# Criteria for Natural Hierarchies

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[Based on arXiv:1402.2658, with Daniel Hernández and Tim M.P. Tait]

Other references most closely related to what I will present (and there are several more):

- A. Casas, J.R. Espinoza, and I. Hidalgo, hep-ph/0410298 and hep-ph/0607279
- M. Farina, D. Pappadopulo, A. Strumia, arXiv:1303.7244

All of the results presented are obtained at the order-of-magnitude level, using naive perturbation theory and dimensional analysis. Much more quantitative results can be found in arXiv:1303.7244.

We only worry about one hierarchy problem at a time (this will become more clear. Landau poles will also be ignored.

# THE HIERARCHY PROBLEM

**Higgs potential:** $V(H) = -\mu^2|H|^2 + \lambda|H|^4$

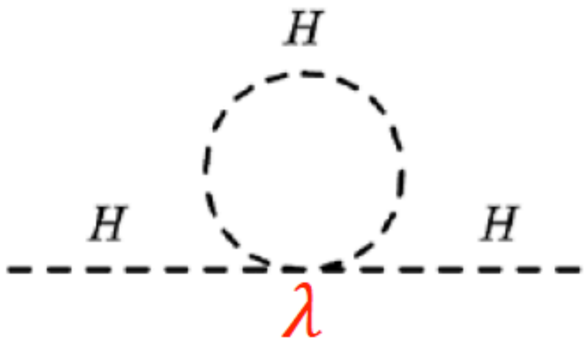
**Classically:** $\mu$ is equivalent to the mass of the Higgs

*Quantum mechanics introduces corrections to this simple relation*

$$\mu^2 \to \mu^2 + \delta\mu^2$$

# THE HIERARCHY PROBLEM

Quantum corrections correspond to loop Feynman diagrams




$$\delta\mu^2 \propto \lambda\Lambda^2$$

unknown mass scale (regulator)

- This correction is infinite when $\Lambda \to \infty$
- It depends on the regulation procedure
- Renormalization gets rid of these

However, this may not be the right way to look at this. Quadratic divergences are not observable – they can be renormalized away. **They don't even show up if one uses dimensional regularization!** Some level of interpretation, often implied, is required in order to state that the presence of quadratic divergences implies that the weak scale is unstable.

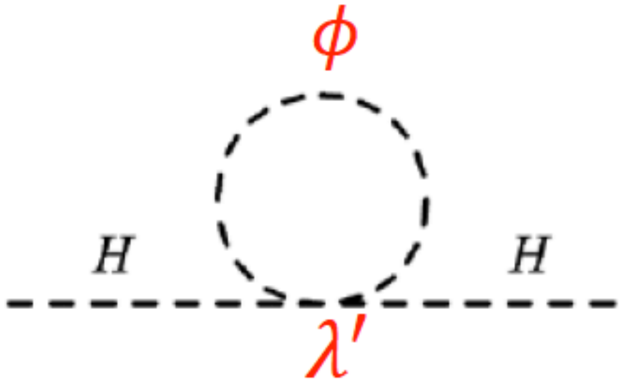
By itself, the Standard Model is a **one-mass-scale theory**. The weak scale, at the tree-level or the one-loop level, is the weak scale. And you can't predict it, it has to be measured.

Life is quite different if, on top of the weak scale, there is **another mass scale** $M_{\rm new}$. Is this case, there are finite corrections to the Higgs mass-squared parameter. These may de-stabilize the weak scale. We use this as the definition of the hierarchy problem:

Can the Weak Scale Co-Exist with Another Mass Scale?

The key point is that the answer **depends on the new physics** and **how it talks to the Higgs boson**. Estimating it from Standard Model parameters may be dangerous.

# THE HIERARCHY PROBLEM (Finite corrections)




Known mass scale

$$\delta\mu^2 \propto \lambda' m_\phi{}^2$$

- **Finite for $\Lambda \to \infty$. Hierarchy Problem when $m_\phi \gg \mu$.**
- **It is independent of the regulator**

