Confinement and extreme conditions in lattice QCD

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Topics

- **QCD phase diagram at finite temperature and quark density**
  (Paolo Cea, L.C., Alessandro Papa)

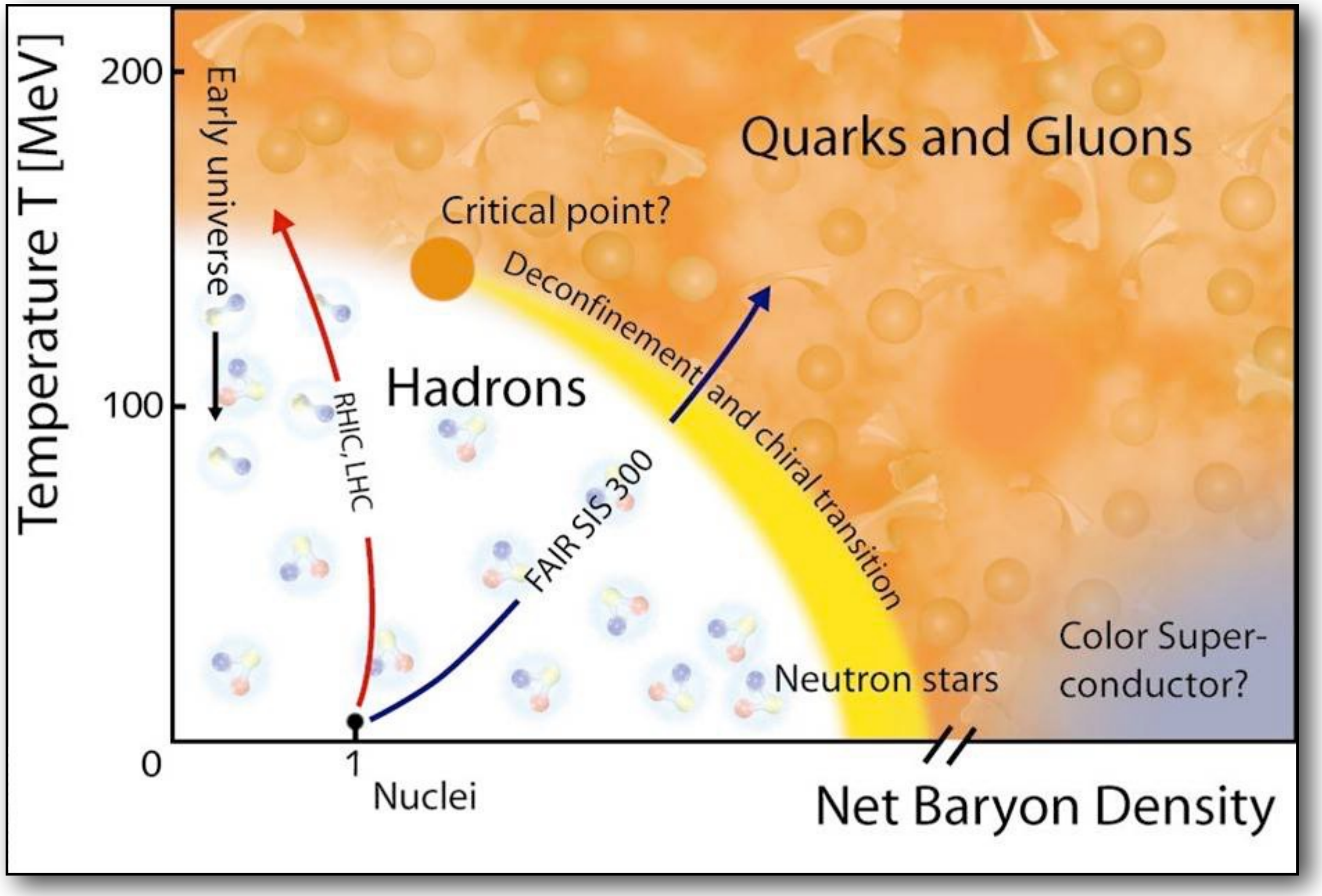
- **Flux tubes in the QCD vacuum**
  (Paolo Cea, L.C., Francesca Cuteri, Alessandro Papa)

- **QCD in external background fields**
  (Paolo Cea, L.C.)

QCD phase diagram at finite temperature and quark density

(Paolo Cea, L.C., Alessandro Papa)

- The study of the QCD phase diagram has become a topic of wide interest in recent years.
- A **transition** or rapid **crossover** is thought to exist from a low temperature hadronic phase to a high temperature quark-gluon plasma phase.



- The determination of the **QCD (pseudo)critical line** (*exact location and nature of the transition*) is related to many important theoretical and phenomenological issues.

For example:

- ✦ the physics of the early universe (***high T*** and ***low baryon*** density region)
- ✦ the physics of the interior of some compact astrophysical objects (***low T*** and ***high baryon*** density region)

- The QCD (pseudo)critical line can be parameterized by a lowest order Taylor expansion in the baryon chemical potential:

$$\frac{T(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T(\mu_B)} \right)^2$$

- Lattice QCD can be used to locate the QCD (pseudo)critical line.

  BUT the ***"sign problem"*** prevents us to do simulations at **real** nonzero baryon chemical potential.

- Possible way out: ***analytic continuation*** from an ***imaginary*** chemical potential (other methods: reweighting from the ensemble at $\mu_B=0$, the Taylor expansion method, the canonical approach, the density of states method).

- Estimate of the (pseudo)critical line by the method of analytic continuation of (2+1) flavor QCD using the HISQ/tree action.

Lattice setup and numerical simulation

- ***Highly improved staggered quark action*** with ***tree level improved Symanzik gauge action*** (**HISQ/tree**) with 2+1 flavors as implemented in the **MILC** code (http://www.physics.utah.edu/~detar/milc/).

- Calculations on a ***line of constant physics*** (*): as the gauge coupling is increased the bare quark masses have been adjusted such that the values of hadron masses in physical units (evaluated at zero temperature) stay approximately constant. The ***light-quark mass*** has been fixed at $m_l = m_s/20$. (M_π = 160 MeV)

  (*) as determined in A. Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012))

