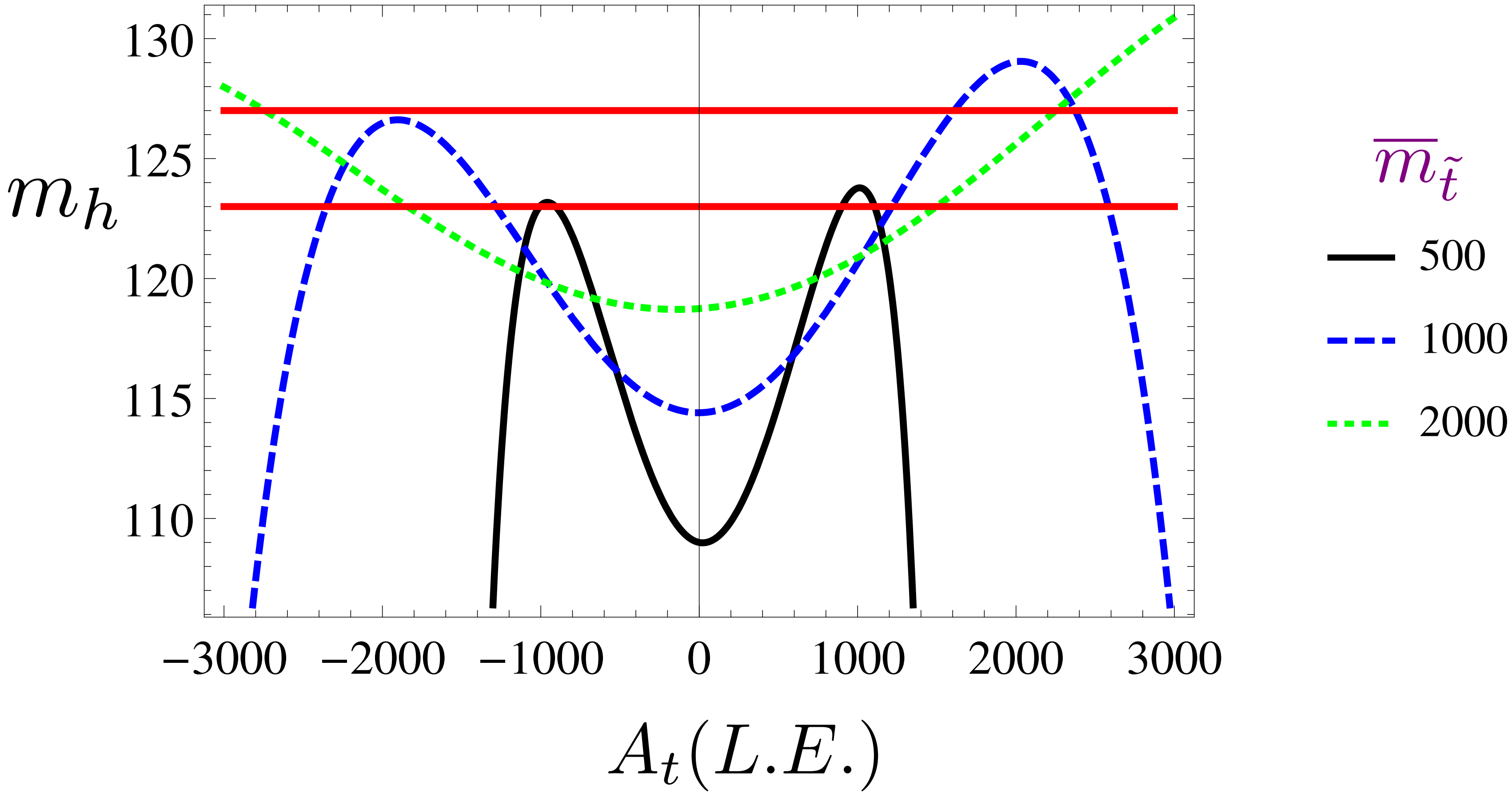
$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + \frac{3}{2\pi^2}\frac{m_t^4}{v^2}\left[\log\frac{m_{\tilde{t}}}{M_t} + f(A_t)\right] + \cdots$$

$m_{\tilde{t}} = 500 - 600\ \mathrm{GeV}$

⇒ A_t close to maximal

⇒ fine-tuning

Fine-tunings left aside (in common analyses)



Fine-tunings left aside (in common analyses)

☆ Fine-tuning to get large $\tan\beta$

$$\tan\beta \simeq \frac{m_{H_d}^2 + m_{H_u}^2 + 2\mu^2}{B\mu} = \frac{m_A^2}{B\mu} \qquad (LE)$$

should be small

$$B\mu|_{LE} = B\mu + \Delta_{\rm rad}(B\mu)$$

potential fine-tuning required

If these additional fine-tunings are significant, they should be combined with the EW fine-tuning

Let us now focus on the electroweak fine-tuning

The EW fine-tuning

At tree-level and large $\tan\beta$

$$-\frac{1}{8}(g^2+g'^2)v^2 = \boxed{-\frac{M_Z^2}{2} = \mu^2 + m_{H_2}^2}$$

1-loop corrections:

$$\lambda(Q_{\text{SUSY threshold}}) = \frac{1}{8}(g^2+g'^2) = \frac{2M_Z^2}{v^2} \longrightarrow \lambda(Q_{\text{EW}}) \simeq \frac{2m_h^2}{v^2}$$

$$\boxed{-\frac{m_h^2}{2} = \mu^2 + m_{H_2}^2}$$

How to measure of the EW fine-tuning

Most used and popular criterion:

$$\frac{\partial m_h^2}{\partial \theta_i} = \Delta_{\theta_i} \frac{m_h^2}{\theta_i} , \qquad \Delta \equiv \mathrm{Max} \; |\Delta_{\theta_i}|$$