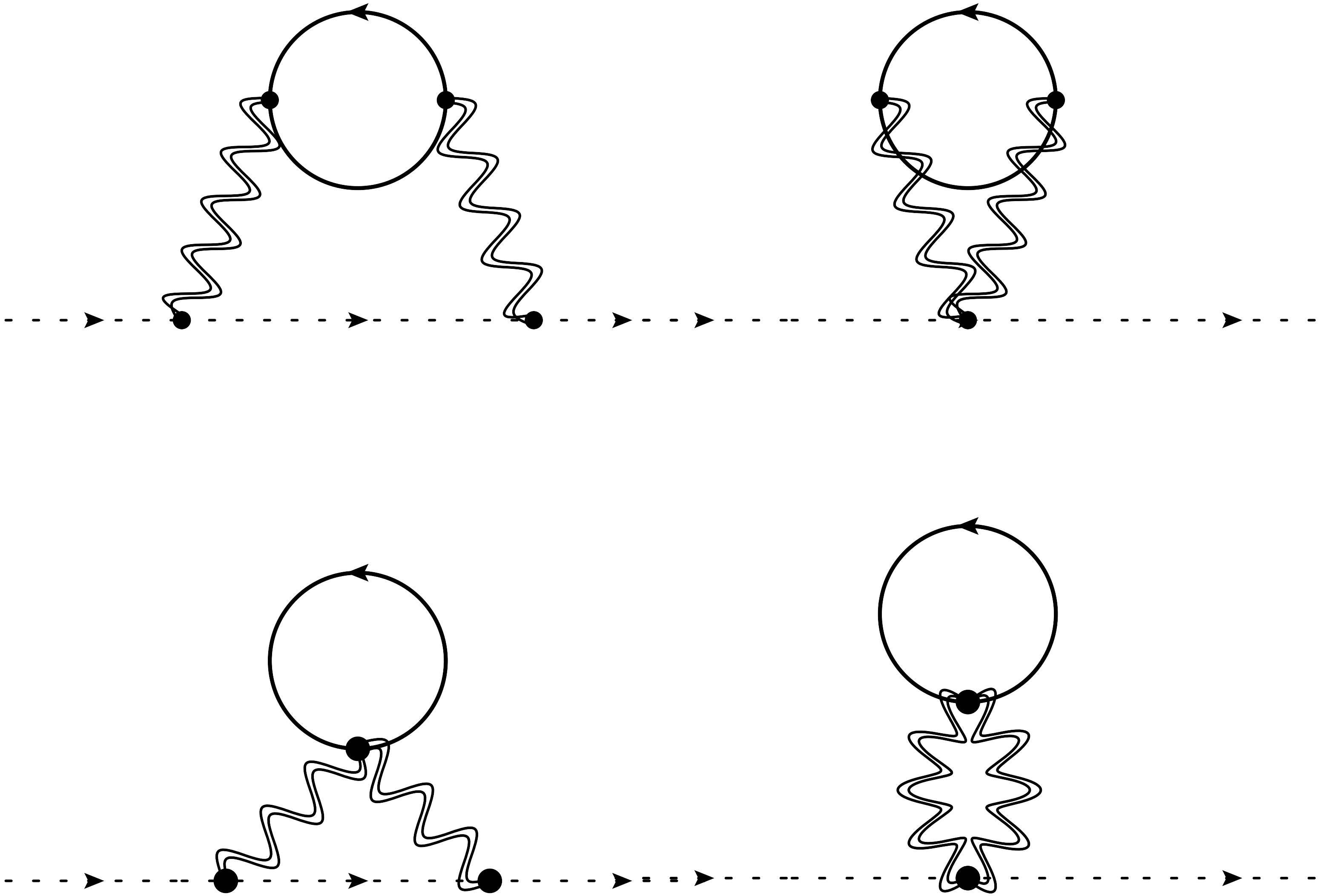
## Case Study: New Fermion $\Psi$, uncoupled to the Standard Model

Complaint: Gravity exists, and the weak scale is way lower than the Planck scale. There is no way these two mass scales can co-exist.

Answer: I don't know how to compute quantum gravity corrections to the Higgs mass-squared. I do know that, perturbatively and at low-energies (below the Planck scale), corrections are tiny since the coupling goes like $1/M_{Pl}^2$:

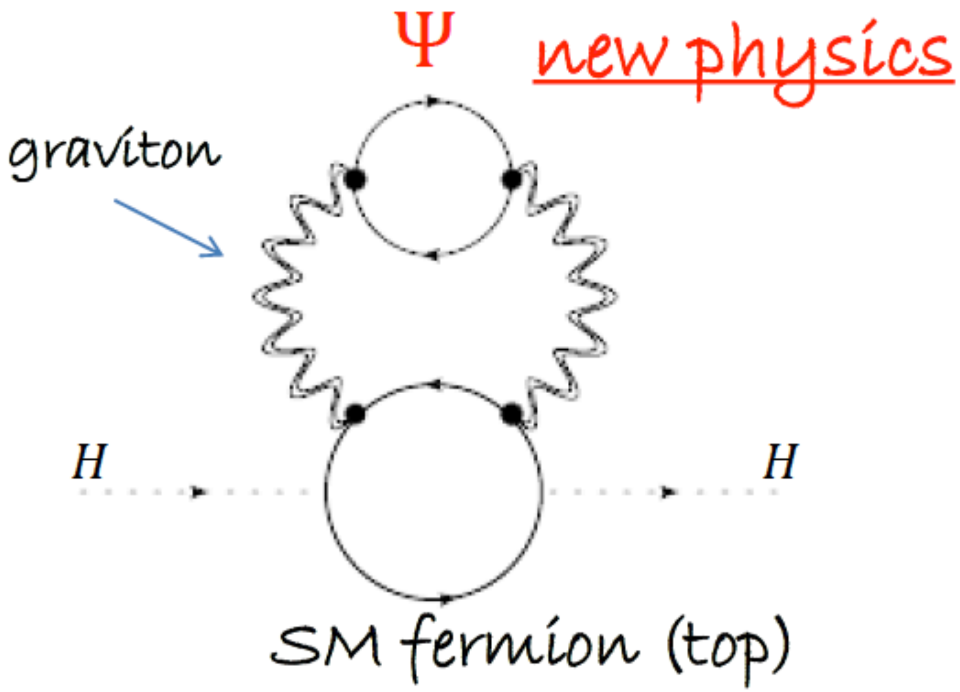
$$\delta\mu^2 \sim \mu^2 \left(\frac{\mu^2}{M_{Pl}^2}\right)^N$$

Toy-model: Add a new vector-like fermion with mass $M_\Psi$ that does not couple to the Standard Model at all, except through gravity.



$$\delta\mu^2 \sim \frac{1}{(16\pi^2)^2}\frac{M_\Psi^4}{M_{Pl}^4} \;\times\; \mu^2.$$

# Uncoupled:
(except for gravity)




$$\delta\mu^2 \sim \frac{\lambda_t^2}{(16\pi^2)^3}\frac{M_\Psi^4}{M_{Pl}^4} \times M_\Psi^2$$

$$\delta\mu^2 \lesssim 100^2 \text{ GeV}^2 \quad \Rightarrow \quad M_\Psi \lesssim 10^{14} \text{ GeV}$$

## Other Case Studies with a new Fermion

Fermion charged under $SU(2)\times U(1)$:

$$\delta\mu^2 = \left(\frac{g^2}{16\pi^2}\right)^2 \times F\left(\frac{M^2_{W,Z}}{M^2_\Psi}\right) \times M^2_\Psi,$$

(two loops). Get in trouble for $M_\Psi$ above tens of TeV.

Yukawa coupled ($y_{\rm new}(\psi H)\Psi$):

$$\delta\mu^2 \sim C\,\frac{y^2_{\rm new}}{16\pi^2} \times M^2_\Psi.$$

(one loop). Get in trouble for $M_\Psi$ above ...? Depends on the Yukawa coupling!

# Hints for New Mass Scales

We know there is new physics beyond the standard model!

1. Nonzero neutrino masses. New mass scale $\to$ maybe. Not necessarily very high.
2. Dark matter. New mass scale $\to$ most likely. Not necessarily very high.
3. Gauge coupling unification. New mass scale $\to$ certainly. Most likely very high.
4. Flavor symmetries. New mass scale $\to$ certainly. Probably pretty high.
5. Inflation. New mass scale $\to$ most likely. Not necessarily very high (?).
6. Baryogenesis. New mass scale $\to$ probably. Not necessarily very high.
7. Dark Energy. New mass scale $\to$ ???