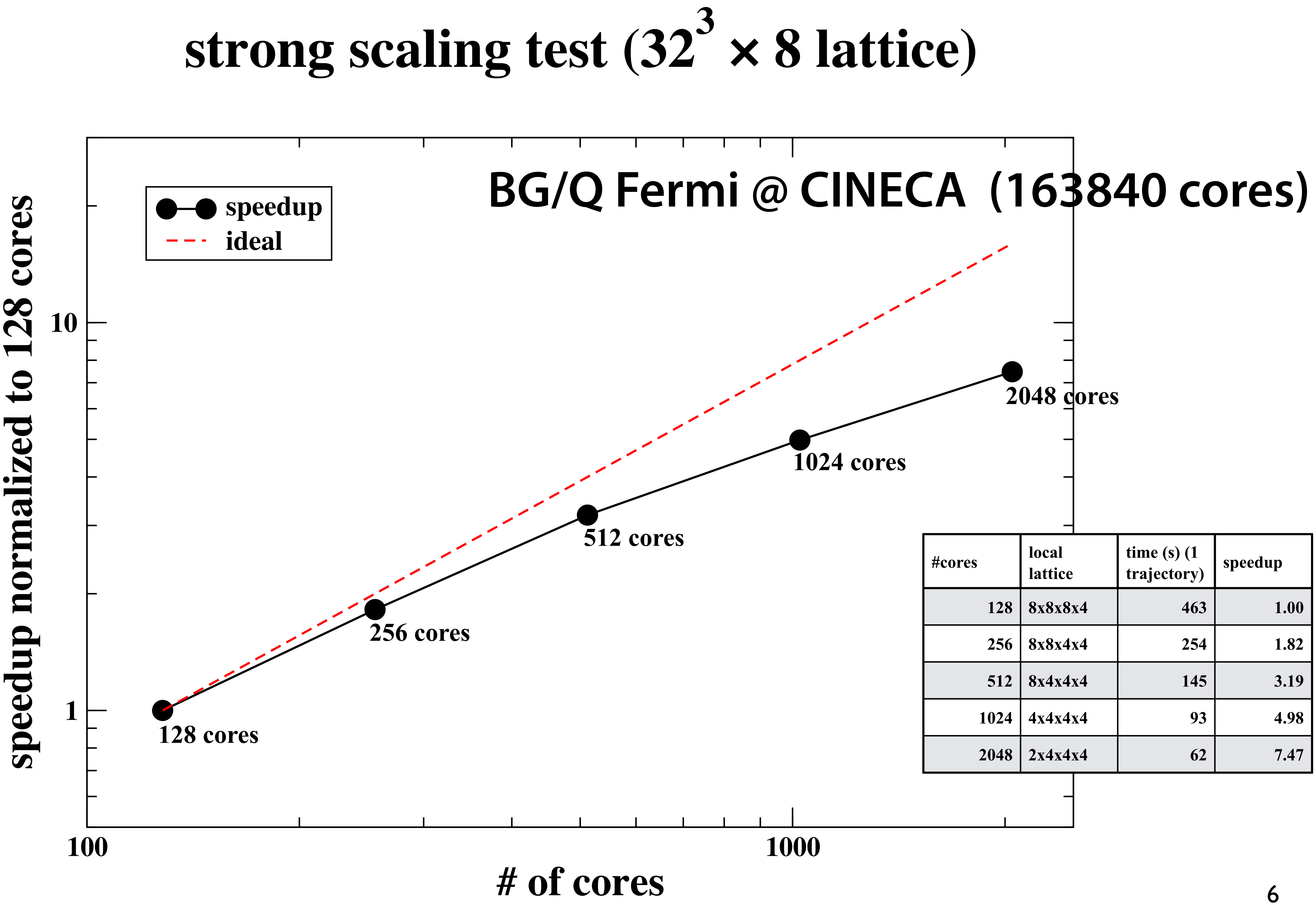
- In the present study we assign the same quark chemical potential to the three quark species: $$\mu_l = \mu_s \equiv \mu = \mu_B/3$$

- To perform numerical simulations we used the **MILC** code suitably modified in order to introduce an imaginary quark chemical potential $\mu=\mu_B/3$.

  That has been done by multiplying ***all forward and backward temporal links*** entering the discretized Dirac operator by ***exp(iaμ)*** and ***exp(–iaμ)***, respectively.

- All simulations make use of the rational hybrid Monte Carlo (RHMC) algorithm. (The length of each RHMC trajectory has been set to 1.0 in molecular dynamics time units.)

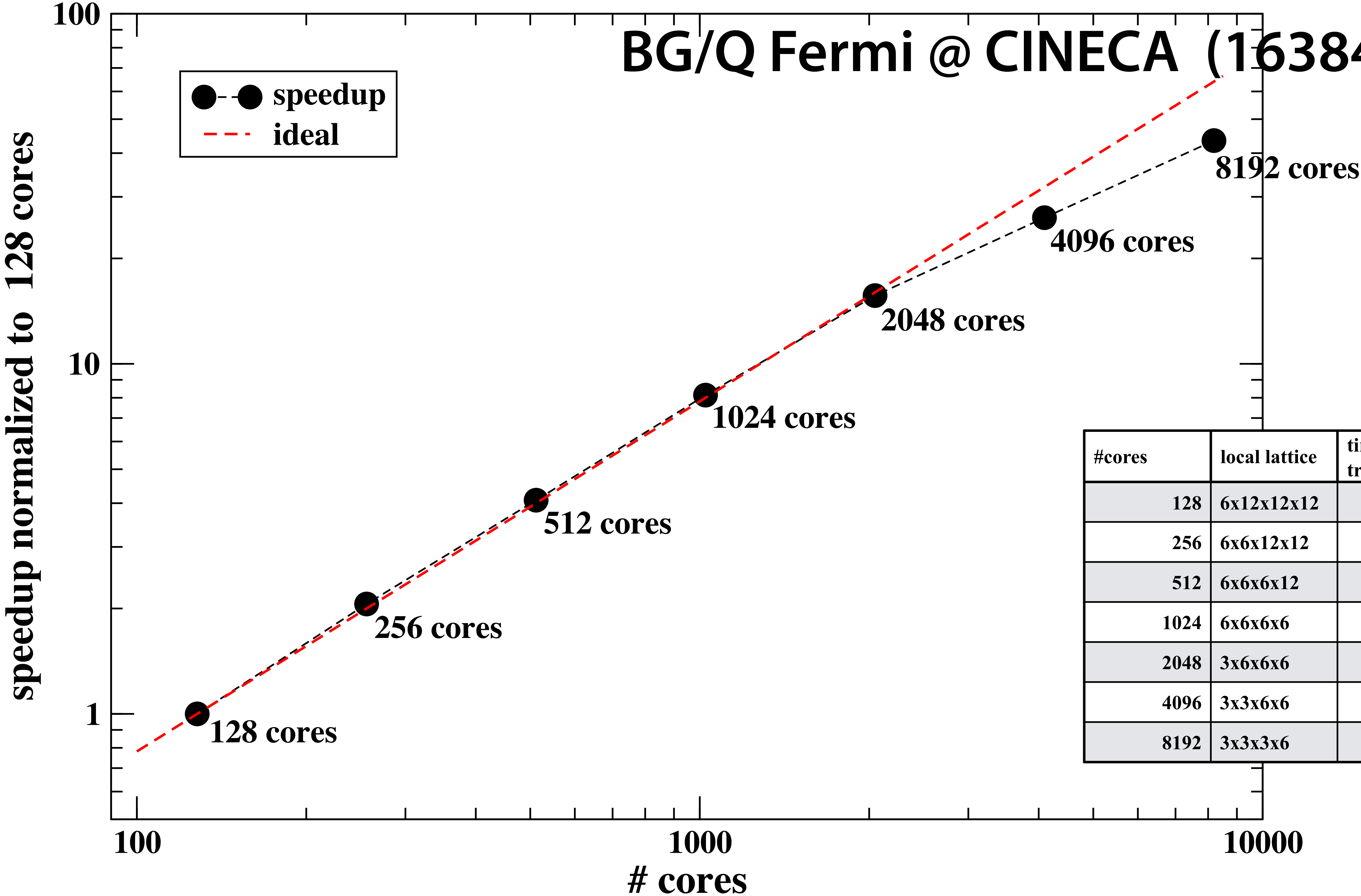
Strong scaling test (MILC code with imaginary quark chemical potential)



#cores	local lattice	time (s) (1 trajectory)	speedup
128	8x8x8x4	463	1.00
256	8x8x4x4	254	1.82
512	8x4x4x4	145	3.19
1024	4x4x4x4	93	4.98
2048	2x4x4x4	62	7.47

Strong scaling test (MILC code with chromomagnetic background field)

strong scaling test $48^3 \times 12$ lattice



#cores	local lattice	time (s) (1 trajectory)	speedup
128	6x12x12x12	6722	1.00
256	6x6x12x12	3260	2.06
512	6x6x6x12	1649	4.08
1024	6x6x6x6	826	8.14
2048	3x6x6x6	429	15.67
4096	3x3x6x6	257	26.16
8192	3x3x3x6	155	43.37