Ellis, Enqvist, Nanopoulos & Zwirner' 86

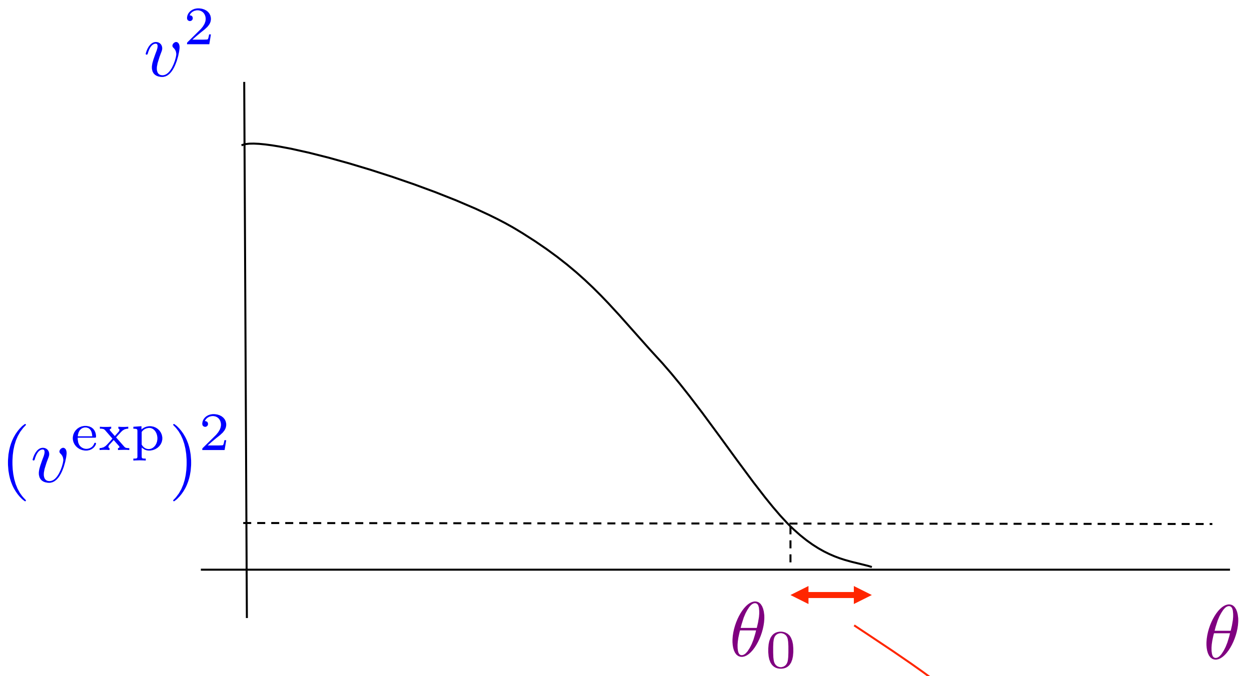
Barbieri & Giudice' 88

$\theta_i \equiv$ independent parameters of the model

$\Delta = 100$ means $\sim 1\%$ fine-tuning, etc.

Δ_{θ_i} admits an statistical interpretation

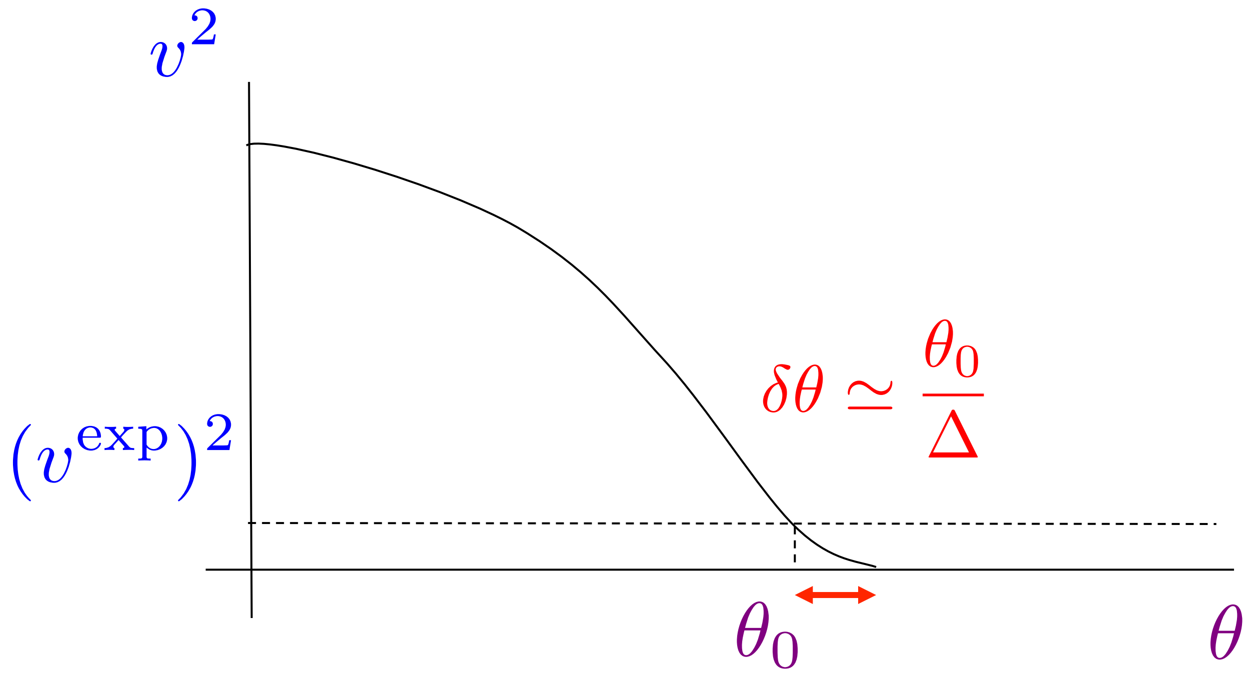
Ciafaloni & Strumia' 97



only in this $\delta\theta_0$ range, $v^2 \leq (v^{\exp})^2$

Δ_{θ_i} admits an statistical interpretation

Ciafaloni& Strumia' 97



$$\mathcal{P}[v^2 \leq (v^{\exp})^2] = \frac{\delta\theta_0}{\theta_0} \simeq \Delta^{-1} \quad \equiv \quad p - \text{value}$$

There are two implicit assumptions behind this statistical interpretation

- Range of $\theta \sim [0,\ \theta_0]$
- Prior $p(\theta) = \mathrm{flat}$

Reasonable, but can be inappropriate in particular theoretical scenarios

These issues become more clear using a

Bayesian approach

Density of probability in the parameter space

Likelihood

Prior

$$p(\theta_i|\text{data}) = \frac{p(\text{data}|\theta_i)\ p(\theta_i)}{p(\text{data})}$$

parameters of the model

Norm. constant

Treat M_Z as another exp. data

Likelihood associated to the other observables

- Approximate the likelihood as

$$p(\mathrm{data}|M_1, M_2, \cdots, \mu) \simeq \delta(M_Z - M_Z^{\mathrm{exp}})\, \mathcal{L}_{\mathrm{rest}}$$

- Use the delta to marginalize any parameter, e.g. μ

$$p(M_1, M_2, \cdots | \; \mathrm{data}) = \int d\mu \; p(M_1, M_2, \cdots, \mu|\mathrm{data})$$

$$\propto \; \mathcal{L}_{\mathrm{rest}} \underbrace{\left|\frac{d\mu}{dM_Z}\right|_{\mu_Z}}_{\left|\frac{\mu}{\Delta_\mu}\right|_{\mu=\mu_Z}} p(M_1, M_2, \cdots, \mu_Z)$$

consistent with the interpretation of Δ^{-1} as probability

Cabrera, JAC & Ruiz de Austri ‘08

The analogy is complete if $p(\mu_Z) \sim \dfrac{1}{\mu_Z}$

prior at $\mu = \mu_Z$

$$p(M_1, M_2, \cdots \,|\, \mathrm{data}) \;\propto\; p(\mu_Z) \left|\frac{\mu}{\Delta_\mu}\right|_{\mu=\mu_Z} \propto \Delta_\mu^{-1}$$