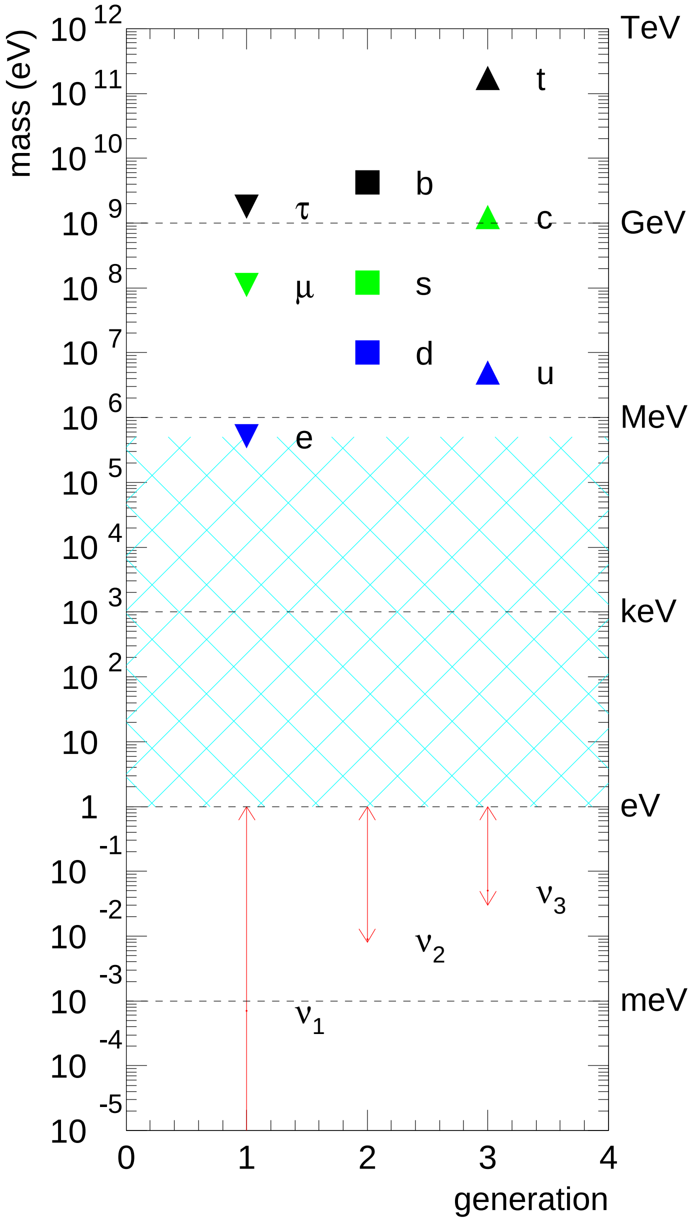
## Some Comments on Dark Matter

In the context of this talk, there are three relevant questions:

1. Does Dark Matter Imply a New Mass Scale?
2. Is the New Mass Scale Much Higher/Lower than the Weak Scale?
3. How Does the Dark Matter "Talk" to the Higgs?

Short answer: we don't know the answer to any of the questions! More detail (speculation required)

1. Probably. For example, new chiral fermions are "ruled out" as the Dark Matter (see, for example, AdG, Wei-Chih Huang, Jennifer Kile, 1207.0510).
2. Probably not very high, could be very low. For example, standard thermal WIMPS masses are below $\sim 100$ TeV. Right-handed neutrinos as DM have masses around 1 keV.
3. In the case of vanilla thermal relics, couplings are, to some extent, known. In other cases, not known or constrained by most anything.




## New Physics, Known Since 1998:

**⇐ NEUTRINOS HAVE TINY MASSES**

**⇓ LEPTON MIXING IS "WEIRD" ⇓**

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & \mathbf{0.2} \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \qquad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

**What Does It Mean?**

## Neutrino Masses: Most* "Palpable" Evidence of Physics Beyond the Standard Model

The SM we all learned in school predicts that neutrinos are strictly massless. Hence, massive neutrinos imply that the the SM is incomplete and needs to be replaced/modified.

Furthermore, the SM has to be replaced by something qualitatively different.

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* There is only a handful of questions our model for fundamental physics cannot explain (these are personal. Feel free to complain).

- What is the physics behind electroweak symmetry breaking? (Higgs ✓).
- What is the dark matter? (not in SM).
- Why is there more matter than antimatter in the Universe? (not in SM).
- Why does the Universe appear to be accelerating? Why does it appear that the Universe underwent rapid acceleration in the past? (not in SM).

## What is the New Standard Model? [$\nu$SM]

The short answer is – WE DON'T KNOW. Not enough available info!

$$\Updownarrow$$

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the $\nu$SM candidates can do. [are they falsifiable?, are they "simple"?, do they address other outstanding problems in physics?, etc]

We need more experimental input.

## Neutrino Masses, Higgs Mechanism, and New Mass Scale of Nature

The LHC has revealed that the minimum SM prescription for electroweak symmetry breaking — the one Higgs double model — is at least approximately correct. What does that have to do with neutrinos?

The tiny neutrino masses point to three different possibilities.

1. Neutrinos talk to the Higgs boson very, very **weakly**;
2. Neutrinos talk to a **different Higgs** boson – there is a new source of electroweak symmetry breaking!;
3. Neutrino masses are small because there is **another source of mass** out there — a new energy scale indirectly responsible for the tiny neutrino masses, a la the seesaw mechanism.

We are going to need a lot of experimental information from all areas of particle physics in order to figure out what is really going on!