Numerical results

$$T_c(\mu) = \frac{1}{a(\beta_c(\mu))L_t}$$

The (pseudo)critical line ***β***

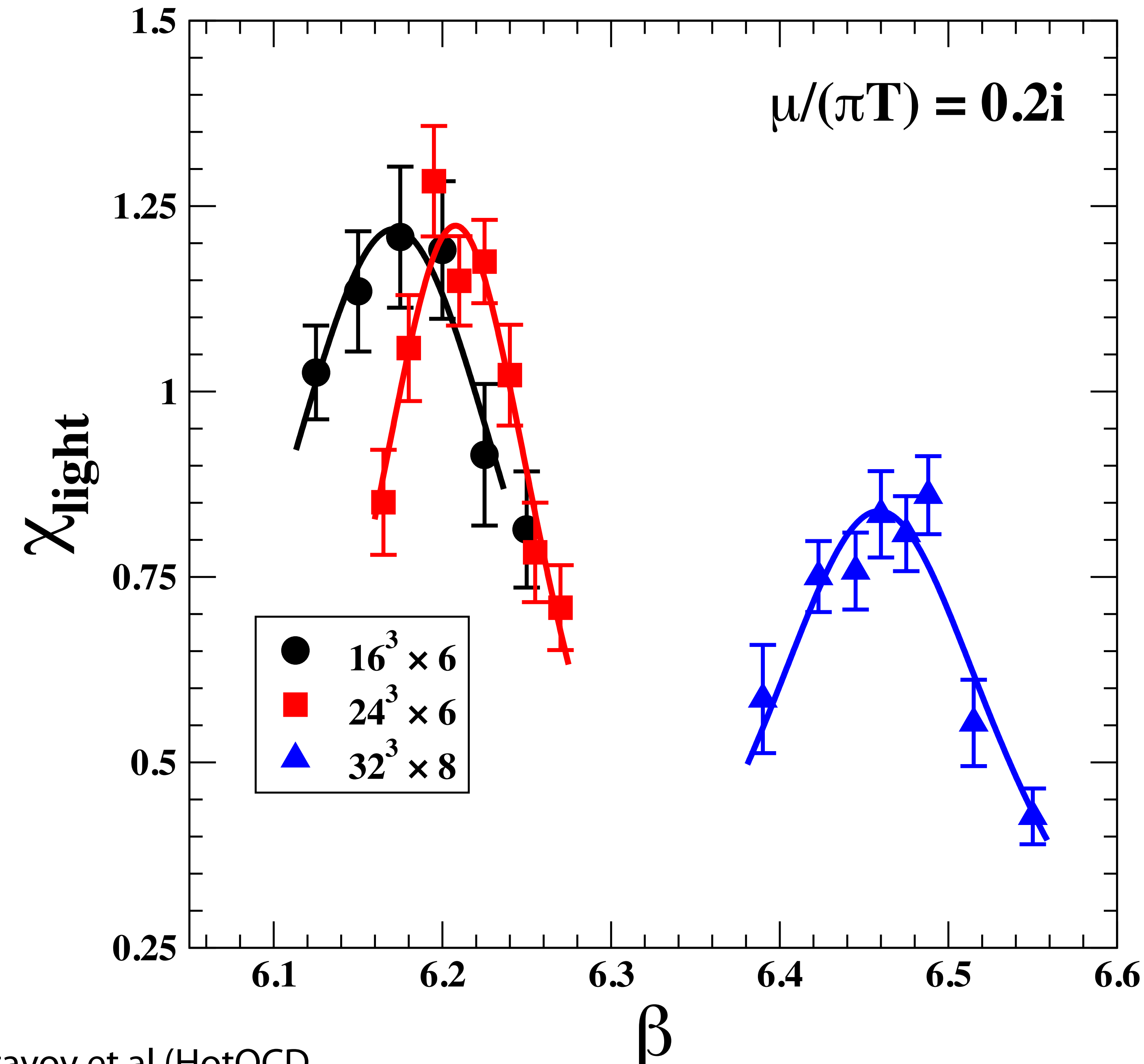
The (pseudo)critical line $\beta_c(\mu^2)$ has been determined as the value for which the **disconnected susceptibility of the light quark chiral condensate** exhibits a peak

$$\chi_{q,disc} = \frac{n_f^2}{16 N_\sigma^3 N_\tau} \left\{ \langle \left(\mathrm{Tr} D_q^{-1} \right)^2 \rangle - \langle \mathrm{Tr} D_q^{-1} \rangle^2 \right\}$$

To localize the peak, a Lorentzian fit has been used:

$$\frac{a_1}{1 + a_2(\beta - \beta_c)^2}$$

Lattice	$\mu/(\pi T)$	β_c
$16^3 \times 6$	0.	6.102(8)
	$0.15i$	6.147(10)
	$0.2i$	6.171(12)
	$0.25i$	6.193(14)
$24^3 \times 6$	0.	6.148(8) (*)
	$0.2i$	6.208(5)
$32^3 \times 8$	0.	6.392(5) (*)
	$0.2i$	6.459(9)



(*) fit to data taken from Table X and Table XI of A.Bazavov et al (HotQCD Collaboration) arXiv:1111.1710 Phys. Rev. D 85, 054503 (2012)

Setting the lattice scale

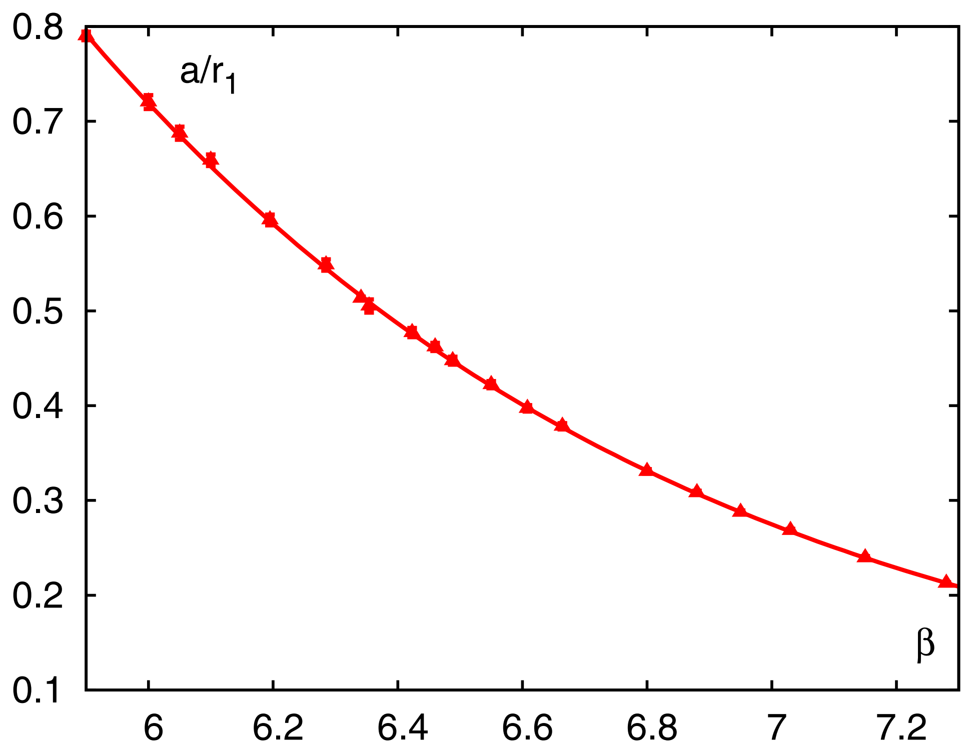
The lattice spacing can be determined using the slope of the static quark-antiquark potential on zero-temperature lattices (we use results of MILC and HotQCD collaborations (*)).