This occurs if

- Range of $\mu \;\sim\; [0,\; \mu_Z]$ and $p(\mu) \;\sim\; \mathrm{flat}$
- or $p(\mu) \;\sim\; \mathrm{logarithmic},\; \mathrm{i.e.}\; p(\mu) \propto 1/\mu$

The modification of F-T criterion for other cases is straightforward

In summary, the “standard fine-tuning measure” is reasonable in many cases

The Bayes analysis tells the implicit assumptions for its validity

If a particular theoretical model does not fulfill them, the “standard fine-tuning criterion” is inappropriate and should be consistently modified

"Program" to evaluate naturalness bounds on the SUSY spectrum:

1.- Express m_h^2 in terms of the initial parameters, θ_i , given at a certain scale, $M_{\rm HE}$

2.- Evaluate $$\Delta_{\theta_i} = \frac{\theta_i}{m_h^2}\frac{\partial m_h^2}{\partial \theta_i} \leq \Delta^{\rm max}$$

$\rightarrow$ limits on θ_i

3.- Translate limits on θ_i into limits on the SUSY spectrum

1.- Express m_h^2 in terms of the initial parameters, θ_i, given at a certain scale, $M_{\rm HE}$

For M_{HE}, M_{LE} fixed, consistency requires

$$m_{H_u}^2(LE) = \quad c_{M_3}M_3^2 + c_{M_2}M_2^2 + c_{M_1}M_1^2 + c_{A_t}A_t^2 + c_{A_tM_3}A_tM_3 + c_{M_2}M_2^2 + \cdots$$
$$+c_{M_3M_2}M_3M_2 + \cdots + c_{m_{H_u}}m_{H_u}^2 + c_{m_{\tilde{Q}_3}}m_{\tilde{Q}_3}^2 + c_{m_{\tilde{t}_R}}m_{\tilde{t}_R}^2 + \cdots$$

1.- Express m_h^2 in terms of the initial parameters, θ_i , given at a certain scale, $M_{\rm HE}$

E.g. for $M_{HE} = M_X, \; M_{LE} = 1 \text{ TeV}$

$$m_{H_u}^2(LE) = -1.603 M_3^2 + 0.285 A_t M_3 + 0.203 M_2^2 - 0.109 A_t^2 - 0.134 M_3 M_2$$
$$+0.068 A_t M_2 + 0.631 m_{H_u}^2 - 0.367 m_{\tilde{Q}_3}^2 - 0.290 m_{\tilde{t}_R}^2 + \cdots$$

$$\mu(LE) = 1.002 \; \mu$$

Fits of this kind are quite common in the literature, but we have obtained it in a very careful way:

- 2-loop RGEs in two steps (important for α_i, y_t):

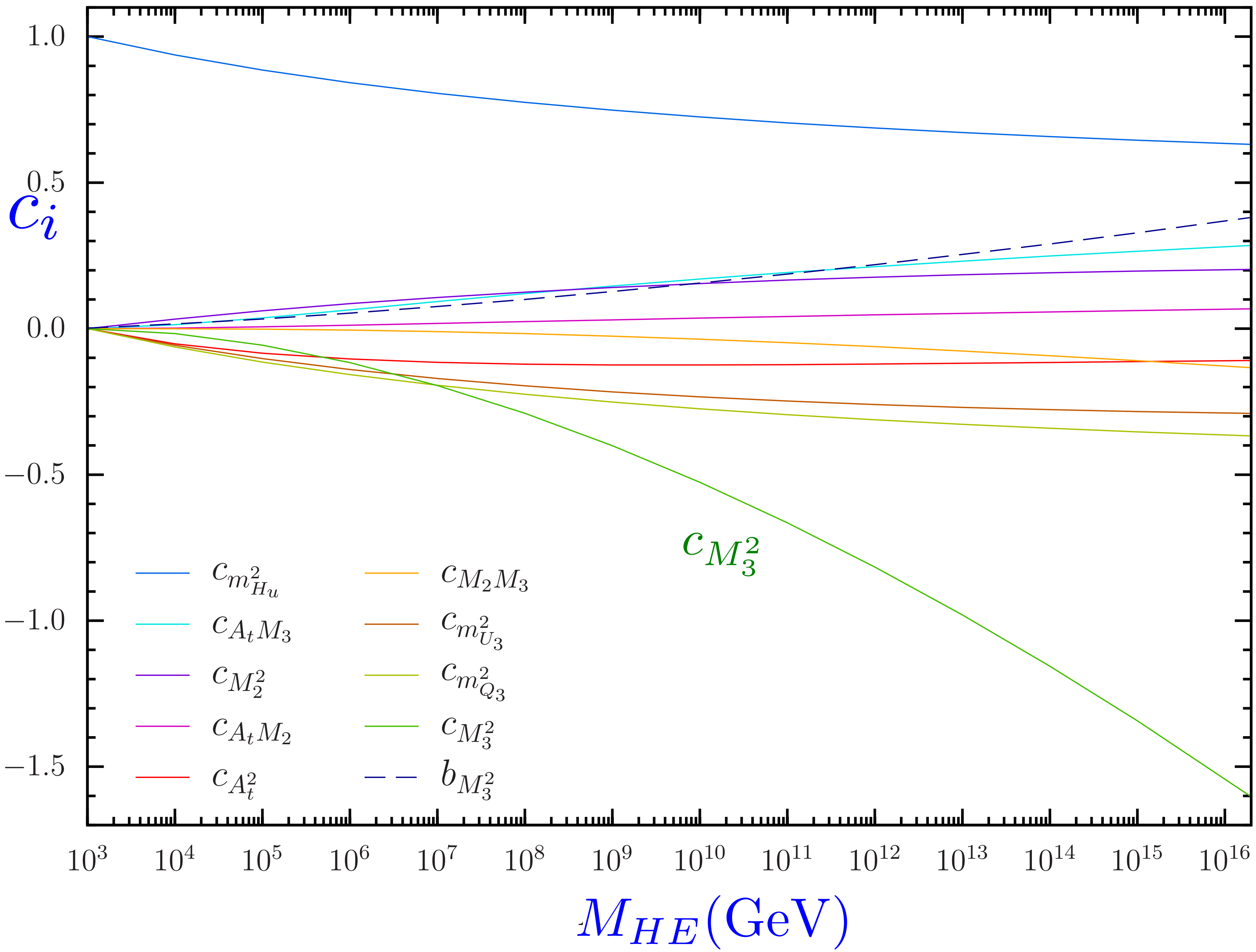
$$M_Z \longrightarrow M_{LE} \longrightarrow M_{HE}$$

- M_{LE}, M_{HE} free parameters

$$c_i(M_{LE}) \simeq c_i(1\ \mathrm{TeV}) + b_i \ln \frac{M_{LE}}{1\ \mathrm{TeV}}$$