# One Candidate $\nu$SM

SM as an effective field theory – non-renormalizable operators

$$\mathcal{L}_{\nu\mathrm{SM}} \supset -y_{ij}\frac{L^i H L^j H}{2\Lambda} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) + H.c.$$

There is only one dimension five operator [Weinberg, 1979]. If $\Lambda \gg 1$ TeV, it leads to only one observable consequence...

$$\text{after EWSB: } \mathcal{L}_{\nu\mathrm{SM}} \supset \frac{m_{ij}}{2}\nu^i\nu^j; \quad m_{ij} = y_{ij}\frac{v^2}{\Lambda}.$$

- Neutrino masses are small: $\Lambda \gg v \rightarrow m_\nu \ll m_f$ ($f = e, \mu, u, d$, etc)
- Neutrinos are Majorana fermions – Lepton number is violated!
- $\nu$SM effective theory – not valid for energies above *at most* $\Lambda/y$.
- Define $y_{\max} \equiv 1 \Rightarrow$ data require $\boxed{\Lambda \sim 10^{14}\text{ GeV.}}$

What else is this "good for"? Depends on the ultraviolet completion!

Is This Finely Tuned? Depends on the ultraviolet completion!

# Type-I Seesaw

$$\mathcal{L}_{SM} + \bar{N}_i \bar{\sigma}^\mu \partial_\mu N^i - \frac{M_R^{ij}}{2} N_i N_j - y_{ij} L^i N^j H + H.c.,$$

where $i, j$ are family indices, $M_R$ is the right-handed neutrino mass-matrix, and $y$ is the neutrino Yukawa coupling matrix.

$$\delta\mu^2 = -\frac{1}{4\pi^2} \sum_{ij} |y_{ij}|^2 \times M_j^2 \quad \rightarrow M < 10^4 \text{ TeV}.$$

The weak scale can co-exist with the right-handed neutrino mass-scale if the right-handed neutrino masses are below 10 TeV. Is this a problem? No, except that vanilla leptogenesis does not work.

( 

## Why are Neutrino Masses Small in the $M \neq 0$ Case?

If $\mu \ll M$, below the mass scale $M$,

$$\mathcal{L}_5 = \frac{LHLH}{\Lambda}.$$

Neutrino masses are small if $\Lambda \gg \langle H \rangle$. Data require $\Lambda \sim 10^{14}$ GeV.

In the case of the seesaw,

$$\Lambda \sim \frac{M}{\lambda^2},$$

so neutrino masses are small if either

- they are generated by physics at a very high energy scale $M \gg v$ (high-energy seesaw); or
- they arise out of a very weak coupling between the SM and a new, hidden sector (low-energy seesaw); or
- cancellations among different contributions render neutrino masses accidentally small ("fine-tuning").

)

## On Very Small Couplings

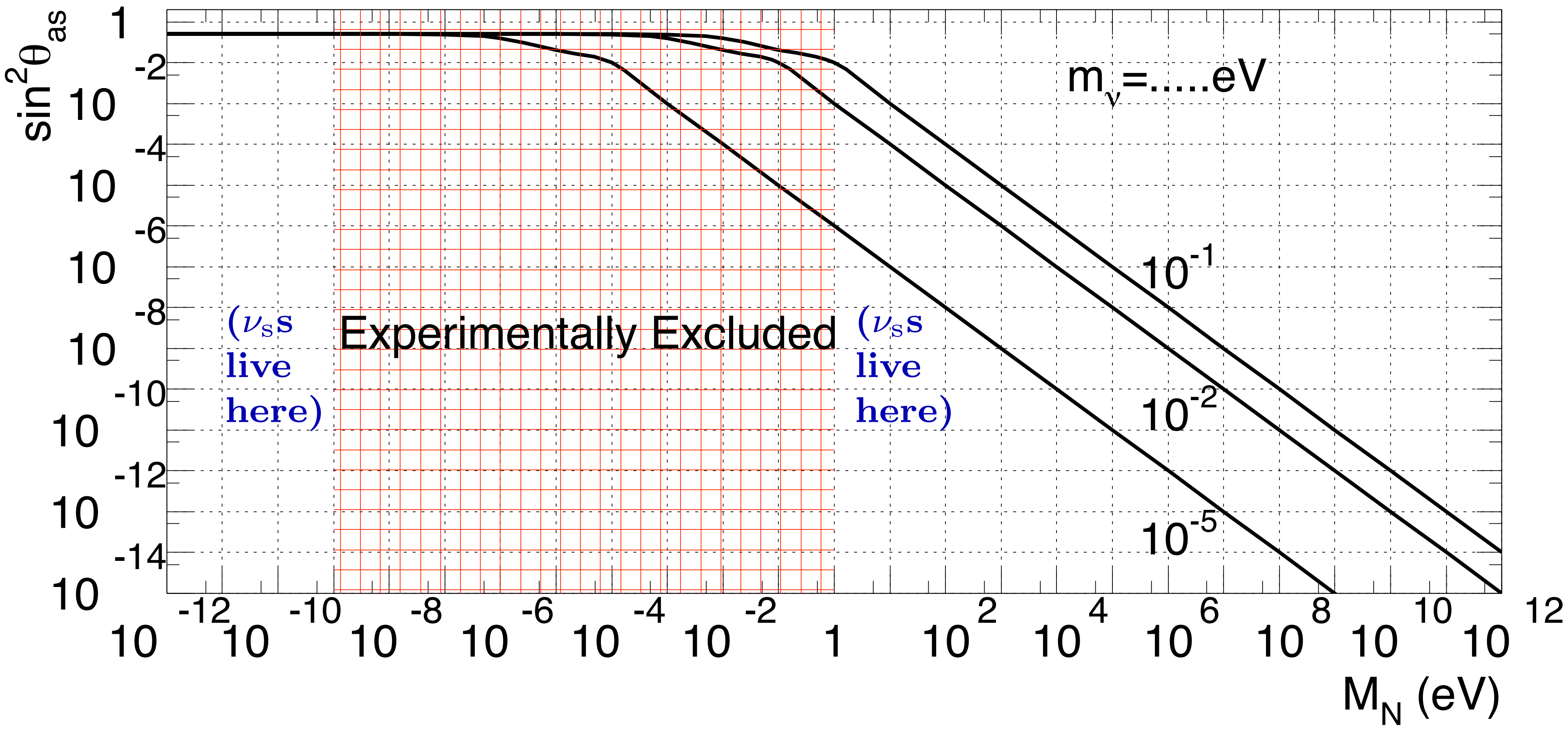
We would like to believe that coupling constants should naturally be of order one.