$$a(\beta)|_{m_l=0.05m_s} = r_1 \frac{c_0 f(\beta) + c_2(10/\beta) f^3(\beta)}{1 + d_2(10/\beta) f^2(\beta)}$$

$$\begin{aligned} r_1 &= 0.3106\ \mathrm{fm} \\ c_0 &= 44.06 \\ c_2 &= 272102 \\ d_2 &= 4281 \end{aligned}$$

$$f(\beta) = (b_0(10/\beta))^{-b_1/(2b_0^2)} \exp(-\beta/(20b_0))$$

b_0, b_1 coefficients of the universal two-loop beta function



$$T_c(\mu) = \frac{1}{\boxed{a(\beta_c(\mu))} L_t}$$

(*) as discussed in Appendix B of A. Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012)

The *critical temperature* vs. imaginary quark chemical potential

Lattice	$\mu/(\pi T)$	β_c	$T_c(\mu)/T_c(0)$
$16^3 \times 6$	0.	6.102(8)	1.000
	$0.15i$	6.147(10)	1.045(13)
	$0.2i$	6.171(12)	1.070(15)
	$0.25i$	6.193(14)	1.093(17)
$24^3 \times 6$	0.	6.148(8)	1.000
	$0.2i$	6.208(5)	1.060(10)
$32^3 \times 8$	0.	6.392(5)	1.000
	$0.2i$	6.459(9)	1.068(11)

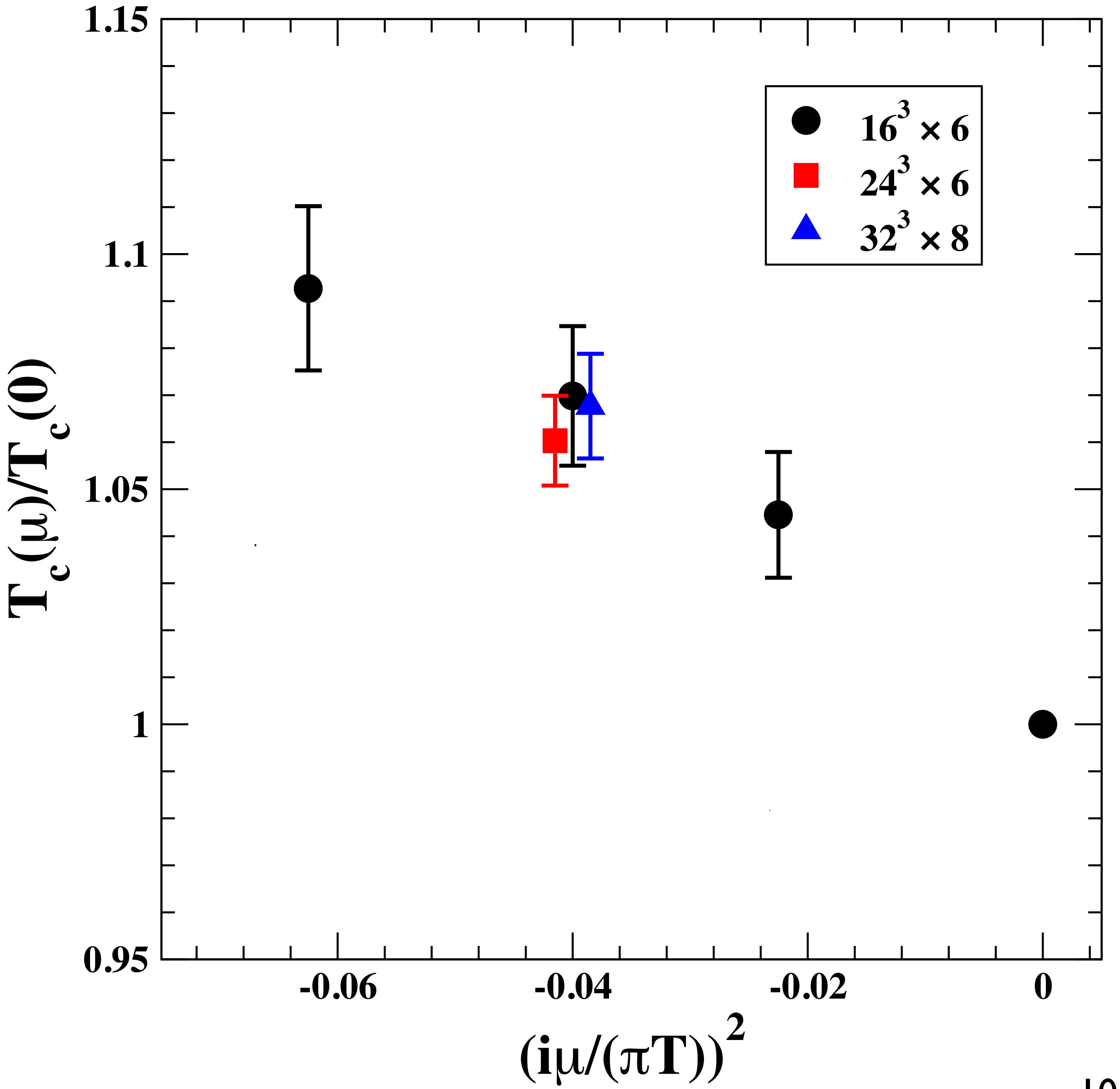
$$T_c = \frac{1}{a(\beta_c) L_t}$$

$$\frac{T_c(\mu)}{T_c(0)} = \frac{a(\beta_c(0))}{a(\beta_c(\mu))}$$

$$\frac{a}{r_1}(\beta)_{m_l=0.05m_s} = \frac{c_0 f(\beta) + c_2(10/\beta) f^3(\beta)}{1 + d_2(10/\beta) f^2(\beta)}$$