HE	$m^2_{H_u}(LE)$	
	c	b
M_3^2	-1.603	0.381
$m^2_{H_u}$	0.631	0.019
$m^2_{Q_3}$	-0.367	0.018
$m^2_{U_3}$	-0.290	0.017
$A_t M_3$	0.285	-0.024
M_2^2	0.203	0.006
$M_2 M_3$	-0.134	0.021
A_t^2	-0.109	-0.006
$A_t M_2$	0.068	0.000
$m^2_{U_{1,2}}$	0.054	-0.001
$m^2_{H_d}$	0.026	-0.001
$m^2_{E_{1,2}}$	-0.026	0.001
$m^2_{E_3}$	-0.026	0.001
$m^2_{L_{1,2}}$	0.025	-0.001
$m^2_{L_3}$	0.025	-0.001
$m^2_{Q_{1,2}}$	-0.025	0.000
$m^2_{D_{1,2}}$	-0.025	0.000
$m^2_{D_3}$	-0.024	0.000
$M_1 M_3$	-0.020	0.002
$A_t M_1$	0.012	0.000
M_1^2	0.006	0.002
$M_1 M_2$	-0.005	0.000
$A_b M_3$	-0.002	0.000
A_b^2	0.001	0.000
$A_b M_2$	–	–
A_τ^2	–	–
$A_\tau M_2$	–	–
$A_b A_t$	–	–
$A_\tau M_1$	–	–

$$M_{\mathrm{HE}} = M_{\mathrm{X}}$$



c_i
$M_{HE}(\mathrm{GeV})$
1.0
0.5
0.0
−0.5
−1.0
−1.5
10^3
10^4
10^5
10^6
10^7
10^8
10^9
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
10^{16}
$c_{M_3^2}$
$c_{m_{H_u}^2}$
$c_{A_t M_3}$
$c_{M_2^2}$
$c_{A_t M_2}$
$c_{A_t^2}$
$c_{M_2 M_3}$
$c_{m_{U_3}^2}$
$c_{m_{Q_3}^2}$
$c_{M_3^2}$
$b_{M_3^2}$

- We have done this, not only for $m^2_{H_u}$, but for all quantities

$$m^2_{\tilde{Q}_3},\ m^2_{\tilde{u}_3},\ M_3,\ M_2,\ A_t, \cdots$$

- This will be necessary later to translate F-T bounds on the HE-parameters into constraints on the physical spectrum

E.g.

HE	$m^2_{Q_3}(LE)$		$m^2_{U_3}(LE)$		$m^2_{D_3}(LE)$	
	c	b	c	b	c	b
M_3^2	3.191	−0.563	2.754	−0.462	3.678	−0.672
$m^2_{Q_3}$	0.871	0.007	−0.192	0.013	−0.029	0.002
M_2^2	0.333	−0.008	−0.151	0.017	−0.010	0.002
$m^2_{H_u}$	−0.118	0.006	−0.189	0.011	−0.015	0.000
$m^2_{U_3}$	−0.095	0.005	0.706	0.011	0.032	0.000
M_2M_3	−0.084	0.015	−0.100	0.018	−0.026	0.007
A_tM_3	0.072	−0.003	0.159	−0.010	−0.010	0.003
A_t^2	−0.034	−0.002	−0.070	−0.004	0.001	0.000
A_tM_2	0.020	0.000	0.047	0.000	−0.001	0.000
$m^2_{Q_{1,2}}$	−0.017	0.001	0.030	0.000	−0.025	0.002
$m^2_{D_3}$	−0.015	0.001	0.032	0.000	0.973	0.001
$m^2_{U_{1,2}}$	0.014	0.000	−0.073	0.002	0.031	0.000
$m^2_{D_{1,2}}$	−0.012	0.001	0.032	0.000	−0.021	0.001
M_1M_3	−0.009	0.001	−0.018	0.002	−0.004	0.001
$m^2_{E_{1,2,3}}$	−0.009	0.000	0.034	−0.001	−0.017	0.000
$m^2_{L_{1,2,3}}$	0.008	0.000	−0.034	0.001	0.017	0.000
A_bM_3	0.006	−0.001	−0.001	0.000	0.014	−0.003
M_1^2	−0.006	0.001	0.041	0.001	0.014	0.000
$m^2_{H_d}$	0.005	0.000	−0.034	0.001	0.011	0.000
A_tM_1	0.004	0.000	0.007	0.000	–	–
A_b^2	−0.003	0.000	–	–	−0.006	0.001
M_1M_2	−0.002	0.000	−0.003	0.000	–	–
A_bM_2	0.002	0.000	–	–	0.004	−0.001
A_bA_t	0.001	0.000	–	–	0.001	0.000

2.- Evaluate $$\Delta_{\theta_i} = \frac{\theta_i}{m_h^2}\frac{\partial m_h^2}{\partial \theta_i} \le \Delta^{\max}$$

$$m_{H_u}^2\Big|_{LE} = f(\underbrace{M_3, M_2, \cdots, m_{\tilde{Q}_3}^2, m_{\tilde{u}_3}^2, \cdots, m_{H_u}^2, \cdots, A_t, \cdots}_{\Theta_\alpha})_{HE}$$

$\Theta_\alpha \equiv$ parameters of "unconstrained MSSM"

2.- Evaluate $\Delta_{\theta_i} = \frac{\theta_i}{m_h^2}\frac{\partial m_h^2}{\partial \theta_i} \leq \Delta^{\max}$

$$m_{H_u}^2\big|_{LE} = f(\underbrace{M_3, M_2, \cdots, m_{\tilde{Q}_3}^2, m_{\tilde{u}_3}^2, \cdots, m_{H_u}^2, \cdots, A_t, \cdots}_{\Theta_\alpha})_{HE}$$

$\Theta_\alpha \equiv$ parameters of "unconstrained MSSM"

- "Unconstrained MSSM": $\Delta_{\Theta_\alpha} = \frac{\Theta_\alpha}{m_h^2}\frac{\partial m_h^2}{\partial \Theta_\alpha}$

- Generic scenario defined by θ_i :

$$\Delta_{\theta_i} = \frac{\theta_i}{m_h^2}\frac{\partial m_h^2}{\partial \theta_i} = \frac{\theta_i}{m_h^2}\sum_\alpha \frac{\partial m_h^2}{\partial \Theta_\alpha}\frac{\partial \Theta_\alpha}{\partial \theta_i}$$

The complete knowledge of the c-coefficients allows to evaluate the fine-tuning for any theoretical model, defined at any HE scale, in a very easy way.