Why is that? Perhaps it is because all gauge couplings are $\mathcal{O}(1)$. Is this a red herring? For example, $g_3$ runs towards small values in the UV so why having it be order 1 at the weak scale "natural"?

Nature, however, does not appear to be married to this idea. Of all known fermions, only one (1) has a "natural" Yukawa coupling – the top quark!

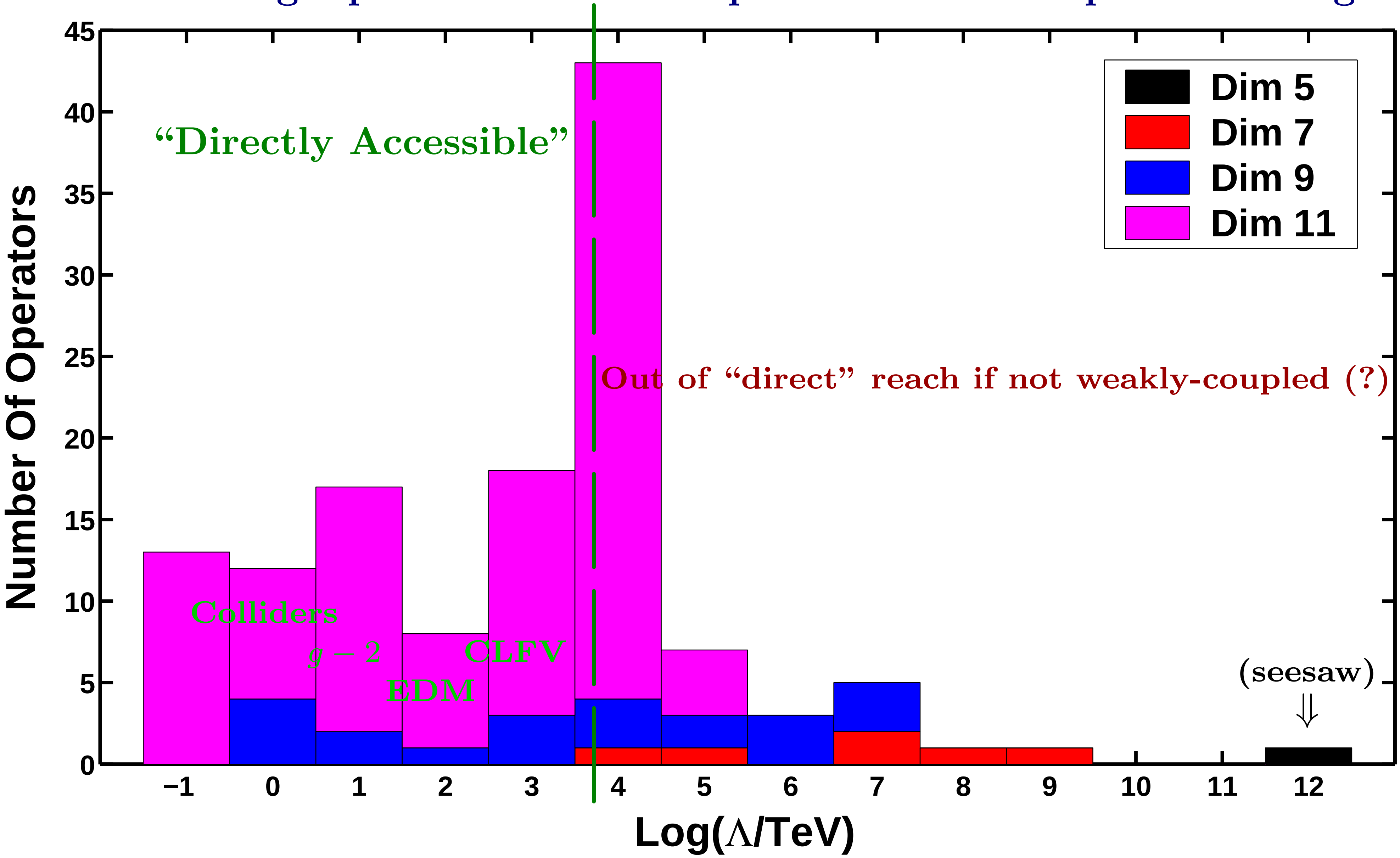
Does that mean anything? We tend to think so, but is it true? Hasn't proven to be especially useful as a paradigm yet...