$$f(\beta) = (b_0(10/\beta))^{-b_1/(2b_0^2)} \exp(-\beta/(20 b_0))$$

$r_1 = 0.3106$ fm
$c_0 = 44.06$
$c_2 = 272102$
$d_2 = 4281$



Linear fit (in μ²) to the data

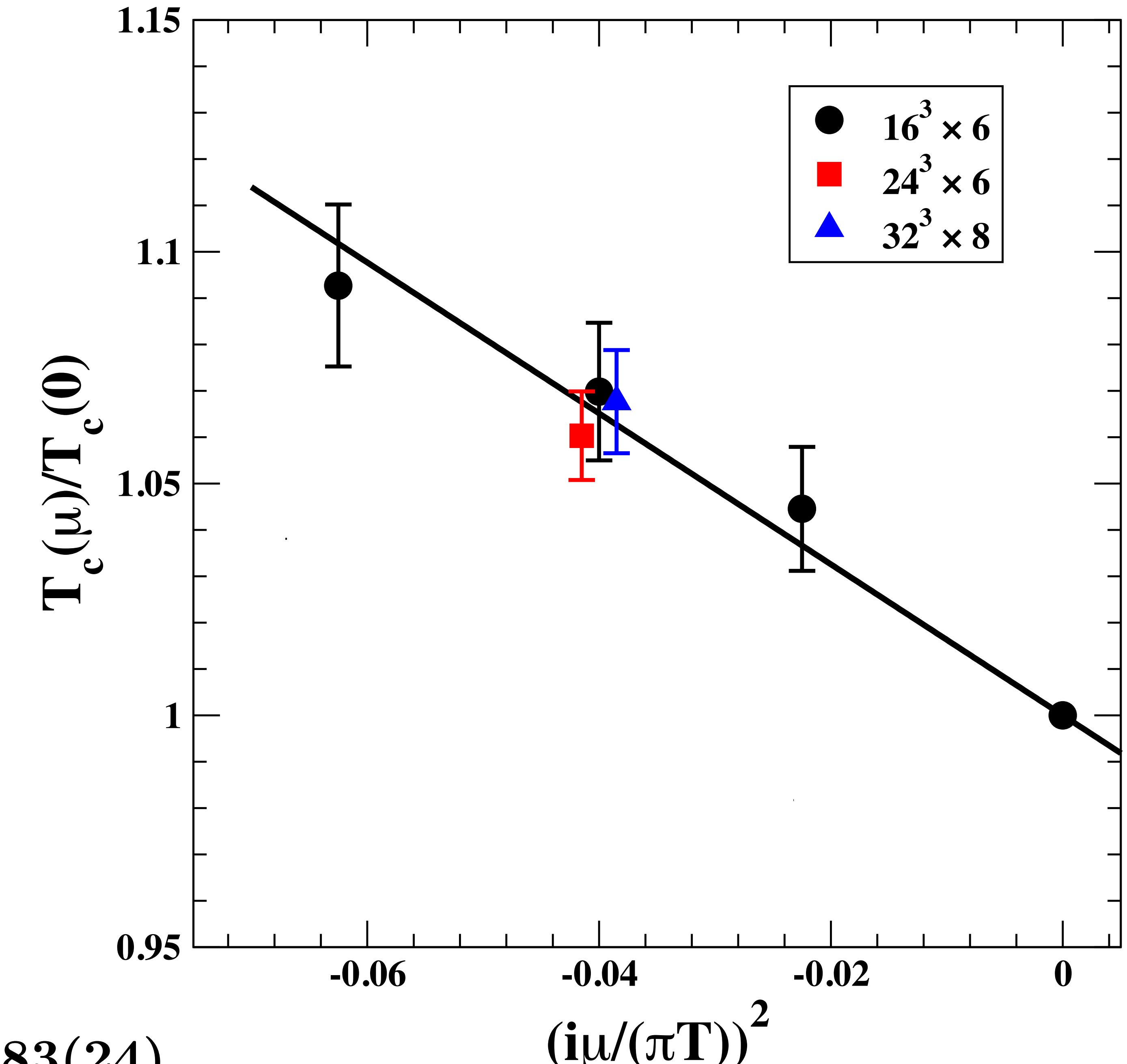
$$\frac{T_c(\mu)}{T_c(0)} = 1 + R_q \left(\frac{i\mu}{\pi T_c(\mu)}\right)^2$$

for the **16³×6** lattice:

$$R_q = -1.63(22)$$
$$\chi^2/\mathrm{d.o.f.} = 0.39$$

curvature of the (pseudo)critical line:

$$\kappa = -\frac{R_q}{(9\pi^2)} = 0.0183(24)$$



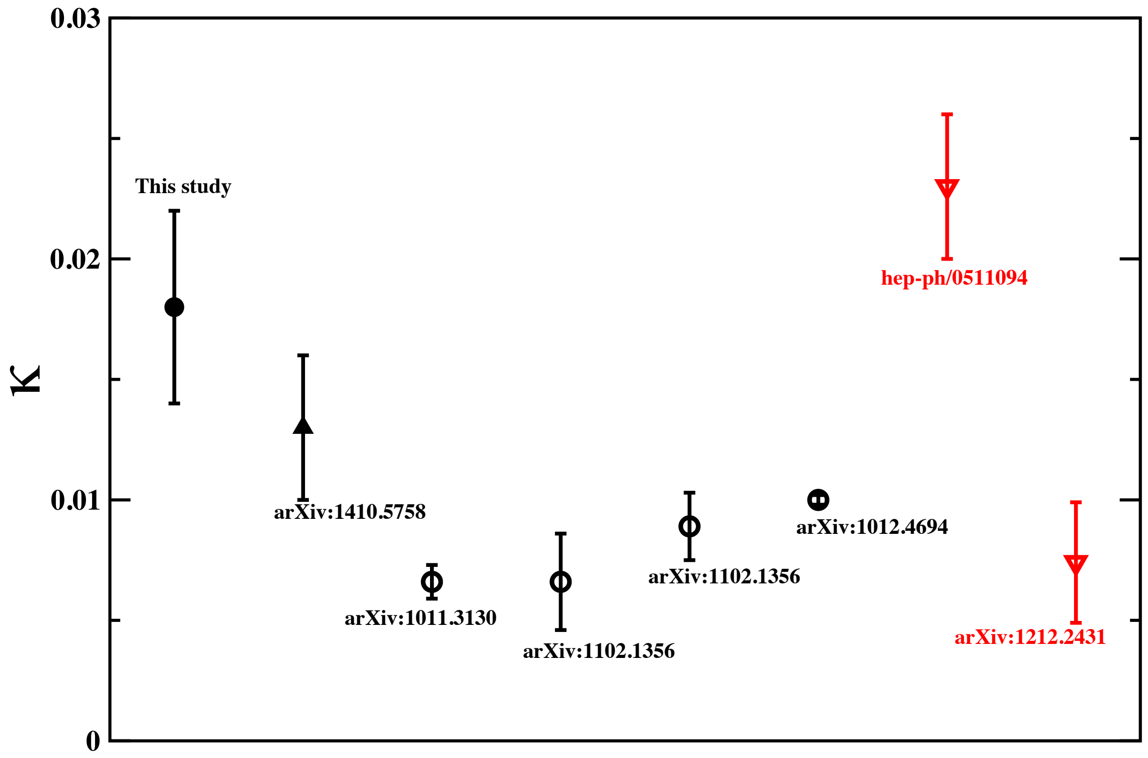
Assuming that linearity still holds on the other lattices:

$$R_q(16^3 \times 6) = -1.63(22)\,, \quad \kappa = 0.0183(24)$$
$$R_q(24^3 \times 6) = -1.51(25)\,, \quad \kappa = 0.0170(28)$$
$$R_q(32^3 \times 8) = -1.70(29)\,, \quad \kappa = 0.0190(32)$$

our estimate:

$$\kappa = 0.018(4)$$

Comparison with other results for the curvature κ



This study P. Cea, L. Cosmai, A. Papa, Phys. Rev. D 89, 074512 (arXiv:1403.0821)

analytic continuation, ***HISQ/tree action***, disconnected chiral susceptibility, $\mu=\mu_l=\mu_s$

arXiv:1410.5758 C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Negro, F. Sanfilippo, Phys.Rev. D90 (2014) 11, 114025

analytic continuation, stout-smeared staggered quarks, $\mu_s=0$, chiral condensate, chiral susceptibility

arXiv:1011.3130 O. Kaczmarek, F. Karsch, E. Laermann, C. Miao, S. Mukherjee, P. Petreczky, C. Schmidt, W. Soeldner, and W. Unger, Phys.Rev. D83 (2011) 014504

Taylor expansion, p4-action, chiral susceptibility

arXiv:1102.1356 G. Endrődi, Z. Fodor, S. D. Katz, and K. K. Szabó, JHEP 1104 (2011) 001