Prototype model: Unconstrained MSSM

☆ Dimension-2 parameters ($m^2_{\tilde{Q}_3}, \; m^2_{\tilde{u}_3}, \; m^2_{H_u} \; \cdots$) do not get mixed in the Δs

e.g.

$$\left|\Delta_{m_{\tilde{u}_3}}\right| = \left| 2c_{m_{\tilde{u}_3}} \frac{m^2_{\tilde{u}_3}}{m^2_h} \right| \leq \Delta^{(\max)}$$

$$\Rightarrow$$

$$m^2_{\tilde{t}_R} \lesssim \left| \frac{1}{2c_{m_{\tilde{t}_R}}} \right| \Delta^{\max} \; m^2_h \simeq 1.72 \; \Delta^{\max} \; m^2_h$$

Prototype model: Unconstrained MSSM

☆ Dimension-1 parameters ($M_3, \; M_2, \; A_t, \cdots$) do mix

e.g. $$|\Delta_{M_3}| = \frac{2}{m_h^2}\left|2c_{M_3}M_3^2 + c_{M_3M_2}M_3M_2 + c_{M_3A_t}M_3A_t\right| \leq \Delta^{(\max)}$$

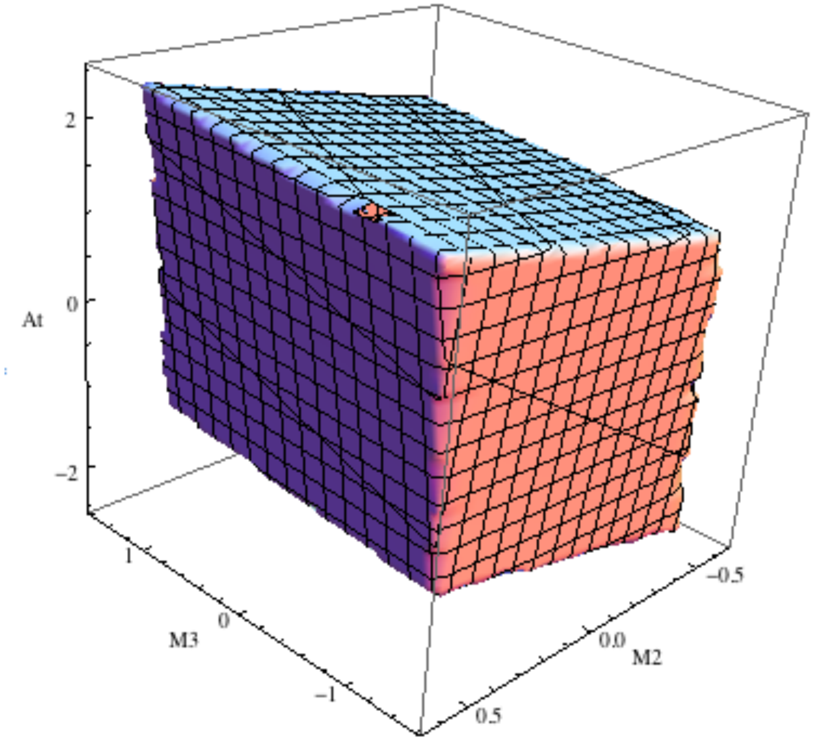
$$|\Delta_{M_2}| = \frac{2}{m_h^2}\left|2c_{M_2}M_2^2 + c_{M_3M_2}M_3M_2 + c_{M_2A_t}M_2A_t\right| \leq \Delta^{(\max)}$$

$$|\Delta_{A_t}| = \frac{2}{m_h^2}\left|2c_{A_t}A_t^2 + c_{M_3A_t}M_3A_t + c_{M_2A_t}M_2A_t\right| \leq \Delta^{(\max)}$$

$$\left|6.41M_3^2 - 0.57A_tM_3 + 0.27M_3M_2\right| \lesssim \Delta^{\max}\ m_h^2$$

$$\left|-0.81M_2^2 + 0.14A_tM_2 + 0.27M_3M_2\right| \lesssim \Delta^{\max}\ m_h^2$$

$$\left|0.44A_t^2 - 0.57A_tM_3 + 0.14A_tM_2\right| \lesssim \Delta^{\max}\ m_h^2$$



Absolute bound:

$$|M_i| < \frac{m_h}{2}\sqrt{\frac{\Delta^{\max}}{|c_{M_i}|}}\left(1+\sum_{i\neq j}\frac{1}{4}\frac{|c_{M_iM_j}|}{\sqrt{|c_{M_i}c_{M_i}|}}\right)$$

Bounds on the initial (HE) parameters in the Unconstrained MSSM

	$M_{\mathrm{M_{HE}}} = 2 \times 10^{16}$	$M_{\mathrm{M_{HE}}} = 10^{10}$	$M_{\mathrm{M_{HE}}} = 10^{4}$
$M_3^{\max}(M_{\mathrm{HE}})$	660	1 162	5 376
$M_2^{\max}(M_{\mathrm{HE}})$	1 646	1 750	3 500
$M_1^{\max}(M_{\mathrm{HE}})$	8 002	6 100	11 048
$A_t^{\max}(M_{\mathrm{HE}})$	2 504	2 227	3 094
$m_{H_u}^{\max}(M_{\mathrm{HE}})$	1 038	1 046	913
$m_{H_d}^{\max}(M_{\mathrm{HE}})$	6 945	14 472	9 784
$\mu^{\max}(M_{\mathrm{HE}})$	624	640	630
$m_{Q_3}^{\max}(M_{\mathrm{HE}})$	1 458	1 687	3 527
$m_{U_3}^{\max}(M_{\mathrm{HE}})$	1 640	1 828	3 710
$m_{D_3}^{\max}(M_{\mathrm{HE}})$	5 682	7 812	20 277
$m_{Q_{1,2}}^{\max}(M_{\mathrm{HE}})$	5 601	7 693	19 288
$m_{U_{1,2}}^{\max}(M_{\mathrm{HE}})$	3 818	5 254	13 975
$m_{D_{1,2}}^{\max}(M_{\mathrm{HE}})$	5 613	7 722	19 764
$m_{L_{1,2,3}}^{\max}(M_{\mathrm{HE}})$	5 557	7 664	20 278
$m_{E_{1,2,3}}^{\max}(M_{\mathrm{HE}})$	5 524	7 607	20 278

$$\Delta^{\max} = 100$$

3.- Translate limits on θ_i into limits on the SUSY spectrum

Unfortunately, there is not a one-to-one correspondence between the physical masses and the soft-parameters and μ−term at high-energy.

The only approximate exception are the gaugino and Higgsino masses.

$$
\begin{aligned}
M_{\tilde{g}} &\simeq M_3(M_{\rm LE}) \simeq 2.22 M_3 \\
M_{\tilde{W}} &\simeq M_2(M_{\rm LE}) \simeq 0.81 M_2 \\
M_{\tilde{B}} &\simeq M_1(M_{\rm LE}) \simeq 0.43 M_1 \\
M_{\tilde{H}} &\simeq \mu(M_{\rm LE}) \simeq 1.002 \mu \ ,
\end{aligned}
$$

$M_{HE} = M_X, \ M_{LE} = 1 \text{ TeV}$

The average stop mass has also an easy-to-handle form

$$\overline{m}_{\tilde{t}}^2 \simeq \frac{1}{2}(5.94 M_3^2 + 0.68 m_{\tilde{t}_L}^2 + 0.62 m_{\tilde{t}_R}^2 + 0.18 M_2^2 - 0.31 m_{H_u}^2 \cdots) + m_t^2$$

$M_{HE} = M_X, \ M_{LE} = 1 \text{ TeV}$

Bounds on the physical masses in the Unconstrained MSSM

	$M_{\mathrm{MHE}} = 2 \times 10^{16}$	$M_{\mathrm{MHE}} = 10^{10}$	$M_{\mathrm{MHE}} = 10^{4}$
$M_{\tilde{g}}^{\mathrm{max}}$	1 440	1 890	5 860
$M_{\tilde{W}}^{\mathrm{max}}$	1 303	1 550	3 435
$M_{\tilde{B}}^{\mathrm{max}}$	3 368	4 237	10 565
$M_{\tilde{H}}^{\mathrm{max}}$	626	610	620
$\overline{m}_{\tilde{t}}^{\mathrm{max}}$	1 650	1 973	4 140
$m_{H^0}^{\mathrm{max}}$	7 252	14 510	9 900

$$\Delta^{\mathrm{max}} = 100$$

Normally, the bound on the gluino mass is both stronger and more robust than the bound on the stops.