## Constraining the Seesaw Lagrangian




[AdG, Huang, Jenkins, arXiv:0906.1611]

# Weinberg Operator Just the Tip of the "Model-Space" Iceberg



## Piecing the Neutrino Mass Puzzle

Understanding the origin of neutrino masses and exploring the new physics in the lepton sector will require unique **theoretical** and **experimental** efforts …

- understanding the fate of lepton-number. Neutrinoless double beta decay!
- A comprehensive long baseline neutrino program. (On-going T2K and $NO\nu$A. LBNF and HyperK next steps towards the ultimate "superbeam" experiment.)
- The next-step is to develop a qualitatively better neutrino beam – e.g. muon storage rings (neutrino factories).
- Different baselines and detector technologies a must for both over-constraining the system and looking for new phenomena.
- Probes of neutrino properties, including neutrino scattering experiments.
- Precision measurements of charged-lepton properties ($g-2$, edm) and searches for rare processes ($\mu \to e$-conversion the best bet at the moment).
- Collider experiments. The LHC and beyond may end up revealing the new physics behind small neutrino masses.
- Neutrino properties affect, in a significant way, the history of the universe (Cosmology). Will we learn about neutrinos from cosmology, or about cosmology from neutrinos?

# Flavor Model Example

$$\mathcal{L} \supset i\overline{D}\partial\!\!\!/ D - M_D\overline{D}D + y_1\overline{Q}_3 DH + y_2\overline{D}b_R\phi + H.c.,$$

where $Q_3$ is the third generation quark doublet, $t_R$ and $b_R$ are the quark singlets, $\phi$ is the scalar whose VEV breaks the $U(1)$ global symmetry, and $D$ is a vector-like quark with the same SM gauge interactions as $b_R$.

$b$-Yukawa coupling only after integrating out $D$,

$$y_b^{\rm eff} = y_1 y_2 \frac{\langle\phi\rangle}{M_D} \ .$$

$$\delta\mu^2 = -\frac{6|y_1|^2}{8\pi^2} \times M_D^2, \rightarrow M_D < \frac{900~\rm GeV}{|y_1|} \ .$$

(Quantum corrections to $\mu^2$ proportional to $M_\phi^2$ are also generically expected. We discuss these momentarily)

Flavor scale low, unless new Yukawa couplings are small. Defeats the purpose of the flavor model? “All the same but small” still better than hierarchical couplings?

## Case Study: New Scalar $\Phi$

Scalars are qualitatively different, since the marginal coupling

$$\mathcal{L}_{SM+\Phi} \supset \lambda_{\rm new}|H|^2|\Phi|^2.$$

is always allowed. At the one-loop level, this interaction allows the mass scale $M_\Phi$ to contribute to the Higgs boson mass-squared,

$$\delta\mu^2 \sim \frac{\lambda_{\rm new}}{16\pi^2} \times M_\Phi^2.$$

## Grand Unified Theories

$$\delta\mu^2 = \frac{C}{16\pi^2} \times M^2_{\rm GUT},$$

where $C$ is a coefficient of order (at least) the known gauge couplings that depends on the detailed physics at the GUT scale. Since limits from proton decay require $M_{\rm GUT} \sim 10^{16}$ GeV, it is clear that this correction to $\mu^2$ is highly unnatural, and requires a seemingly magical cancellation between the tree-level Higgs mass-squared parameter and all of its higher order quantum corrections.