Taylor expansion, stout action, chiral condensate

Taylor expansion, stout action, strange quark number susceptibility

arXiv:1012.4694 R. Falcone, E. Laermann, M.P. Lombardo, PoS LATTICE2010 (2010) 183

analytic continuation, p4-action, Polyakov loop

hep-ph/0511094 J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys.Rev. C73 (2006) 034905

freeze-out curvature, analysis based on the standard statistical hadronization model

arXiv:1212.2341 F. Becattini, M. Bleicher, Th. Kollegger, T. Schuster, Jan Steinheimer, and Reinhard Stock, Phys.Rev.Lett. 111 (2013) 082302

freeze-out curvature, revised analysis

Estimate of the (pseudo)critical line

From our estimate of the curvature

$$\kappa = 0.018(4)$$

and

$$T_c(\mu_B) = a - b\mu_B^2$$

$$a = T_c(0)$$

$$b = \frac{\kappa}{T_c(0)}$$

$$T_c(0) = 154(9)\,\mathrm{MeV} \quad (*)$$

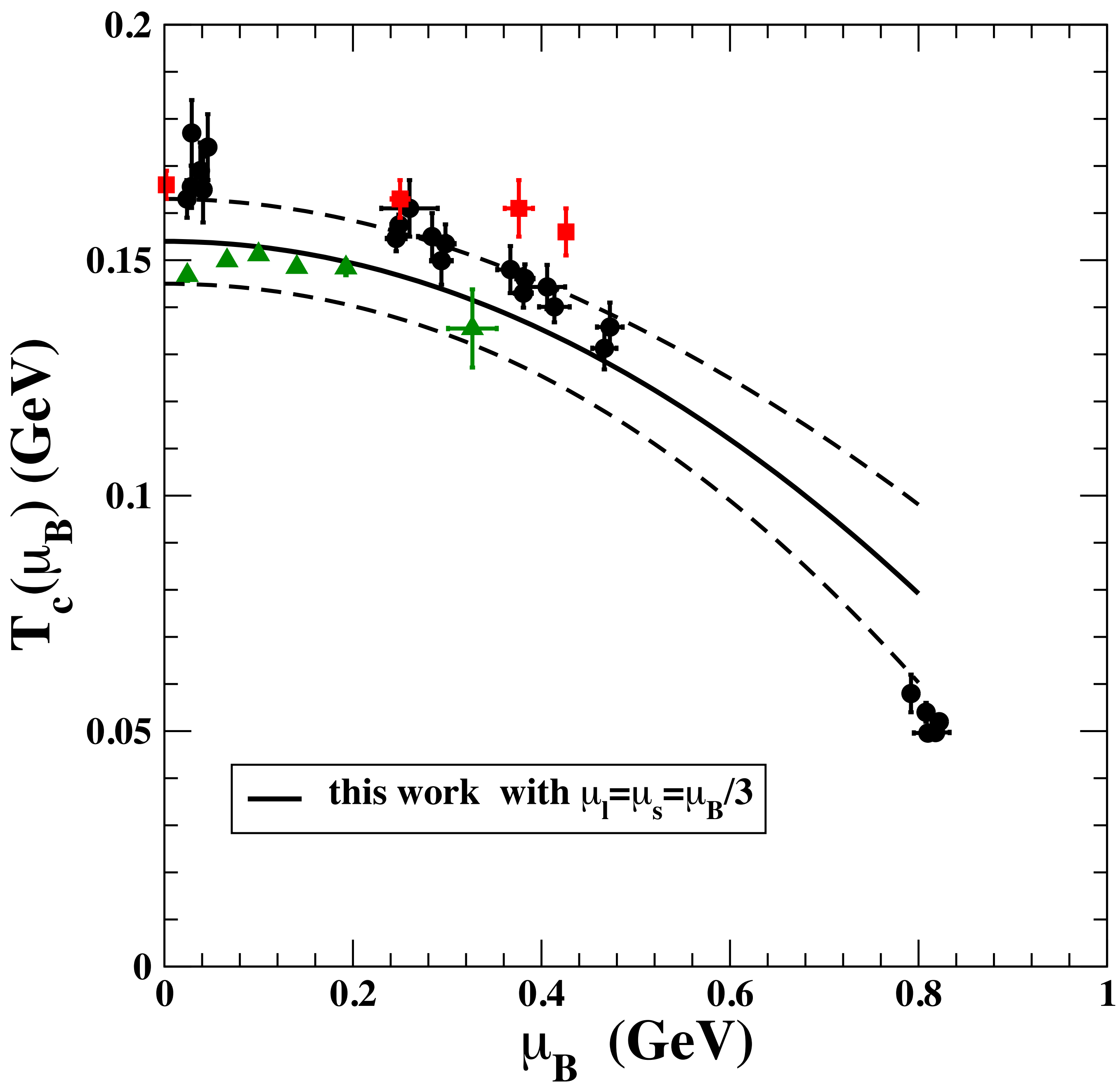
we get:

$$b = 0.117(27)\,\mathrm{GeV}^{-1}$$

to be compared with:

$$b = 0.139(16)\,\mathrm{GeV}^{-1}$$

hep-ph/0511094 J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys.Rev. C73 (2006) 034905



● J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys.Rev. C73 (2006) 034905 (hep-ph/0511094)

■ F. Becattini, M. Bleicher, Th. Kollegger, T. Schuster, Jan Steinheimer, and Reinhard Stock, Phys.Rev.Lett. 111 (2013) 082302 (arXiv:1212.2341)

▲ P. Alba, W. Alberico, R. Bellwied, M. Bluhm, V. Mantovani Sarti, M. Nahrgang, C. Ratti, (arxiv:1403.4903)

(*) A. Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012)

Flux tubes in the QCD vacuum

(Paolo Cea, L.C., Francesca Cuteri, Alessandro Papa)

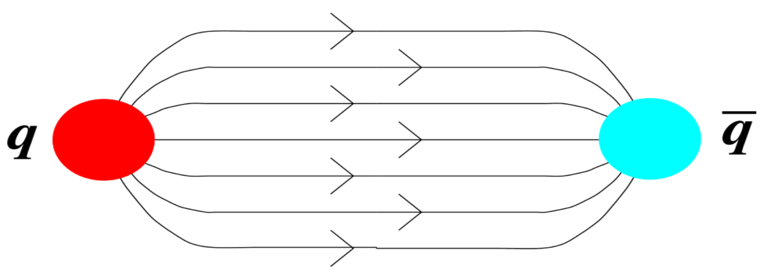
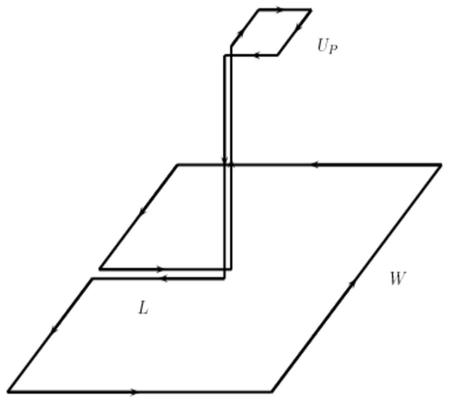
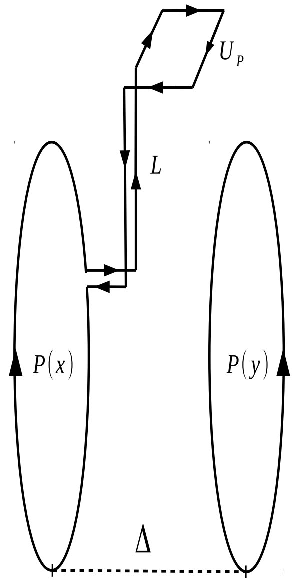