However, since it comes from a two-loop effect, it becomes weaker as the high-energy scale decreases.

The bounds on Higgsinos are, by far, the most model-independent ones.

In specific MSSM scenarios there are correlations between the soft terms at M_{HE}

E.g. suppose $\{m^2_{H_u}, m^2_{Q_3}, m^2_{U_3}\} = \{a_{H_u}, a_{Q_3}, a_{U_3}\}\, m_0^2$

$$\Rightarrow \quad \left|\Delta_{m_0^2}\right| = \left| -2\frac{m_0^2}{m_h^2} \underbrace{\left(c_{m^2_{H_u}} a_{H_u} + c_{m^2_{Q_3}} a_{Q_3} + c_{m^2_{U_3}} a_{U_3} \right)}_{\text{if small, then reduced fine-tuning}} \right| \lesssim \Delta^{\max}$$

In specific MSSM scenarios there are correlations between the soft terms at MHE

E.g. suppose $\{m^2_{H_u}, m^2_{Q_3}, m^2_{U_3}\} = \{a_{H_u}, a_{Q_3}, a_{U_3}\}\, m_0^2$

$$\Rightarrow \quad \left|\Delta_{m_0^2}\right| = \left| -2\frac{m_0^2}{m_h^2} \underbrace{\left(c_{m^2_{H_u}} a_{H_u} + c_{m^2_{Q_3}} a_{Q_3} + c_{m^2_{U_3}} a_{U_3} \right)}_{\text{if small, then reduced fine-tuning}} \right| \lesssim \Delta^{\max}$$

This happens for $a_{H_u} = a_{Q_3} = a_{U_3}$ & $M_{\rm HE} = M_{\rm X}$

$$\Rightarrow \quad \overline{m_{\tilde{t}}} \lesssim 4 \text{ TeV}$$ (focus-point regime)

In specific MSSM scenarios there are correlations between the soft terms at MHE

Similarly, if $\{M_1, M_2, M_3, A_t\} = \{a_1, a_2, a_3, a_t\}\, M_{1/2}$

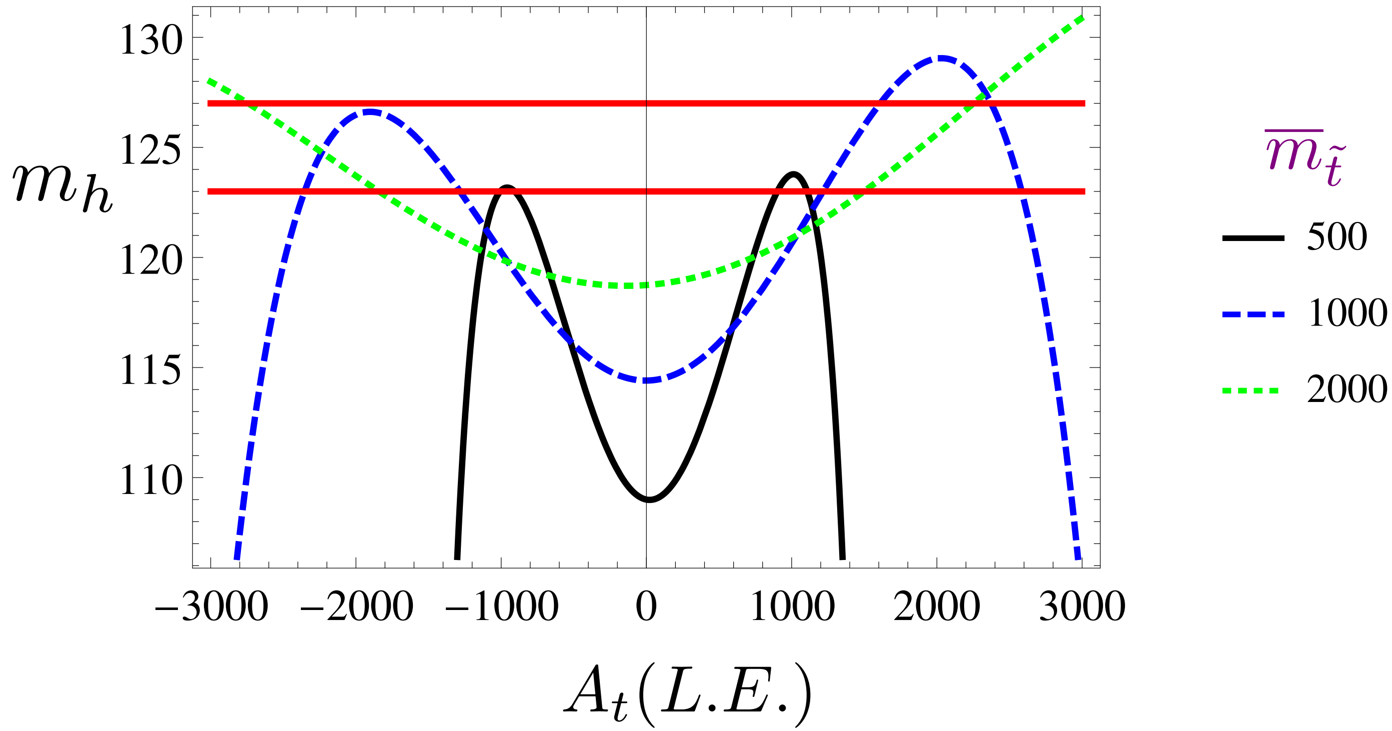
then for $\dfrac{a_2}{a_3} = -2.50,\ 3.16 \ ; \quad a_t = 0$ & $M_{\rm HE} = M_{\rm X}$