## Higgs Portal Dark Matter

While there is a new mass scale, it is of order the weak scale (so you get the relic density right). Not problem here.

## Case Study: New Real Scalar $\Phi$, $|H|^2\Phi$ Coupling

A gauge-singlet scalar can also couple singly to a pair of Higgs fields via

$$\mathcal{L}_{SM+\Phi} \supset \kappa_{\rm new}|H|^2\Phi,$$

where $\kappa_{\rm new}$ is a coupling constant with dimensions of mass. In the limit $\kappa_{\rm new} \to 0$, there is an enhanced $Z_2$ symmetry, indicating that any value of $\kappa_{\rm new}$ is natural in the sense of 't Hooft.

$$\delta\mu^2 \sim -\frac{\kappa_{\rm new}^2}{16\pi^2} \times \log\left(\frac{M_\Phi^2}{|\mu^2|}\right),$$

whose size is characterized by $\kappa_{\rm new}$, and depends only logarithmically on $M_\Phi$. Obviously the theory will be finely tuned unless $\kappa_{\rm new} < \text{TeV}$. However, the super-renormalizable interaction **shields the Higgs mass from the heavy mass scale** $M_\Phi$, despite allowing for relatively large coupling between the Higgs and $\Phi$ sectors.

($\lambda_{\rm new}\Phi^2|H|^2$ is still around, but its contribution is negligible if $\lambda_{\rm new} \ll 1$. This is not pretty, but technically natural.)

## Case Study: Mixed Fermion/Scalar (Scalar Portal)

$$\mathcal{L}_{SM+\Phi+\Psi} \supset \kappa_{\rm new}|H|^2\Phi + y\Phi\bar{\Psi}\Psi,$$

and

$$\delta\mu^2 = \frac{y^2\kappa^2_{\rm new}M^2_{\Psi}}{(16\pi^2)^2M^2_{\Phi}}\ F\left(\frac{M^2_{\Psi}}{M^2_{\Phi}}\right)$$

where $F(M^2_{\Psi}/M^2_{\Phi})$ is $\mathcal{O}(1)$. The finite corrections proportional to $M^2_{\Psi}$ do not destabilize the weak scale as long as $M^2_{\Psi} < M^2_{\Phi}$ (provided $\kappa_{\rm new} <$ TeV and $\lambda_{\rm new} \ll 1$). This feature extends to diagrams with any number of loops, since in the limits $\kappa_{\rm new} \to 0$ or $M_{\Phi} \to \infty$, the SM and $\Psi$ must decouple $\to$ all contributions to $\delta\mu^2$ equal to $\kappa^2_{\rm new}$ multiplied by an analytic function of the ratio $M^2_{\Psi}/M^2_{\Phi}$.

Natural model (?) of heavy fermionic dark matter (played by $\Psi$ in the discussion above) which communicates primarily with the Higgs via exchange of $\Phi$. At low energies, $\Phi$ exchange results in an operator of the form $|H|^2\bar{\Psi}\Psi$. This is relatively free from fine-tuning.

## Some Final Remarks

- We advocate a "QFT-only" approach to the hierarchy problem. It is less ambitious but very concrete. And keep in mind that the standard interpretation [quadratic divergences imply new physics at the weak scale] may be a red herring.
- The standard model by itself has only one mass scale. It is "natural." So we are addressing the following question: **can the weak scale "co-exist" with other new physics scales?**
- The answer depends dramatically on how the new physics talks to the Higgs boson! Beware of small couplings. There is nothing wrong with them (remember that most couplings we know are small) and they allow different scales to co-exist.
- Standard model parameters play no or a limited role as far as deciding whether an extension of the standard model is natural. $\rightarrow$ Maybe the only thing special about the top quark is that it is the heaviest one!

- Quantum gravity. Perturbatively and at low energies, graviton loops are safe (whatever that means). A more definitive answer requires a concrete model.
- Nature has already revealed that there is physics beyond the standard model. Dark matter and nonzero neutrino masses require new degrees of freedom and, **perhaps**, new mass scales.
- More indirect hints like the unification of gauge couplings and the fermion particle content (GUTs), the need for a mechanism of baryogenesis, the strong CP problem, and the flavor puzzle also suggest the existence of new, usually very heavy, new states.
- SUSY allows different mass scales to co-exist. Broken SUSY, of course, brings about its own naturalness problem (a bunch of new fermions and bosons with mass $M_{\rm SUSY}$ and SM couplings). This appears to be the only way forward if GUTs are real. Same applies for many models that require ultra-high mass scales.