Figure: $q\bar{q}$ pair at distance R in the QCD vacuum

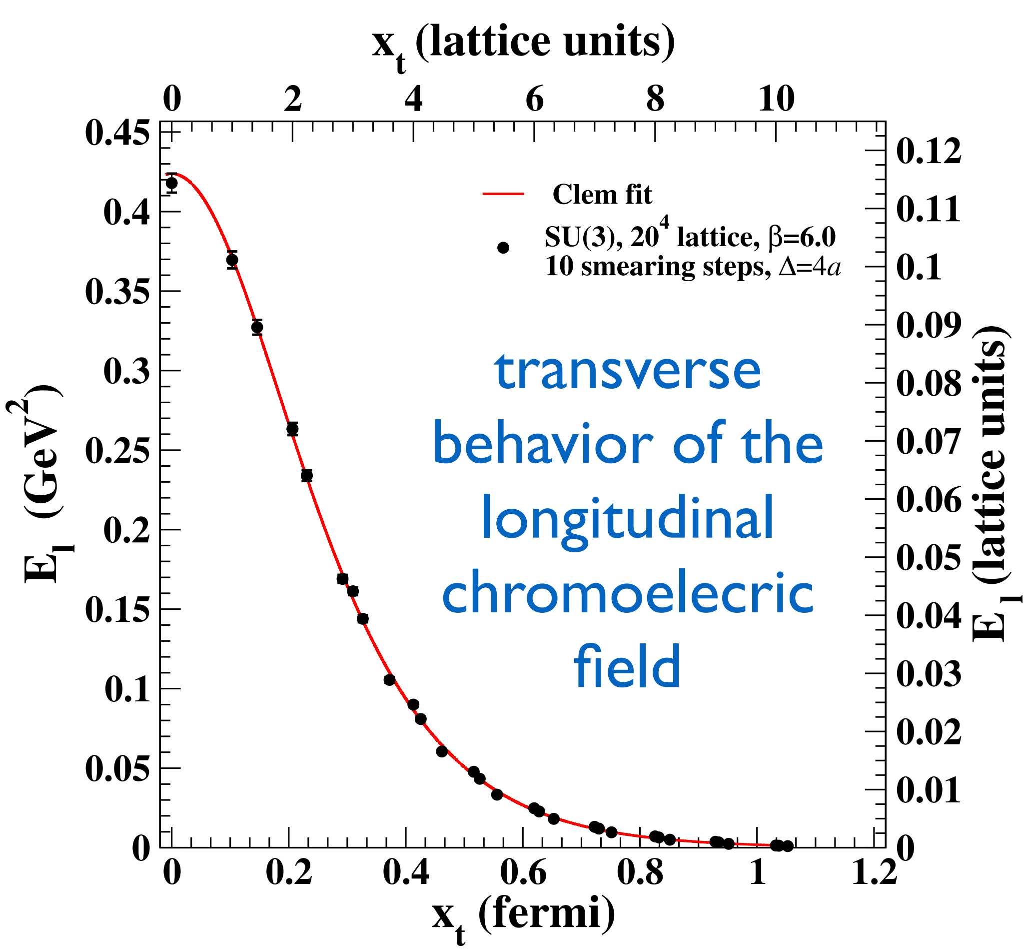




$$\rho_W^{\rm conn} = \frac{\langle {\rm tr}\,(W L U_P L^\dagger)\rangle}{\langle {\rm tr}(W)\rangle} - \frac{1}{N}\frac{\langle {\rm tr}(U_P){\rm tr}(W)\rangle}{\langle {\rm tr}(W)\rangle}$$

$$\rho_P^{\rm conn} = \frac{\langle {\rm tr}\,(P(x)\,L U_P L^\dagger)\,{\rm tr}P(y)\rangle}{\langle {\rm tr}\,(P(x))\,{\rm tr}\,(P(y))\rangle} - \frac{1}{3}\frac{\langle {\rm tr}\,(P(x))\,{\rm tr}\,(P(y))\,{\rm tr}\,(U_P)\rangle}{\langle {\rm tr}\,(P(x))\,{\rm tr}\,(P(y))\rangle}$$

$$\rho \xrightarrow{a\to 0} a^2 g \left[\langle F_{\mu\nu}\rangle_{q\bar{q}} - \langle F_{\mu\nu}\rangle_0\right]$$



Clem fit: [J.R.Clem, J.Low.Temp.Phys. 18,427(1975)]

$$E_l(x_t) = \frac{\phi}{2\pi}\frac{\mu^2}{\alpha}\frac{K_0[(\mu^2 x_t^2+\alpha^2)^{1/2}]}{K_1[\alpha]} \qquad x_t \ge 0,$$

$$R = \sqrt{x_t^2+\xi_v^2}, \qquad \mu = \frac{1}{\lambda}, \qquad \frac{1}{\alpha} = \frac{\lambda}{\xi_v}, \qquad \kappa = \frac{\lambda}{\xi} = \frac{\sqrt{2}}{\alpha}\left[1 - K_0^2(\alpha)/K_1^2(\alpha)\right]^{1/2}.$$

$\lambda = 0.1750(63)$ fm — *penetration length*

$\xi = 0.983(121)$ fm — *coherence length length*

P. Cea, L.Cosmai, F.Cuteri, A. Papa, Phys.Rev. D89 (2014) 094505 (arXiv:1404.1172)

QCD in external background fields

(Paolo Cea, L.C.)

Background fields on the lattice can be implemented by means of the gauge invariant lattice Schrödinger functional

$$\mathcal{Z}[U_k^{\mathrm{ext}}] = \int_{U_k(L_t,\vec{x})=U_k(0,\vec{x})=U_k^{\mathrm{ext}}(\vec{x})} \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi} e^{-(S_G+S_F)}$$

spatial lattice links belonging to a fixed time slice (and to spatial boundaries) are constrained

Abelian Chromomagnetic Background Field:

$$\vec{A}_a^{\mathrm{ext}}(\vec{x}) = \vec{A}^{\mathrm{ext}}(\vec{x})\delta_{a,\tilde{a}}\,, \quad A_k^{\mathrm{ext}}(\vec{x}) = \delta_{k,2}x_1 H\,. \qquad a^2\frac{gH}{2} = \frac{2\pi}{L_1}n_{\mathrm{ext}}$$

$$U_1^{\mathrm{ext}}(\vec{x}) = U_3^{\mathrm{ext}}(\vec{x}) = 1 \qquad U_2^{\mathrm{ext}}(\vec{x}) = \begin{bmatrix} \exp(i\frac{agHx_1}{2}) & 0 & 0 \\ 0 & \exp(-i\frac{agHx_1}{2}) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

static constant abelian chromomagnetic field directed along spatial direction ˆ3 and direction a˜ in the color space

Magnetic Background Field:

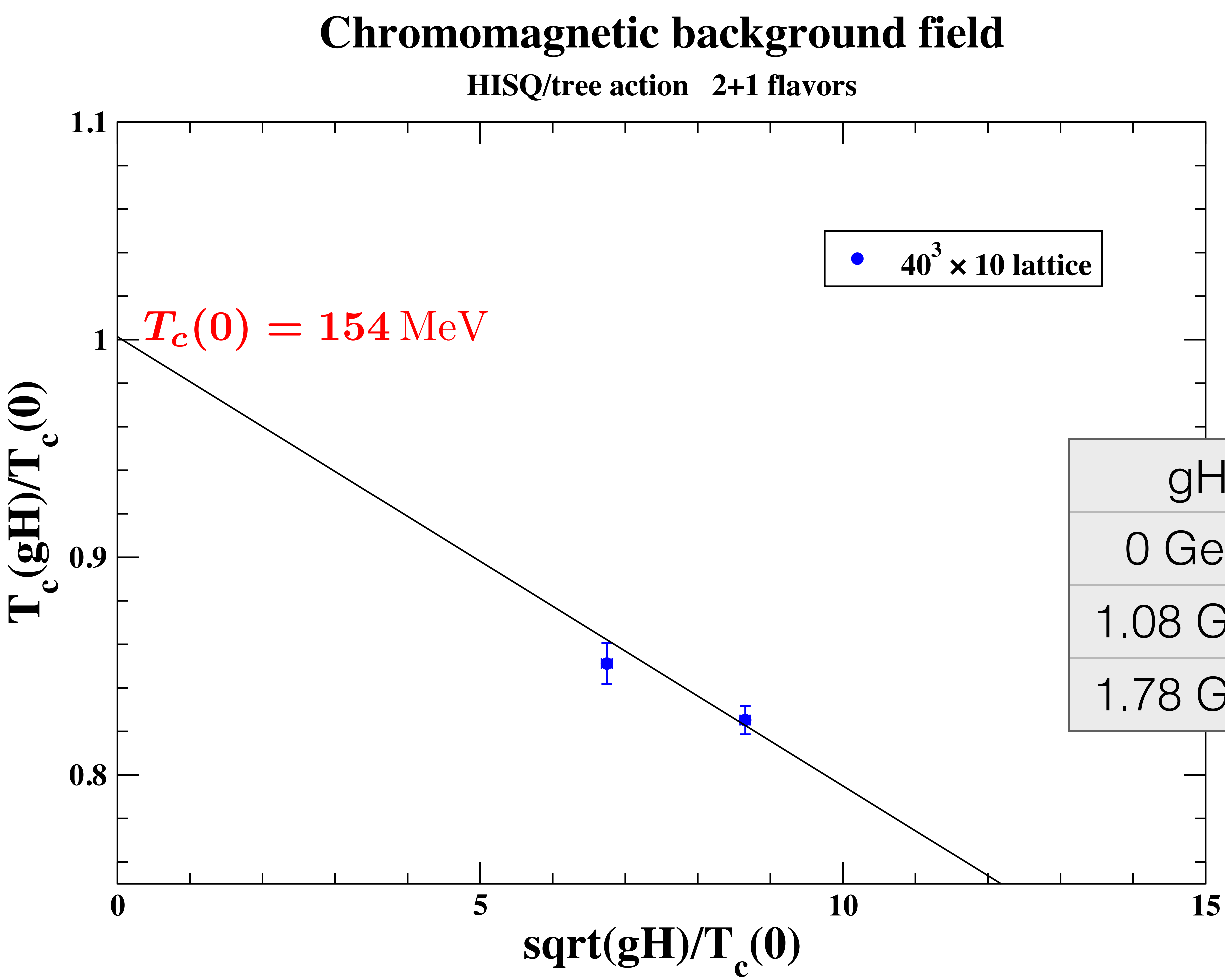
$$A_k^{\mathrm{ext}}(\vec{x}) = \delta_{k,2}x_1 H\,. \qquad U_2^{\mathrm{ext}}(\vec{x}) = \cos(q_f eHx_1) + i\sin(q_f eHx_1)$$

$$U_1^{\mathrm{ext}}(\vec{x}) = U_3^{\mathrm{ext}}(\vec{x}) = 1 \qquad a^2 q_f eH = \frac{2\pi}{L_1}n_{\mathrm{ext}}$$

constant magneetic field directed along the x_3 direction

Chromomagnetic background field   (2+1) flavor QCD

We work on a ***line of constant physics*** ***(M_π = 160 MeV)***

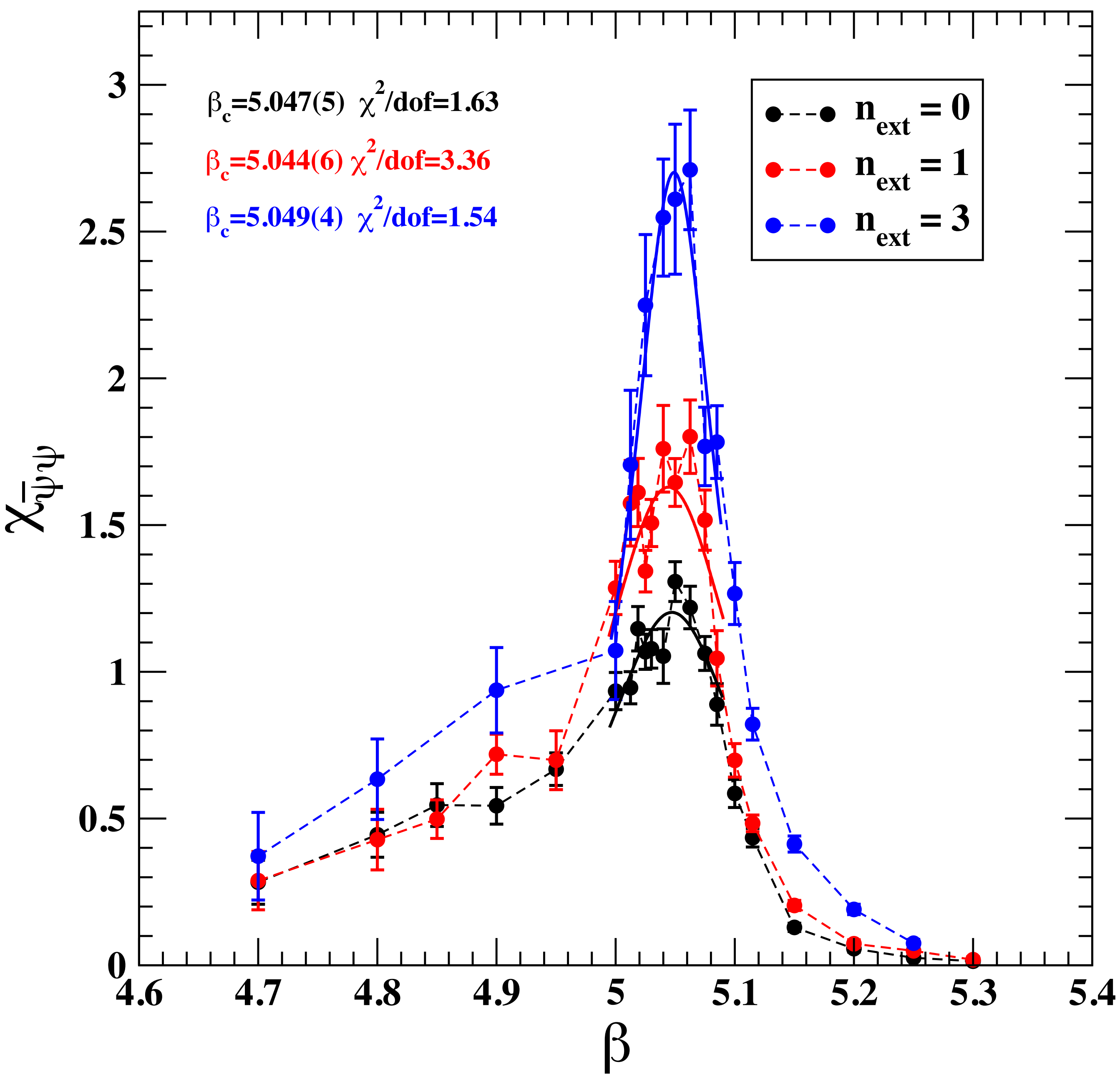


gH	T_c(gH)
0 GeV^2	154 MeV
1.08 GeV^2	131 MeV
1.78 GeV^2	127 MeV

The deconfinement temperature *depends* on the strength of the chromomagnetic background field

Magnetic background field - 1 flavor QCD

staggered fermions + Wilson gauge action, $24^3 \times 4$ lattice, am=0.025



The deconfinement temperature *seems to be independent* of the strength of the magnetic background field

In progress:

- EOS with magnetic background field
- study with HISQ fermions

Conclusions & Acknowledgements

- All simulations have been done mostly using the computational resources made available under the INFN SUMA project.

- Most of the topics that I have discussed could not be addressed without the new computational resources that the SUMA project has provided to the INFN computational community.

- Hope for a renewed support of computational theoretical physics at INFN.