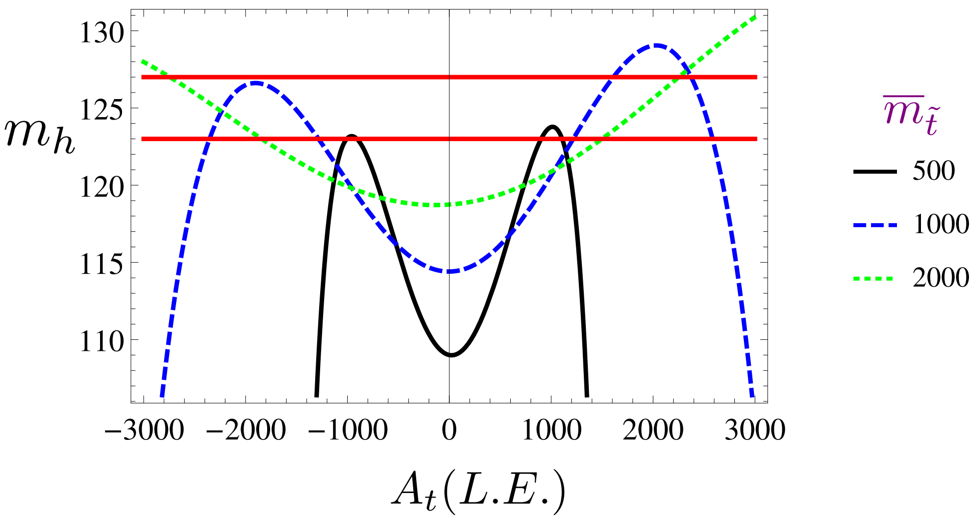
⇨ $M_{1/2}$ essentially unconstrained

It is easy to explore in this way the existence of other focus-point scenarios

Fine-tuning to get $m_h \simeq 125\ \mathrm{GeV}$

Recall

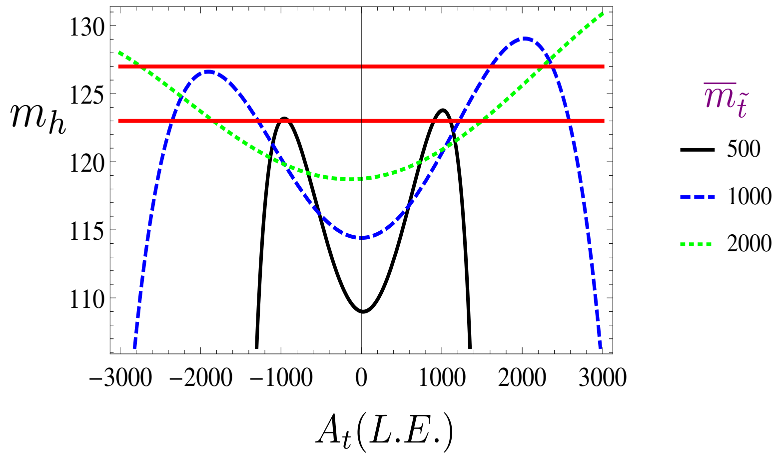




Clearly, if stops are light, there is an additional, independent, fine-tuning, whose p-value has to be multiplied with the EW one (thus increasing the total F-T enormously)

Roughly speaking, for $\overline{m_{\tilde{t}}} \gtrsim 800\ \mathrm{GeV}$ there is no need of F-T. So naturalness prefers not-too-light stops !

Furthermore....

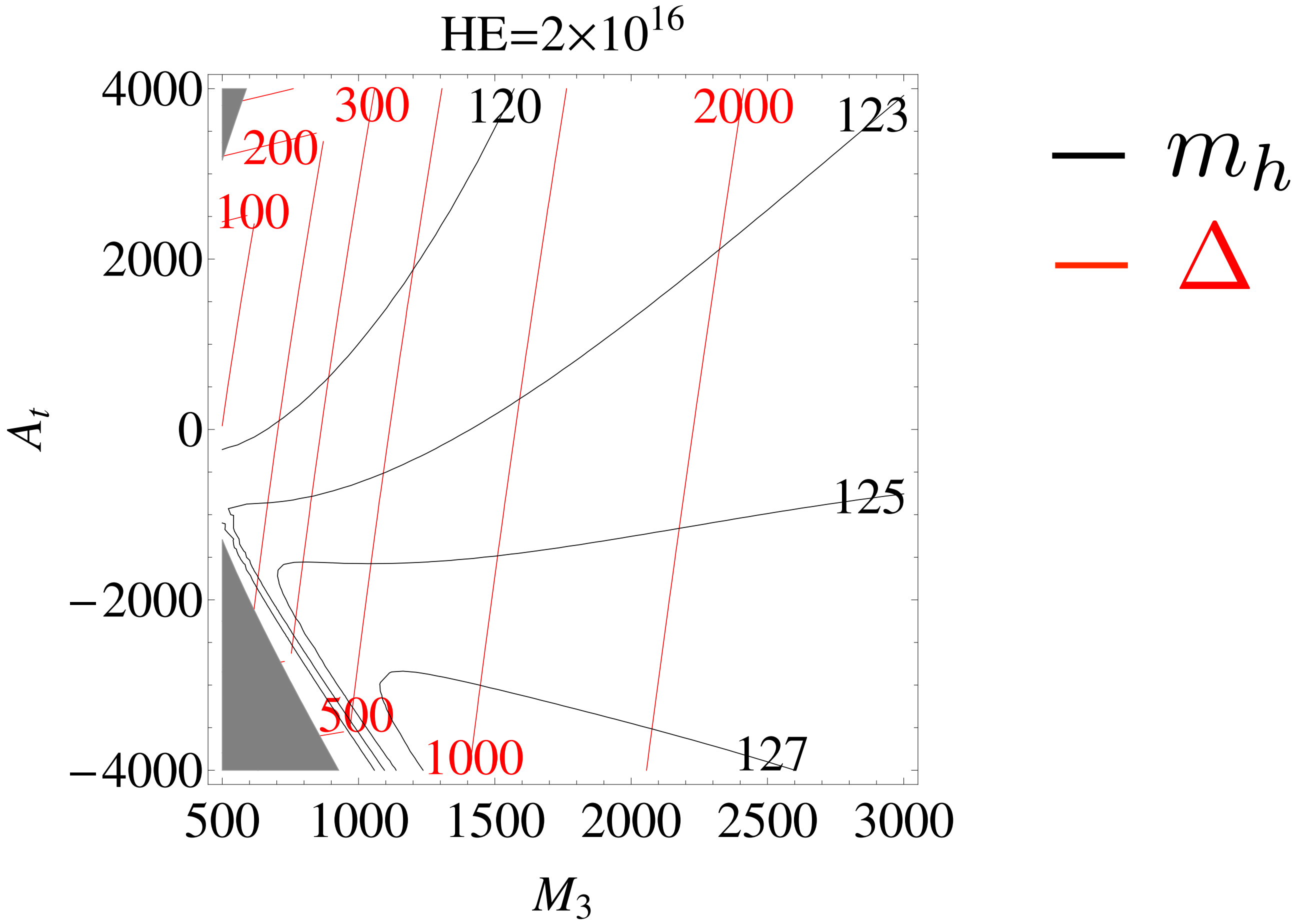


A sizeable $A_t(LE)$ is generically required

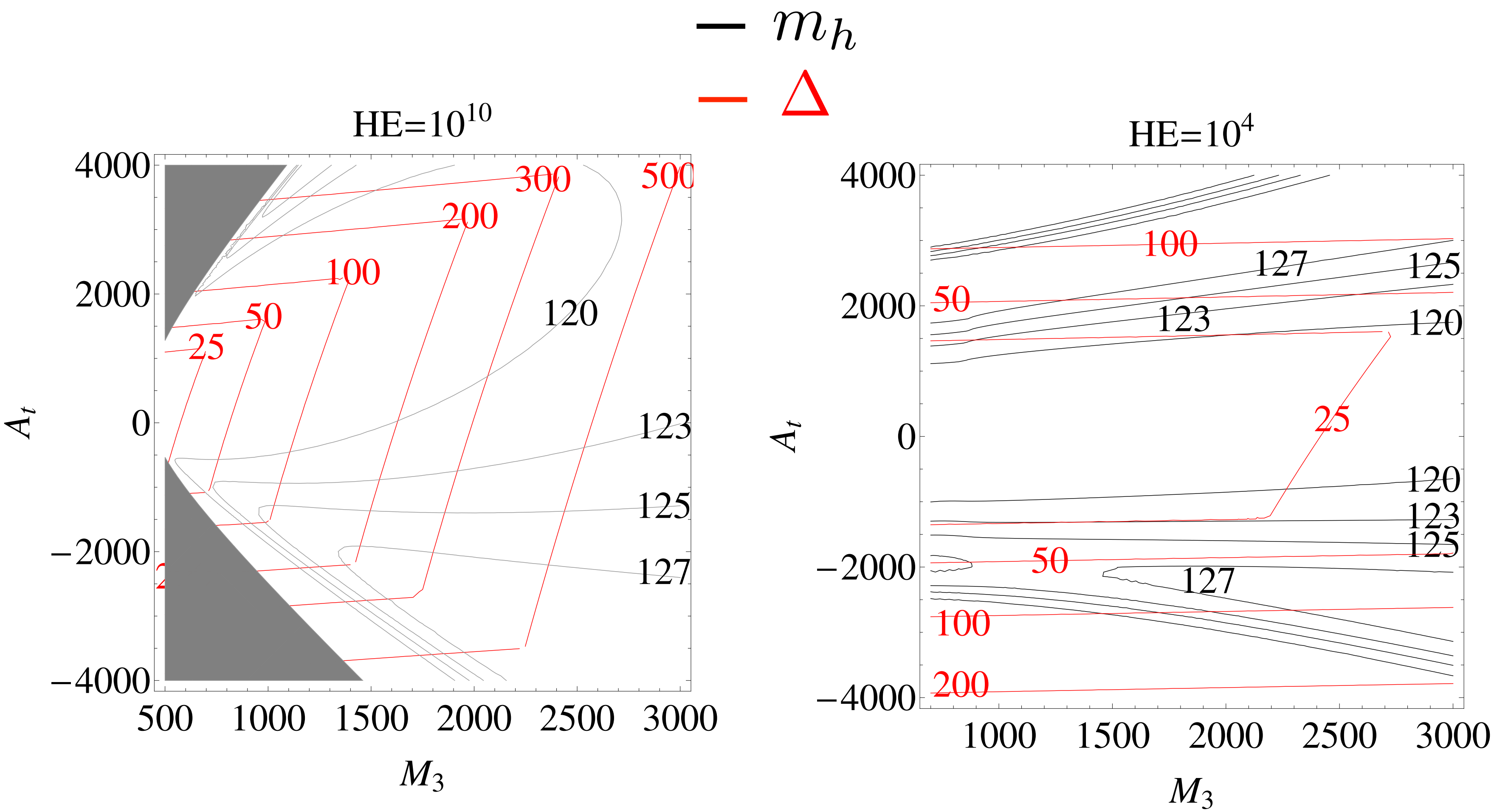
Since A_t is driven to negative values along the running, the only way to achieve a sizeable A_t (without an enormous F-T price) is to start with negative A_t.

But a negative sign for A_t increases the F-T associated to the gluino!

$$\left|6.41 M_3^2 - 0.57 A_t M_3 + 0.27 M_3 M_2\right| \lesssim \Delta^{\max}\ m_h^2$$



This effect decreases for smaller M_{HE}



Fine-tuning to get large $\tan\beta$

Recall

$$\tan\beta \simeq \frac{m_{H_d}^2 + m_{H_u}^2 + 2\mu^2}{B\mu} = \frac{m_A^2}{B\mu}$$

$$\left|\Delta_\theta^{(\tan\beta)}\right| \simeq \tan\beta \left|\frac{\theta}{m_A^2}\frac{d(B\mu)}{d\theta}\right|$$

Using

$$B\mu|_{\rm low} \simeq B\mu + 0.46 M_3\mu - 0.35 M_2\mu - 0.34 A_t\mu - 0.03 M_1\mu + \cdots$$

$(M_{HE} = M_X,\ M_{LE} = 1\ \text{TeV})$

$$\left|\Delta^{(\tan\beta)}_{\{B,M_3,M_2,A_t\}}\right| \simeq \tan\beta \left|\frac{\mu}{m_A^2}\{B,\ 0.46M_3,\ 0.35M_2,\ 0.34A_t\}\right|$$

Typically, $\Delta^{(\tan\beta)} \gtrsim 5-10$ for $\tan\beta \gtrsim 15-30.$

Conclusions

- The Run I of LHC has not seen any serious hint of BSM

  the results create a moderate tension with the Naturalness (Hierarchy Problem) arguments.

- However, the motivation to search BSM at LHC remains.

  It is perhaps too soon to think that the LHC results are in conflict with the Naturalness argument.

- SUSY remains a well-motivated candidate for BSM.

Conclusions II

- The unconstrained version of the MSSM (with M_{HE}=M_X) is fine-tuned at ~ 1% (due to the gluino)

  The fine-tuning is substantially less severe if M_{HE}< M_X

- There is really no solid reason based on naturalness to expect light stops (in particular lighter than gluino)

  - If the stops are heavy because m_0 is large, then there is no fine-tuning price

  - The fine-tuning due to stops also decreases if M_{HE} < M_X

Conclusions III

- The most robust prediction from Natural SUSY is, by far,

$$m_{\tilde{H}} \lesssim 0.7 \text{ GeV}$$

- SUSY is in good shape, though somewhat fine-tuned

  "Natural" SUSY (the less fine-tuned version of the MSSM without "fooling" the LHC) is 1%-10% fine-tuned

  Going beyond the MSSM, i.e. NMSSM, BMSSM, RPV,... could reduce the fine-tuning as well

- If naturalness arguments are sound and SUSY is true, we could be about seeing SUSY (or perhaps other BSM) in LHC-14

Since fine-tuning seems to be the main (actually the only) problem with SUSY, a reasonable guide to explore SUSY scenarios is to look for as little fine-tuning as possible

Natural SUSY $\equiv$ MSSM as natural (non-fine-tuned) as possible